Box 2
New Estimates of the Neutral Interest Rate in Colombia

Anderson Grajales-Olarte
José David Pulido*

Identifying the monetary policy stance is of vital importance to the decisions households and businesses make with respect to consumption and investment. For example, when monetary policy is expansive, agents expect a boost in aggregate demand. This can generate inflationary pressures, depending on the state of the output gap. Therefore, having the means to recognize the stance of monetary policy allows households and companies to make better-informed decisions.

One way to infer the monetary policy stance is by contrasting the level of the policy rate set by *Banco de la República* with the level of the neutral interest rate in the economy. The neutral interest rate, a concept attributed to Wicksell (1936), is defined as the rate that does not exert pressure on the output gap or on inflation. Accordingly, the monetary stance is considered expansive when the policy rate is lower than the neutral rate. It is considered contractive when that level is above the neutral rate.

However, a practical difficulty arises when making that comparison; namely, the extent of the neutral rate cannot be known with certainty. The neutral interest rate is not observable and, therefore, its level and trend must be estimated. Irremediably, its estimated level poses a considerable degree of uncertainty that is inherent in any estimation process, but is associated with the choice of the estimating methodology.

There are several methodologies outlined in the economic literature that can be used to estimate the neutral interest rate. Each of them has advantages and disadvantages, making it difficult to reach a consensus on which of the different methods to estimate the neutral interest rate is better (Magud and Tsounta, 2012). Taking the foregoing into account, this box presents a series of estimates of the neutral real interest rate - developed with different methodologies - that expands and updates those presented in a previous *Inflation Report* (Amador and Beltrán, 2015). These include static approaches (the neutral interest rate is estimated as a constant value of steady state) and dynamic ones (the neutral interest rate can change over time).[1]

Static methods are based on the first order conditions of consumption smoothing models. Dynamic methods include three methodologies: 1) the HP filter; 2) methods based on compliance with a macroeconomic equilibrium equation (for example, uncovered rate parity), and 3) maximum likelihood (ML) estimates of simultaneous macroeconomic equations. Within this last group, the reference methodology in the literature is Laubach and Williams (2003), who jointly estimate a dynamic IS, a Phillips curve and a neutral real interest rate that depends on potential growth and other shocks.[2] In this note, the extension to Laubach and Williams (2003) is used for an open economy proposed by Armelius et al. (2018), which includes the real exchange rate channel.

Other dynamic models include a Taylor rule, an estimate of the neutral rate as a common factor between short and long-term interest rates, and the so-called short-run, long-term and long-run, long-term models proposed by Roberts (2018). In these last two models, the neutral rates are estimated based on the real rates of long-term instruments, specifically the rates on the 10-year TES. As pointed out by Mishkin (1996) and Roberts (2018), neutral long-term rates may be more important to spending decisions than short-term ones. To make their level comparable to other methodologies, these rates are normalized using the term-premium for the period under consideration.

Table B2.1 shows the neutral real interest rate estimates for 2018, developed with the methodologies mentioned earlier. A brief description of each methodology is provided below. The results report considerable uncertainty about the precise level of the neutral real interest rate. It should be noted that the amplitude of the range found using all the methods (between 1.08% and 4.60%) is due to the extreme value produced by the consumption smoothing model, which is known in the literature[3] for generating a puzzle. A more reasonable range (between 1.12% and 2.0%) is obtained by discarding the maximum and the minimum, thus generating a new average of 1.50%. This value is similar to

* The authors are junior researchers with the Macroeconomic Models Department. The opinions and possible errors or omissions found herein are not binding on *Banco de la República* or its Board of Directors.

1 The methods considered here are outlined in Giammarioli and Valla (2004), Basdevant *et al.* (2004), Sources and Gredig (2007), Magud and Tsounta (2012) and Armelius *et al.* (2018).

2 This technique calculates the neutral real interest rate as that consistent with an output level equal to its potential and with stable inflation.

3 It is the equity premium puzzle. This refers to the fact that Euler's condition in a consumption smoothing model usually generates very high interest rates for reasonable risk aversion coefficients (see Cochrane, 2001).

Table B2.1
Estimates of the Neutral Real Interest Rate for 2018

Model	Estimate (2018)
Dynamic models: ML estimates	
Armelius et al. (2018)	1.44
Long-run, "long term"[a/]	2.00
Common factor short and long term rates	1.16
Dynamic Taylor rule	1.72
Dynamic models: equilibrium equations	
Uncovered interest rate parity	1.46
Short-run, "long term"[a/]	1.08
Dynamic models: Statistical filters	
HP filter	1.59
Statistical models	
Consumption smoothing	4.60
Consumption smoothing with habits	1.12
Range	1.08 - 4.60
Range excluding outliers	1.12 - 2.00
Average	1.80
Average excluding outliers	1.50
Median	1.46

Observation: The shaded value pertains to the reference methodology in the literature, which is an extension of Laubach and Williams (2003) for a small and open economy. The other methods are explained in Magdu and Tsounta (2012) and in Roberts (2018).
a/ The estimated values are normalized by the term-premium of the 10-year TES.
Source: Calculations by *Banco de la República*

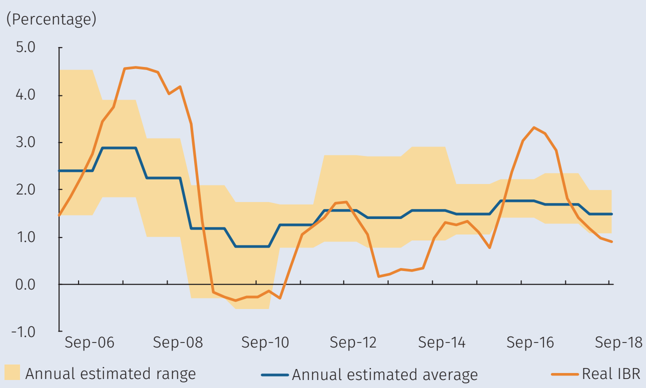
the estimate obtained with the model developed by Armelius *et al.* (2018), which is the reference model in the literature. Graph B2.1 offers a comparison between the temporal evolution of the range estimated with the dynamic models for each year of the sample of the average of those estimates and the real interbank rate (real IBR).[4] This contrast allows us to infer how the monetary policy stance evolves over time.

1. A Brief Description of the Methodologies

1.1 Armelius *et al.* (2018): Laubach and Williams (2003) with the Exchange Rate Channel

This estimate is based on the approach developed by Laubach and Williams (2003), but adjusted for application to small and open economies. Accordingly, it takes the real exchange rate channel into account. This method simultaneously models output *(y)*, the real interest rate *(r)*, the real exchange rate *(q)* and inflation *(π)*, variables that are

Graph B2.1
Temporal Evolution of the Different Dynamic Estimates of the Neutral Real Interest Rate



Source: calculations by *Banco de la República.*

linked through a Phillips curve. As in the case of factor models for non-observed factors, it is assumed that output, the interest rate and the exchange rate have a long-term component and a gap component. In turn, long-term components are presumed to have an autoregressive term. Specifically, we have:

$$x_t = x_t^* + \tilde{x}_t \text{ para } x = y,q,r$$
$$r_t^* = cg_{t-1} + z_{t-1}$$

4 The range shown in Chart B2.1 was constructed by taking into account the maximum and minimum values produced with the different dynamic methodologies. The range does not consider the uncertainty associated with the point estimate of each of the methodologies.

$$
\begin{gathered}
z_t = z_{t-1} + \varepsilon_t^z, \\
q_t^* = q_{t-1}^* + \varepsilon_t^q, \\
y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^y, \\
g_t = g_{t-1} + \varepsilon_t^g, \\
\tilde{x}_t = \varphi_{x1} \tilde{y}_{t-1} + \varphi_{x2} \tilde{r}_{t-1} + \varphi_{x3} \tilde{q}_{t-1} + \varepsilon_t^{\tilde{x}} \text{ for } x = y, q, r \\
\pi_t = \delta_\pi \pi_{t-1} + \delta_q \Delta q_{t-1} + \delta_{\tilde{y}} \tilde{y}_t + \epsilon_t^\pi
\end{gathered}
$$

where *c* and the different phis and deltas are parameters to be estimated; variables with an asterisk represent long-term values, and with a virgule they represent gaps; *g* is the growth in potential GDP and the epsilons represent residuals. Bayesian methods are applied in the estimation, using four observed variables: a real interest rate (+ IBR deflated using the expectations of analysts), real GDP, inflation and a real exchange rate (RERI-T-CPI). [5]

1.2 *Long-run, Long-term and Short-run, Long-term Interest Rates*

This estimate is based on Roberts (2018). With this methodology, the term *run* refers to the horizon at which the interest rate stabilizes output. Accordingly, a short-run rate stabilizes output, period by period, while a long-run rate stabilizes it in the long term. *Term* refers to the maturity of the instrument associated with the interest rate. Thus, a short-term rate is calculated, for example, using the IBR; a long-term rate uses, for example, the 10-year TES rate.

The short-run, long-term neutral real interest rate is calculated based on an IS curve for a closed economy, given by:

$$\text{gap}_t = \eta gap_{t-1} - (\sigma R_t - R_t^n),$$

and, therefore,

$$R_t^n = R_t + \frac{gap_t - \eta gap_{t-1}}{\sigma},$$

where *R* is the real rate on 10-year TES (deflated using 10-year inflation expectations); *gap* is that of output, *η=0.4697* y *σ=0.75.* [6]

The long-run component of the long-term rate is calculated using a Kalman filter in which it is assumed the long-term component follows a random walk, while the cyclic component is first-order autoregressive:

$$
\begin{gathered}
R_t^n = \bar{R}_t^n + cyc_t, \\
\bar{R}_t^n = \bar{R}_{t-1}^n + \varepsilon_t, \\
cyc_t = \alpha cyc_{t-1} + \upsilon_t
\end{gathered}
$$

5 The *priors*, both their distribution and their moments, were constructed on the basis of Armelius *et al.* (2018).

6 The value for σ is that used in Roberts (2018) and corresponds to estimated values in several DSGE models for the United States. In the case of η, it corresponds to an estimate of the persistence of the output gap in Colombia.

In order to make these long-term rates comparable to the short-term rates, they are normalized by subtracting the 10-year term-premium.

1.3 The Common Factor of Short and Long-term Rates

In this estimate, which follows Basdevant *et al.* (2004), it is assumed there is a common trend between the short (r_t) and long (R_t) term nominal rates, which represents the neutral interest rate (r^*_t). Therefore, it is possible to write:

$$
\begin{gathered}
r_t = r_t^* + \pi_t^e + \varepsilon_{rt} \\
R_t = r_t^* + \alpha_t + \pi_t^e + \varepsilon_{Rt}
\end{gathered}
$$

where α_t represents the term-premium and $\pi_t e$ represents expected inflation. The behavior of the common trend and the term-premium are modeled as first-order autoregressive processes:

$$
\begin{gathered}
r_t^* = r_{t-1}^* + \upsilon_{r,t} \\
\alpha_t = \lambda_0 + \lambda_1 \alpha_{t-1} + \upsilon_{t,2}
\end{gathered}
$$

The system is estimated with a Kalman filter, using the IBR, the 10-year TES and the inflation expectations from *Banco de la República's* survey.

1.4 Dynamic Taylor Rule

In this model, with specifications following Magud and Tsounta (2012), the neutral interest rate is construed from a policy rule in which the intervention rate (r_t) responds to the output gap ($\tilde{y}_t$) and to deviations in inflation (π_t) with respect to the target ($\bar{\pi}_t$). The neutral rate, in turn, follows a random walk with a drift (s_t) that varies over time. Therefore, the state-space representation is given by:

$$
\begin{gathered}
r_t = r_t^* + \beta(\pi_t - \bar{\pi}_t) + \theta \tilde{y}_t + \varepsilon_t \\
r_t^* = r_{t-1}^* + s_{t-1} \\
s_t = s_{t-1} + \upsilon_t
\end{gathered}
$$

The system is estimated with a Kalman filter, using the IBR, observed non-food inflation, the quarterly target and the output gap estimated by the Programming and Inflation Department at *Banco de la República*.

1.5 Uncovered Interest Rate Parity

In this estimate, the domestic neutral real rate equals the sum of the external neutral real interest rate, the trend risk premium and the trend real depreciation rate. For the external neutral rate, the estimate made by Laubach and

Williams (2003) for the United States, which is updated regularly, is used as a proxy. The trend risk premium corresponds to an estimate of the medium-term trend in the spread on five-year credit default swaps for Colombia. It is assumed that trend real depreciation is 0

1.6 Consumption Smoothing Models

In these models, the neutral interest rate is obtained as a steady state value based on log-linearization of the Euler condition of a consumption smoothing model. The following is obtained for a model without consumption habits:

$$r^{*}= -ln\beta + \gamma E(\Delta lny_{t+1}) - (\gamma^{2}/2)VAR(\Delta lny_{t+1})$$

where *r** is the neutral interest rate, *β* is the assumed discount factor between 0.97 and 0.99, and the aversion coefficient relative to the risk is presumed to be between 1 and 2. For its part, the rate of expected growth in output per capita $E(\Delta lny_{t+1})$ and its variance $VAR(\Delta lny_{t+1})$ approximate the average and the variance in the potential rate of GDP growth per capita, estimated by the Programming and Inflation Department at *Banco de la República*.

The following is obtained with log-linearization for the model without consumption habits:

$$r^{*}= -ln\beta + \gamma E(\Delta lny_{t+1}) - (\gamma/2)(1-\varphi)$$

where *φ*, a parameter that measures habits in consumption, is assumed to be between 0.94 and 0.96. The calibration of the parameters in both models follows Sources and Gredig (2007), and the estimated neutral interest rate pertains to the average value found in the parameter grid.

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