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Abstract

High non-wage labor costs, particularly payroll taxes, are a major obstacle to job creation and labor market formalization in developing countries. This paper assesses the macroeconomic implications of Colombia's 2012 tax reform, which reduced payroll taxes by 13.5 percentage points to foster formal employment. To this end, we develop and estimate a dynamic stochastic general equilibrium (DSGE) model in which informal employment coexists with unemployment and labor force inactivity, highlighting the search frictions and regulatory constraints faced by the formal sector relative to the informal sector. Our findings show that lower payroll taxes strengthens firms' incentives to hire and retain formal workers, leading to a substantial reallocation towards the formal sector. As a result, formal employment increases, informality declines, wages rise across both sectors, and unemployment falls modestly. This reallocation also enhances aggregate productivity and boosts GDP. Consistent with the Colombian evidence, our results suggest that payroll tax reductions can be an effective policy for promoting formalization without adverse effects on unemployment. Moreover, the reform is largely self-financing through higher economic activity and a broader tax base, although complementary financing measures are required to address fiscal challenges.

Keywords: tax reform, payroll tax, informality, labor markets in developing countries

JEL classification: E24, E62, H24, J46, O17

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Reformas tributarias y mercado laboral en una economía de mercado emergente*

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Resumen

Los altos costos laborales no salariales, especialmente los impuestos sobre la nómina, son un obstáculo para la creación de empleo y la formalización del mercado laboral en los países en desarrollo. Este documento evalúa las implicaciones macroeconómicas de la reforma tributaria colombiana de 2012, que redujo los impuestos sobre la nómina en 13,5 puntos porcentuales con el fin de fomentar el empleo formal. Para ello, se desarrolla y estima un modelo de equilibrio general dinámico estocástico en el que el empleo informal coexiste con el desempleo y la población fuera de la fuerza laboral, resaltando las fricciones de búsqueda y las restricciones regulatorias que enfrenta el sector formal en relación con el informal. Los hallazgos muestran que impuestos sobre la nómina más bajos incentivan a las empresas a contratar y retener trabajadores formales, lo que conduce a una reasignación sustancial hacia el sector formal. Como resultado, el empleo formal aumenta, la informalidad disminuye, los salarios suben en ambos sectores y el desempleo disminuye modestamente. Esta reasignación también mejora la productividad agregada e incrementa el PIB. Estos hallazgos son consistentes con la evidencia empírica y sugieren que reducir el impuesto sobre la nómina puede ser una política eficaz para promover la formalización sin efectos adversos sobre el desempleo. Además, la reforma es en gran medida autofinanciada mediante una mayor actividad económica y una base impositiva más amplia, aunque se requieren medidas complementarias para abordar los desafíos fiscales.

Palabras clave: reformas tributarias, impuestos sobre la nómina, informalidad, mercados laborales en países en desarrollo

Clasificación JEL: E24, E62, H24, J46, O17

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1 Introduction

Economists widely agree that high non-wage labor costs pose a significant obstacle to job creation and the formalization of employment in developing countries. Payroll taxes, in particular, have been central to this debate due to their effects on formal employment and wages, as well as their potential role in sustaining the high levels of informality that characterize these economies.

This paper contributes to the discussion by evaluating the macroeconomic implications of Colombia's 2012 tax reform, which was designed to promote labor formalization. The reform introduced a 13.5 percentage point (pp) reduction in the payroll tax rate and a partial shift of the tax base from labor to corporate income. To assess its impact, we develop a dynamic stochastic general equilibrium (DSGE) model with a labor market structure representative of developing countries.

In particular, the model aligns with the so-called dualistic approach, which views labor markets as comprising two distinct sectors that are interlinked. The formal sector is characterized by search and matching frictions and is subject to labor market regulations, including income and payroll taxes, as well as hiring and firing costs. In contrast, the informal sector operates outside labor legislation, avoiding these frictions and costs. As a result, informal activities are not taxed, and workers do not receive social security benefits.

In addition, labor market flows are modeled in a way that allows individuals to obtain formal employment through search while either unemployed or informally employed. Specifically, unemployed job-seekers may search directly for formal jobs or transition into informal employment, and, in the informal sector, search for formal-sector opportunities with varying intensity.

Furthermore, the separation rate is determined endogenously, following Mortensen and Pissarides (1994), so that fluctuations in formal employment in response to aggregate and policy shocks arise from changes in both vacancy creation and job destruction. This feature enables the model to better capture the observed volatility of key labor market variables, particularly unemployment, which has been proven to be substantial in empirical studies

(Fujita and Ramey, 2009, 2012).[1]

We incorporate endogenous labor force participation into this framework and embed it within an otherwise standard small open economy real business cycle (SOE-RBC) model. In this respect, our approach is closely related to that of Leyva and Urrutia (2020, 2023), who also consider the coexistence of unemployment, informality, and labor force inactivity.

The model is calibrated to the pre-reform steady state of the Colombian economy and estimated to match key business-cycle moments over the 2006IV-2019IV period. It successfully reproduces the cyclical comovements of formal and informal employment, unemployment, and labor market tightness. Importantly, the model generates a negative correlation between unemployment and output, overcoming a limitation of previous frameworks with endogenous labor force participation, in which unemployment tends to be procyclical (see, e.g., Leyva and Urrutia, 2020, 2023).[2] Overall, it provides a good fit to the data, particularly along the dimensions most relevant for policy analysis.

We then use the estimated model to evaluate the effects of the 2012 tax reform on macroeconomic and labor market outcomes. The results indicate that a 13.5 pp reduction in payroll taxes induces a substantial reallocation of labor from the informal to the formal sector, increasing formal employment and reducing informality. The reform also raises formal wages, suggesting a sizable pass-through of lower payroll taxes to workers' earnings in the Colombian labor market.

This paper contributes to the literature that develops and estimates structural models to assess the impact of labor tax reforms. While most existing studies rely on reduced-form methods, their findings on the implications of payroll tax cuts for employment and earnings remain mixed. Some papers find no employment effects but substantial wage pass-through, implying that workers bear most of the tax incidence (Cruces et al., 2010). In contrast, more recent evidence documents positive employment effects with little or no response in earnings

[1] Seminal RBC models with labor market matching typically assume exogenous job destruction and a constant separation rate (Andolfatto, 1996; Merz, 1995). Subsequent studies, including den Haan et al. (2000) and Christiano et al. (2011), show that endogenous job separations substantially improve the ability of DSGE models to match unemployment dynamics. For emerging economies, Guerra-Salas et al. (2021) finds that endogenous separations amplify employment responses to shocks in a DSGE model estimated on Chilean data.

[2] Nonetheless, the model understates the volatility of unemployment, reflecting the well-known unemployment volatility puzzle documented in standard search-and-matching frameworks (see Shimer, 2005).

(Lobel, 2022). Other studies report intermediate results, with only partial pass-through of payroll tax reductions to wages (Kugler and Kugler, 2009).

Furthermore, much of the structural literature on the impact of payroll tax policies on informality relies on equilibrium search-and-matching models (see, for instance, Ulyssea, 2010; Charlot et al., 2015; Garlati-Bertoldi, 2020). Because these frameworks are partial equilibrium in nature, they cannot capture broader macroeconomic effects and are therefore ill-suited to evaluating policy consequences beyond the labor market.

Our findings are broadly consistent with the evidence on Colombia's 2012 tax reform, which is based primarily on difference-in-differences estimates (Bernal et al., 2017; Fernandez and Villar, 2017; Kugler et al., 2017) and, to a lesser extent, calibrated search models (Garlati-Bertoldi, 2020). However, these approaches cannot quantify the reform's effects on outcomes such as government revenue. This limitation underscores the value of a dynamic general equilibrium framework, which complements reduced-form and search-based analyses by capturing economy-wide interactions and their fiscal implications.

Moreover, the model reproduces a particularly challenging policy outcome, namely, that a reduction in payroll taxes can lower informality without increasing unemployment. This represents an improvement over equilibrium search-and-matching frameworks, which generally find such policies ineffective in achieving both objectives simultaneously (Ulyssea, 2010; Charlot et al., 2015). However, the results also point to a decline in the formal wage premium, in contrast to previous studies that find payroll tax cuts either raise the premium (Charlot et al., 2015) or have only modest effects on wage differentials (Ulyssea, 2010).

The remainder of the paper is organized as follows. Section 2 reviews Colombia's 2012 tax reform and the evidence on its effects. Section 3 introduces the model, and Section 4 describes the calibration and estimation procedures. Section 5 presents the reform simulations, examines the channels through which the policy affects labor market and macroeconomic outcomes, and evaluates alternative financing scenarios. Section 6 concludes the paper.

2 Background

The Colombian labor market has been characterized by high rates of unemployment and informality relative to OECD and Latin American and Caribbean standards. In particular, informality in labor was exceptionally high in the early 2000s, regardless of the measurement criteria used, ranging from 56 to 67 percent of the total workforce (see Bernal, 2009; Mondragón-Vélez et al., 2010).

Also, non-wage labor costs have typically been high in Colombia. Before the 2012 tax code reform, these accounted for more than 60 percent of the average wage rate and were among the highest in Latin America (Hernández, 2012). Specifically, high payroll taxes have often been blamed for employers' high labor costs and for discouraging formal employment (Kugler and Kugler, 2009).

To reduce labor costs and promote formal job creation, Colombia implemented a tax reform in December 2012 that reduced payroll taxes by 13.5 percentage points. The most significant change was the elimination of employer non-wage costs, which included contributions to health, job training programs, and childcare, representing 8.5 percent, 2.0 percent, and 3.0 percent of wages, respectively. Although the reform affected only payments made by employers of two or more workers earning up to 10 times the minimum wage, it did not change the amount of taxes or contributions payable by workers.

To compensate for the lost revenue, a new tax, known as the CREE (for its initials in Spanish), was introduced. For practical purposes, the CREE tax was equivalent to a corporate income tax (henceforth CIT) of 8 percent.[3] To avoid increasing the tax burden of firms, the CIT was simultaneously decreased from 33 to 25 percent. Overall, this reform implied a partial shifting of the tax base from labor to corporate income, but left the CIT rate approximately constant.

Colombia's 2012 tax reform has been relatively well studied. Most of the papers addressing this tax reform focus on identifying the causal effects of the payroll tax cut on a limited set of labor market variables, particularly formal and informal employment and wages, by

[3] The CREE was central to the original 2012 reform design. Its rate was set at 9% between 2013 and 2015, reduced to 8% in 2016, and ultimately eliminated in 2017.

computing reduced-form estimates through difference-in-differences strategies (Bernal et al., 2017; Fernandez and Villar, 2017; Kugler et al., 2017) or panel regressions (Morales and Medina, 2017). Table 1 summarizes their main results.[4]

Table 1: Summary of studies evaluating the effects of Colombia's tax reform of 2012

Study	Effect on		
	formality rate* (pp)	informality rate (pp)	formal wage (%)
Antón (2014)	3.55	-3.15***	4.85
Bernal et al. (2017)	4.26**	–	2.69
Fernandez and Villar (2017)	–	-4.78	–
Kugler et al. (2017)	6.80	–	–
Morales and Medina (2017)	2.92	–	0.42
Garlati-Bertoldi (2020)	–	-5.40	–

Notes: * For details on the calculation of these effects, see Bonilla et al. (2023, Table 8). ** Effect on formal employment as a fraction of the working age population. *** Effect on informal employment as a fraction of the working age population.

Unlike the studies mentioned above, only a few papers have addressed the effects of the 2012 tax reform using structural models. In this regard, Garlati-Bertoldi (2020) shows that the reform implied a reduction in informality of between 5.4 pp and 9.6 pp by performing simulations on a calibrated search model of the labor market. Lastly, Antón (2014) develops and calibrates a dynamic general equilibrium model with full employment[5] to suggest that the reform led to a significant reallocation of labor between the formal and informal sectors, along with an increase in the net wage of formal workers, the latter result being consistent with a significant pass-through effect on the Colombian labor market.

[4] For details on the calculation of the effects on the formality rate, see Bonilla et al. (2023, Table 8).

[5] This is in contrast to Hernández (2012), who, before the reform, analyzed the effects of eliminating the so-called *parafiscales* taxes—payroll surcharges earmarked for social programs then accounting for 9 percent of the non-wage labor costs—using a computable general equilibrium model with involuntary unemployment and informality. The results show that abolishing *parafiscales* contributions would lead to a 1 pp decrease in the unemployment rate in the absence of alternative financing. However, if other taxes substitute for the *parafiscales*, there might be no effect on unemployment at all, due to additional distortions in the economy.

3 The model

In this section, we introduce an aggregate dynamic general equilibrium model of a small open emerging economy that allows for alternative scenarios regarding government finances. Our model is somewhat similar to the one developed by Leyva and Urrutia (2020, 2023), which features a rich labor market structure characterized by the coexistence of unemployment, informality, and inactivity.

However, our model differs from these studies in that it characterizes labor market flows in a way that allows an individual to obtain formal employment by incurring a search effort while being unemployed or informally employed.[6] To be specific, unemployed job-seekers may directly search to get a job in the formal sector or transition to informal employment, prior acceptance of an offer received at an endogenous rate, and search for a formal job with variable intensity.[7]

Also, the model allows for endogenous determination of the separation rate as in Mortensen and Pissarides (1994), so that it depends on the value of a match for the entrepreneur. This feature aims to capture the contribution of separations to unemployment volatility, which has been documented to be substantial in empirical studies, and to generate more realistic dynamic interrelationships as compared to model specifications with a constant separation rate (Fujita and Ramey, 2009, 2012).

In the model, individuals make labor participation decisions, modeled as a choice between work (formal or informal) and leisure (inactivity). Formal employment faces search and matching frictions, leading to equilibrium unemployment. In addition, firms in the formal sector incur costs from the regulatory framework, which take the form of income and payroll taxes, as well as firing costs. The informal sector, in turn, avoids the frictions and labor costs associated with formal sector regulations, although it is less productive.

To keep the analysis simple, we consider the household side of the economy as an infinitely

[6] Leyva and Urrutia (2020, 2023) assume that the only way for an individual to get a formal job is to become unemployed and exert a search effort therein.

[7] This characterization of labor market flows also differs from those of Ulyssea (2010) and Charlot et al. (2015), in which the labor market is not segmented. In these papers, in particular, it is assumed that both formal and informal sectors are subject to matching frictions and that they share the same pool of unemployed job-seekers, who search for jobs in either sector with the same intensity.

lived representative family that consists of (formally and informally) employed workers, unemployed job-seekers, and individuals out of the labor force at each point in time. Family members insure one another perfectly against variations in income, so every member enjoys the same level of consumption and has equal value of leisure regardless of his occupational status. In addition, preferences exhibit internal habit formation and there are adjustment costs of investment.

3.1 The labor market

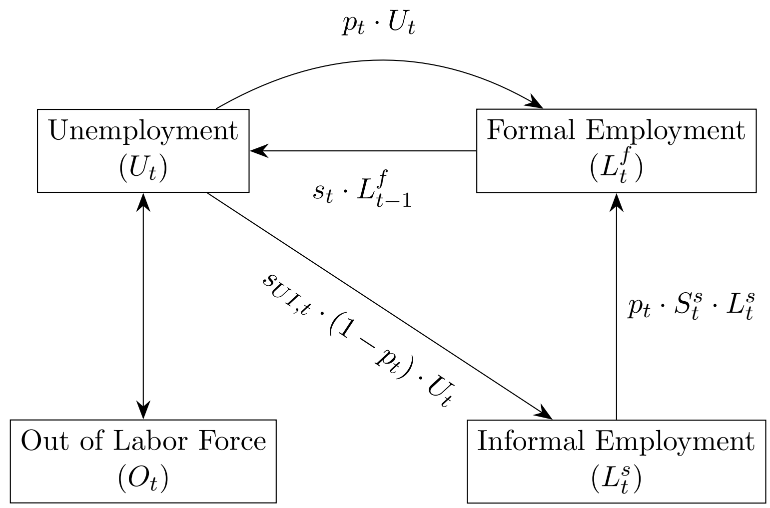
In the model, the labor market is characterized by the so-called dualistic approach, which views the labor market as comprising two distinct sectors that are linked to each other. The formal sector features matching frictions as in Merz (1995), Andolfatto (1996), and Boz et al. (2015), and includes, as part of the institutional setting, income and payroll taxes rebated to consumers as a lump-sum transfer and both hiring and firing costs. In contrast, the informal sector avoids search costs in hiring and the burden of labor and tax regulations, and does not entitle workers to social security benefits.

Individuals decide whether or not to participate in the labor force every period. In addition to the dual labor market structure, the intersectoral linkages and the endogenous participation decision lead to wage dualism (Fields, 2009), characterized by unemployment and inactivity. Therefore, the labor market comprises four occupational categories as follows: formal employment L_t^f, informal employment L_t^s, unemployment U_t, and out of the labor force O_t.

Figure 1 illustrates the flows of the labor market that characterize the model, in which each occupational category represents a state. Arrows indicate the transitions between these states. Thus, the transition from unemployment to formal employment occurs with probability $p_t U_t$, where p_t is the job finding rate. In addition, the dissolution of a match results in the loss of a formal job and the return to unemployment, a transition given by $s_t L_{t-1}^f$, where s_t is the probability of separation in the formal sector.

An unemployed individual receives a job offer from the informal sector every period at a

Figure 1: Labor market flows in the model



rate $s_{UI,t}$ given by

$$s_{UI,t} = \bar{s}_{UI} \left(\frac{w_t^s}{\bar{w}^s} \right)^{\phi_{UI}}, \tag{1}$$

where $\bar{s}_{UI}$ is the value of $s_{UI,t}$ in the steady state and the elasticity ϕ_{UI} governs the responsiveness of the job offer arrival rate to deviations of the informal wage w_t^s from its steady-state value $\bar{w}^s$. Such deviations in turn serve as a price signal that summarizes informal sector conditions: w_t^s rises when the informal sector contracts and falls when it expands, hence capturing the relative scarcity or abundance of informal job opportunities.[8]

If the unemployed job-seeker accepts the offer from the informal sector, he transitions to informal employment and searches for a job in the formal sector while being informally employed. Hence the flow of new informal workers is represented by $s_{UIt}(1-p_t)U_t$, while that of workers transitioning from informal to formal employment is given by $p_t S_t^s L_t^s$, where S_t^s denotes the search intensity of informal workers at time t.[9] Accordingly, the laws of motion

[8]The intuition underlying Equation (1) is consistent with evidence that informal labor markets in Latin America exhibit significant wage-driven dynamics and that transitions between unemployment and informality respond to sector-specific conditions (Mondragón-Vélez et al., 2010). For Colombia, in particular, Fernández and Villar (2016) document substantial flows between unemployment and informal employment that vary systematically with labor market conditions, which we deem provides further support for our endogenous specification of the informal job offer arrival rate.

[9]The transition from informal to formal employment aims to capture the aspiration of informal workers to secure formal jobs. In particular, the search intensity term S_t^s reflects the responsiveness of these workers to economic incentives and labor market conditions, which ultimately results in varying opportunities to find formal employment. Thus, the incorporation of this term reflects informal workers' active efforts to improve

of formal and informal employment are given by

$$L_t^f = (1 - s_t)L_{t-1}^f + p_t(U_t + S_t^s L_t^s), \tag{2}$$

$$L_t^s = (1 - p_{t-1}S_{t-1}^s)L_{t-1}^s + s_{UI,t}(1 - p_t)U_t. \tag{3}$$

The first term on the right side of Equation (2) accounts for the retention of formal workers. In contrast, the second captures the inflow of new formal workers from both unemployment and informal employment. Similarly, Equation (3) aims to model the persistence of informal employment and the inflow of new informal workers from unemployment. The first term on the right side reflects the continued presence of informal workers who have not transitioned to formal employment. The flow of new informal workers, denoted by the second term, aligns with the observation that many unemployed individuals accept informal jobs while continuing to search for formal employment.[10] This dual job search behavior is prevalent in developing economies where formal jobs are scarce and highly sought after.

Finally, movements of individuals between unemployment and inactivity occur freely, which is consistent with the labor market behavior in developing economies, where individuals frequently move into and out of the labor force due to various factors.

3.1.1 Labor supply

Households have a constant endowment of labor $\bar{L} = 1$ each period, which can be allocated to the four occupational categories mentioned above as follows:

$$\underbrace{L_t^f + L_t^s}_{\text{employed}} + \underbrace{U_t + O_t}_{\text{non-employed}} = 1. \tag{4}$$

Formal and informal workers face the same utility cost of foregone leisure while earning labor income. In contrast, unemployed and informal workers may search for formal employment

their occupational status in response to relatively favorable developments in the formal sector.

[10]In this sense, the transition from unemployment to informal employment can be seen as a passive one: While unemployed job-seekers have to search actively to meet potential employers, those who transition to informality face a positive meeting rate $s_{UI,t}$ without explicit efforts.

without incurring a direct utility cost. However, changes in the search intensity of informal workers entail an adjustment cost. Successful job search results in a transition to formal employment within the same period, as specified in Equation (2). Formal workers are assumed not to engage in on-the-job search.

Preferences Households' preferences are described by the lifetime utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} \left(C_t - h_c C_{t-1}\right)^{1-\sigma} + \frac{\varphi}{1+\nu} O_t^{1+\nu} \right\}, \tag{5}$$

where C_t represents consumption in period t and O_t denotes leisure defined as inactivity. The specification of the period utility function is borrowed from Cairó et al. (2022). Note that preferences are written in terms of the utility of leisure (not in terms of the disutility of participation). The parameter $\beta \in (0,1)$ denotes the subjective discount factor, h_c measures the degree of internal habit formation, φ governs the utility of leisure, and ν is the extensive-margin (inverse) elasticity of nonparticipation.

Savings Households have access to a one-period foreign bond B_t that earns a return $r_t^* = (1+i_t^*)\Theta(B_t)$, where i_t^* follows the stochastic process

$$\log(1+i_t^*) = \rho_i \log(1+i_{t-1}^*) + (1-\rho_i)\log(1+i^*) + \varepsilon_t^i,$$

in which disturbances ε_t^i are iid with mean zero and variance σ_i^2. We introduce $\Theta(B_t)$ as an endogenous risk premium with a small elasticity to the net foreign asset position to eliminate the unit root commonly present in small open economy models (see Schmitt-Grohe and Uribe, 2003).

Budget constraint Each period, households face the following budget constraint:

$$(1+\tau_c)C_t + (1+r_{t-1}^*)B_{t-1} = (1-\tau_w)w_t L_t^f + (1-S_t^s)w_t^s L_t^s + B_t + \kappa_t s_t L_{t-1}^f + \Pi_t + T_t + C_t^{ev} - \Xi_t^S, \tag{6}$$

where Π_t denotes entrepreneurs' profits and T_t represents lump-sum government transfers. The term C_t^{ev} corresponds to the reimbursement of concealment expenditures incurred by final-good firms, as defined below. Since these outlays reflect payments for domestically supplied services, they ultimately accrue to the representative household as income. Finally, Ξ_t^S denotes the adjustment costs associated with changes in the search intensity of informal workers (see Appendix A).

Upon separation, the firm pays the worker a severance payment, κ_t, defined as a fixed number of weeks of the economy-wide average formal-sector wage:

$$\kappa_t = \kappa_{tg} \bar{w}_t, \tag{7}$$

where $\bar{w}_t$ denotes the average wage in the formal sector. Because individual firm–worker pairs take this aggregate wage as given during bargaining, severance payments are independent of the match-specific wage.[11]

3.1.2 Matching and labor demand in the formal sector

While the current unemployed U_t and informal workers L_t^s, with search intensity S_t^s, search for jobs in the formal sector, formal entrepreneurs post vacancies V_t. Formal job matches F_t result from a Cobb-Douglas matching technology $F_t = D(U_t + S_t^s L_t^s)^{\phi} V_t^{1-\phi}$, where D governs the matching efficiency and ϕ is the elasticity of the matching function.

Job finding and vacancy filling probabilities From the matching function, the probabilities p_t of a searching worker finding a job and q_t of an employer succeeding in filling a vacancy are given by

$$p_t = D\left(\frac{U_t + S_t^s L_t^s}{V_t}\right)^{\phi-1} \quad \text{and} \quad q_t = D\left(\frac{U_t + S_t^s L_t^s}{V_t}\right)^{\phi}, \tag{8}$$

where the ratio $\theta_t \equiv \left(\frac{V_t}{U_t + S_t^s L_t^s}\right)$ is the labor market tightness.

Law of motion of formal employment Formal employment is a long-term relationship between workers and firms. Once a match is formed, it remains active until separation occurs. Let s_t denote the separation rate realized between periods $t-1$ and t. Formal employment

[11] This specification preserves the standard Nash bargaining framework. In equilibrium, where $\bar{w}_t = w_t$, severance payments are proportional to formal-sector wages, consistent with statutory requirements.

evolves according to

$$L_t^f = (1 - s_t)L_{t-1}^f + q_t V_t. \tag{9}$$

We assume that matches active at the beginning of period t generate output throughout the period, while separations occur between periods and affect future continuation values. Consequently, formal employment dynamics depend on the realized separation rate s_t, whereas continuation values are based on the expected separation rate s_{t+1}.

Value of a match in the formal sector An active match produces one unit of the formal intermediate input and employs one worker. The entrepreneur's value of a match is defined recursively as

$$J_t = \left[\Omega p_t^{M,f} - (1+\tau)w_t\right]\lambda_t^C + \beta E_t\left[(1 - s_{t+1})J_{t+1} - \kappa_t s_{t+1}\lambda_{t+1}^C\right], \tag{10}$$

where λ_t^C denotes the marginal utility of consumption. This expression incorporates two labor market policy instruments: a payroll tax, τ, levied by the government, and a firing cost, κ_t, modeled as a severance payment to the worker (see Equation (7)). The severance payment is incurred when a match is dissolved before production takes place in period $t+1$, implying that the continuation value depends on κ_t rather than κ_{t+1}. Finally, the firm's outside option is assumed to be zero.

On the other hand, the utility value for the household of having a member employed in the formal sector λ_t^{Lf} is defined recursively by

$$\lambda_t^{Lf} = (1 - \tau_w)w_t\lambda_t^C - \varphi\left(O_t\right)^{\nu} + \beta E_t[(1 - s_{t+1})\lambda_{t+1}^{Lf} + \kappa_t s_{t+1}\lambda_{t+1}^C]. \tag{11}$$

Notice that the informal wage w_t^s is the outside option for formal workers.

Endogenous separation Following Mortensen and Pissarides (1994), we assume that the separation rate depends endogenously on the value of a match for the entrepreneur. Let the normalized value of a match be $\tilde{J}_t = J_t/\lambda_t^C$; then, the separation rate is given by

$$s_t = \tilde{s}\frac{1}{1+\exp\left(\frac{\tilde{J}_t}{\sigma_J}\right)}, \tag{12}$$

where $\tilde{s}$ is a scale parameter and σ_J governs the dispersion of match-specific shocks. This logistic specification ensures $s_t \in (0, \tilde{s})$ is monotonically decreasing in $\tilde{J}_t$ and allows permanent changes in factors affecting J_t (e.g., firing costs or taxes) to generate permanent changes in the separation rate.[12]

Wage determination The formal sector wage w_t is determined by period-by-period bargaining using the Nash protocol. Every period, after observing the shocks, the matched entrepreneur-worker pair negotiates over the current wage, taking the severance payment κ_t as predetermined. Thus, the bargaining problem is

$$w_t = \arg\max\left\{\left(\lambda_t^{Lf}\right)^{\gamma}(J_t + \kappa_t U_{c,t})^{1-\gamma}\right\},$$

where γ is the worker's bargaining power, and the second term captures the entrepreneur's value of keeping a match, net of the alternative option of closing it.

The Nash-bargained wage is implicitly defined by the standard sharing rule given by

$$(1-\gamma)\lambda_t^{Lf} = \gamma\left(J_t + \kappa_t U_{c,t}\right).$$

Zero-profit condition for vacancy posting Formal entrepreneurs can post vacancies at a constant cost η, which covers the advertisement of jobs and the implementation of screening and review processes. A vacancy lasts for one period only. If the vacancy meets a worker, with probability q_t, the match becomes active in the current period. Assuming competitive

[12] More specifically, the choice of such a logistic function serves two purposes: (i) it preserves the intuition of the endogenous destruction mechanism in Mortensen and Pissarides (1994), wherein matches dissolve when their value falls below a threshold; and (ii) it avoids the need to explicitly model heterogeneous firms, thus reducing computational complexity without sacrificing realism.

entrepreneurs, the following zero-profit condition for vacancy posting holds:

$$\eta\lambda_t^C = q_t J_t. \tag{13}$$

3.1.3 The informal sector

The representative firm in the informal sector hires informal workers for one period. In contrast to formal firms, this decision is static. Perfect competition in the informal sector implies that the wage rate of informal workers satisfies

$$w_t^s = \chi p_t^{M,s}, \tag{14}$$

where $p_t^{M,s}$ denotes the price of informal intermediate goods relative to the final consumption goods bundle, and χ is a parameter denoting the productivity of informal workers in units of the informal intermediate good (see below). Although we assume that informal firms are less productive than formal matches $\chi < \Omega$, this disadvantage is compensated for by their ability to avoid payroll taxes and the lack of entry or hiring costs. Notice also that the informal sector is excluded by assumption from financial markets, so interest rates have no first-order effects on its dynamics.

3.2 Final and intermediate goods production

There are three technologies in the economy. One is to produce a final good using intermediate goods and private and public capital. Intermediate goods are made by formal or informal workers using linear production functions that depend only on labor. We assume that the intermediate inputs produced in the formal and informal sectors are imperfect substitutes for each other. This structure allows us to derive a simple expression for the aggregate production function in which TFP has an endogenous component.

Furthermore, informal input production entails real resource costs for the final-good producing firms, yet these are typically unreported for tax purposes. This leads us to distinguish between *economic profits*, which net out all real costs, and *taxable income*, which only nets out deductible, verifiable items.

Moreover, these firms are allowed to understate taxable income by introducing an endogenous compliance rate. Deviations from full compliance entail concealment costs, reflecting the fact that firms spend real resources to hide informal activities and tax evasion.[13] For the sake of simplicity, these costs are assumed increasing in both undeclared earnings and the *traceability* of the input mix, as we define later on.

3.2.1 Final goods production

Intermediate inputs M_t and private and public capital are combined to produce a final good Y_t using a constant returns to scale technology

$$Y_t = A_t K_{g,t-1}^{\alpha_1} K_{t-1}^{\alpha} M_t^{1-\alpha}, \tag{15}$$

where K_{t-1} represents private capital, $K_{g,t-1}$ captures the role of public capital in affecting private output, and each type of capital has a share in production $\alpha, \alpha_1 \in (0,1)$. In this specification, A_t is an aggregate technology shock following the stochastic process given by

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_t^A,$$

in which $\rho_A \in (0,1)$ governs the persistence of the shock and ε_t^A is and iid innovation with mean zero and variance σ_A^2.

Also, the process of capital accumulation displays adjustment costs, so that investing I_t units of the final good increases the capital stock according to the law of motion

$$K_t = (1-\delta)K_{t-1} + \zeta_t I_t \left(1 - \frac{\vartheta_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right), \tag{16}$$

where $\delta \in (0,1)$ denotes the capital depreciation rate and the investment adjustment cost function $\frac{\vartheta_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ ensures that adjustment costs are null in the steady state. The term

[13]In reality, such concealment costs typically comprise hiring accountants and lawyers and, particularly, paying bribes to public officials in order to avoid inspections (see Johnson et al., 1997).

ζ_t, in turn, represents an investment efficiency shock that follows the stochastic process

$$\log \zeta_t = (1-\rho_\zeta)\log \bar{\zeta} + \rho_\zeta \log \zeta_{t-1} + \varepsilon_t^\zeta,$$

where $\rho_\zeta \in (0,1)$ and ε_t^ζ is iid with mean zero and variance σ_ζ^2.

Lastly, note that public capital is modeled as a positive externality that complements private inputs.

3.2.2 Intermediate goods production

The aggregate intermediate good is a composite of inputs produced in the formal sector (M_t^f) and in the informal sector (M_t^s) according to the CES aggregator

$$M_t = \left\{ (M_t^f)^{\frac{\epsilon-1}{\epsilon}} + (M_t^s)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}}, \tag{17}$$

with elasticity of substitution ϵ. Formal and informal workers produce their intermediate goods using linear deterministic technologies $M_t^f = \Omega L_t^f$ and $M_t^s = \chi(1-S_t^s)L_t^s$, where S_t^s indicates the search effort of informal employees at time t.

The demand for formal and informal intermediate inputs, and hence their relative prices, are obtained from the final-good firm's cost minimization problem, taking into account corporate taxation and concealment costs. The resulting first-order conditions are reported in Appendix A.

From the CES aggregator (17), the relative price of formal intermediate goods is

$$p_t^{M,f} = (1-\alpha)A_t K_{g,t}^{\alpha_1} \left(\frac{K_t}{M_t}\right)^{\alpha} \left(\frac{M_t}{\Omega L_t^f}\right)^{\frac{1}{\epsilon}}, \tag{18}$$

and the relative price of informal intermediate goods is

$$p_t^{M,s} = p_t^{M,f} \left(\frac{\Omega L_t^f}{\chi(1-S_t^s)L_t^s}\right)^{\frac{1}{\epsilon}}. \tag{19}$$

3.2.3 Corporate income taxation with endogenous compliance and traceability

Economic profits from final-good production subtract all real input expenses, including those for informal operations. Thus, it is defined as

$$\Pi_t \equiv Y_t - p_t^{M,f} M_t^f - p_t^{M,s} M_t^s - I_t. \tag{20}$$

Taxable income Final good firms are assessed a tax on their corporate income at a fixed statutory rate τ^r. However, they are allowed to deduct both depreciation and the expenses associated with formal operations $p_t^{M,f} M_t^f$ from calculation of their tax base. Those expenses associated with informal operations $p_t^{M,s} M_t^s$, in contrast, are *non-deductible* real costs due to their lack of verifiable documentation.

In addition to tax avoidance, final good firms choose to conceal a fraction $(1 - \varphi_t) \in (0, 1]$ of their activities from the tax authority. Accordingly, the reported tax base is

$$\mathcal{B}_t \equiv \varphi_t \left(Y_t - p_t^{M,f} M_t^f - \delta K_{t-1} \right) \tag{21}$$

while corporate tax liabilities are defined as

$$T_t^r \equiv \tau^r \max\{\mathcal{B}_t, 0\}, \tag{22}$$

where the non-negativity operator rules out negative tax obligations in low-profit states.[14]

Traceability and the cost of under-reporting Deviations from full compliance entail concealment costs that are assumed to scale with output and to be an increasing function of both undeclared earnings and the *traceability* of the input mix.

Let the formal share of intermediate inputs be defined as

$$m_t^f \equiv \frac{M_t^f}{M_t}. \tag{23}$$

[14] To avoid non-differentiation issues, we assume that both the calibration and shock sizes guarantee $\mathcal{B}_t > 0$ along the equilibrium path, so that corporate tax liabilities are always positive.

Then the tax evasion cost is given by

$$C_t^{ev} \equiv \frac{\kappa_\varphi}{2}(1-\varphi_t)^2\, Y_t\, (m_t^f)^\psi, \qquad \kappa_\varphi > 0,\ \psi > 0, \tag{24}$$

where, following Chen (2003), the unit cost of evasion takes the convex form $\frac{\kappa_\varphi}{2}(1-\varphi_t)^2$. The last term on the right side aims to convey the notion of *traceability*; that is, when production relies more heavily on formal inputs (higher m_t^f), a larger fraction of total output is observable to the tax authority, making noncompliance more costly on the margin. Conversely, when the firm relies relatively more on informal inputs (lower m_t^f), transactions are more likely to occur off the books, reducing the information content available for enforcement and thereby lowering the marginal cost of under-reporting. The parameter ψ governs how sensitive enforcement is to the formal share of intermediate inputs.

Final-good firm problem We assume that the firms that produce the final good are ultimately owned by the household and thus discount expected future profits using the same stochastic discount factor used by the household, defined by $\tilde{\beta}^j = \beta^j \frac{\lambda_{t+j}^C}{\lambda_0^C}$.

The problem of these firms is

$$\max_{\{K_t, I_t, M_t^f, M_t^s, \varphi_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ Y_t - p_t^{M,f} M_t^f - p_t^{M,s} M_t^s - I_t - T_t^r - C_t^{ev} \right\},$$

subject to the restrictions (15), (17) and (16).

Recall that concealment expenditures C_t^{ev} are rebated to the representative household through lump-sum transfers. As both firms and households take these transfers as given, they do not affect optimal behavior.

3.2.4 Aggregation

Combining the final good production function (15) and the intermediate good technology (17), we obtain the aggregate production function for the economy

$$\underbrace{Y_t}_{\text{GDP}} = \underbrace{\left\{ A_t K_{g,t}^{\alpha_1} \left[(\Omega(1-l_t^s))^{\frac{\epsilon-1}{\epsilon}} + (\chi l_t^s)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon(1-\alpha)}{\epsilon-1}} \right\}}_{\text{TFP}} (K_t)^{\alpha} (L_t)^{1-\alpha}, \tag{25}$$

where the term in brackets represents measured total factor productivity (TFP) and includes both an exogenous (A_t) and an endogenous component, the latter depending on public capital $K_{g,t}$ and the informality rate $l_t^s \equiv \frac{(1-S_t^s)L^s}{L_t} = \frac{(1-S_t^s)L_t^s}{L_t^f + (1-S_t^s)L_t^s}$, i.e., the share of informal workers in total employment.

3.3 Fiscal policy

The government balances a budget constraint that equals revenues with expenditures. More specifically, the government receives revenues from oil Y_t^p and tax collection Tax_t and uses them to finance government expenditures G_t consisting of public consumption C_t^g, public investment I_t^g, and household transfers T_t. Denoting government revenues by G_t^r, the government budget constraint is

$$G_t = G_t^r, \tag{26}$$

where

$$G_t^r = Tax_t + Y_t^p, \tag{27}$$

$$G_t = C_{g,t} + I_{g,t} + T_t, \tag{28}$$

$$Tax_t = \tau_c C_t + T_t^r + (\tau + \tau_w) w_t L_t^f. \tag{29}$$

It can be seen in Equation (29) that tax collection from households takes the form of consumption taxes at a rate τ_c and (formal) labor income taxes at a rate τ_w. Similarly, the government raises taxes on formal intermediate goods producers by levying payroll taxes at a rate τ and corporate income taxes at a rate τ^r (see Equation (22)).

Furthermore, government investment determines public capital accumulation, as dictated by the following law of motion:

$$K_{g,t} = (1-\delta_g)K_{g,t-1} + I_{g,t}\left(1 - \frac{\vartheta_g}{2}\left(\frac{I_{g,t}}{I_{g,t-1}} - 1\right)^2\right), \tag{30}$$

where δ_g denotes the public capital depreciation rate.

Moreover, public consumption and public investment follow auto-regressive smoothing rules that revert to their long-run targets $\bar{C}_g$ and $\bar{I}_g$; that is,

$$C_{g,t} = (C_{g,t-1})^{\rho_{c_g}}(\bar{C}_g)^{1-\rho_{c_g}}\exp(\varepsilon_t^{C_g}), \tag{31}$$

$$I_{g,t} = (I_{g,t-1})^{\rho_{I_g}}(\bar{I}_g)^{1-\rho_{I_g}}\exp(\varepsilon_t^{I_g}), \tag{32}$$

where $\rho_{c_g}, \rho_{I_g} \in (0,1)$ govern the persistence of expenditure adjustments, and $\varepsilon_t^{C_g}$ and $\varepsilon_t^{I_g}$ are expenditure shocks.[15]

Given the absence of government debt, lump-sum transfers T_t adjust residually to satisfy the government budget constraint (26)-(28) period by period.

Finally, we assume that oil revenues are given by

$$Y_t^p = P_t^b y_t^p, \tag{33}$$

where y_t^p is the volume of exported oil and P_t^b its international price. We assume that both y_t^p and P_t^b follow AR(1) stochastic processes, as detailed in Appendix A.

3.4 Market clearing

In each period, the market for the final good clears so that

$$C_t + I_t + (1+r_{t-1}^*)B_{t-1} = Y_t + B_t - \eta V_t + Y_t^p - C_{g,t} - I_{g,t} - \Xi_t^S.$$

[15]The long-run targets $\bar{C}_g$ and $\bar{I}_g$ in turn are defined as fractions of aggregate gross income in steady state, $\bar{C}_g = \varsigma_c\left(\bar{Y} + \bar{Y}^p\right)$ and $\bar{I}_g = \varsigma_i\left(\bar{Y} + \bar{Y}^p\right)$, where ς_c and ς_i are the shares of public consumption and public investment in GDP. Further details on these target values can be found on Table B.1 in Appendix B.

Also, the law of motion of foreign debt is given by

$$B_t = (1 + r^*_{t-1})B_{t-1} - NX_t, \tag{34}$$

and hence one obtains the resource constraint

$$C_t + I_t + C_{g,t} + I_{g,t} + \eta V_t + \Xi^S_t + NX_t = Y_t + Y^p_t. \tag{35}$$

This constraint indicates that the final good is allocated to private and public consumption, investment, net exports, formal-sector vacancy posting costs, and search-intensity adjustment costs.

It is worth noting that concealment expenditures C^{ev}_t do not enter Equation (35). Although these expenditures represent a genuine private cost that affects final-good firms' compliance and input-mix decisions, they do not constitute a net loss of resources at the aggregate level and are therefore excluded from the economy's resource constraint. Further details are provided in Appendix A.

4 Fitting the model

The model is fitted to the Colombian economy using information on labor market outcomes and macroeconomic aggregates. The fitting strategy consists of two steps. First, the steady state is calibrated to replicate pre-reform averages of key labor market and macroeconomic indicators. Second, the parameters governing business-cycle fluctuations are estimated using the generalized method of moments (GMM) on quarterly data from 2006IV to 2019IV.

Note that the estimation relies on data extending beyond the reform period because the length of the sample provides sufficient time-series variation to discipline labor-market comovements.[16] Even so, policy analysis is conducted around the pre-reform steady state.

[16] This is especially true for the comovements that help identify the elasticity of substitution between formal and informal inputs.

4.1 Steady-state calibration

The calibrated parameters are divided into two groups: those whose values are assigned and those whose values are calibrated. The first group of parameters is listed in Table 2. Some of the parameter values in this group are common in the DSGE literature, such as the discount factor (β), the habit formation coefficient (h_c), the depreciation rate (δ), and the capital share (α); or assigned arbitrarily, as in the case of the elasticity of the concealment cost to traceability (ψ).[17] Other parameter values are taken from studies or estimated using data from the Colombian economy. Thus, the value of the risk aversion coefficient (σ) comes from Prada and Rojas (2010) and the elasticity of the matching function (ϕ) results from applying the estimation methodology of Blanchard and Diamond (1990) on series of hires and vacancies that, in turn, were computed by Morales et al. (2019) with GEIH microdata.

Table 2: Fixed parameters

Parameter	Value	Source
Discount factor (β)	0.99	Annual interest rate of 4%
Risk aversion coefficient (σ)	2.35	Prada and Rojas (2010)
Habit formation in consumption (h_c)	0.65	DSGE literature
Depreciation rate (δ)	0.025	DSGE literature
Capital share (α)	0.36	DSGE literature
Traceability elasticity of concealment cost (ψ)	1	Arbitrary
Elasticity of matching function (ϕ)	0.44	Estimation based on Blanchard and Diamond (1990)*
Payroll tax rate (τ)	0.471	Payroll costs

Note: * Series of hires and vacancies used in this estimation are computed from GEIH data using the methodology of Morales et al. (2019).

The second group of parameters comprises the utility of leisure (φ), the steady-state value of the informal job offer arrival rate ($\bar{s}_{UI}$), the bargaining power of the worker (γ), the productivity of the formal sector (Ω), the productivity of the informal sector (χ), the scale of the matching function (D), the firing cost (κ_{tg}), the separation rate scale parameter ($\tilde{s}$), and the cost of posting a vacancy (η). While the productivity of the formal sector is normalized to one, the remaining parameters are calibrated to match the following steady-state target

[17] In this case, we set $\psi = 1$ with the aim to impose a transparent benchmark in which traceability increases concealment costs proportionally, thus avoiding a further strong curvature in the absence of direct micro evidence to discipline this elasticity.

statistics: (i) informal employment (as a share of the working age population) of 34.80%; (ii) fraction of out of the labor force in POP of 33.31%; (iii) share of unemployment in POP of 7.39%; (iv) wage premium in the formal sector (relative to the informal sector) of 89%; (v) labor market tightness of 0.29; (vi) separation cost equivalent to 10 weeks of the average wage; (vii) long-term separation rate of 8.8%; and (viii) ratio of recruitment expenses to GDP of 0.01. Table 3 summarizes the calibrated parameters and the steady-state targets.

Table 3: Calibrated parameters

Parameter description	Value	Target	Data	Source
Utility of leisure (φ)	0.122	Out of the labor force (% POP)	33.31	GEIH – National aggregate
Informal job offer arrival rate ($\bar{s}_{UI}$)	0.129	Informal employment (% POP)	34.80	GEIH – National aggregate
Worker's bargaining power (γ)	0.653	Unemployment (% POP)	7.39	GEIH – National aggregate
Productivity of informal sector (χ)	0.044	Formal wage premium (%)	89	GEIH – National aggregate
Scale of matching function (D)	0.477	Labor market tightness	0.29	GEIH – 23 main cities (fas)[a,b]
Firing cost (κ_{tg})	2.146	Firing cost (weeks of wages)	10	PILA, labor code[b]
Separation rate scaling ($\tilde{s}$)	2.695	Steady-state separation rate (%)	8.8	Morales et al. (2019)
Cost of posting a vacancy (η)	2.812	Recruitment expenses/GDP	0.01	Boz et al. (2015)

Notes: [a] Series of vacancies are computed using the methodology of Morales et al. (2019).
[b] Computing procedures described in Bonilla et al. (2024).

Note that the first four target values correspond to quarterly averages calculated using GEIH data for the national aggregate during the 2005I-2012IV period. In turn, labor market tightness is calculated as the ratio of vacancies to unemployment, with vacancies computed from GEIH data for formal wage earners in 23 cities using the methodology of Morales et al. (2019). Further, the long-term separation rate and the recruitment expenses to GDP ratio are taken from Morales et al. (2019) and Boz et al. (2015), respectively.

Finally, using PILA data, we find that the average tenure of a worker who is separated from his job in Colombia is approximately 3 years. An employee dismissed unjustly with such seniority, according to the labor code, must receive severance pay equal to 70 days of wages; therefore, the target number of weeks is set at 10. This number, in turn, is weighted by the percentage of separations that are effectively dismissals in the country (44%). Further details on the computation of the target values (v) and (vi) are described in Bonilla et al. (2024).

4.2 Generalized method of moments

In addition to the steady-state calibration, we estimate the values of the following parameters: the substitution elasticity between formal and informal inputs (ϵ), the elasticity of nonparticipation (ν), the dispersion of match-specific shocks (σ_J), the unemployment-to-informality transition elasticity (ϕ_{UI}), and the investment adjustment cost (ϑ_k). We also estimate the persistence coefficients and the standard deviations of the exogenous auto-regressive processes: the technology shock (ρ_A, σ_A), the government expenditure shock (ρ_{c_g}, σ_{c_g}), the investment efficiency shock (σ_{z_I}), and the foreign interest rate shock (ρ_i, σ_i).

To this end, we use GMM estimation to match the observed cyclical behavior of several macroeconomic and labor market indicators. Our intuition is that output, government expenditure, and foreign interest rate moments inform aggregate shock propagation ($\rho_A, \sigma_A, \rho_{c_g}, \sigma_{c_g}, \rho_i, \sigma_i$); investment moments provide information regarding both the adjustment cost ϑ_k and the investment efficiency shock σ_{z_I}; formality-to-informality comovements help pin down the substitution elasticity ϵ; moments of the separation rate and labor market tightness discipline firm-level dispersion σ_J; the unemployment-to-inactivity comovement helps identify the nonparticipation elasticity ν; and the unemployment-to-informality comovement informs the transition elasticity ϕ_{UI}. Details on data treatment and estimation procedures are described in Appendix B.

Table 4 reports the parameter estimates along with their associated standard errors and t-statistics, while Table B.2 contrasts the estimated moments with their targeted counterparts. Overall, the model, together with the chosen set of moment conditions, provides an adequate fit to the data. In particular, the estimated model replicates the joint behavior of output growth, investment, government consumption, and several labor-market variables with remarkable accuracy.

Note that the model is highly over-identified, with roughly 1.5 times as many moments as parameters. Nevertheless, the null hypothesis of model validity cannot be rejected: the J-test of over-identifying restrictions yields a statistic of 3.073 with a p-value of 0.800. Finally, both the parameter estimates and the corresponding diagnostic statistics exhibit robustness across

Table 4: Results from GMM estimation

	Estimate	Std. error	t-stat
ϵ	1.2772	0.0127	100.3778
ν	-1.4534	0.5894	-2.4661
σ_J	0.8307	0.0343	24.1921
ϕ_{UI}	1.3433	0.5314	2.5279
ϑ_k	7.2235	1.4965	4.8269
ρ_A	0.2415	0.1109	2.1781
ρ_{c_g}	0.6053	0.1648	3.6732
ρ_i	0.8739	0.0210	41.6436
σ_A	0.0068	0.0007	9.6694
σ_{c_g}	0.0215	0.0022	9.9910
σ_{z_I}	0.1960	0.0400	4.9008
σ_i	0.0024	0.0003	8.6421

alternative specifications (see Table B.3 in Appendix B).

As regards the labor market, the estimated model matches the cyclical comovements of formal and informal employment, non-employment, and labor market tightness quite well. Table 5 shows that, indeed, the model successfully reproduces the negative correlations observed in the data between both informal employment and unemployment with GDP,[18] suggesting that these variables exhibit countercyclical behavior. Importantly, the model's ability to generate negative comovement between unemployment and output represents a clear improvement over earlier efforts to incorporate endogenous participation decisions into SOE-RBC frameworks with search frictions (e.g., Leyva and Urrutia, 2020, 2023), in which unemployment has been found to be procyclical.

Nonetheless, the baseline model fails to account for the cyclical variability of unemployment. As shown in Table 5, the standard deviation of unemployment implied by the estimated model is substantially smaller than that observed in actual data. Consequently, the model inherits the well-known unemployment volatility puzzle present in standard search-and-matching frameworks (Shimer, 2005). This shortcoming is also evident in other SOE-RBC

[18]Note that, as described in Appendix B, the estimation procedure demeans each data series by its empirical mean and each model moment by its theoretical mean. Thus, while the estimation results reported in Table 4 and in Tables B.2-B.3 are consistent with this procedure, both the empirical and theoretical moments contrasted in Table 5 are computed from HP-filtered series with a smoothing parameter of 1600.

Table 5: Business cycle moments of labor market: data vs model*

	Data	Model
Standard deviation		
$\sigma(U)$	4.086	0.484
$\sigma(\theta)$	1.894	2.059
Correlation		
Corr(L^f, Y)	0.401	0.892
Corr(L^s, Y)	-0.182	-0.573
Corr(U, Y)	-0.683	-0.564
Corr$(O + U, Y)$	-0.638	-0.680
Corr(θ, Y)	0.446	0.389
Corr(L^f, L^s)	-0.432	-0.743
Corr(U, L^s)	0.255	0.679

Note: * Moments computed from HP-filtered series with a smoothing parameter of 1600.

models with labor market frictions, such as Boz et al. (2015), even though both settings depart from the canonical Mortensen-Pissarides model along several dimensions.

5 Quantitative analysis

5.1 Simulating the tax reform

Once the model has been fitted to data, we examine the quantitative implications of the payroll tax component of Colombia's 2012 tax reform. To this end, we conduct a policy experiment consisting of a permanent reduction from 47.1% to 33.6% (13.5 pp) in the payroll tax rate (τ) and analyze how this reduction affects the steady-state values of several labor market and macroeconomic indicators. As described in Section 2, the payroll tax reduction was accompanied by the creation of the CREE tax and a simultaneous reduction in the statutory CIT rate. Since this arrangement did not imply a permanent increase in the combined statutory corporate tax rate, our experiment keeps the CIT rate constant.[19]

Table 6 presents the long-run effects of this payroll tax reduction and compares the model-

[19] Because the CREE-related increase in the combined statutory corporate tax rate was small and non-lasting, incorporating it as a transitory CIT shock would not affect the post-reform steady state analyzed in the simulation exercise.

based simulation with the observed performance of the Colombian economy five and ten years after the 2012 reform. The observed post-reform figures are intended as descriptive benchmarks rather than causal estimates, since they may also reflect other policy changes and aggregate shocks over the period. The column labeled ‘Baseline Colombia’ reports the model’s steady-state values, which serve as the reference point. For both the simulated and observed outcomes, the corresponding ‘Value’ columns display levels of each variable, while the ‘Variation’ columns show deviations from the baseline. Depending on the variable, these deviations are expressed in percentage points or as percentages relative to the baseline steady state.

Table 6: Long-run effects of Colombia’s 2012 tax reform: simulation vs actual data

Variable	Baseline	Simulated 13.5 pp payroll tax cut		Observed post-reform outcomes			
				5 years later		10 years later	
	Colombia	Value	Change	Value	Change	Value	Change
Employment rate (%)	59.3	59.3	0.01 pp	59.5	0.22 pp	56.5	-2.83 pp
Formal employment (% POP)	24.5	27.6	3.14 pp	25.3	0.79 pp	27.0	2.50 pp
Informal employment (% POP)	34.8	31.7	-3.13 pp	34.2	-0.55 pp	29.5	-5.33 pp
Informality rate (%)	58.7	53.4	-5.28 pp	57.5	-1.16 pp	52.2	-6.49 pp
Participation rate (%)	66.7	65.9	-0.76 pp	66.0	-0.66 pp	63.4	-3.34 pp
Unemployment rate (%)	11.1	10.0	-1.04 pp	9.8	-1.24 pp	10.9	-0.22 pp
Probability of finding a job	0.19	0.20	1.08 pp	–	–	–	–
Labor market tightness	0.29	0.33	3.92 pp	0.31	2.28 pp	0.38	8.77 pp
Formal wage	6.28	6.59	4.81 %	–	–	–	–
Formal wage premium	0.89	0.79	-11.51 %	0.73	-18.09 %	0.91	2.66 %
TFP	2.09	2.15	2.68 %	–	–	–	–
GDP	6.07	6.34	4.50 %	–	–	–	–

The simulation results suggest that the payroll tax reduction leads to a substantial reallocation of labor across sectors, reducing informal employment by 3.13 pp and increasing formal employment by 3.14 pp, and to a decline in the unemployment rate of 1.04 pp. In addition, the net wage for formal workers rises by 4.81%, indicating a pronounced pass-through effect of the payroll tax cut on the labor market. Overall, these findings are in line with Colombia’s observed post-reform behavior, particularly over a five-year horizon.

Furthermore, the simulation results are broadly consistent with the existing evidence on the effects of the 2012 tax reform. As reviewed in Section 2, empirical studies report increases in formality rates between 2.92 and 6.80 pp and reductions in informality rates ranging from -5.40 to -3.15 pp (see Table 1). The findings in Table 6, with a 5.28 pp decline in informality, fall well within these ranges, suggesting that the model provides a reliable approximation of the reform's effects. Likewise, the predicted 4.81% increase in formal wages is compatible with the literature, which reports gains ranging from 0.42% to 4.85%.

Moreover, the model predicts a 1.04 pp decrease in the unemployment rate, highlighting its ability to replicate a particularly challenging result, namely, payroll tax cuts can simultaneously reduce informality and unemployment. This represents a clear improvement over earlier studies on the impact of payroll tax reductions—especially those based on models with matching frictions in both formal and informal sectors—which typically conclude that such policies are ineffective at reducing informality and enhancing labor market performance at the same time (Charlot et al., 2015; Ulyssea, 2010).

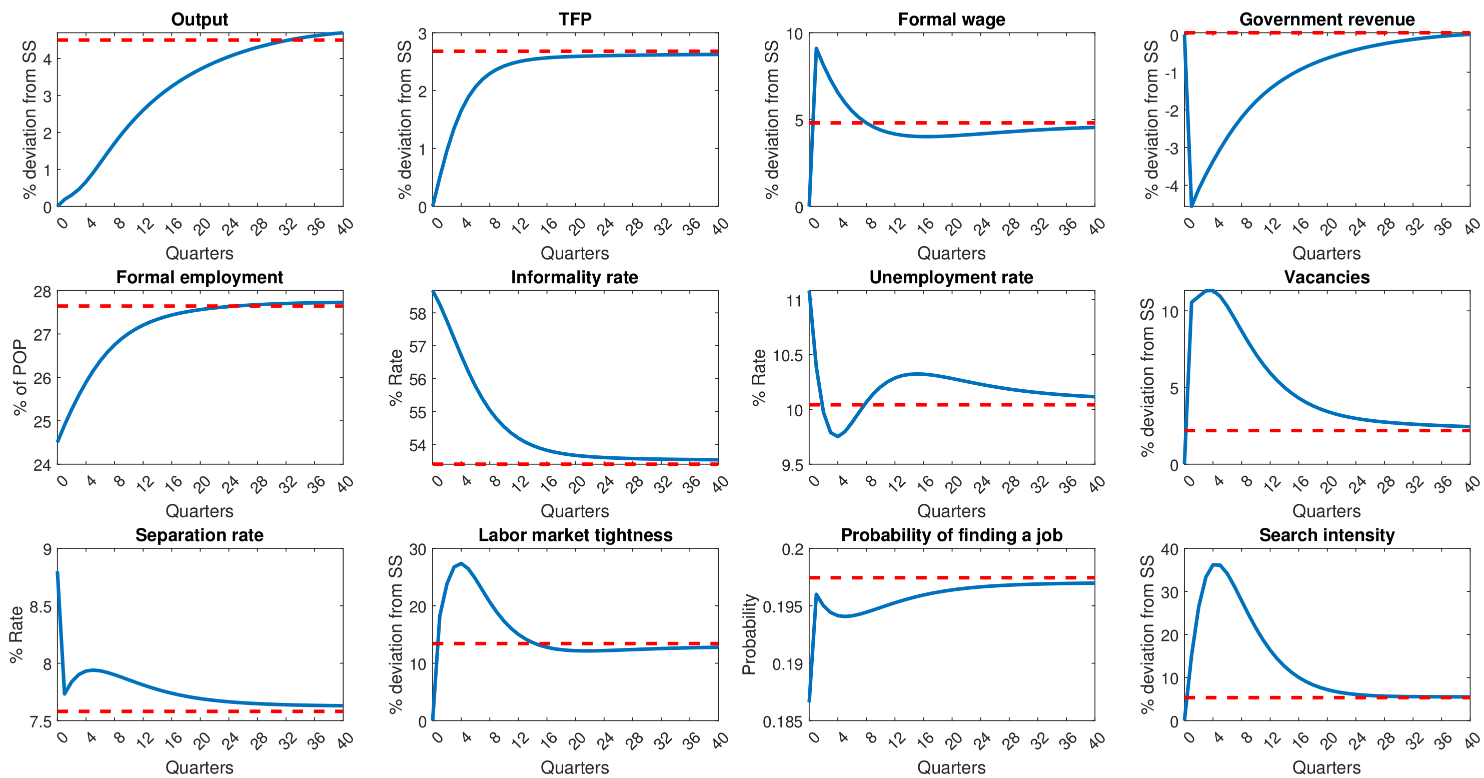
Finally, the model predicts an 11.5% decline in the formal wage premium. While this estimate appears reasonably accurate over a five-year horizon, it is not supported by the data a decade after the reform. Moreover, this result contrasts with the existing literature, which suggests that reductions in payroll taxes may either increase the formal wage premium—by lowering relative wages in the informal sector (Charlot et al., 2015)—or have limited long-term effects on wage differentials (Ulyssea, 2010).

5.2 Inspecting the mechanism

Figure 2 illustrates the mechanism associated with the simulated tax reform. In the figure, the blue lines depict the trajectories of the model variables from the initial steady state at quarter zero to their post-reform long-run levels, indicated by the red dashed lines. Depending on the variable, some trajectories are expressed as percentage deviation from the initial steady-state values while others are reported as rates, working-age population shares, or probabilities.

The underlying mechanism is as follows: A lower payroll tax rate increases the value of a match for employers by reducing non-wage labor costs. This discourages firms from firing

Figure 2: Transitional dynamics of the simulated payroll tax reduction



current employees, thereby lowering the separation rate and reducing unemployment. By the same token, firms are eager to hire new workers, leading to increased demand for salaried labor, which in turn raises formal wages. Thus, as in Antón (2014), formal workers bear a considerable share of the payroll tax reduction in the form of higher wages.

As vacancies become more attractive, the labor market tightens and the probability of finding a job increases. This situation encourages unemployed job seekers and informal workers to search more intensively, which translates into an increase in formal employment. Now, given that the supply of informal labor decreases, and since informal jobs are not subject to regulations or wage bargaining, the informal wage rises by a larger proportion, leading to a decline in the formal wage premium. Lastly, when earnings improve in both sectors, individuals value leisure more, and hence participation diminishes.

Note that the change in the composition of employment has a positive impact on productivity, and thereby on output. As the informality rate decreases, the endogenous component of TFP improves, resulting in higher overall productivity and growth. Yet the increase in output is barely enough to offset the lower payroll tax rate, so government revenues do not change substantially. Consequently, reductions in formal-sector taxation affect not only the labor market but also macroeconomic performance as a whole.

5.3 Alternative financing scenarios

In both the baseline model and the policy experiment, all tax rates other than the payroll tax remain at their pre-reform levels. Similarly, public consumption and public investment converge to their steady-state values, $\bar{C}_g$ and $\bar{I}_g$, following the expenditure rules described in Section 3. As lump-sum transfers T_t adjust residually to satisfy the government budget constraint in every period, the payroll tax reduction is implicitly financed through corresponding changes in transfers.

This section examines how the effects of the payroll tax reduction vary across alternative financing instruments. To ensure comparability, each instrument is calibrated on an ex ante revenue-equivalent basis. That is, when evaluated at the pre-reform steady state and holding behavior constant, the static revenue gain from the financing mechanism exactly offsets the static revenue loss associated with the payroll tax cut. Government consumption and investment are kept at their pre-reform levels throughout the analysis.[20]

Table 7 presents simulation results for alternative approaches to financing the payroll tax reduction that admit an interior solution.[21] Unlike Table 6, which reports steady-state outcomes, this table summarizes average outcomes over the first sixteen quarters of the perfect-foresight transition—the period most relevant for evaluating the reform. For comparison, the final row presents the corresponding long-run effects.

Changes in employment and unemployment are expressed in percentage points relative to the pre-reform equilibrium, while changes in wages, GDP, consumption, government revenue, public consumption, and public investment are reported as percentage deviations from their pre-reform levels. The 'Recovery' column shows the ex-post change in total revenue as a share of the static revenue loss. A value of 100% therefore denotes full ex-post revenue neutrality, whereas lower values indicate greater tax-base erosion.[22]

The 'Transfers adjustment' row reports the transition dynamics of the baseline policy experiment, namely a payroll tax reduction without an explicit offsetting fiscal measure. Under this scenario, all other tax rates and expenditure targets are held constant, while transfers adjust to absorb the resulting fiscal imbalance. Formal employment increases by an average of 1.89 pp during the first four years following the reform, compared with 3.14 pp in the new steady state. These findings suggest that the reform's effects on labor formalization materialize gradually, rather than immediately.

Three results deserve particular attention. First, the reform is largely self-financing. Even in the absence of offsetting measures, the expansion of formal employment and consumption recovers 74.7% of the static revenue loss over the first four years of the transition and nearly

[20]Note that we do not impose ex-post revenue neutrality. Since informality adjusts endogenously, general-equilibrium effects erode tax bases, making full ex-post compensation infeasible for all fiscal instruments considered.

[21]In the cases of value-added and labor-income taxes, the higher tax burden on formal activity reduces the value of a formal match, increasing the separation rate. This shortens match duration and further lowers match values. Beyond a threshold, this feedback mechanism eliminates the interior steady state with an active formal sector.

[22]Further details on the construction of the ex ante revenue-equivalent instruments and the recovery measure are provided in Appendix C.

Table 7: Effects of Payroll Tax Reduction under Alternative Ex-Ante Revenue-Equivalent Financing

Scenario	Δ instr.	Δl^s (pp)	ΔL^f (pp)	ΔU (pp)	Δw (%)	Δw^s (%)	ΔY (%)	ΔC (%)	ΔG^r (%)	ΔC_g (%)	ΔI_g (%)	Recovery (%)
Transfers adjustment	–	-3.00	1.89	-0.92	5.13	6.92	1.55	3.86	-2.28	0.00	0.00	74.7
CIT financing	11.3 pp	0.21	-0.11	0.89	-0.58	-3.75	-0.71	-2.50	-2.22	0.00	0.00	75.4
Revenue-max. VAT rate	2.7 pp	-2.66	1.49	0.24	5.92	6.20	1.35	1.70	0.92	0.00	0.00	110.2
C_g-financed	-18.4 %	-3.05	1.49	-1.41	5.62	7.60	0.35	6.56	-2.66	-15.61	0.00	70.4
Baseline	–	-5.28	3.14	-1.04	4.81	10.81	4.50	5.93	0.05	0.00	0.00	–

Notes: All scenarios implement a 13.5 pp reduction in payroll taxes. Reported effects are averages over the first 16 quarters of the perfect-foresight transition, relative to the pre-reform equilibrium. Unless used as the financing instrument, C_g and I_g are held at their pre-reform levels. Financing instruments are calibrated to be ex ante revenue-equivalent, while 'Recovery' denotes the ex post change in total government revenue relative to the static loss in payroll-tax revenues. The last row reports the long-run effects of the reform without compensatory financing, as shown in Table 6. Appendix C provides further details on the design and calibration of the financing scenarios.

all of it in the long run. While informality typically erodes the tax bases available to finance compensating policies, the opposite occurs when the reform itself promotes the formalization of economic activity.

Second, none of the tax instruments provides a sufficiently strong source of permanent financing once general-equilibrium base erosion is taken into account. CIT financing would require an ex ante increase of 11.3 pp and would substantially diminish the reform's intended benefits: informality rises by 0.21 pp, while formal employment and GDP decline. Furthermore, total revenue recovers 75.4% of the static loss, only slightly above the 74.7% recovery achieved without any financing instrument.

Moreover, raising the VAT to its revenue-maximizing rate can more than offset the estimated static revenue loss during the transition period. Under this scenario, the reform package recovers about 110% of the static loss over the first four years while preserving most of the labor-market gains. This comes at the cost of a modest increase in unemployment and considerably smaller consumption gains. These additional revenues, however, are temporary. As economic activity gradually shifts toward the informal sector, tax-base erosion absorbs much of the initial revenue increase. Consequently, in the long run, the VAT hike plays only a limited role in fiscal recovery, covering roughly one-tenth of the static loss beyond the gains attributable to formalization alone.

Third, the expenditure side offers limited scope for financing the reform. Financing the reform through lower public consumption would require a cut of 18.4% and would largely preserve the labor-market gains, while raising private consumption by 6.56% as resources are reallocated to households. However, output gains would be smaller than under the transfer-financing scenario, as the associated wealth effect dampens labor supply and labor-force participation.[23]

Overall, the financing analysis leads to a clear conclusion: in economies with large informal sectors and endogenous compliance, conventional tax instruments provide limited scope for financing a payroll tax reduction. Higher corporate taxes may ultimately undermine revenue

[23]Financing the reform through cuts in public investment is not feasible, as the required adjustment would exceed the entire pre-reform level of public investment (132.6%).

by eroding the tax base, while the revenue-raising capacity of consumption taxes is constrained by the size of the formal economy. Consequently, the reform's formalization dividend emerges as the most credible financing mechanism, complemented, where necessary, by expenditure-side adjustments.

6 Conclusions

This paper investigates the macroeconomic implications of Colombia's 2012 tax reform using a DSGE framework. The reform combined a substantial reduction in payroll taxes with a partial shift in the tax base from labor to corporate income. To assess its impact, we extend a standard SOE-RBC model by incorporating search-and-matching frictions. The resulting framework provides a richer characterization of labor market dynamics in developing countries, where high levels of informal employment coexist with unemployment and labor force inactivity.

The findings suggest that reducing payroll tax rates promotes formal employment and decreases informality without adverse effects on unemployment. By lowering non-wage labor costs, the reform strengthens firms' incentives to hire and retain formal workers, leading to higher formal-sector wages and lower unemployment. The increased attractiveness of formal employment induces unemployed and informal workers to transition into formal jobs, contributing to the decline in informality.

Beyond labor market effects, the reallocation of workers from informal to formal employment raises productivity and output. Although economic activity expands, these gains only partially offset the decline in payroll tax revenues, leaving overall government revenues largely unchanged. Consistent with the empirical evidence, the findings highlight the reform's effectiveness not only in Colombia but potentially in other countries with high non-wage labor costs that are considering eliminating payroll taxes or replacing them with alternative revenue sources. At the same time, the results underscore the need for complementary policy measures to address transitional fiscal challenges.

While our framework can be applied to a broad range of policy-relevant questions, including labor market regulation in developing countries, it has important limitations. In particular, the model assumes a non-binding minimum wage. This assumption is especially restrictive in the Colombian context, where evidence suggests that the minimum wage is binding (see, for example, Bell, 1997). Extending the framework to incorporate a binding minimum wage remains an important direction for future research.

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A Model details

A.1 Households and workers

Given initial conditions B_0, L^f_{-1}, L^s_{-1}, prices w_t, w^s_t, job finding rate p_t, profits Π_t, and the stochastic processes for aggregate shocks, the representative household chooses contingent plans for aggregate variables $\{C_t, B_t, L^f_t, L^s_t, U_t, O_t, S^s_t\}_{t=0}^{\infty}$ to solve

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} \left[C_t - h_c C_{t-1}\right]^{1-\sigma} + \frac{\varphi}{1+\nu} O_t^{1+\nu} \right)$$

subject to

$$(1+\tau_c)C_t + (1+r^*_{t-1})B_{t-1} = (1-\tau_w)w_t L^f_t + (1-S^s_t)w^s_t L^s_t + B_t + \kappa_t s_t L^f_{t-1} + \Pi_t + T_t + C^{ev}_t - \Xi^S_t, \qquad (\beta^t \lambda^C_t)$$

$$L^f_t = (1-s_t)L^f_{t-1} + p_t(U_t + S^s_t L^s_t), \qquad (\beta^t \lambda^{L^f}_t)$$

$$L^s_t = (1-p_{t-1}S^s_{t-1})L^s_{t-1} + s_{UI,t}(1-p_t)U_t, \qquad (\beta^t \lambda^{L^s}_t)$$

$$L^f_t + L^s_t + O_t + U_t = 1. \qquad (\beta^t \lambda^O_t)$$

where Ξ^S_t denotes an aggregate adjustment cost associated with changes in the search intensity of informal workers as follows:

$$\Xi^S_t = \frac{\vartheta_s}{2}\left(\frac{S^s_t}{S^s_{t-1}} - 1\right)^2 w^s_t.$$

The Lagrangian of this problem is

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{1}{1-\sigma} \left[C_t - h_c C_{t-1}\right]^{1-\sigma} + \frac{\varphi}{1+\nu} \left(O_t\right)^{1+\nu} \\
& + \lambda_t^C \Big((1-\tau_w) w_t L_t^f + (1 - S_t^s) w_t^s L_t^s + B_t + \kappa_t s_t L_{t-1}^f + \Pi_t + T_t \\
& \qquad - (1+\tau_c) C_t - (1 + r_{t-1}^*) B_{t-1} - \Xi_t^S \Big) \\
& + \lambda_t^{Lf} \left((1 - s_t) L_{t-1}^f + p_t (U_t + S_t^s L_t^s) - L_t^f \right) \\
& + \lambda_t^{L^s} \left((1 - p_{t-1} S_{t-1}^s) L_{t-1}^s + s_{UI,t} (1 - p_t) U_t - L_t^s \right) \\
& + \lambda_t^O \left(1 - L_t^f - L_t^s - O_t - U_t \right) \Bigg\},
\end{aligned}
$$

with first-order conditions:

$$
\begin{aligned}
& \frac{\partial \mathcal{L}}{\partial C_t} : \lambda_t^C (1+\tau_c) = U_{c,t} - h_c \beta E_t \left\{ U_{c,t+1} \right\}, \quad U_{c,t} \equiv \left[C_t - h_c C_{t-1}\right]^{-\sigma}, \\
& \frac{\partial \mathcal{L}}{\partial B_t} : \lambda_t^C = \beta E_t \left[\lambda_{t+1}^C (1 + r_t^*) \right], \\
& \frac{\partial \mathcal{L}}{\partial L_t^f} : \lambda_t^{L^f} = \lambda_t^C (1-\tau_w) w_t + \beta E_t \left[(1 - s_{t+1}) \lambda_{t+1}^{L^f} + \kappa_t s_{t+1} \lambda_{t+1}^C \right] - \lambda_t^O, \\
& \frac{\partial \mathcal{L}}{\partial L_t^s} : \lambda_t^{L^s} = \lambda_t^C (1 - S_t^s) w_t^s - \lambda_t^O + p_t S_t^s \lambda_t^{L^f} + \beta E_t \left[(1 - p_t S_t^s) \lambda_{t+1}^{L^s} \right], \\
& \frac{\partial \mathcal{L}}{\partial O_t} : \lambda_t^O = U_{O,t}, \quad U_{O,t} \equiv \varphi \left(O_t\right)^{\nu}, \\
& \frac{\partial \mathcal{L}}{\partial U_t} : \lambda_t^O = p_t \lambda_t^{L^f} + s_{UI,t} (1 - p_t) \lambda_t^{L^s}, \\
& \frac{\partial \mathcal{L}}{\partial S_t^s} : \lambda_t^C w_t^s L_t^s = p_t L_t^s \left(\lambda_t^{L^f} - \beta E_t \lambda_{t+1}^{L^s} \right) - \lambda_t^C \vartheta_s \left(\frac{S_t^s}{S_{t-1}^s} - 1 \right) \frac{w_t^s}{S_{t-1}^s} \\
& \qquad + \beta E_t \left[\lambda_{t+1}^C \vartheta_s \left(\frac{S_{t+1}^s}{S_t^s} - 1 \right) w_{t+1}^s \frac{S_{t+1}^s}{(S_t^s)^2} \right].
\end{aligned}
$$

When $\vartheta_s = 0$, the search-effort condition collapses to

$$
\lambda_t^C w_t^s = p_t \lambda_t^{L^f} - \beta p_t E_t \left[\lambda_{t+1}^{L^s} \right],
$$

after dividing by $L_t^s > 0$.

From the FOCs with respect to O_t and U_t, we obtain

$$\varphi(O_t)^{\nu} = p_t \lambda_t^{L^f} + s_{UI,t}(1 - p_t)\lambda_t^{L^s}.$$

When $\vartheta_s = 0$, combining the FOCs with respect to L_t^s and S_t^s yields the simpler expression

$$\lambda_t^{L^s} = \lambda_t^C w_t^s - \lambda_t^O + \beta E_t \lambda_{t+1}^{L^s}.$$

With $\vartheta_s > 0$, this expression is modified by the adjustment-cost terms in the search-effort condition above.

Finally, condition $\frac{\partial \mathcal{L}}{\partial L_t^f}$ can be rewritten as a recursive expression for the marginal value of a formal worker for the household is as follows:

$$\lambda_t^{L^f} = (1 - \tau_w)\lambda_t^C w_t - \lambda_t^O + \beta E_t[(1 - s_{t+1})\lambda_{t+1}^{L^f} + \kappa_t s_{t+1}\lambda_{t+1}^C].$$

A.2 Final goods production

The final goods producing firms are ultimately owned by the household, and hence discount expected future profits using the stochastic discount factor of the household defined by

$$\tilde{\beta}^j = \beta^j \frac{\lambda_{t+j}^C}{\lambda_0^C}.$$

Normalizing $\lambda_0^C = 1$, their problem is to choose $\{K_t, I_t, M_t^f, M_t^s, \varphi_t\}_{t \geq 0}$ to solve

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^C \left(Y_t - p_t^{M,f} M_t^f - p_t^{M,s} M_t^s - I_t - T_t^r - C_t^{ev} \right)$$

subject to

$$
\begin{aligned}
Y_t &= A_t K_{g,t-1}^{\alpha_1} K_{t-1}^{\alpha} M_t^{1-\alpha}, \\
M_t &= \left[\left(M_t^f \right)^{\frac{\epsilon-1}{\epsilon}} + (M_t^s)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \\
K_t &= (1-\delta)K_{t-1} + \zeta_t I_t \left(1 - \frac{\vartheta_k}{2} \left(\frac{I_t}{It-1} - 1 \right)^2 \right). \qquad (\beta^t \lambda_t^K)
\end{aligned}
$$

Let the reported tax base be

$$
\mathcal{B}_t \equiv \varphi_t \Big(Y_t - p_t^{M,f} M_t^f - \delta K_{t-1} \Big), \qquad \varphi_t \in (0,1],
$$

while tax payments are

$$
T_t^r \equiv \tau^r \mathcal{B}_t.
$$

Now, define $m_t^f \equiv \frac{M_t^f}{M_t}$; then, the cost of under-reporting is

$$
C_t^{ev} \equiv \frac{\kappa_\varphi}{2} (1-\varphi_t)^2 \, Y_t \, (m_t^f)^{\psi}, \qquad \kappa_\varphi > 0, \;\; \psi > 0.
$$

Lastly, denote $x_t \equiv \frac{I_t}{I_{t-1}}$ and $\Gamma_t \equiv 1 - \frac{\vartheta_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \equiv 1 - \frac{\vartheta_k}{2} (x_t - 1)^2$. The Lagrangian is

$$
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \lambda_t^C \left[Y_t - p_t^{M,f} M_t^f - p_t^{M,s} M_t^s - I_t - T_t^r - C_t^{ev} \right] + \lambda_t^K \left[(1-\delta) K_{t-1} + I_t \Gamma_t - K_t \right] \Big\}.
$$

A.2.1 Useful derivatives

Before getting to the first-order conditions, it is useful to consider the following derivatives: Let

$$
p_t^M \equiv \frac{\partial Y_t}{\partial M_t} = (1-\alpha) \frac{Y_t}{M_t}, \qquad r_t \equiv \frac{\partial Y_t}{\partial K_{t-1}} = \alpha \frac{Y_t}{K_{t-1}}.
$$

The derivatives of the intermediate input (CES aggregator) with respect to M_t^f and M_t^s are

$$
\frac{\partial M_t}{\partial M_t^f} = \left(\frac{M_t}{M_t^f} \right)^{1/\epsilon}, \qquad \frac{\partial M_t}{\partial M_t^s} = \left(\frac{M_t}{M_t^s} \right)^{1/\epsilon}.
$$

Hence

$$\frac{\partial Y_t}{\partial M_t^f} = p_t^M \left(\frac{M_t}{M_t^f}\right)^{1/\epsilon}, \qquad \frac{\partial Y_t}{\partial M_t^s} = p_t^M \left(\frac{M_t}{M_t^s}\right)^{1/\epsilon}.$$

Likewise, the derivatives of the traceability share are

$$\frac{\partial m_t^f}{\partial M_t^f} = \frac{1}{M_t}\left[1 - m_t^f \left(\frac{M_t}{M_t^f}\right)^{1/\epsilon}\right], \qquad \frac{\partial m_t^f}{\partial M_t^s} = -\frac{m_t^f}{M_t}\left(\frac{M_t}{M_t^s}\right)^{1/\epsilon}.$$

Finally, the derivatives of the concealment cost function are

$$\frac{\partial C_t^{ev}}{\partial z} = \frac{\kappa_\varphi}{2}(1-\varphi_t)^2 \left[(m_t^f)^\psi \frac{\partial Y_t}{\partial z} + Y_t\, \psi\, (m_t^f)^{\psi-1} \frac{\partial m_t^f}{\partial z}\right], \quad z \in \{M_t^f, M_t^s\},$$

and

$$\frac{\partial C_t^{ev}}{\partial K_{t-1}} = \frac{\kappa_\varphi}{2}(1-\varphi_t)^2 (m_t^f)^\psi\, r_t. \tag{36}$$

A.2.2 First-order conditions

The first-order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial M_t^f} &: \big(1-\tau^r \varphi_t\big)\bigg(\frac{\partial Y_t}{\partial M_t^f} - p_t^{M,f}\bigg) - \frac{\partial C_t^{ev}}{\partial M_t^f} = 0, \text{ where } \frac{\partial \mathcal{B}_t}{\partial M_t^f} = \varphi_t\bigg(\frac{\partial Y_t}{\partial M_t^f} - p_t^{M,f}\bigg),\\
\frac{\partial \mathcal{L}}{\partial M_t^s} &: \big(1-\tau^r \varphi_t\big)\frac{\partial Y_t}{\partial M_t^s} - p_t^{M,s} - \frac{\partial C_t^{ev}}{\partial M_t^s} = 0,\\
\frac{\partial \mathcal{L}}{\partial \varphi_t} &: -\tau^r \frac{\partial \mathcal{B}_t}{\partial \varphi_t} - \frac{\partial C_t^{ev}}{\partial \varphi_t} = 0,\\
&\quad -\tau^r \left(Y_t - p_t^{M,f} M_t^f - \delta K_{t-1}\right) + \kappa_\varphi (1-\varphi_t) Y_t (m_t^f)^\psi = 0,\\
\frac{\partial \mathcal{L}}{\partial I_t} &: \lambda_t^C = \lambda_t^K \frac{\partial (I_t \Gamma_t)}{\partial I_t} + \beta E_t \lambda_{t+1}^K \frac{\partial (I_{t+1}\Gamma_{t+1})}{\partial I_t},\\
&\quad = \lambda_t^K \big[\Gamma_t - \vartheta_k (x_t - 1) x_t\big] + \beta E_t \lambda_{t+1}^K \vartheta_k (x_{t+1}-1)\left(\frac{I_{t+1}}{I_t}\right)^2,\\
\frac{\partial \mathcal{L}}{\partial K_t} &: \lambda_t^K = \beta E_t \bigg\{(1-\delta)\lambda_{t+1}^K + \lambda_{t+1}^C \bigg[(1-\tau^r \varphi_{t+1})\frac{\partial Y_{t+1}}{\partial K_t} + \tau^r \varphi_{t+1}\,\delta - \frac{\partial C_{t+1}^{ev}}{\partial K_t}\bigg]\bigg\},\\
&\quad = \beta E_t \bigg\{(1-\delta)\lambda_{t+1}^K + \lambda_{t+1}^C \bigg[\bigg(1 - \tau^r \varphi_{t+1} - \frac{\kappa_\varphi}{2}(1-\varphi_{t+1})^2 (m_{t+1}^f)^\psi\bigg) r_{t+1} + \tau^r \varphi_{t+1}\,\delta\bigg]\bigg\}.
\end{aligned}$$

A.3 Entrepreneur or formal intermediate goods producer

The household also owns firms in the formal sector. Thus, given initial condition L^f_{-1}, prices $p^{M,f}_t$ and w_t, vacancy filling rate q_t, and the stochastic processes for aggregate shocks, the entrepreneur chooses contingent plans for aggregate variables $\{L^f_t, V_t\}_{t=0}^{\infty}$ to solve

$$\max\ \mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \overbrace{\left[p^{M,f}_t M^f_t - (1+\tau)w_t L^f_t - \kappa_t s_t L^f_{t-1} - \eta V_t\right]}^{\Pi_t}$$

$$\text{s.t.}\quad L^f_t = (1-s_t)L^f_{t-1} + q_t V_t. \qquad (\tilde{\beta}^t \lambda^F_t)$$

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \tilde{\beta}^t \left\{ p^{M,f}_t \left(\Omega L^f_t\right) - (1+\tau)w_t L^f_t - \kappa_t s_t L^f_{t-1} - \eta V_t + \lambda^F_t \left((1-s_t)L^f_{t-1} + q_t V_t - L^f_t\right)\right\}$$

with first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial L^f_t}: \quad \lambda^C_t \left[p^{M,f}_t \Omega - (1+\tau)w_t - \lambda^F_t\right] + \beta E_t \lambda^C_{t+1}\left[-\kappa_t s_{t+1} + \lambda^F_{t+1}(1-s_{t+1})\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial V_t}: \quad \eta \lambda^C_t = \lambda^C_t \lambda^F_t q_t$$

Letting $J_t = \lambda^C_t \lambda^F_t$, condition $\frac{\partial \mathcal{L}}{\partial L^f_t}$ can be rewritten as a recursive expression for the value of a match to the entrepreneur as follows:

$$J_t = \left[\Omega p^{M,f}_t - (1+\tau)w_t\right]\lambda^C_t + \beta E_t\left[(1-s_{t+1})J_{t+1} - \kappa_t s_{t+1}\lambda^C_{t+1}\right],$$

and, in turn, condition $\frac{\partial \mathcal{L}}{\partial V_t}$ provides an expression for the zero-profit condition for posting vacancies as follows:

$$\eta \lambda^C_t = q_t J_t.$$

A.4 Wage setting

The formal wage w_t is determined by Nash bargaining between the entrepreneur and the worker. Every period, after observing the shocks, the matched entrepreneur-worker pair negotiates over w_t, taking the severance payment κ_t as predetermined. The bargaining problem is

$$w_t = \arg\max \left\{ \left(\lambda_t^{Lf}\right)^{\gamma} (J_t + \kappa_t U_{c,t})^{1-\gamma} \right\},$$

where γ is the worker's bargaining power. The corresponding sharing rule is

$$(1-\gamma)\lambda_t^{Lf} = \gamma \left(J_t + \kappa_t U_{c,t}\right).$$

In contrast, earnings in the informal sector are determined competitively:

$$w_t^s = \chi p_t^{M,s},$$

where χ is informal labor productivity and $p_t^{M,s}$ is the relative price of informal intermediate goods.

A.5 Goods market clearing

Taking the household budget constraint

$$(1+\tau_c)C_t + (1+r_{t-1}^*)B_{t-1} = (1-\tau_w)w_t L_t^f + w_t^s(1-S_t^s)L_t^s + B_t + \kappa_t\ s_t L_{t-1}^f + \Pi_t + T_t + C_t^{ev} - \Xi_t^S$$

and substituting the expression for profits Π_t, the final-good firm's dividend, and the government budget constraint, one obtains the market-clearing condition for goods

$$
\begin{aligned}
(1+\tau_c)C_t + (1+r^*_{t-1})B_{t-1} =& (1-\tau_w)w_tL_t^f + (1-S_t^s)w_t^sL_t^s + B_t + \kappa_t\ s_tL_{t-1}^f + C_t^{ev} - \Xi_t^S \\
& + \overbrace{\left[p_t^{M,f}M_t^f - (1+\tau)w_tL_t^f - \kappa_t\ s_tL_{t-1}^f - \eta V_t\right]}^{\Pi_t} \\
& + \overbrace{Y_t - p_t^{M,f}M_t^f - p_t^{M,s}M_t^s - I_t - \tau^r\varphi_t\left(Y_t - p_t^{M,f}M_t^f - \delta K_{t-1}\right) - C_t^{ev}} \\
& + \overbrace{\tau_cC_t + \tau^r\varphi_t\left(Y_t - p_t^{M,f}M_t^f - \delta K_{t-1}\right) + (\tau+\tau_w)w_tL_t^f + Y_t^p - C_{g,t} - I_{g,t}}^{T_t = Tax_t + Y_t^p - C_{g,t} - I_{g,t}} \\
=& w_t^s(1-S_t^s)L_t^s + B_t + Y_t - p_t^{M,s}M_t^s - \eta V_t - I_t + \tau_cC_t + Y_t^p - C_{g,t} - I_{g,t} \\
& - \Xi_t^S,
\end{aligned}
$$

where the rebate C_t^{ev} received by the household exactly cancels against the concealment outlay $-C_t^{ev}$ in the final-good firm's dividend, so concealment costs do not enter the aggregate constraint.

Given that $w_t^s(1-S_t^s)L_t^s = p_t^{M,s}M_t^s$, one obtains the resource constraint

$$C_t + I_t + (1+r^*_{t-1})B_{t-1} = Y_t + B_t - \eta V_t + Y_t^p - C_{g,t} - I_{g,t} - \Xi_t^S,$$

where the law of motion of foreign debt is given by

$$B_t = (1+r^*_{t-1})B_{t-1} - NX_t,$$

and hence

$$C_t + I_t + C_{g,t} + I_{g,t} + \eta V_t + \Xi_t^S + NX_t = Y_t + Y_t^p.$$

A.6 Summarizing

In equilibrium, all households consume identical quantities. Thus, individual consumption equals average consumption across households $C_t = \tilde{C}_t$. The equilibrium is characterized by the following system of equations.

$$\{\lambda_t^C, \lambda_t^{L^f}, \lambda_t^{L^s}, \lambda_t^O, U_{c,t}, Y_t, C_t, I_t, M_t, K_{t+1}, B_{t+1}, C_t^{ev}, \Xi_t^S, L_t^f, L_t^s, U_t, O_t, V_t, J_t, p_t, q_t, p_t^{M,f},$$
$$S_{UI,t}, s_t, S_t^s, w_t, w_t^s, r_t, \varphi_t, \lambda_t^K, C_{g,t}, I_{g,t}, K_{g,t+1}, G_t^r, Tax_t, T_t, Y_t^p, A_t, r_t^\star, y_t^p, P_t^p, \zeta_t\}$$

1. $U_{c,t} = [C_t - h_c C_{t-1}]^{-\sigma}$, marginal utility of consumption

2. $\lambda_t^C = \frac{1}{1+\tau_c}\left(U_{c,t} - h_c\beta E_t\left\{U_{c,t+1}\right\}\right)$ Consumer's FOC

3. $\lambda_t^C = \beta E_t\left[\lambda_{t+1}^C(1+r_t^*)\right]$ Consumer's FOC

4. $\lambda_t^{L^f} = \lambda_t^C(1-\tau_w)w_t + \beta E_t\left[(1-s_{t+1})\lambda_{t+1}^{L^f} + \kappa_t s_{t+1}\lambda_{t+1}^C\right] - \lambda_t^O$, formal employment FOC

5. $\lambda_t^{L^s} = \lambda_t^C(1-S_t^s)w_t^s - \lambda_t^O + p_t S_t^s \lambda_t^{L^f} + \beta E_t\left[(1-p_t S_t^s)\lambda_{t+1}^{L^s}\right]$, informal employment FOC

6. $\lambda_t^O = \varphi\left(O_t\right)^\nu$, leisure FOC

7. $\lambda_t^O = p_t\lambda_t^{L^f} + s_{UI,t}(1-p_t)\lambda_t^{L^s}$ unemployment FOC

8. $\lambda_t^C w_t^s L_t^s = p_t L_t^s\left(\lambda_t^{L^f} - \beta E_t\lambda_{t+1}^{L^s}\right) - \lambda_t^C\vartheta_s\left(\frac{S_t^s}{S_{t-1}^s} - 1\right)\frac{w_t^s}{S_{t-1}^s} + \beta E_t\left[\lambda_{t+1}^C\vartheta_s\left(\frac{S_{t+1}^s}{S_t^s} - 1\right)w_{t+1}^s\frac{S_{t+1}^s}{(S_t^s)^2}\right]$, Search effort FOC

9. $L_t^f + L_t^s + U_t + O_t = 1$ Labor market clearing

10. $L_t^f = (1-s_t)L_{t-1}^f + q_t V_t$ Evolution of formal employment

11. $L_t^s = (1-p_{t-1}S_{t-1}^s)L_{t-1}^s + s_{UI,t}(1-p_t)U_t$ Evolution of informal employment

12. $Y_t = A_t K_{g,t-1}^{\alpha_1}(K_{t-1})^\alpha(M_t)^{1-\alpha}$ Final goods producer (can be written in terms of M^f and M^s) $M^f = \Omega L_t^f$ and $M^s = \chi(1-S_t^s)L_t^s$

13. $K_t = (1-\delta)K_{t-1} + I_t\left(1 - \frac{\vartheta_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$ Evolution of capital

14. $\lambda_t^C = \lambda_t^K\left(1 - \frac{\vartheta_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \vartheta_k\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right) + \beta\lambda_{t+1}^K\vartheta_k\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$,

15. $-\tau^r\left(Y_t - p_t^{M,f}M_t^f - \delta K_{t-1}\right) + \kappa_\varphi(1-\varphi_t)Y_t(m_t^f)^\psi = 0$,

16. $\lambda_t^K = \beta E_t \Big\{ (1-\delta)\lambda_{t+1}^K + \lambda_{t+1}^C \Big[\Big(1 - \tau^r \varphi_{t+1} - \frac{\kappa_\varphi}{2}(1-\varphi_{t+1})^2 (m_{t+1}^f)^\psi \Big) r_{t+1} + \tau^r \varphi_{t+1}\,\delta \Big] \Big\}$

17. $M_t = \left\{ \left(\Omega L_t^f \right)^{\frac{\epsilon-1}{\epsilon}} + (\chi(1-S_t^s)L_t^s)^{\frac{\epsilon-1}{\epsilon}} \right\}^{\frac{\epsilon}{\epsilon-1}}$ Intermediate goods producer

18. $r_t = \alpha A_t K_{g,t-1}^{\alpha_1} \left(\frac{K_{t-1}}{M_t} \right)^{\alpha-1}$ Marginal product of capital

19. $p_t^{M,f} = \frac{\partial Y_t}{\partial M_t^f} - \frac{1}{1-\tau_r \phi_t} \frac{\partial C_t^{ev}}{\partial M_t^f}$, FOC of the final goods producers wrt M_t^f $(\frac{\partial \mathcal{L}}{\partial M_t^f})$

20. $w_t^s = \chi p_t^{M,s}$

21. $p_t^{M,s} = \left(1 - \tau^r \varphi_t\right) \frac{\partial Y_t}{\partial M_t^s} - \frac{\partial C_t^{ev}}{\partial M_t^s}$, FOC of the final goods producer wrt M_t^s $(\frac{\partial \mathcal{L}}{\partial M_t^s})$

22. $J_t = \left[\Omega p_t^{M,f} - (1+\tau) w_t \right] \lambda_t^C + \beta E_t[(1-s_{t+1})J_{t+1} - \kappa_t\ s_{t+1}\lambda_{t+1}^C]$ Value of the match for the entrepreneur (formal intermediate goods producer)

23. $q_t J_t = \eta \lambda_t^C$ Free entry condition (ZPC for a vacancy)

24. $(1-\gamma)\lambda_t^{Lf} = \gamma\left[J_t + \kappa_t U_{c,t}\right]$ Nash bargaining

25. $s_t = \tilde{s} \frac{1}{1+\exp\left(\frac{J_t/\lambda_t^C}{\sigma_J} \right)}$, endogenous separation.

26. $p_t = D \left(\frac{U_t + S_t^s L_t^s}{V_t} \right)^{\phi-1}$ Job finding rate

27. $q_t = D \left(\frac{U_t + S_t^s L_t^s}{V_t} \right)^{\phi}$ Vacancy filling rate

28. $B_t = (1+r_{t-1}^*)B_{t-1} - Y_t - Y_t^p + C_t + I_t + C_{g,t} + I_{g,t} + \eta V_t + \Xi_t^S$, Goods market clearing

29. $C_t^{ev} \equiv \frac{\kappa_\varphi}{2}(1-\varphi_t)^2\, Y_t\, (m_t^f)^\psi$, cost of under-reporting (rebated lump sum to the household)

30. $\Xi_t^S = \frac{\vartheta_s}{2} \left(\frac{S_t^s}{S_{t-1}^s} - 1 \right)^2 w_t^s$, aggregate adjustment cost associated with changes in the search intensity of informal workers

31. $G_t^r = C_{g,t} + I_{g,t} + T_t$ Government budget constraint

32. $G_t^r = Tax_t + Y_t^p$ Government revenues

33. $Tax_t = \tau_c C_t + \tau^r \varphi_t \left(Y_t - p_t^{M,f} M_t^f - \delta K_{t-1} \right) + (\tau + \tau_w) w_t L_t^f$ Tax collection

34. $K_{g,t} = (1-\delta_g)K_{g,t-1} + I_{g,t}\left(1 - \frac{\vartheta_g}{2}\left(\frac{I_{g,t}}{I_{g,t-1}} - 1\right)^2\right)$ Evolution of public capital

35. $C_{g,t} = e^{\varepsilon_t^{C_g}}(C_{g,t-1})^{\rho_{c_g}}(\bar{C}_g)^{1-\rho_{c_g}}$, Public consumption rule

36. $I_{g,t} = e^{\varepsilon_t^{I_g}}(I_{g,t-1})^{\rho_{I_g}}(\bar{I}_g)^{1-\rho_{I_g}}$, Public investment rule

37. $Y_t^p = P_t^b y_t^p$ Oil income

38. $\log(A_t) = (1-\rho_A)\log(\bar{A}) + \rho_A \log(A_{t-1}) + \varepsilon_t^A$, with $\varepsilon_t^A \sim N(0, \sigma_A^2)$ AR(1) process for aggregate technology shock

39. $\log(1+i_t^*) = \rho_i \log(1+i_{t-1}^*) + (1-\rho_i)\log(1+i^*) + \varepsilon_t^i$, with $\varepsilon_t^i \sim N(0, \sigma_i^2)$ AR(1) process for foreign interest rate shock

40. $y_t^p = (y_{t-1}^p)^{\rho^p}(y_{ss}^p)^{1-\rho^p}\exp(\varepsilon_t^p)$, with $\varepsilon_t^p \sim N(0, \sigma_p^2)$ Volume of exported oil

41. $P_t^b = (P_{t-1}^b)^{\rho^b}(P_{ss}^b)^{1-\rho^b}\exp(\varepsilon_t^b)$, with $\varepsilon_t^b \sim N(0, \sigma_b^2)$ International relative price of oil

42. $\log\zeta_t = (1-\rho_\zeta)\log\bar{\zeta} + \rho_\zeta \log\zeta_{t-1} + \varepsilon_t^\zeta$, with $\varepsilon_t^\zeta \sim N(0, \sigma_\zeta^2)$ AR(1) process for the investment efficiency shock

B Calibration and estimation

B.1 Calibrated parameters

Table B.1: Remaining parameter and target values

Parameter	Value	Description
τ^r	0.330	Effective CIT rate
s_{w_s}	3.325	Normalization parameter of w_s
κ_φ	4.472	Scale of compliance/evasion cost
ρ_{z^I}	0.000	Persistence of investment efficiency shock
$\bar{\Omega}$	1.000	Steady-state formal productivity
$\bar{C}_g$	1.129	Steady-state public consumption target
$\bar{I}_g$	0.157	Steady-state public investment target
b/Y	0.360	Annualized debt-to-GDP target
$\bar{Y}^p$	0.200	Steady-state oil income
$\bar{P}^b$	1.000	Steady-state oil price
y_p/Y	0.033	Oil income share parameter
ς_c	0.180	Public consumption share in GDP
ς_i	0.025	Public investment share in GDP
τ_c	0.118	Consumption tax
τ_w	0.225	Labor income tax
α_1	0.010	Public capital elasticity
ρ_b	0.850	Persistence of oil price
ρ_p	0.850	Persistence of oil volume
δ_g	0.020	Public capital depreciation
ϑ_g	5.000	Public investment adjustment cost
s_c	0.489	Govt consumption share in revenue
s_i	0.068	Govt investment share in revenue
ρ_{I_g}	0.850	Persistence of public investment rule
θ	0.001	Risk premium elasticity
$\bar{i}^\star$	0.010	Steady-state foreign interest rate
$\bar{b}$	9.029	Steady-state debt level
$\bar{A}$	0.982	Steady-state technology

B.2 Generalized method of moments

B.2.1 Data and sample

We estimate the model using quarterly data from 2006IV to 2019IV. As discussed in Section 4, this extended sample is used to improve the identification of dynamic parameters governing labor-market comovements, while the steady-state calibration remains anchored to pre-reform averages.

Macroeconomic aggregates (output, consumption, and government consumption) are taken from the National Statistics Department, seasonally adjusted and expressed in real terms, and the corresponding observables enter the moment conditions in log-difference form. In turn, labor market series (unemployment, out of the labor force, formal and informal employment, and labor market tightness) are constructed from the same sources as in Section 4.1.

For estimation, we use centered moments by demeaning each observable over the estimation sample, which aligns second moments with the model's deterministic steady state and avoids confounding mean differences with dynamic fit (see Adjemian et al., 2024).

B.2.2 Moment conditions

Let $\widehat{m}$ denote the vector of empirical moments and $m(\theta)$ the counterpart implemented by the model. We match a number of second moments that jointly discipline the volatility, persistence and comovements of several macroeconomic and labor market indicators. Table B.2 reports the data and model moments in the estimated parameter vector.

B.2.3 GMM criterion and baseline HAC weighting

We estimate θ by minimizing the quadratic GMM criterion

$$\widehat{\theta} \;=\; \arg\min_{\theta} Q(\theta) \;=\; \arg\min_{\theta} \left[\widehat{m} - m(\theta)\right]' W \left[\widehat{m} - m(\theta)\right].$$

Our baseline follows Dynare's staged weighting scheme `IDENTITY`→`DIAGONAL`→`OPTIMAL`. In the final stage, `OPTIMAL` constructs the optimal weighting matrix $W = \widehat{S}^{-1}$ using a HAC

Table B.2: Comparison of moments: data vs model (GMM)

Moment	Data	Model
$E\left[\Delta\log(Y)_t^{obs} \times \Delta\log(Y)_t^{obs}\right]$	0.9762807	1.0589219
$E\left[\Delta U_t^{obs} \times \Delta U_t^{obs}\right]$	1.0319099	1.0380892
$E\left[\Delta\log(Y)^{obs}{}_t \times \Delta U_t^{obs}\right]$	-0.1625660	-0.2456975
$E\left[s_t^{obs} \times s_t^{obs}\right]$	0.9777778	1.0061835
$E\left[s_t^{obs} \times s_{t-1}^{obs}\right]$	-0.5349780	-0.2934939
$E\left[\theta_t^{obs} \times \theta_t^{obs}\right]^*$	0.9782609	1.0200706
$E\left[U_t^{obs} \times \theta_t^{obs}\right]$	-0.8861032	-0.6853083
$E\left[L_t^{s,obs} \times L_t^{f,obs}\right]$	-0.3034341	-0.2680344
$E\left[L_t^{f,obs} \times L_{t-1}^{s,obs}\right]$	-0.2316307	-0.2696563
$E\left[U_t^{obs} \times L_t^{s,obs}\right]$	0.1189321	0.2127329
$E\left[L_t^{s,obs} \times U_{t-1}^{obs}\right]$	0.2093623	0.2295965
$E\left[\Delta U_t^{obs} \times \Delta O_t^{obs}\right]$	-0.3190772	-0.3539938
$E\left[\Delta\log(C_g)_t^{obs} \times \Delta\log(C_g)_t^{obs}\right]$	0.9434866	0.8278484
$E\left[\Delta\log(C_g)_t^{obs} \times \Delta\log(C_g)_{t-1}^{obs}\right]$	-0.1846262	-0.1633595
$E\left[\Delta\log(Y)_t^{obs} \times \Delta\log(I)_{t-1}^{obs}\right]$	0.2758785	0.2464185
$E\left[\Delta\log(I)_t^{obs} \times \Delta\log(I)_t^{obs}\right]$	1.0850944	0.9252956
$E\left[i_t^{\star,obs} \times i_{t-1}^{\star,obs}\right]$	1.1276667	1.0599763
$E\left[i_t^{\star,obs} \times i_t^{\star,obs}\right]$	1.2058083	1.2129519

* θ_t^{obs} corresponds to the observed labor market tightness.

(Newey- West-type with a Bartlett kernel) estimate $\widehat{S}$ of the long-run covariance matrix of the moment conditions. This corresponds to a two-step iterated GMM procedure with heteroskedasticity- and autocorrelation-robust inference (Adjemian et al., 2024).

All estimation results are generated from a single frozen estimation run. The optimizer is initialized at the calibrated values given the data file and the optimization settings (`mode_compute=5` with an additional optimization step). The procedure is deterministic.

B.2.4 Robustness

We conduct a number of robustness checks considering alternative choices for the GMM estimation while keeping the underlying model unchanged, as follows: (i) replacing the final optimal HAC weighting matrix with a diagonal weighting matrix; (ii) replacing the covariance between unemployment and labor market tightness, $E\left[u_t^{obs}\theta_t^{obs}\right]$, with the first-order autocovariance of labor market tightness, $E\left[\theta_t^{obs}\theta_{t-1}^{obs}\right]$; and (iii) disabling the additional optimizer step (single-step optimization).

The results are reported in Table B.3. It can be seen that the over-identifying restrictions are not rejected in any specification, with p-values ranging from 0.77 to 0.81. The persistence and volatility of the shocks are essentially unaffected by the estimation choices, and the parameters governing labor supply and capital accumulation are also stable: the curvature of the utility of leisure lies between -1.57 and -1.44, and the capital adjustment cost between 6.7 and 7.4. The elasticity of substitution between formal and informal labor and the unemployment-informality transition parameter vary moderately across specifications, remaining within roughly 10 and 15 percent of their baseline values, respectively.

The dispersion of idiosyncratic profitability, σ_J, is the parameter most sensitive to the estimation choices. Under the diagonal weighting matrix, which weights each moment by its own variance only and ignores the covariance structure of the moment conditions, the optimizer pushes σ_J to the upper bound of its search interval and ρ_A to its lower bound, whereas the alternative-moment and single-step specifications yield interior estimates below the baseline value. We read the corner solutions produced by the diagonal scheme as an argument for the

Table B.3: Robustness of the GMM estimation

	Baseline	W diagonal	Alternative moment	One-step optimizer
Weighting	Identity→Diagonal→Optimal (HAC)	Identity→Diagonal	Identity→Diagonal→Optimal (HAC)	Identity→Diagonal→Optimal (HAC)
Optimizer	mode=5 + steps	mode=5 + steps	mode=5 + steps	mode=5 only
Objective Q	0.0580	7.6632	0.0564	0.0583
J test statistic	3.0735	3.3366	2.9882	3.0899
J test p value	0.7996	0.7656	0.8103	0.7975
J test df	6	6	6	6
ϵ	1.2772	1.1756	1.4000	1.3637
ν	-1.4534	-1.5743	-1.4418	-1.4631
σ_J	0.8307	2.0000	0.5439	0.5930
ϕ_{UI}	1.3433	1.4838	1.1558	1.5234
ϑ_k	7.2235	6.7245	7.1137	7.4123
ρ_A	0.2415	0.2000	0.2419	0.2158
ρ_{c_g}	0.6053	0.5827	0.6328	0.5999
ρ_i	0.8739	0.8781	0.8716	0.8722
σ_A	0.0068	0.0067	0.0067	0.0067
σ_{c_g}	0.0215	0.0227	0.0215	0.0219
σ_{z_I}	0.1960	0.1832	0.2096	0.2029
σ_i	0.0024	0.0024	0.0025	0.0025

two-step HAC-optimal procedure adopted in the baseline rather than as evidence of fragility. Finally, note that the values of the objective function Q are not comparable across columns estimated with different weighting matrices.

C Tax reform financing scenarios

We describe the construction of the financing scenarios reported in Table 7. In each scenario, the payroll tax rate is permanently reduced from its pre-reform value, $\tau^{pre} = 0.471$, to its post-reform value, $\tau^{post} = 0.336$, implying a payroll tax cut of $\Delta\tau \equiv \tau^{pre} - \tau^{post} = 0.135$.

C.1 Static revenue loss from the payroll tax reduction

The ex-ante fiscal cost of the payroll tax reduction is calculated under the assumption that economic behavior remains at its pre-reform steady-state levels. Let w^{pre} and $L^{f,pre}$ denote the pre-reform formal wage and formal employment, respectively. The associated static revenue loss is

$$\mathcal{L}^{static} = \Delta\tau \, w^{pre} L^{f,pre}.$$

This measure serves as the benchmark for calibrating the alternative financing instruments considered in the analysis. Because it keeps tax bases unchanged at their pre-reform levels, it captures only the direct mechanical effect of the tax cut and does not account for general equilibrium adjustments in employment, wages, consumption, output, or tax compliance.

Pre-reform tax revenue is obtained by evaluating Equation (29) at the initial steady state. The corresponding tax bases are given by

$$\begin{aligned}
\mathcal{B}^{VAT} &= C^{pre}, \\
\mathcal{B}^{PIT} &= w^{pre} L^{f,pre}, \\
\mathcal{B}^{CIT} &= \varphi^{pre} \left(Y^{pre} - p^{M,f,pre} \Omega L^{f,pre} - \delta K^{pre} \right).
\end{aligned}$$

C.2 Ex-ante revenue-equivalent instruments

Each financing instrument is calibrated to generate the same ex-ante static revenue effect, evaluated at the pre-reform steady state. Specifically, holding behavior fixed, the instrument-specific adjustment is chosen so that the associated static revenue gain is $\mathcal{L}^{static}$.

For tax-based financing instruments, the required rate changes satisfy

$$\Delta\tau_c = \frac{\mathcal{L}^{static}}{\mathcal{B}^{VAT}}, \quad \Delta\tau_w = \frac{\mathcal{L}^{static}}{\mathcal{B}^{PIT}} = \Delta\tau, \quad \Delta\tau_r = \frac{\mathcal{L}^{static}}{\mathcal{B}^{CIT}}.$$

Consequently, the VAT-, PIT-, and CIT-financed scenarios increase the corresponding tax rates from their pre-reform levels $(\tau_c^{pre}, \tau_w^{pre}, \tau_r^{pre})$ by $\Delta\tau_c, \Delta\tau_w$ and $\Delta\tau_r$, respectively.

For expenditure-based instruments, the ex-ante revenue-equivalent changes are achieved through reductions in the long-run expenditure targets:

$$\Delta C_g = -\mathcal{L}^{static}, \quad \Delta I_g = -\mathcal{L}^{static}.$$

Accordingly, government-consumption financing sets

$$\bar{C}_g^{new} = \bar{C}_g^{pre} - \mathcal{L}^{static},$$

while public-investment financing sets

$$\bar{I}_g^{new} = \bar{I}_g^{pre} - \mathcal{L}^{static}.$$

A spending-based financing option is deemed ex-ante infeasible whenever the required adjustment exceeds the corresponding pre-reform expenditure level.

C.3 Fiscal closure and transitions

All scenarios are solved as perfect-foresight transitions from the pre-reform steady state. At the beginning of the transition, the payroll tax is permanently reduced to τ^{post}, and the selected financing instrument is adjusted according to the ex-ante revenue-equivalent rule described above. Except for the financing instrument under consideration, the fiscal closure is identical across scenarios. Public consumption and public investment follow the rules described in Section 3 and gradually converge to their long-run targets. Unless either expenditure category serves as the financing margin, $\bar{C}_g$ and $\bar{I}_g$ remain fixed at their calibrated benchmark levels. Lump-sum transfers T_t adjust residually to satisfy the government budget constraint in Equations (26)-(29) in every period.

For each scenario, we first compute the associated post-reform steady state. To enhance numerical stability, the payroll tax and the corresponding financing instrument are gradually moved from their initial values to their scenario-specific levels using a homotopy procedure. Each intermediate steady state serves as the starting point for the next step. Once the final steady state is obtained, we solve the perfect-foresight transition path from the pre-reform equilibrium to the new long-run steady state.

C.4 Reported statistics and recovery rate

Table 7 reports average outcomes over the first T quarters of the transition, with $T = 16$. For any variable X_t, the transition average is defined as

$$\bar{X}^{(T)} = \frac{1}{T} \sum_{t=1}^{T} X_t.$$

Let $G^{r,pre}$ denote pre-reform government revenue and $\bar{G}_j^{r,(T)}$ the average government revenue over the first T quarters under scenario j. The last column of Table 7 reports the ex-post recovery rate,

$$Recovery_j = 100 \left[1 - \frac{G^{r,pre} - \bar{G}_j^{r,(T)}}{\mathcal{L}^{static}} \right].$$

This measure captures the share of the static payroll-tax revenue loss recovered during the transition. A value of 100 implies full recovery, whereas values below 100 indicate that general-equilibrium effects erode the tax base and limit the extent to which the static loss is recouped. Values above 100 imply that the scenario generates sufficient additional revenue to more than offset the initial static loss.

Unlike ex-ante revenue equivalence, which is computed mechanically using pre-reform tax bases, the recovery rate is an ex-post measure that incorporates endogenous responses in employment, wages, consumption, output, and tax compliance.

C.5 Non-existence and revenue-maximizing VAT scenarios

Some tax instruments do not admit an interior equilibrium at the ex-ante revenue-equivalent level. This arises when formal employment vanishes before the tax rate required to offset the loss of payroll-tax revenue can be reached. As tax wedges increase, formal matches become less profitable, raising the separation rate and reducing both expected match duration and match values. These effects reinforce one another and may eventually eliminate the possibility of an interior equilibrium. Results for such cases are therefore omitted.

For the VAT, we additionally consider the tax rate that maximizes government revenue within the set of feasible equilibria. Specifically, τ_c is allowed to vary between its pre-reform level and the highest value consistent with the existence of an interior equilibrium. The revenue-maximizing VAT rate is defined as

$$\tau_c^{\max} = \arg\max_{\tau_c} G^r(\tau_c),$$

subject to the existence of an interior equilibrium. This scenario represents the maximum revenue that can be raised through the consumption tax while maintaining a feasible interior equilibrium in the formal sector.