

BORRADORES DE ECONOMÍA



Designing the Future of Money:
The Case for Multiple CBDCs

By:
Julian A. Parra-Polania
Constanza Martínez-Ventura

No. 1311
2025



Designing the Future of Money: The Case for Multiple CBDCs

Julian A. Parra-Polania* and Constanza Martínez-Ventura*

The opinions contained in this document are the sole responsibility of the authors and do not commit *Banco de la República* or its Board of Directors

Abstract

We examine the optimal design of central bank digital currencies (CBDCs) by focusing on two key features: the anonymity-security trade-off and the remuneration (i.e., interest rate). Building on the extended model by Agur et al. (2022), which accounts for potential negative externalities associated with the anonymity of payment methods, we incorporate the possibility of multiple CBDCs into the framework. Our findings reveal that with optimally designed CBDCs and when anonymity costs are significant, a cashless economy is the preferred choice for the central bank. Furthermore, irrespective of anonymity costs, an economy with cash and one or more CBDCs is welfare dominated by a cashless economy with one additional CBDC. These results underscore the flexibility and welfare-enhancing potential of CBDCs compared to cash in modern payment systems.

Keywords: CBDC; optimal design; anonymity; security; digital currency; cashless economy.

JEL Classification: D60, E41, E42, E43, E58, G21

*Banco de la República (Central Bank of Colombia). E-mails: jparrapo@banrep.gov.co (corresponding author); amartive@banrep.gov.co. We appreciate the comments provided by Carlos Arango, Juan E. Carranza, Clara Machado, Andres Murcia, and the participants at Banco de la Republica's internal seminar held in February 2025. Any error is the exclusive responsibility of the authors.

Diseñando el futuro del dinero: el caso de múltiples CBDCs

Julian A. Parra-Polania* and Constanza Martínez-Ventura*

Las opiniones contenidas en el siguiente documento son responsabilidad exclusiva de los autores y no comprometen al *Banco de la República* ni a su Junta Directiva

Resumen

Analizamos el diseño óptimo de las monedas digitales de los bancos centrales (CBDC), centrándonos en dos características clave: la disyuntiva entre anonimato versus seguridad y la remuneración (tasa de interés). Con base en el modelo extendido de Agur et al. (2022), que considera las posibles externalidades negativas asociadas con el nivel de anonimato de los métodos de pago, incorporamos la posibilidad de múltiples CBDCs. Nuestros hallazgos revelan que con CBDCs óptimamente diseñadas, y cuando los costos de la anonimato son relativamente altos, una economía sin efectivo es la opción preferida del banco central. Adicionalmente, independientemente de los costos de la anonimato, una economía con efectivo y una o más CBDCs es menos preferida, en términos de bienestar social, que una economía sin efectivo y con una CBDC adicional. Estos resultados resaltan la flexibilidad y el potencial de mejora del bienestar de las CBDCs en comparación con el efectivo en los sistemas de pago modernos.

Palabras clave: CBDC; diseño óptimo; anonimato; seguridad; monedas digitales; economía sin efectivo.

Clasificación JEL: D60, E41, E42, E43, E58, G21

*Banco de la República (Colombia). Correos electrónicos: jparrapo@banrep.gov.co (autor para contactos); amartive@banrep.gov.co. Agradecemos los comentarios de Carlos Arango, Juan E. Carranza, Clara Machado, Andres Murcia y los participantes del seminario interno del Banco de la República realizado en febrero de 2025. Cualquier error es responsabilidad exclusiva de los autores.

1 Introduction

The digitalization of transactions (online banking and other financial services, and the development of e-commerce), coupled with the rise of unregulated private payment solutions (e.g., stablecoins) in the retail payments market have led central banks to consider the issuance of digital currencies (CBDCs). Like cash, CBDCs would be a form of fiat money accessible to individuals and firms, and like deposits, they could be used in both online and in-person transactions. Central banks face critical decisions regarding the design and role of CBDCs within monetary systems. These decisions involve trade-offs between user preferences for anonymity and security, the efficiency of payment systems, and the broader social implications of reducing or eliminating cash.

In line with the related literature, this paper examines the economic impact of CBDCs by modeling competition among different forms of money—cash, bank deposits, and CBDCs—at the retail level. To this end, we build upon the model developed by Agur et al. (2022), extending it to analyze monetary equilibria with multiple CBDCs. As in their baseline model, we focus on two key design dimensions: the position of CBDCs along the anonymity-security spectrum and their interest rates. Additionally, following their extended model (presented in their appendix), we incorporate the externalities associated with anonymity, which can generate social costs. Specifically, highly anonymous payment methods may facilitate tax evasion and hinder the detection of illicit activities such as money laundering, drug and human trafficking, extortion, corruption, terrorist financing, and other financial crimes (see Chiu and Davoodalhosseini, 2023).

It is important to note that there is an ongoing policy debate regarding the necessity and relevance of issuing a CBDC, as many of its potential benefits could be achieved through alternative policies that may be easier to implement, such as tokenized deposits or a fast payment system (see, for instance, Aurazo et al., 2024, and Banco de la Republica, 2024). This paper does not take a stance in that debate; rather, it focuses on analyzing the optimal design of CBDCs. Moreover, this is a theoretical work that focuses on two factors that may be different from the ones that are regarded as the important ones by policymakers in their decision on whether or not issuing a CBDC.

Features such as interest rates, levels of anonymity, and holding and spending limits are crucial for the design of CBDCs. These features are also key determinants of their adoption and usage. For example, as explained above, a completely anonymous CBDC, like cash, could impose negative externalities on society. On the contrary, a CBDC that requires a high degree of user identification would be a safe form of money and, if in addition offers some remuneration to its holders, could become a close substitute for bank deposits, putting some pressure on banks' lending activities (see Keister and Sanches, 2023).¹ Substitution between bank deposits and CBDCs could also depend on other factors, such as transaction speed and access to digital markets (see Garratt, Jiaheng and Haoxiang, 2022). Therefore, while cash and cash-like CBDCs are, in theory, near-perfect substitutes in payments, the elasticity of substitution between deposits and a deposit-like CBDCs may be lower

¹An interest-bearing CBDC could raise bank funding costs, which may in turn constrain bank lending activities, leading to a reduction in investment and output (see Agur et al., 2022).

(see Chiu and Davoodalhosseini, 2023; Bacchetta and Perazzi, 2022).² The contrasting effects of different CBDC designs unveil a trade-off between anonymity and security. Anonymity in existing forms of money and CBDCs has been examined in previous theoretical works. Agur et al. (2022) study the demand of consumers for types of money under diverse preferences for anonymity, whereas Ahnert, Hoffmann, and Monnet (2022) and Cheng and Izumi (2023) focus on firm behavior.

In this context, this paper addresses the following central research question: What is the optimal design of CBDCs? Agur et al. (2022) provide insights into a similar question within an environment where households choose among cash, bank deposits, and a single CBDC for their transactions. This selection crucially depends on household preferences regarding design features of each type of money, especially along the anonymity-security spectrum. Given the heterogeneity of these preferences, ensuring diversity in payment instruments becomes essential for maximizing social welfare. Recognizing a potential limitation in their approach, Agur et al. (2022) acknowledge that considering only one CBDC may be suboptimal and suggest that introducing multiple CBDCs is socially optimal in the context of their framework.

Building on this idea, we extend their model by incorporating the possibility of multiple CBDCs. As in the baseline model, we consider an economy where the heterogeneity in household preferences over anonymity versus security plays a key role, and where the demand for each type of money - determining its share of users- depends on its degree of anonymity, remuneration on money holdings, and network effects. By introducing a spectrum of CBDCs ranging from cash-like to deposit-like, we capture variations in household decision-making regarding money use and examine how these CBDCs can be optimally designed to accommodate diverse user preferences.³ Other properties that may explain heterogeneity, such as differences in the money settlement times, are not modelled in this framework but are considered a promising avenue for future research.⁴

The main contributions of our analysis are related to the following two questions: (1) Should central banks introduce multiple CBDCs and what should their design features be? (2) Is transitioning to a cashless society beneficial in terms of welfare, particularly when anonymity externalities are negligible or significant? Our main findings indicate that, under the assumption that it is possible to issue a CBDC that represents a good substitute for cash (which in practice may not be feasible), a cashless economy is generally preferred by the central bank, especially when the cost of anonymity is significant. Moreover, regardless of the anonymity cost, a cashless economy with $n + 1$ optimally designed CBDCs consistently outperforms an economy with cash and n optimally

²The potential disintermediation problem could be avoided by the implementation of a hybrid retail CBDC in which the issuance is the responsibility of the central bank while commercial banks are in charge of the distribution. In the present paper, as in Agur et al. (2022), we will consider the disintermediation problem as if the central bank were in charge of both issuance and distribution.

³Throughout this document we refer to multiple CBDCs to differentiate the options and conditions available to users along the anonymity-security spectrum. Each CBDC imposes distinct requirements regarding the information users must provide. However, beyond these conditions, CBDCs -like cash- would remain accessible to all individuals and business.

⁴Differences in settlement times can also influence households' and firms' decisions about what type of money to use in retail transactions. When cash is used, the balances of the payer (sender) to the payee are immediately transferred, once the latter receives the respective amount of money. This settlement process is deferred when using bank deposits and, depending on banking processes, its length varies from a few minutes to some business days.

designed CBDCs, underscoring the relative advantages of transitioning to a cashless society with well-designed CBDCs.

Previous works considering the effects on introducing a digital fiat currency for retail transactions include Chiu and Davoodalhosseini (2023), who show that a cash-like CBDC will be more effective than a deposit-like CBDC in promoting consumption and welfare. For Agur et al. (2022) if network effects matter, the optimal design will be an interest-bearing CBDC as it will allow to preserve the different types of money in the economy; otherwise, it will be optimal to consider a non-remunerated CBDC. Keister and Sanches (2023) show that while a CBDC may represent potential gains in exchange, it may also reduce bank deposits, increase funding costs, and decrease investment. Cheng and Izumi (2023) show that the anonymity of CBDCs should be designed to eliminate the equilibrium multiplicity and therefore data-sharing is highly desirable. However, if it is not feasible to implement, the solution is to raise the degree of anonymity: The central bank has to give up some efficiency to eliminate a highly inefficient lending equilibrium. Our study builds on this literature by addressing the optimal design and welfare implications of multiple CBDCs and the transition to a cashless economy.

2 The Model and Results

The model economy consists of households, banks, firms, and a central bank. Each household chooses one of the $n + 2$ possible forms of money: bank deposits, one of the n CBDCs, or cash. Deposits are used by banks to offer loans to firms, and in turn firms, in a perfectly competitive environment, use these loans to finance a portion of each project that produces a consumption good. The central bank aims to maximize welfare (i.e., the sum of household utilities) by setting the design attributes of each CBDC, namely, its degree of anonymity vs. security and its interest rate. There is a trade-off between anonymity and security such that a unique scale is used for these two attributes, where higher values of the former imply lower values of the latter and vice versa.

The model is entirely based on the one proposed by Agur et al. (2022) in their Appendix C.2, which adds (to the original setup in their main text) negative externalities derived from the anonymity of money forms, due to the potential for illicit activities. We generalize it to consider the possible introduction of multiple CBDCs.

For the reader's convenience, we outline the main formal features of the model below. More details of the model, its motivation, and the solution with only one CBDC and without anonymity externalities can be found in Agur et al. (2022) (or with only one CBDC but including anonymity externalities in their Appendix C.2).

Households

There is a unit continuum of households whose preferences h are uniformly distributed over the anonymity-security scale $[0, 1]$. Let $\theta_j \in [0, 1]$, $j = 0, 1, 2, \dots, n, n+1$, denotes the place of each money type in the scale. Deposits (d or 0) are the type with the lowest level of anonymity (and hence with the highest level of security), i.e., $\theta_d = \theta_0 = 0$, and cash (c) is at the opposite extreme of the scale,

i.e., $\theta_c = \theta_{n+1} = 1$. There are n CBDCs placed at different levels in the scale, θ_j , $1 \leq j \leq n$, such that in terms of its anonymity-security features a CBDC with θ_j closer to zero is more like deposits and a CBDC closer to one is more like cash. Furthermore, anonymity is associated with negative externalities (i.e., in the form of reduced transparency, tax evasion, illicit activities...) and hence every household dislikes anonymity in other households' transactions.

Each household chooses one of the money types to maximize its utility:

$$\max_j U_h(j) = \rho C_j - |\theta_j - h| - \eta_j - \beta \int_h \theta_{j(h)} dh, \quad (1)$$

where C_j is consumption, $|\theta_j - h|$ represents the cost of using a payment method that differs from the household's preference, and η_j captures a network effects cost. For the sake of tractability, Agur et al. (2022) assume that while a money type j is in use $\eta_j = 0$, and then if its share of users falls short of a threshold, such a money type falls out of use. The last term in the equation captures the anonymity externality.⁵ $\beta \in [0, 1]$ and $\rho > 0$ are constant parameters.

Each household starts with the same endowment, equal to one, which is stored in a specific money type j and used to consume at the end of the period. Thus, the budget constraint is

$$C_j = 1 + r_j - T + \pi, \quad (2)$$

where π represents firm profits (the household's share in firms is normalized to 1), and r_j is the net interest earned on money type j ($r_c = r_{n+1} = 0$). The interest paid on CBDCs is financed by a lump-sum tax:

$$T = \int_1^n r_{j(h)} s_{j(h)} dh, \quad (3)$$

where s_j is the share of users of the CBDC j .

Banks

Banks collect deposits d from households at deposit rate r_d (or r_0) and use them to offer funding in the form of loans $l = d$ to firms at loan rate R . Banks are assumed to be risk neutral, and therefore $r_d = R$.

Firms

Firms use loans to finance a portion $k = l$ of a total of k^0 available projects to produce consumption goods with the technology $Y = Ak - k^2/2$, where $A > 1$ is the productivity parameter. The remaining projects ($k^0 - k$) are liquidated at a rate $\phi \in (0, 1)$. Each firm chooses k to maximize its profits:

$$\max_k \pi(k) = Y + \phi(k^0 - k) - (1 + R)k.$$

The first-order condition yields

$$1 + R = A - \phi - k, \quad (4)$$

⁵As each household is infinitesimally small, the space of "the other households" is still the space of h .

and therefore

$$\pi^* = \phi k^0 + \frac{k^2}{2} \quad (5)$$

Central Bank

The central bank aims to maximize the sum of household utilities:

$$\max_{\theta_j, r_j} \int_h U_h(j^*(h)) dh$$

where $U_h(j^*(h))$ denotes the utility level of a household h that has selected money type j subject to the constraint that each type in use must have a minimum share of users ($s_j \geq \underline{s}$).

Equilibrium

In a competitive equilibrium where markets clear:

$$k = l = d = s_d. \quad (6)$$

The last equality follows from the uniform distribution and the fact that each household that selects deposits as its money type starts with a unitary endowment (as every household).

Let us remark again that the abovementioned model nests the one in the main text of Agur et al. (2022) with only one CBDC ($n = 1$) and without anonymity externalities ($\beta = 0$).

2.1 Optimal design in the cash equilibrium

We start by considering the model equilibrium under the assumption that all the n CBDCs, deposits, and cash coexist in the economy, i.e., all the $n + 2$ types of money remain in use and hence none of them are below the threshold \underline{s} (thus $\eta_j = 0, \forall j$). Further formal details about this assumption are provided at the end of this subsection.

Taking into account the abovementioned assumption and the households' utility expression, equation (1), we can deduce that money type j is preferred over money type $j - 1$ when

$$h > \frac{\rho(r_{j-1} - r_j) + \theta_{j-1} + \theta_j}{2} \equiv h_j. \quad (7)$$

Remember that $r_0 = r_d, r_{n+1} = r_c = 0, \theta_0 = \theta_d = 0$, and $\theta_{n+1} = \theta_c = 1$. Furthermore, since h is uniformly distributed then the share of users of a money type j is given by

$$s_j = h_{j+1} - h_j \quad (8)$$

with $s_d = s_0 = h_1$ and $s_c = s_{n+1} = 1 - h_{n+1}$.

The maximization of welfare with respect to both the place of each CBDC in the anonymity-security scale (θ_j) and the interest rate paid on each CBDC (r_j) yields the following results (see

the appendix for derivation details):

$$r_j^* = \frac{n-j+1}{n} r_1^*, \forall j > 1, \quad (9)$$

$$\theta_j^* = \frac{j-1}{n} + \frac{n-j+1}{n} \theta_1^*, \forall j > 1, \quad (10)$$

$$r_1^* = \frac{2n\beta}{2n+\rho+2} \frac{(2-A+\phi)(2+\rho) - 2n(A-\phi-1)}{2(1-\beta)(1+n) + \rho(1+2n) - \beta\rho}, \quad (11)$$

and

$$\theta_1^* = \frac{(2+\rho)(1-\beta) + 2n(A-\phi-1)\rho}{2(1-\beta)(1+n) + \rho(1+2n) - \beta\rho}. \quad (12)$$

By setting $n = 1$, from (11) and (12) we arrive at equations (59) and (58) in the Appendix of Agur et al. (2022), which in turn nest their equations (40) and (36) (in the main text of their article) for $\beta = 0$. Notice that when there is no anonymity externality ($\beta = 0$), the optimal result is that $r_j^* = 0$, for all j . From (11) and (12), it can be seen that r_1^* is decreasing and θ_1^* is increasing in the value of $A - \phi$, that is to say, a high value added from bank intermediation implies, with regard to the optimal design of the first CBDC in the anonymity-security scale, a lower interest rate and a higher place in the scale in order to reduce its attractiveness to deposit users and hence mitigate bank disintermediation. It can also be verified that θ_1^* is decreasing in n , i.e., the greater the number of CBDCs in the economy, the lower the anonymity level (the closer to zero) of the first CBDC in the scale.⁶

2.1.1 Optimal shares of users of each type of money

As mentioned above the share of users of money type j is given by (8). Using (7) to substitute h_j and h_{j+1} and taking into consideration that optimal conditions imply that $r_{j-1}^* - 2r_j^* + r_{j+1}^* = 0$ (see Appendix A.1.), and therefore

$$s_j^* = \frac{\theta_{j+1}^* - \theta_{j-1}^*}{2}, 1 \leq j \leq n \quad (13)$$

which can be rewritten, in terms of θ_1^* , using (10):

$$s_j^* = \frac{1 - \theta_1^*}{n}, 1 \leq j \leq n. \quad (14)$$

The share of users is equal for all CBDCs (it does not depend on j). Furthermore, as the following lemma shows, this share is decreasing in both the value added from bank intermediation ($A - \phi$) and the number of CBDCs (n).

Lemma 1 *The optimal share of users of each CBDC in the cash equilibrium (i.e., equation 14) is decreasing in $A - \phi$ and decreasing in n .*

⁶If we assume that it is possible to introduce at least one CBDC, then, for $n = 1$, $\theta_1^* < 1$ requires $1 - \beta - (A - \phi - 2)\rho > 0$. This is a sufficient condition to have $d\theta_1^*/dn > 0$, from (12).

Proof. The negative relation of s_j^* and $A - \phi$ follows from the fact that the former is decreasing in θ_1^* which, in turn, is increasing in the latter -this is straightforward from equation (12).

Substituting (12) into (14) we obtain

$$s_j^* = \frac{4\rho + 2(1 - \beta) - 2\rho(A - \phi)}{2(1 - \beta)(1 + n) + \rho(1 + 2n) - \beta\rho},$$

from which it is straightforward to see the negative relation of s_j^* and n . ■

Since $s_c^* = 1 - h_{n+1}^* = (1 - \rho r_n^* - \theta_n^*)/2$, using (9) and (10) (when $j = n$) we find that

$$s_c^* = \frac{1 - \theta_1^* - \rho r_1^*}{2n} = \frac{1}{2} \left(s_j^* - \frac{\rho r_1^*}{n} \right). \quad (15)$$

The following lemma shows that under the restrictions for ρ and $A - \phi$ (introduced by Agur et al., 2022 to focus on the well-behaved equilibria), the introduction of additional CBDCs reduces the optimal share of users of cash, s_c^* .

Lemma 2 *Under the restriction $3/4 \leq \rho \leq 3/2$ and $1 < A - \phi \leq 5/2$, the optimal share of users of cash in the cash equilibrium (i.e., equation 15) is decreasing in the number of CBDCs, n .*

Proof. See Appendix A.2 ■

Finally, for the optimal share of users of deposits we have

$$s_d^* = 1 - ns_j^* - s_c^* = \theta_1^* - s_c^*.$$

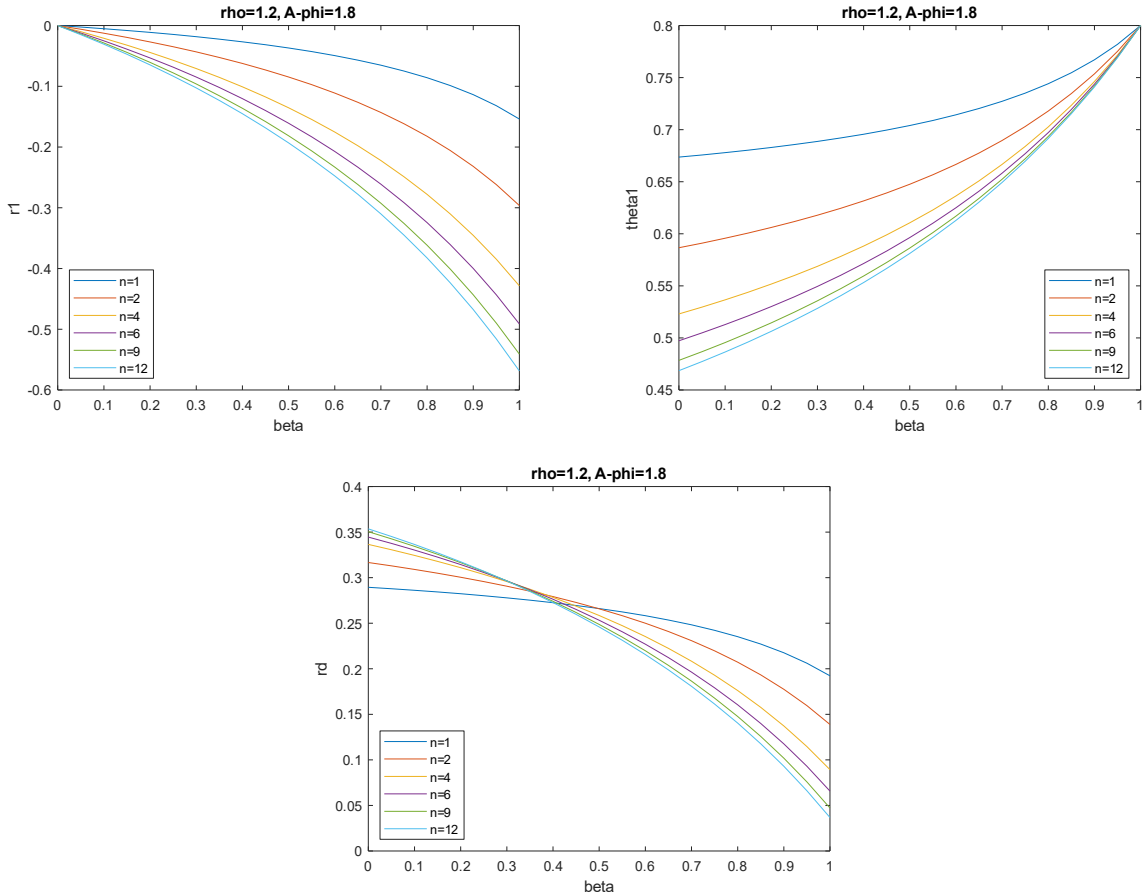
The baseline model by Agur et al. (2022) incorporates network effects across all money types. As noted at the beginning of Section 2.1, these effects imply that for a money type to remain viable it must maintain a minimum share of users, \underline{s} . Therefore we have assumed so far that all $n + 2$ money types (bank deposits, n CBDCs, and cash) satisfy this condition and remain in use (an alternative scenario without cash will be explored in Section 2.2). For each of the n CBDCs, this means that $s_j^* \geq \underline{s}$. As long as $\underline{s} > 0$, there is a limit to the number of CBDCs that can be introduced without breaching this threshold. Additionally, from equation (14), we deduce that $\theta_1^* \leq 1 - n\underline{s}$. Since θ_1^* increases with the value added by bank intermediation (lending to firms), a higher value of $A - \phi$ reduces the number of CBDCs that can coexist in the economy and the need for CBDCs with characteristics similar to bank deposits (in the anonymity-security scale). Regarding cash, so far we have solved the model under the assumption that it remains in circulation, which requires that $s_c^* \geq \underline{s}$.

2.1.2 Numerical example

To illustrate some of the abovementioned results we provide a numerical example with parameter values set within the restrictions introduced by Agur et al. (2022). Particularly, we set $\rho = 1.2$,

$A - \phi = 1.8$, and $\underline{s} = 5\%$. Figure 1 illustrates that as the cost of anonymity externalities increases, the interest rate r_1^* decreases (top-left panel), while the position of the first CBDC on the anonymity-security scale, θ_1^* , increases (top-right panel). From equations (9) and (10), subsequent CBDCs in the scale have progressively higher interest rates (i.e., closer to zero) and occupy higher positions on the anonymity-security scale (r_j^* and θ_j^* , $j > 1$, are not shown in the figure). These results (and the fact that r_1^* is negative) are explained by the fact that, for the parameter values considered, there is a high social value for bank intermediation relative to the value of monetary diversity. As the anonymity cost rises, this relative value grows, prompting the central bank to set r_1^* and θ_1^* in a way that increases the supply of deposits. This, in turn, lowers the corresponding deposit interest rate r_d^* (bottom panel). The resulting increase in the share of users of deposits can be seen in the bottom panel of Figure 2.

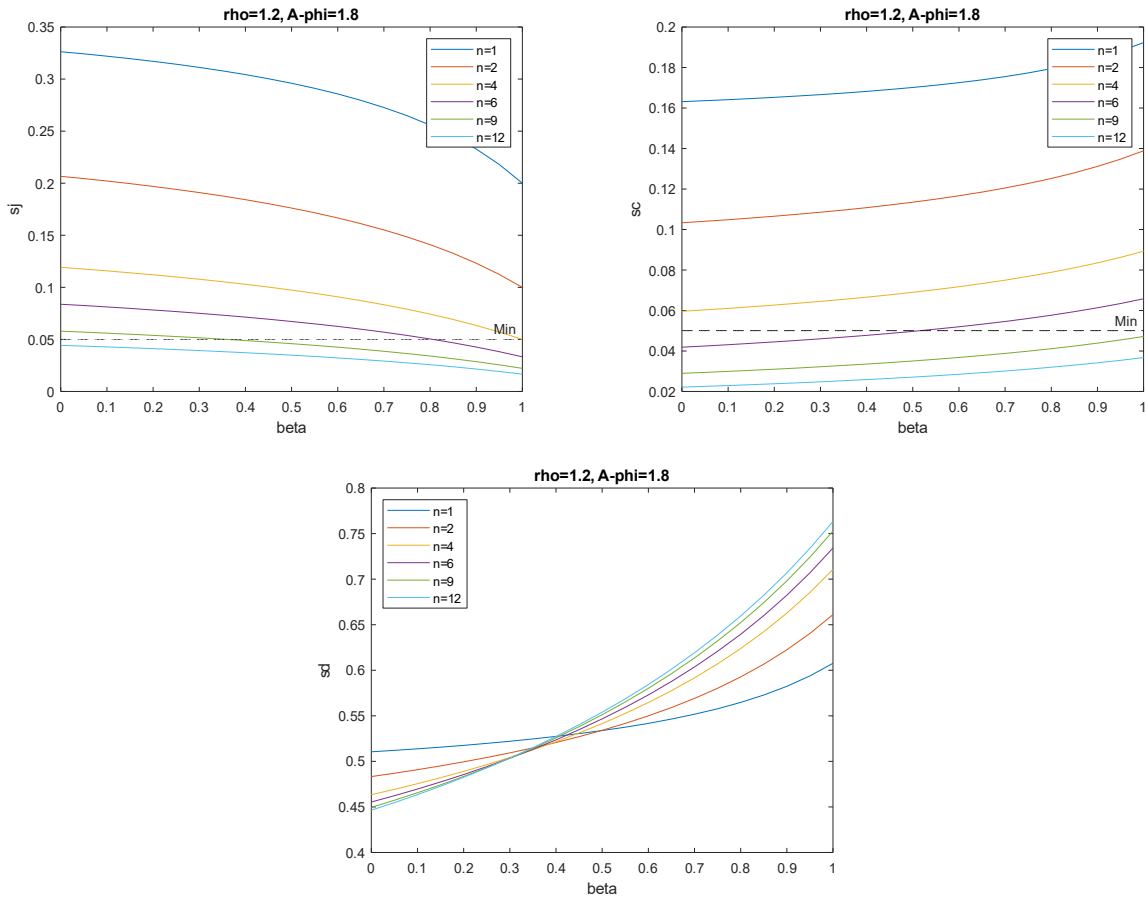
Figure 1. Optimal values of r_1^* , θ_1^* , and r_d^* in the cash equilibrium according to β and n



The top-left panel of Figure 2 shows that s_j^* , the share of users for each CBDC, decreases with the number of CBDCs (as established in Lemma 1) and, for the parameter values considered, also decreases as the cost of anonymity externalities (β) rises. On the other hand, the top-right panel of the same figure shows that s_c^* , the share of cash users, increases with higher values of β , although it

declines as the number of CBDCs grows (as indicated in Lemma 2). Together, these panels suggest that the central bank could introduce up to four or five CBDCs without breaching the (network) threshold of $\underline{s} = 0.05$ for either CBDCs or cash, thereby avoiding the disappearance of the latter. However, as will be discussed in Section 2.3, if the externality cost is high, the central bank prefers the cashless equilibrium over the one with cash. Moreover, regardless of the anonymity cost, the central bank favors the disappearance of cash as long as one additional CBDC can be introduced.

Figure 2. Optimal shares of users (s_j^* , s_c^* , and s_d^*) in the cash equilibrium according to β and n



2.2 Optimal design in the cashless equilibrium

Now we analyze the model equilibrium as if cash were no longer present in the economy. In this case, the users with the highest preference for anonymity will use CBDC number n , i.e., the CBDC with the highest value of θ in the scale. Notice that the elimination of cash implies that $h_{n+1} = 1$.

The maximization of welfare with respect to both the place of each CBDC in the anonymity-security scale (θ_j) and the interest rate paid on each CBDC (r_j) yields the following results (see

Appendix A.3. for derivation details):

$$r_j^* = r_1^* - \frac{2\beta(j-1)(1-\theta_1^*)}{(\beta+2n-1)\rho}, \forall j > 1, \quad (16)$$

$$\theta_j^* = \frac{[1+\beta+2(n-j)]\theta_1^* + 2(j-1)}{\beta+2n-1}, \forall j > 1, \quad (17)$$

$$r_1^* = -\frac{\beta}{\rho}\theta_1^*, \quad (18)$$

and

$$\theta_1^* = \frac{(2+\rho)(1-\beta) + (2n-1+\beta)(A-\phi-1)\rho}{2n(1-\beta+\rho) - \beta^2 + 1}. \quad (19)$$

By setting $n = 1$ and $\beta = 0$, equations (18) and (19) reduce to equations (40) and (37), respectively, in Agur et al. (2022). Notice that in the absence of cash and with the anonymity cost ($\beta > 0$), all interest rates are negative with the intention of mitigating the impact of the CBDCs on banking intermediation.

2.2.1 Optimal shares of users of each type of money

From equations (7), (8), and (16) it is easy to show that (13) still applies and then using (17) we obtain that for all CBDCs

$$s_j^* = \frac{2(1-\theta_1^*)}{2n-1+\beta}, \quad (20)$$

and therefore, as in the cash equilibrium, the share of users is equal for all CBDCs (it does not depend on j).

Since the only remaining money type is deposits we have that

$$s_d^* = 1 - ns_j^* = \frac{2n\theta_1^* - 1 + \beta}{2n - 1 + \beta}.$$

2.2.2 Numerical example

Using the same parameter values set for the cash equilibrium in Section 2.1.2 ($\rho = 1.2$, $A - \phi = 1.8$, $\underline{s} = 5\%$) we calculate a numerical example for the cashless equilibrium. Figure 3 shows that in the absence of cash, the optimal strategy is to position CBDCs further to the right on the anonymity-security scale compared to the cash equilibrium (cf. Figure 1). This adjustment is particularly significant when the anonymity cost is low (top-right panel), as it helps partially compensate for the disappearance of the most anonymous form of money. Simultaneously, lower returns on CBDCs are recommended, with a more pronounced reduction in the case of high anonymity costs (top-left panel), to mitigate the adverse effects on bank intermediation.

Figure 3. Optimal values of r_1^* , θ_1^* , and r_d^* in the cashless equilibrium according to β and n

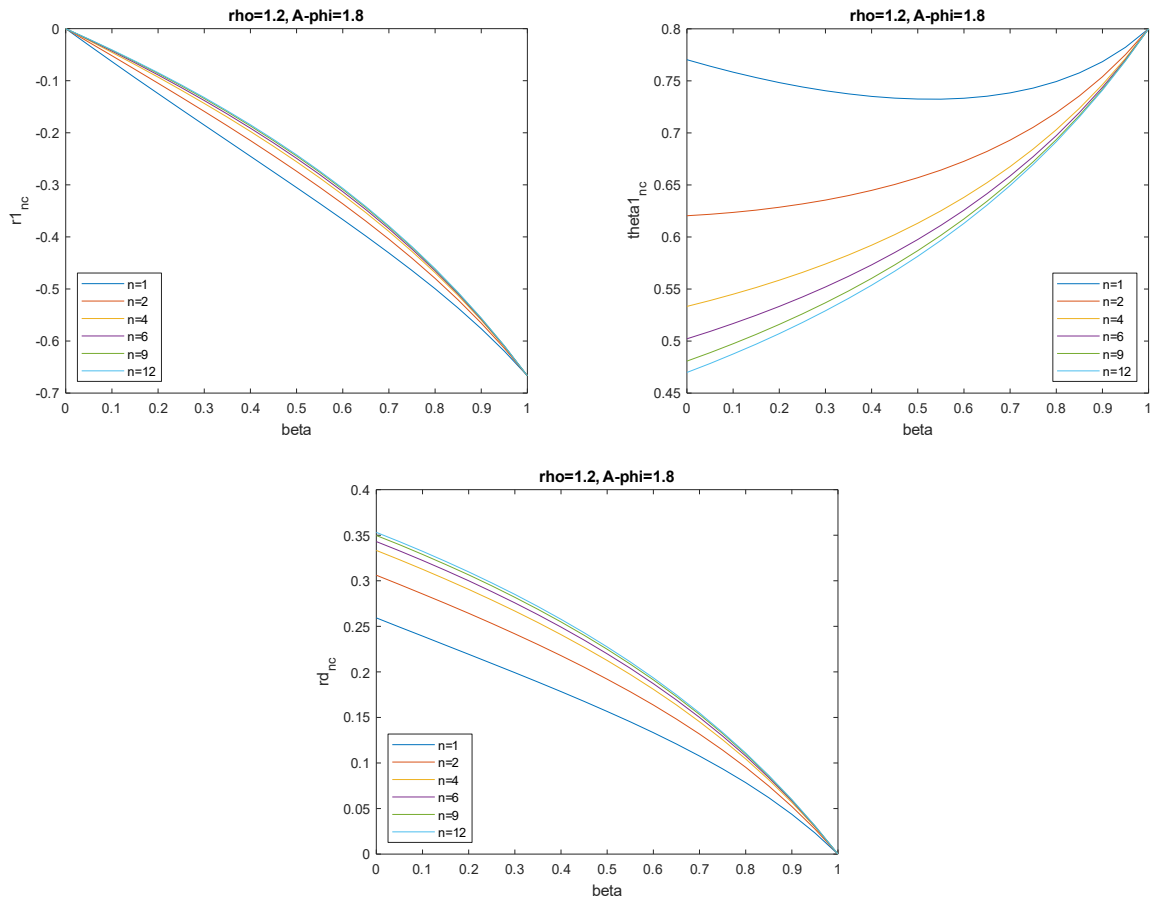


Figure 4. Optimal shares of users (s_j^* and s_d^*) in the cashless equilibrium according to β and n

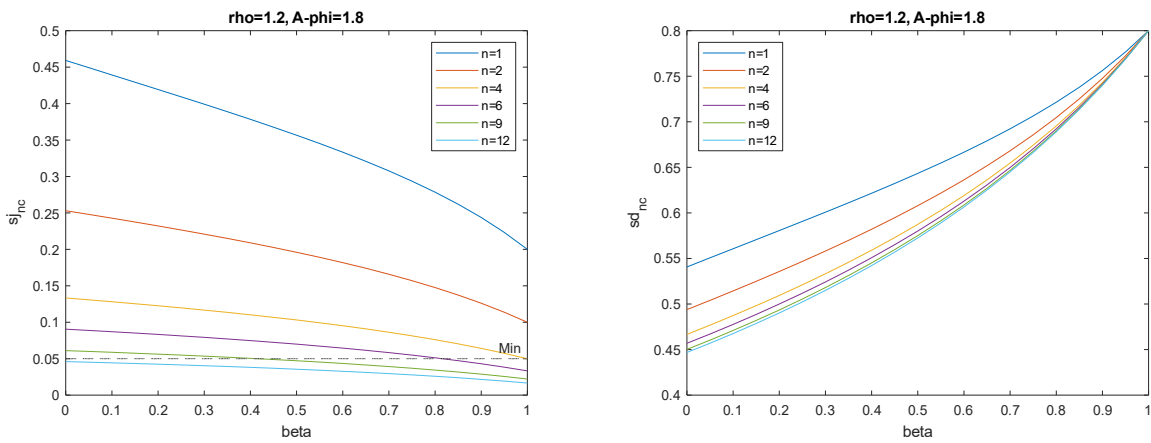


Figure 4 (cf. Figure 2) shows that, as a result of these adjustments, the disappearance of cash leads to higher shares of users for CBDCs (left panel), particularly when the anonymity cost is

low. Additionally, it results in a higher share of deposit users (right panel), especially when the anonymity cost is high. Furthermore, for the parameter values considered, the absence of cash allows for the introduction of up to about nine CBDCs without breaching the network threshold ($\underline{s} = 0.05$) when anonymity costs are relatively low ($\beta < 0.4$), and up to four or five CBDCs when these costs are relatively high ($\beta > 0.8$).

2.3 Aggregate Welfare Analysis

Using the results obtained in the previous subsections we calculate aggregate welfare for both equilibria (with and without cash). While this calculation benefits from the fact that the optimal shares of CBDCs are independent of j in both equilibria,⁷ it still involves complex and unilluminating algebra, which we omit here for brevity. Instead, we focus on presenting the final results.

Proposition 1 demonstrates that in the absence of anonymity costs, the central bank prefers the equilibrium where cash remains in use over the cashless equilibrium. This result extends the one by Agur et al. (2022) to the case of multiple CBDCs, highlighting that, in the absence of anonymity externalities, the preference for cash persists even when additional CBDCs are introduced.

Proposition 1 *in the absence of anonymity externalities (i.e., $\beta = 0$) the welfare of the cash equilibrium, $W^{ce}(\beta = 0, n)$, exceeds that of the cashless equilibrium, $W^{nce}(\beta = 0, n)$.*

Proof. Using equations (7)-(12), to compute $W^{ce}(\beta = 0, n)$, and equations (7)-(8) and (16)-(19), to compute $W^{nce}(\beta = 0, n)$, we obtain

$$W^{ce}(\beta = 0, n) - W^{nce}(\beta = 0, n) = \frac{[\rho(A - \phi - 2) - 1]^2}{4n(\rho + 1)(\rho + 3 + 2n(1 + \rho)) + 2\rho + 4} > 0,$$

(which for $n = 1$ reduces to equation 38 in Agur et al, 2022). ■

Without anonymity externalities ($\beta = 0$), cash's anonymity provides utility without incurring social costs. In addition, the coexistence of multiple money types—including cash and CBDCs—supports a broader range of user preferences and transactional needs, thereby enhancing overall welfare compared to a scenario where cash is eliminated.

In contrast, when the cost of anonymity externalities is at its highest ($\beta = 1$), the central bank prefers the cashless equilibrium over the one in which cash remains in use. This preference arises because the social costs associated with anonymity in cash outweigh its benefits, as demonstrated in the following proposition:

⁷Notice that $\int_h \theta_{j(h)} dh = \sum_{j=1}^{n+1} \theta_j s_j$. In the cash equilibrium, from (10) and (14), $\sum_{j=2}^n \theta_j s_j = \frac{(1-\theta_1^2)(n-1)}{2n}$. In the cashless equilibrium, from (17) and (20), $\sum_{j=2}^n \theta_j s_j = \frac{2(1-\theta_1)(n-1)[n+\theta_1(n+\beta-1)]}{(2n+\beta-1)^2}$.

Another useful result: $\sum_{j=2}^n \left(\int_{h_j}^{\theta_j} (\theta_j - h) dh + \int_{\theta_j}^{h_{j+1}} (h - \theta_j) dh \right)$ (i) in the cash equilibrium (using equations 7, 9, and 10) is equal to $\frac{(n-1)[\rho^2 r_1^2 + (\theta_1 - 1)^2]}{4n^2}$, and (ii) in the cashless equilibrium (using equations 7, 16, and 17) is equal to $\frac{(n-1)(\theta_1 - 1)^2(\beta^2 + 1)}{(2n + \beta - 1)^2}$.

Proposition 2 *when the cost of anonymity externalities is maximal ($\beta = 1$) the welfare of the cashless equilibrium, $W^{nce}(\beta = 1, n)$, dominates that of the cash equilibrium, $W^{ce}(\beta = 1, n)$.*

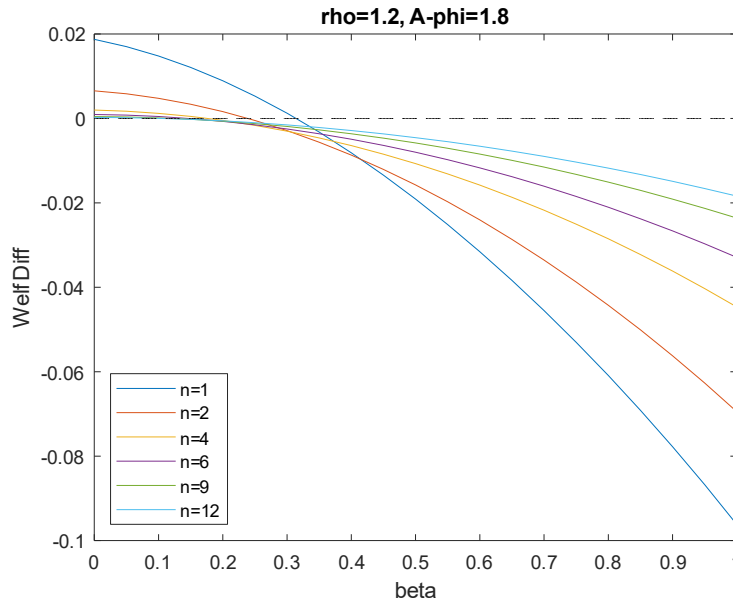
Proof. Using equations (7)-(12), to compute $W^{ce}(\beta = 1, n)$, and equations (7)-(8) and (16)-(19), to compute $W^{nce}(\beta = 1, n)$, we find that

$$W^{ce}(\beta = 1, n) - W^{nce}(\beta = 1, n) = -\frac{1}{2(2n + \rho + 2)} < 0.$$

■

Using the same parameter values as in the numerical examples above ($\rho = 1.2$, $A - \phi = 1.8$) we see, in Figure 5, that for low values of the anonymity cost the cash equilibrium is preferred over the one without cash; however, the opposite occurs when this cost is moderate or high. In these cases, the utility derived from cash's anonymity no longer offsets the negative externalities. Instead, CBDCs can be designed to strike a better balance between anonymity and security while minimizing social costs. These results emphasize the importance of well-designed CBDCs as a substitute that can address these costs while preserving essential monetary functions.

Figure 5. Welfare difference ($W^{ce} - W^{nce}$) according to β and n



Finally, the following proposition shows that the central bank prefers an equilibrium without cash and $n + 1$ optimally designed CBDCs over an equilibrium with cash and n optimally designed CBDCs. Remarkably, this preference holds even in the absence of anonymity externalities ($\beta = 0$):

Proposition 3 *the welfare of the cashless equilibrium with one additional CBDC, $W^{nce}(\beta, n + 1)$, exceeds the welfare of the cash equilibrium, $W^{ce}(\beta, n)$.*

Proof. See Appendix A.4 ■

Intuitively, a monetary system with $n + 1$ CBDCs allows the central bank to better meet the diverse preferences of users compared to a system with n CBDCs and cash. This is because the central bank can optimally adjust the interest rate and position of each CBDC along the anonymity-security spectrum, whereas cash has a fixed remuneration ($r_c = 0$) and a fixed position in the scale ($\theta_c = 1$). In addition, this finding reinforces the case for transitioning to a cashless society, highlighting the critical need to design CBDCs that accommodate a wide range of user preferences and address the challenges posed by anonymity.

Returning to the numerical example (and taking into consideration Proposition 3) we conclude that for the parameter values analyzed, the central bank would introduce at least seven CBDCs—and potentially up to eleven—when anonymity costs are low to intermediate ($\beta < 0.7$). In this scenario, the equilibrium with additional CBDCs and no cash is feasible, as the network threshold for these CBDCs is not breached. Consequently, cash disappears as a result of network effects. For high anonymity costs ($\beta > 0.7$) the central bank can introduce between four and six CBDCs, depending on the specific value of β , without breaching the network threshold. While cash is not eliminated through network effects in this case, the central bank prefers its removal due to the significant cost of anonymity externalities.

The numerical example suggest that the feasibility of these equilibria depends on anonymity costs: low and intermediate costs support a higher number of CBDCs, while high costs limit their introduction but still motivate the removal of cash to mitigate the negative externality.

3 Conclusion

This paper contributes to the ongoing discussion on the optimal design and implications of central bank digital currencies (CBDCs) by extending the framework of Agur et al. (2022) to consider multiple CBDCs. By focusing on key design features—such as positions on the anonymity-security spectrum and interest rates—we analyze the welfare implications of CBDCs in both cash and cashless equilibria.

Our findings highlight that the introduction of CBDCs offers the central bank a powerful tool to balance user preferences, enhance payment system efficiency, and address social costs associated with payment anonymity. Specifically, when anonymity externalities are significant, transitioning to a cashless economy is preferred due to the potential welfare benefits of eliminating cash. Moreover, regardless of anonymity costs, an economy with cash and one or more optimally designed CBDCs is welfare dominated by a cashless economy with one additional CBDC. This underscores the flexibility and adaptability of CBDCs in meeting diverse user needs while mitigating the social cost of anonymity.

The results also demonstrate the importance of CBDC design in shaping monetary equilibria. While cash imposes rigidities due to its fixed position on the anonymity-security scale and lack of remuneration, CBDCs allow the central bank to strategically tailor their characteristics. This

enables better accommodation of user heterogeneity and a more efficient allocation of resources within the monetary system. Our analysis supports the idea that a well-designed suite of CBDCs can achieve a broad range of policy objectives, including mitigating negative externalities, and maintaining the competitiveness of the central bank’s money relative to private alternatives.

While our study advances the understanding of CBDC design and its implications, it also opens avenues for further research. Future studies could explore additional dimensions of CBDC design, such as settlement speed and accessibility, as well as their impact on financial inclusion.

In conclusion, our analysis reinforces the potential of CBDCs to enhance welfare and modernize payment systems. However, realizing this potential requires careful design choices that account for user preferences and externalities associated to some features of money types.

References

Agur, I., Ari, A., Dell’Ariccia, G. (2022). “Designing central bank digital currencies”, *Journal of Monetary Economics* 125 (2022) 62–79.

Ahnert, T., Hoffmann, P., Monnet, C. (2022). “The Digital Economy, Privacy, and CBDC”, CEPR Discussion Paper No. DP17313.

Aurazo, J., Banka, H., Frost, J., Kosse, A., Piveteau, T. (2024). "Central bank digital currencies and fast payment systems: rivals or partners?", BIS Paper No. 151, Bank for International Settlements.

Bacchetta, P., Perazzi, E. (2022). “CBDC as Imperfect Substitute to Bank Deposits: A Macroeconomic Perspective”, Swiss Finance Institute, Research Paper Series No 21-81.

Banco de la Republica (2024). "Relevance and Risks of Issuing a Central Bank Digital Currency in Colombia", Technical Document.

Cheng, Y., Izumi, R. (2023). “CBDC: Banking and Anonymity”, Wesleyan Economics Working Papers 2023-002, Wesleyan University, Department of Economics.

Chiu, J., Davoodalhosseini, M. (2023). “Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-Like Design”, *Management Science, INFORMS*, vol. 69(11), 6708-6730.

Garratt, R., Jiaheng, Y., Haoxiang, Z. (2022). “The case for convenience: how CBDC design choices impact monetary policy pass-through”, BIS Working Paper 1049, Bank for International Settlements.

Keister, T., Sanches, D. (2023). “Should central banks issue digital currency?”, *Review of Economic Studies* 90, 404–431.

Appendix. Derivation of results

Let us start by rewriting some terms that are part of the welfare calculation (i.e. the aggregation over h of equation 1). First, using equations (2)-(6), $r_d = R$, and $r_{n+1} = 0$, we have that:

$$\int_h C_{j(h)} dh = 1 + \phi k^0 + \left(A - \phi - 1 - \frac{s_d}{2} \right) s_d, \quad (\text{A.1})$$

which is equal to equation (26) in Agur et al. (2022).

Second, we rewrite the welfare losses that result from the distance between money types and households' preferences in the following form:

$$\begin{aligned} \int_h |\theta_{j(h)} - h| dh &= \int_0^{h_1} h dh + \int_{h_1}^{\theta_1} (\theta_1 - h) dh + \int_{\theta_1}^{h_2} (h - \theta_1) dh + \dots \\ &+ \int_{h_n}^{\theta_n} (\theta_n - h) dh + \int_{\theta_n}^{h_{n+1}} (h - \theta_n) dh + \int_{h_{n+1}}^1 (1 - h) dh. \end{aligned} \quad (\text{A.2})$$

Third, we rewrite the anonymity externality as:

$$\begin{aligned} \int_h \theta_{j(h)} dh &= \int_{h_1}^{h_2} \theta_1 dh + \int_{h_2}^{h_3} \theta_2 dh + \dots + \int_{h_n}^{h_{n+1}} \theta_n dh + 1 - h_{n+1} \\ &= -h_1 \theta_1 + h_2 (\theta_1 - \theta_2) + \dots + h_{n+1} (\theta_n - 1) + 1 \end{aligned} \quad (\text{A.3})$$

In addition, it will be useful to derive expressions for r_d and s_d in terms of θ_1 and r_1 . From (4), (6), (7) for $j = 1$, and the conditions $R = r_d$ and $s_d = h_1$ we have that

$$r_d = \frac{2(A - \phi - 1) + \rho r_1 - \theta_1}{2 + \rho}, \quad (\text{A.4})$$

and

$$s_d = h_1 = \frac{\rho(A - \phi - 1 - r_1) + \theta_1}{2 + \rho}. \quad (\text{A.5})$$

which are equal to equations (23) and (24), respectively, in Agur et al. (2022).

A.1. Derivation of equations (9)-(12)

We start by deriving welfare with respect to r_j . To that purpose we use equations (A.1)-(A.3) and take into account, from (7), that r_j affects h_j and h_{j+1} : for $j > 1$, $\partial h_j / \partial r_j = -\rho/2$ and $\partial h_{j+1} / \partial r_j = \rho/2$. After deriving and using (7) to substitute h_j and h_{j+1} , we arrive at this condition:

$$\rho(r_{j-1} - 2r_j + r_{j+1}) = \beta(2\theta_j - \theta_{j-1} - \theta_{j+1}), \forall j > 1.$$

Following a similar procedure with respect to θ_j (for $j > 1$, $\partial h_j / \partial \theta_j = \partial h_{j+1} / \partial \theta_j = 1/2$), we obtain

$$\beta\rho(r_{j-1} - 2r_j + r_{j+1}) = 2\theta_j - \theta_{j-1} - \theta_{j+1}, \forall j > 1.$$

These two equations imply that

$$r_{j-1} - 2r_j + r_{j+1} = 0, \forall j > 1, \quad (\text{A.6})$$

$$\theta_{j-1} - 2\theta_j + \theta_{j+1} = 0, \forall j > 1, \quad (\text{A.7})$$

and considering that $r_{n+1} = 0$ and $\theta_{n+1} = 1$, it is easy to see that

$$r_j = \frac{n-j+1}{n-j+2} r_{j-1}, \forall j > 1, \quad (\text{A.8})$$

and

$$\theta_j = \frac{1 + (n-j+1)\theta_{j-1}}{n-j+2}, \forall j > 1, \quad (\text{A.9})$$

which by recursion lead to equations (9) and (10), respectively.

When $j = 1$ we must consider the fact that $s_d = h_1$ is a function of r_1 and θ_1 . From equation (A.5) $\partial h_1 / \partial r_1 = -\rho / (2 + \rho)$ and $\partial h_1 / \partial \theta_1 = 1 / (2 + \rho)$. Deriving the welfare function with respect to r_1 and using (A.5), to substitute h_1 , and (7), to substitute h_2 , we get

$$r_2 = \frac{4 + \rho}{2 + \rho} r_1 - \frac{\beta}{\rho} \left(\theta_2 - \frac{4 + \rho}{2 + \rho} \theta_1 \right). \quad (\text{A.10})$$

Deriving the welfare function with respect to θ_1 and using (A.5) to substitute h_1 and (7) to substitute h_2 we have

$$\begin{aligned} & 2\rho(1 + \beta)(A - \phi - 1) + \theta_2(2 + \rho) - (4 + 3\rho + 2\beta\rho)\theta_1 \\ & = \beta\rho[r_1(\rho + 4) - r_2(\rho + 2)]. \end{aligned} \quad (\text{A.11})$$

Finally, using equations (A.8) and (A.9) for $j = 2$, (A.10), and (A.11), and then solving the system, we obtain equations (11) and (12).

A.2. Proof of Lemma 2

Substituting (11) and (12) into (15) and deriving with respect to n we arrive at

$$-2 \frac{(4\lambda_2 n^2 - \lambda_1 \lambda_3) \lambda_4 + (4\lambda_2 n + \lambda_3 + \lambda_1 \lambda_2) \lambda_5}{(\lambda_1 + 2n)^2 (\lambda_3 + 2\lambda_2 n)^2}$$

where $\lambda_1 = 2 + \rho$, $\lambda_2 = 1 + \rho - \beta$, $\lambda_3 = (2 + \rho)(1 - \beta)$, $\lambda_4 = (1 - \beta)(1 - (A - \phi - 1)\rho) + \rho$, and $\lambda_5 = (1 - \beta)(\rho^2(1 - (A - \phi - 1)) + (3\rho + 2)(1 - (A - \phi - 1)\rho))$. Then, it is sufficient to show that the numerator is always positive. Since it is increasing in n , we set $n = 1$ and derive it with respect to $A - \phi - 1$ to obtain $-\rho(1 - \beta)(\rho + 4)^2(\rho + 1 - \beta)$, which is negative, and hence the numerator is minimized when $A - \phi - 1$ takes its maximum value, i.e., $A - \phi - 1 = 3/2$.

With $n = 1$ and $A - \phi - 1 = 3/2$, the numerator is a convex function of β (the second derivative

is $\rho^3 + 2\rho^2 + 32$) whose minimum value is

$$-\frac{1}{8} \frac{\rho^3 (\rho^5 + 16\rho^4 + 96\rho^3 + 160\rho^2 - 320\rho - 1024)}{\rho^3 + 2\rho^2 + 32},$$

which is strictly positive for $0 < \rho \leq 1.98$ (a range that includes the restriction introduced by Agur et al., 2022).

A.3. Derivation of equations (16)-(19)

The problem is similar to the one in Appendix A.1 (equations A.10 and A.11, for $j = 1$, and A.6 and A.7, for $1 < j < n$, still apply) but now we take into consideration that $h_{n+1} = 1$, and hence the derivatives with respect to r_n and θ_n , respectively, lead to

$$\rho(r_{n-1} - r_n) = \beta(\theta_n - \theta_{n-1}),$$

$$\beta\rho(r_{n-1} - r_n) = 2(\beta - 1)(1 - \theta_n) + \theta_n - \theta_{n-1}.$$

These two equations imply that

$$r_n = r_{n-1} - \frac{2\beta(1 - \theta_n)}{\rho(1 + \beta)}, \quad (\text{A.12})$$

and

$$\theta_n = \frac{(1 + \beta)\theta_{n-1} + 2}{3 + \beta}. \quad (\text{A.13})$$

Using these two equations together with (A.6) and (A.7) we deduce that

$$r_j = r_{j-1} - \frac{2\beta(1 - \theta_j)}{\rho(1 + \beta)}, \quad \forall j > 1, \quad (\text{A.14})$$

$$\theta_j = \frac{[1 + \beta + 2(n - j)]\theta_{j-1} + 2}{\beta + 2(n - j) + 3}, \quad \forall j > 1. \quad (\text{A.15})$$

which by recursion lead to equations (16) and (17), respectively. Then by using (A.10), (A.11), and (A.14), (A.15) for $j = 2$, we obtain equations (18) and (19).

A.4. Proof of Proposition 3

We will prove that $W^{ce}(\beta, n) - W^{nce}(\beta, n + 1)$ is a strictly concave function of $A - \phi - 1$ and that this function is negative at its maximum value, and hence negative in general (for any $\beta \in (0, 1)$). Then we show that it is also true when $\beta = 0$ and $\beta = 1$.

The second derivative $\frac{\partial^2(W^{ce}(\beta, n) - W^{nce}(\beta, n + 1))}{\partial(A - \phi - 1)^2}$ is equal to

$$-\frac{\rho^2(1 - \beta)(1 + \beta)^2}{\Psi_1 n^2 + (\Psi_1 + 2(1 - \beta)\Psi_2)n - (1 - \beta)(2 + \rho)\Psi_3}, \quad (\text{A.16})$$

where $\Psi_1 \equiv 4(1 - \beta + \rho)^2$, $\Psi_2 \equiv -\beta^2 - 2\beta + \rho^2 + 4\rho + 3$, and $\Psi_3 \equiv \beta(2 + \beta) - 2\rho - 3$. Thus to demonstrate that $W^{ce}(\beta, n) - W^{nce}(\beta, n + 1)$ is a strictly concave function of $A - \phi - 1$ we show

that the denominator is strictly positive.

The second derivative of the denominator with respect to n is $8(1 - \beta + \rho)^2$, which is strictly positive and hence the denominator of (A.16) is a strictly convex function of n , which is minimized when

$$n = \frac{\beta^2 + \beta(4 + \rho) - 3\rho - 5}{4(1 - \beta + \rho)}.$$

This expression implies a negative value of n (for any value of $\beta \in [0, 1]$).⁸ This is not a feasible value of n but, since the denominator of (A.16) is a strictly convex function of n , we know that to the right of $n = 0$ it is increasing in n . For $n = 0$, the denominator of (A.16) is equal to $(1 - \beta)(2 + \rho)(3 - \beta(2 + \beta) + 2\rho)$, which is strictly positive and thus we know that the denominator of (A.16) is positive for any $n > 0$. This proves that (A.16) is negative, i.e., $W^{ce}(\beta, n) - W^{nce}(\beta, n + 1)$ is a strictly concave function of $A - \phi - 1$ (for any $\beta \in [0, 1]$).

$W^{ce}(\beta, n) - W^{nce}(\beta, n + 1)$ is maximized when $A - \phi - 1 = (1 - \beta + \rho)/\rho$, which implies that

$$W^{ce}(\beta, n) - W^{nce}(\beta, n + 1) = -\frac{1}{2} \frac{\beta^2}{2n + \rho + 2} < 0, \text{ for } \beta > 0$$

and then it must be the case that for any $\beta \in (0, 1)$, $W^{ce}(\beta, n) - W^{nce}(\beta, n + 1) < 0$.

With $\beta = 0$, $W^{ce}(\beta = 0, n) - W^{nce}(\beta = 0, n + 1) = -\frac{[\rho(A - \phi - 2) - 1]^2}{[2n(1 + \rho) + \rho + 2][4n(1 + \rho) + 4\rho + 6]} < 0$.

With $\beta = 1$, $W^{ce}(\beta = 1, n) - W^{nce}(\beta = 1, n + 1) = -(2(2n + \rho + 2))^{-1} < 0$. (Notice that $W^{nce}(\beta = 1, n) = W^{nce}(\beta = 1, n + 1)$).

⁸Notice that the expression is increasing in β , and that even at the maximum value of β (i.e., $\beta = 1$), it is negative, $n = -1/2$.