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Abstract

We study the behaviour of three quantitative sample surveys and a non sample inflation expectation report for Colombia. We found that expectations in Colombia; (i) are not strongly, i.e. a la Muth, rational because they show cross-section disagreement, (ii) expectations, however, show some features of weak rationality, (iii) expectations disagreement is time varying and relate to inflation, inflation changes and the output gap, thus suggesting a staggered information flow to agents, (iv) the forecast error loss function employed by agents is not symmetric and increasingly penalizes higher expectations than finally observed inflation as the horizon grows, and (v) this fact also explains the stylised fact that observed expectation share with theoretical rational expectations that *expectations look like lagged versions of inflation that dampen with the horizon*. The latest finding also arises from a very general econometric set up we develop in this paper. These results imply that the effect of weakening the rational expectations assumption in Colombian monetary policy models should be assessed, especially when compared to sticky information and heterogeneous agents choosing non Mean Square forecast Error losses.

Keywords: Inflation Expectations, expectation disagreement, near unit root, weak and strong rationality, non symmetric loss function.
JEL: C53, C82, E31, E37

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Expectativas de Inflación: Racionalidad, Desacuerdo y el Papel de la Función de Pérdida en Colombia[1]

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Resumen

Analizamos tres encuestas cuantitativas muestrales y un reporte no muestral de expectativas de inflación para Colombia. Encontramos que las expectativas en Colombia:(i) no son fuertemente, a la Muth, racionales debido a que exhiben descuerdo en cada corte transversal; (ii) sin embargo, muestran características de racionalidad débil; (iii) el desacuerdo es tiempo variante y se relaciona con la inflación, sus cambios y la brecha del PIB, sugiriendo un flujo escalonado de la información para formularlas; (iv) la función de pérdida ante errores de expectativas no es simétrica y penaliza de forma creciente las expectativas más altas que la inflación observada en la medida que se extiende el horizonte; y (v) este resultado explica también el hecho estilizado que comparten las expectativas observadas y las teóricas que *las expectativas parecen versiones rezagadas de la inflación observada que se suavizan con el horizonte.* Este hallazgo surge también de un esquema econométrico muy general que desarrollamos en este artículo. Estos resultados implican que se debe establecer el efecto de debilitar el supuesto de expectativas racionales en los modelos para la política monetaria, especialmente cuando se comparan con modelos con flujos escalonados de información y agentes heterogéneos que escogen funciones de pérdida distintas al Error Cuadrático Medio de pronósticos.

Palabras Clave: Expectativas de inflación, desacuerdo de las expectativas, cercanía a una raíz unitaria, Racionalidad débil y fuerte, función de pérdida asimétrica.
JEL: C53, C82, E31, E37

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Executive Summary

- We extend the results in Iregui-Bohórquez et al. (2021) in several ways. First, We prolong the sample to include both the COVID-19 period and its aftermath. Second, We approach rationality in a more formal way by distinguish between their weak and strong forms. Third, We compare the behaviour of observed expectations with the features of theoretical rational expectations derived from a very general econometric set-up that We develop in this paper. Fourth, We establish how expectations arise from a formal decision problem where the optimal decision ends up depending on the shape of the forecast error loss function. Fifth, under Mean Square Error loss, MSE, We take into account that inflation and expectations have unit roots for weak RE tests. And sixth, We explore weak rationality under the assumption that agents choose non-symmetric forecast error losses, and determine the degree and direction of loss asymmetry.
- According to Tesfatsion (2019), agent i expectations are *weakly rational* if they satisfy two conditions. First, they are based on both, her *subjective knowledge* (about the *objectively true* Data Generating Process, DGP, of the economy) and her *information set* at the survey date. And second, her expectation equals the expected future inflation based on the *objectively true DGP*, conditional on her information set, plus a zero mean idiosyncratic error. Weak RE reduces to the assumption that agents make "optimal use of whatever information they have", Tesfatsion (2019).
- In turn, expectations are said to be *strongly rational*, i.e. a la (Muth, 1961), if they are weakly rational agent i information set "contains all the information known to the modeller at the beginning of period t", Tesfatsion (2019). According to this author this RE form reduces to "a coordinating device that permits the construction of a "representative agent" having "representative expectations"". Therefore, under strong form RE agents share a great amount of information and, thus, show no disagreement at every cross section, i.e. all agents elicit the same expectation at every survey date for a fixed horizon. Since real word sample survey expectations show cross-section disagreement, strong rationality may be rejected, a-priori.
- However, agent's expectations arise as the solution to a decision problem, i.e. the minimisation of the expected value of the agent's *forecast loss function* conditional on her *information set* at the survey date, calculated employing her *subjective knowledge*. Two distinct situations arise from this fact: First, Under Mean Square forecast Error, MSE, loss, the *optimal* expectation is the expected future inflation conditional on her *information set* at the survey date, employing her *subjective knowledge*. And second, when the loss is not MSE, or more specifically, when it is non-symmetric, the *optimal*

expectation is the conditional expectation above plus *bias* term that depends on the shape of the loss function and the second and higher conditional moments of inflation at the expectation target date.

- The traditional treatment of expectations, such as the RE definitions above, along with the associated RE tests do not take into account the role of the forecast error loss function, thus defaulting to the MSE loss. As a result, two different econometric approaches are required; one under MSE and the other when a non-symmetric loss function is suspect.
- Under MSE, weak rationality is equivalent to the null that the coefficients of the Mincer and Zarnowitz (1969) regression are $(\beta_0, \beta_1) = (0, 1)$[5], and that the errors are uncorrelated up to the h-th lag, where h is the expectation horizon.
- However, if inflation and expectations contain unit roots, under MSE, the Mincer and Zarnowitz (1969) regression becomes a co-integration relationship with a co-integrating vector $(0, 1)$ and co-integration errors equal to the expectation errors. Thus, co-integration techniques are required to test weak rationality under MSE.
- Finally, under non symmetric loss, the approach above leads to excess rationality rejection and bias. Under these circumstances *the efficient use of information* implies a set of moments conditions that may be readily tested, and the degree and direction of loss asymmetry may be estimated.

Our results summarize as follows:

- We found that BR-ETE, BR-EME and FE-SR median expectations as well as FE-LR reports are not accurate and their inaccuracy increases with the expectation horizon[6]. Inaccuracy reflects the information loss agents face as they are not able to anticipate the shocks arising along the forecast horizon. Inaccuracy, in turn, increases with the horizon as the information loss, i.e. number of shocks, accumulates.
- However, BR-ETE, BR-EME and FE-SR sample survey results might wrongly be perceived as precise[7]. As a matter of fact, Colombian inflation expectations look wrongly precise because disagreement (We measure through the 5-95% Inter Quantile Rank, IQR), is substantially smaller than the 90% forecast confidence interval in Banco de la República's inflation and monetary policy reports at similar hori-

[5]The Mincer and Zarnowitz (1969) regression is $\Pi^h_{i,t} = \beta_0 + \beta_1 \Pi_t + U^h_{i,t}$ where $\Pi^h_{i,t}$ is i-th agent expectation at the survey date $t - h$ for inflation at the target date t, Π_t is the observed inflation at the target date, and $U^h_{i,t}$ is a zero mean noise.

[6]Accuracy has to do with how close median expectations/reports are to the finally realized inflation.

[7]Precision has to do with the cross section uncertainty of sample survey expectations.

zons. This discrepancy arises from *expectations heterogeneity* that leads dispersion measures based on cross section sample survey expectations not to reflect *the true* uncertainty, Mankiw, Reis, and Wolfers (2004). To grasp expectations uncertainty, direct information about it should be gather from agents.

- We found that theoretical rational expectations and observed expectations coincide in the fact that they, both, look like lagged versions of the inflation observed at the survey date that smooth out with the horizon. This resemblance arises from expectation errors depending on the shocks that come up along the expectation horizon, which are unknown to the agent at the survey date. As the horizon extends, the information loss increases, leading observed expectations to behave as theoretical RE expectation in these respects.
- Furthermore, Colombian sample survey expectations are not strongly rational because of the simple fact that they display cross-section disagreement. According to Mankiw et al. (2004) disagreement is a result of heterogeneity and may be "key to macroeconomic analysis". These authors suggest, also, that disagreement arises from a staggered flow of information to agents.
- We found that disagreement in Colombian expectations is time varying and relates to inflation, inflation changes, expectations and the output gap at the survey date. Similar results for the US gave Mankiw and Reis (2002) and Mankiw et al. (2004) evidence of staggered information, which led these authors to formulate a sticky information model for the US that mimics closely expectations disagreement and the Volker disinflation.
- We found also evidence of unit roots as Colombian inflation has been subject to slow transitions between steady states, such as the one started in 1991, and sudden drops, such as the 1998-1999 fall. We also found strong evidence of co-integration between expectations and the finally observed inflation rate. Furthermore, We found that the $(0,1)$ hypothesis is not rejected, and the co-integration error has zero mean. As a result, under MSE we found evidence in favour of weak rationality.
- Furthermore, we found strong evidence for weak rationality with agents employing asymmetric forecast error losses with a higher loss when expectations are higher than the final inflation result. We found that the degree of asymmetry increases with the horizon, which may explain the fact that expectations variation dampen with the horizon.

1 Introduction and Motivation

1.1 Issues

Inflation expectations play several roles in the tool-kit of Inflation Targeting Central Banks, ITCBs, such as Banco de la República, BR. On one hand, inflation expectations determine current and future inflation outcomes. In fact, expectations are taken to be self-fulfilling prophesies as they affect the current decisions of households and firms. For instance, immediately after a sudden rise in expectations, households may anticipate consumption that would otherwise be postponed, thus driving current consumption and inflation up. Households may, furthermore, engage in wage bargaining to maintain the purchasing power they lose due to a higher future inflation, and if households are successful, this leads firms to raise their current prices to compensate for higher labour costs. As a result, forward looking decisions may potentially shift current as well as future economic outcomes.

On the other hand, it is widely agreed that central bank policy decisions and communication affect inflation expectations. As a matter of fact, when inflation expectations swerve out of target, central banks exert a great effort in communicating their policies to sway expectations back and prevent the realisation of the self-fulfilling prophesy. For instance, when inflation expectations increase above target over a three to five year horizon, ITCBs usually interpret that inflation is adrift and may consider raising the interest rates to keep inflation under control. To avoid the costs associated with interest rates hikes, ITCBs carefully cater their communications to the public about the effect of its policies to turn expectations back on track.

However, for communication to affect expectations, ITCBs must enjoy a great deal of credibility, another crucial tenet of Inflation Targeting, IT. As a result, ITCBs spare no effort in maintaining high credibility to keep expectations on track through carefully designed communication of their policies. In other words, ITCBs emphasise on *forward inflation guidance.*

Underlying the mechanisms through which ITCBs understand that (i) expectations transform into current or future outcomes and (ii) ITCBs communication steer market expectations are the assumptions that agents decisions are forward looking and that expectations are rational. As a matter of fact, the Phelps-Friedman expectations augmented Phillips curve establishes that inflation variation arise, in part, from expectations changes. Furthermore, one of the most important ITCBs tenets is that the real effect of pushing inflation up depends on the extent that the rise was anticipated. Under a strict form of

Rational Expectations, RE, in which market participants input all the relevant information into the best model available to forecast inflation, only the unexpected component of the inflation rise will have a GDP or unemployment effect. Furthermore, current ITCBs models rely on micro-economic foundations of optimising agents whose decisions are forward looking and subject to some form of imperfect competition and price stickiness. As a result, firm's forward looking behaviour play a key role in policy design and analysis in ITCBs. See Gali, Gertler, and Lopez-Salido (2001), for instance.

However, central bank communication is not the only determinant of expectations. According to Sargent (2023), "[t]he influences between expectations and outcomes flow both ways", and thus current inflation, especially when it has remained persistently low/high for an extended period of time, influences expectations markedly.

Recognising the importance of expectations, ITCBs track households, firms and experts expectations at different horizons through sample surveys, and consider indirect expectations measures as well. For instance, BR conducts two quantitative surveys; a quarterly survey of economic expectations, ETE, on a sample of firms from different economic sectors, and a monthly survey on a sample of economic analysts, EME[8]. Furthermore, Focus Economics, FE, surveys the expectations on a growing sample of local and international financial and consulting firms as well as think tanks on a monthly basis. FE reports, also, non sample long run expectations. Finally, BR has also considered using break even inflation indirect expectations measures, acknowledging they have important drawbacks[9]. As a result, BR emphasises on the analysis of price setting firms expectations as those are the ones that more likely determine the current and future path of inflation.

Inflation expectations, as a result, help the following purposes. First, they should Granger cause inflation, that is help improve inflation forecasts. Second, expectations should play a crucial role in the historical decomposition of inflation shocks, i.e. they help explain particular circumstances in economic history. And third, expectations provide ITCBs with a gauge of policy soundness, credibility and the effectiveness of its communication.

Loosely speaking, expectations are rational if two conditions are met (i) economic agents input the best information available to them into the best accessible model to make their decisions, and (ii) agents learn from past mistakes. "In its stronger forms,

[8]ETE and EME correspond to their acronym in Spanish.

[9]Break even inflation is the price of risk in government debt issued in Colombian Pesos, COP, trades, i.e. the difference between nominal and real yields. The advantage of these measurements is that traders have a natural incentive to include their best information regarding future inflation in their prices. However, filtering out the price of other risks such as liquidity and interest rate is still a challenge in Colombia.

[RE,] operates as a coordination device that permits the construction of a "representative agent" having "representative expectations"", while in its weaker form "the concept of RE essentially reduces to an assumption that agents make optimal use of whatever information they have to form their expectations", Tesfatsion (2019).

More formally, the expectations of a particular agent are said to be *weakly* rational if they met two conditions. On one hand, her expectations are based on both, her *subjective knowledge* (about the economy's true Data Generating Process, DGP), and her (public or private) *information set*. And on the other, her expectation equals the expected value of future inflation based on the *objectively true* DGP of the economy, conditional on her information set plus a zero mean idiosyncratic forecast error, see Tesfatsion (2019). Another form of weak rationality, Pesaran and Weale (2006), relates to a weighted average expectation rather than individual expectation, and assumes that the mixture (i.e. the weighed average) of the individual distributions conditional to their corresponding information sets equals the density of future inflation derived from the objectively true knowledge conditional to the public information set.

Likewise, expectations are said to be *strongly rational* if they are weakly rational and the agent's information set "contains all the the information known to the modeller at the beginning of period t", Tesfatsion (2019)[10]. This strong RE form is consistent with Muth (1961) RE, Pesaran and Weale (2006). Stronger forms of the RE hypothesis may also be considered. For instance, expectations may be *perfect foresight rational* when they are strongly rational and no shocks arise along the expectation horizon, thus agent's expectation are correct, but in a different manner than perfect foresight.

Strong form RE has two distinct implications. First, private information play no role in expectation formation. And second, all agents share the same subjective knowledge (about the economy's GDP) and information set as the modeller. The latter implies that expectations *show no cross-section disagreement* between agents, which is at odds with sample survey results data. See Mankiw et al. (2004) and Capistrán and Timmermann (2009).

As a result, by imposing an unbelievable strong requirement on agents knowledge,

[10] According to this author, "this information includes: (a) Equations, variable classification, and admissibility conditions for the model, including the actual decision rules used by any other agent (private or public) appearing in the model to generate their actions and/or expectations; (b) The true values for all deterministic exogenous variables for the model; (c) All properties of the probability distributions governing stochastic exogenous variables that are known by the modeller at the beginning of period t; (d) Realized values for all endogenous variables and stochastic exogenous variables as observed by the modeller through the beginning of period t."

the strong RE hypothesis yields models that are "mathematically more tractable than alternatives that relax one or another of the [strong RE] framework's assumptions", Carroll (2001), at the cost of the lack of cross section expectations disagreement. However, according to Tesfatsion (2019) strong RE assumptions are more acceptable when viewed as limit cases "for the expectations of boundedly rational agents with limited information who engage in learning in successive time periods".

In turn, weaker RE forms allow, purposefully, for disagreement among agents expectations. In fact, Mankiw et al. (2004) propose that the cross section disagreement in expectation surveys "may be a key to macroeconomic analysis". These authors found that inflation expectation disagreement is time varying and relates to other macro variables of interest. They also found that the Mankiw and Reis (2002) sticky-information model predicts some of the disagreement features observed in the Livingston and Michigan inflation expectations surveys for the US, and mimic the cross sectional distribution dynamics during the Volcker disinflation. See Capistrán and Timmermann (2009) also.

Furthermore, weaker RE forms entail that agents' expectations reflect the *efficient* use of the information, and thus they should display the features of theoretically optimal forecast errors, see (Hamilton, 1994, pp. 422). Under RE the simplest form of efficiency is unbiasedness, i.e. the cross section average expectation is unbiased with respect to the optimal forecast derived from the "objectively-true" DGP based conditional on all public information. A stronger form of efficiency has to do with the unpredictability of forecast errors with respect to publicly available information, particularly to past expectations and expectation errors.

However, expectations efficiency is meaningless when traditional RE hypothesis is tested on agents that optimise their expectations with respect to asymmetric forecast error losses, see Elliott, Komunjer, and Timmermann (2008). As a matter of fact, the traditional economic discussion about rationality overlooked the role the loss function play in expectations formation, thus employing the Mean Square forecast Error, MSE, default. Under MSE loss the optimal forecast is the expected future inflation consistent with the agent's subjective knowledge and conditional to her information set, which ensures that traditional RE holds. However, under an asymmetric loss the optimal predictor becomes the expected value above plus a term that depends on future inflation moments of order two and higher and the shape of the loss function itself. This optimal expectation is biased with respect to the expected future inflation under traditional RE.

As a result, traditional RE test may be biased when agents employ non symmetric forecast error losses. As a matter of fact, Elliott et al. (2008, Prop. 1.) prove for a particular asymmetric loss function that RE may be falsely rejected. Furthermore, under non symmetric loss large efficiency losses may not be detected leading to excess false non traditional RE rejection. Under asymmetric loss the procedure to test traditional RE is to detect whether agents' expectations are optimal, that is, if they minimize the expected loss. Since optimality involves the minimization of the expected loss, RE tests can be performed through the moment conditions implied by the first order conditions for the expected loss minimization combined with the efficiency regressions. See Elliott, Timmermann, and Komunjer (2005), also.

In this paper we study the behaviour of three quantitative inflation sample surveys and one long run non sample report for Colombia. The first two surveys belong to BR: BR-EME started in September 2003, and records the expectations of financial agents and think tanks about future inflation at fixed horizons of $h = 1, 12, 24$ months as well as variable horizons for December of the current and following year. BR-ETE, in turn, started in 2000Q1 and surveys the expectations about future inflation of several economic sectors, academics and consulting firms, on a quarterly basis, at fixed horizons of $h = 1, 2, 3, 4, 8$ quarters. The third set belongs to FE, and surveys each month, since September 1999, the expectations of an assorted and growing set of financial agents, consulting firms and think tanks both local and international about Short Run, FE-SR, inflation at the end of the current and next years. Finally, FE reports an individual Long Run expectations report since August 2000, from the end of the current to the end the fifth year from now.

We confirm, first, that sample survey inflation expectations show a great deal of disagreement, which is at odds with strong form RE. Disagreement, however, does not equate uncertainty, and thus further direct information from agents is required to assess the latter. Furthermore, we found that disagreement relates to the output gap, current inflation, current inflation change and expectations, suggesting that expectations are subject to information rigidities. We observe, also, that sample survey expectations results are precise, i.e. have a narrow range of response, but inaccurate with respect to the inflation observed at the target date. Inaccuracy arises because agents cannot predict the effect of shocks arising along the expectation horizon. Furthermore, since the number of shocks increase with the horizon, so does inaccuracy.

We confirm, also, that observed and theoretical RE (derived from a very general setup We put forward in this paper) share a common feature; they are both lagged versions of the inflation observed at the survey date that smooth out with the horizon. However, this does not mean that expectations are adaptive.

We found evidence that inflation and expectations have a unit root or near unit root behaviour, each, related to (i) smooth transitions between steady states such as the one started in 1991 and (ii) sudden drops such as the one in 1998-1999. As a result, weak RE is to be tested employing co-integration techniques over the Mincer and Zarnowitz (1969) regression. We found evidence in favour of weak RE under MSE as the null $(\beta_0, \beta_1) = (0, 1)$ is not rejected and the co-integration errors are zero mean. Thus, the simplest form of efficiency, expectations unbiasedness, is reached.

However, we found evidence of weak RE under non symmetric forecast error loss. This asymmetry penalizes more positive expectation errors than negative ones, and the degree of asymmetry increases with the horizon.

We conclude that it is important to assess the effect of weakening the strong RE assumption in favour of information rigidities, bounded rationality and agents choosing non symmetric losses in current DSGE models in Colombia. These alternatives may not only explain disagreement and its dynamics, but also explain specific events in Colombian inflation history.

1.2 Bibliography Review

The literature on expectations, both theoretical and empirical, is dauntingly big, and we will not attempt to review all in this paper. Instead, along the body we will refer to the international works we consider more related to our work, and in this subsection we review local references, only.

Previous works for Colombia emphasize the empirical features of inflation expectations. González, Jalil, and Romero (2011) study the forecast ability, rationality and learning of a subset of Colombian expectations employing monthly data between Sep-2004 and Dec-2009. The expectations set employed by these authors include break even inflation, BR-EME for $h = 12$ months, and BR-ETE for $h = 4$ quarters. They found that expectations show persistence, fail some rationality features, Granger-cause inflation but are not "good inflation forecasts", and expectations depend on real measures. In turn, Zárate, Katherine, and Marín (2011) transform qualitative expectations reports to quantitative ones employing different quantification methods, and study their forecasting ability and rationality. They found that expectations are rational employing four rationality criteria. Furthermore, Huertas, González, and Ruíz (2015) study the rationality, adaptiveness, and credibility of inflation expectations in Colombia. For this, these authors analyse BR-EME $h = 12$ months, BR-ETE $h = 4$ quarters and break even inflation at $h = 12, 24$ months, employing a sample starting in Sep-2003, Sep-2000 and Jan-2004, respectively, and ending in Oct-2015. These authors found evidence of unit roots in expectations and inflation, and found that expectations are both adaptive and rational at the same time. More recently, Iregui-Bohórquez et al. (2021) compiled several expectation studies in a single volume. This volume addresses several questions about expectations such as their precision, efficiency, bias, whether they are anchored, help improve other variables forecasts, employ all public information available and whether price setting agents forecast prices better than non setters. Regarding the topics we address in this paper, this in this volume it was found that expectations and inflation have unit roots, the Mincer and Zarnowitz (1969) regression is estimated through co-integration techniques employing a three-dimensional panel from which RE departures were found. It was also found a low percentage of weak efficiency and a substantial difference of strong efficiency between periods of market quiet and turmoil.

1.3 Contribution

Although this paper share some elements with Iregui-Bohórquez et al. (2021), our paper differs from these studies in several ways. First, We include the latest data available, especially the COVID-19 period and its aftermath. Second, because this format allows for greater formality in comparison to Iregui-Bohórquez et al. (2021), We distinguish between their weak and strong forms. Third, We compare the behaviour of observed expectations with the features of theoretical rational expectations derived from a very general econometric set-up that We develop in this paper. Fourth, We establish how expectations arise from a formal decision problem where the optimal decision ends up depending on the shape of the forecast error loss function. Fifth, under Mean Square Error loss, MSE, We take into account that inflation and expectations have unit roots for weak RE tests. And sixth, We explore weak rationality under the assumption that agents choose non-symmetric forecast error losses, and determine the degree and direction of loss asymmetry.

1.4 Paper Organization

Apart from this introduction, the paper is organized as follows. In section 2 we establish the notation and the conceptual framework for an agent forecasting in a formal way, the formal definition and implications of weak and strong RE, traditional RE testing, the relationship between RE testing and non symmetric losses, the source and effect of inflation and expectations near unit root behaviour on traditional RE testing, and describe the datasets under analysis. The third section contains the results as follows; We establish some stylized facts of theoretical RE based on a simple but general econometric set up We put forward int this paper and compare them to observed expectations facts, We show evidence and discuss disagreement and its implications, We show evidence in favour of inflation and expectations unit roots, and partial evidence of rationality under MSE and unit roots, as well as under general non symmetric loss. In section four We conclude.

2 Notation, Conceptual Framework, Data and Methodology

2.1 Notation

Let us denote Π_t the, unless otherwise specified, Year on Year, YoY, CPI inflation rate at a given period of time $t \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers. Let $\{\Pi_t\}_{t\in\mathbb{Z}}$ be the corresponding stochastic (inflation) process, $\{\pi_t\}_{t=1}^{T}$ be the realised sample path over a sample period $t = 1, 2, \ldots, T$ with sample size T, and π_t the realised rate of inflation at t.

The economy, we assume, consists of a (perhaps huge but) finite dimension vector process $\boldsymbol{Y}_t$, which contains all the information at any time t, within which lays the inflation rate Π_t. Furthermore, let us assume that the dynamics of this economy is governed by its *objectively true* DGP we denote as P, which is, in turn, assumed to be causal, ergodic and stationary so that laws of large numbers and central limit theorems hold. See Brockwell and Davis (1987), for instance.

Let, additionally, $\mathfrak{F}_t$ be the set of *all* the information regarding the economy up to time t, and $\boldsymbol{\varepsilon}_t$ be the one period ahead best linear predictor error of $\boldsymbol{Y}_t$ based on $\mathfrak{F}_{t-1}$. In other words, $\boldsymbol{\varepsilon}_t$ is the multivariate Wold error for $\boldsymbol{Y}_t$. A standard result establishes that $\mathfrak{F}_t = \{\boldsymbol{Y}_\tau | \tau \leq t\} = \{\boldsymbol{\varepsilon}_\tau | \tau \leq t\}$. See Brockwell and Davis (1987).

Let $f_{\Pi_t|\mathfrak{F}_{t-h}}(\pi_t)$ be the probability density of inflation at time t, conditional on all the information available at $t-h$ in the economy, $\mathfrak{F}_{t-h}$, whose conditional expectation is

$$\Pi_t^h = E_P[\Pi_t | \mathfrak{F}_{t-h}] \tag{1}$$

where the subscript P under the expected value sign indicates that the expectation is calculated employing the objectively true DGP of the economy, P.

2.2 Conceptual Framework

2.2.1 The Agent's Formal Forecast Procedure

Economic agents build their expectations by solving a statistical decision problem. At the *survey date*, $t-h$, the i-th agent ($i = 1, 2, 3, \ldots, N$), chooses an *optimal* expectation or forecast (i.e., a non-trivial function $m = m(\mathfrak{F}_{i,t-h})$ of the information set available to her at $t-h$, $\mathfrak{F}_{i,t-h}$), by minimising the expected value of her own forecast loss function, $L_i(.)$ over the set $\mathfrak{B}$ of all the non-trivial functions of the information set available to her as follows

$$\tilde{m}_i^h = \underset{m \in \mathfrak{B}}{\arg\min}\, E_{P_i}[L_i(e_m)|\mathfrak{F}_{i,t-h}]$$

where her loss $L_i(e_m) = L_i(m(\mathfrak{F}_{i,t-h}) - \Pi_t)$ depends on the forecast error $e_m = m(\mathfrak{F}_{i,t-h}) - \Pi_t$ associated to a particular decision $m \in \mathfrak{B}$, and $E_{P_i}[.]$ is the expected value calculated employing agent i's subjective knowledge, P_i. In this notation h is the *forecast horizon* and t is the expectation's *target date*. See Elliott and Timmermann (2008), for instance.

The loss function accounts for the (not necessarily financial) cost associated to choosing a particular decision. In typical RE discussions the loss function is not discussed, thus defaulting to MSE. However, non MSE or asymmetric losses may naturally arise in individual agents' and institutions' forecasts. For instance, a central bank pursuing inflation forecast targeting, Svensson (1997), faces a non-symmetric reputation loss when forecasting inflation. A small but positive forecast error, $m(\mathfrak{F}_{i,t-h}) > \Pi_t$ inflicts a smaller reputation loss (or even a gain) to the central bank than a negative one, $\Pi_t > m(\mathfrak{F}_{i,t-h})$, where a sudden higher than expected inflation is finally realised. In a similar way, asymmetric forecast error loss functions arise in individual households or firms, especially when the forecast task is delegated to a third party in a moral hazard like situation. Finally, asymmetric losses come out naturally in experiments. See Elliott et al. (2008, pp. 123), for instance.

More formally, the i-th agent's expectation, $\tilde{m}_i^h$, depends on the shape of both, the conditional density of future inflation derived from her subjective knowledge, $f_{i,\Pi_t|\mathfrak{F}_{i,t-h}}(\pi_t)$, and her loss function $L_i(.)$. As a matter of fact, Christoffersen and Diebold (1997, Proposition 2) showed that if $\Pi_t|\mathfrak{F}_{i,t-h}$ has mean $\Pi_{i,t}^h$ and a vector (of possibly time varying) conditional moments of order two and higher, $\boldsymbol{\lambda}_{t|t-h}$, and $L(e_m)$ is *any* loss function defined on the h periods ahead forecast error, e_m, then the optimal expectation is

$$\tilde{m}_i^h = \Pi_{i,t}^h + \alpha_{t|t-h} \tag{2}$$

where $\alpha_{t|t-h}$ depends only on the shape of the loss function and $\boldsymbol{\lambda}_{t|t-h}$.

More specifically, when the MSE loss (i.e. a quadratic), loss is assumed, the optimal expectation and its properties are completely known. As a matter of fact, Granger (1969) showed that if the loss $L_i(.)$ is quadratic, $L_i(e_m) = c{e_m}^2$ for $c > 0$, i.e. the widely known MSE loss, the i-th agent expectation takes the form

$$\tilde{m}_i^h = \Pi_{i,t}^h = E_{P_i}[\Pi_t|\mathfrak{F}_{i,t-h}] \tag{3}$$

that is, the *expected future inflation* based on the agent's subjective knowledge, conditional on her information set.

Furthermore, under MSE loss the i-th agent's optimal expectation, $\Pi_{i,t}$, has two key properties. On one hand, since the forecast error $e_{\Pi_{i,t}^h}$ has conditional expectation

$$E_{P_i}\left[e_{\Pi_{i,t}^h}|\mathfrak{F}_{i,t-h}\right] = E_{P_i}\left[(E_{P_i}[\Pi_t|\mathfrak{F}_{i,t-h}] - \Pi_t)|\mathfrak{F}_{i,t-h}\right] = 0 \tag{4}$$

it follows that the optimal i-th agent expectation is *unbiased.* And on the other, since

$$\text{Cov}\left[e_{\Pi_{i,t}^1}, e_{\Pi_{i,t-k}^1}|\mathfrak{F}_{i,t-h}\right] = 0 \qquad\qquad k = 1, 2, \ldots \tag{5}$$

and

$$\text{Cov}\left[e_{\Pi_{i,t}^h}, e_{\Pi_{i,t-k}^h}|\mathfrak{F}_{i,t-h}\right] = 0 \qquad\qquad k = 1, 2, \ldots, h-1 \tag{6}$$

it follows that *one period ahead expectation errors show no auto correlation, and h periods ahead expectation errors have no auto correlation up to the $h-1$ order.*

Together, Equations (4), (5) and (6) are known as the *expectations efficiency* condition[11].

As a result, unless very strong restrictions be imposed on $L(e_m)$ and $\boldsymbol{\lambda}_{t|t-h}$, we find that $E\left[(\tilde{m}_i^h - \Pi_t)|\mathfrak{F}_{i,t-h}\right] \neq 0$, that is, the optimal expectation (2) is biased with respect to $\Pi_{i,t}^h = E\left[\Pi_t|\mathfrak{F}_{i,t-h}\right]$. Therefore, under non symmetric loss, the optimal expectation does not seem efficient.

However, this inefficiency does not arise from the ineffective use of information or wrong beliefs, but from the fact that the cost of different outcomes is not quadratic. As a result new rationality tests are required when the forecast loss is not quadratic.

[11] However, Granger (1969) showed that loss symmetry around zero together with the symmetry of the conditional density of future inflation $f_{i,\Pi_t|\mathfrak{F}_{t-h}}(\pi_t)$ around its mean, is not enough to ensure the result in Equation (3). For this, either the derivative of the loss is "strictly monotonically increasing" for $-\infty < e_m < \infty$, or the conditional density is continuous and uni-modal. These conditions, however, do not exhaust all the possible situations where the result in Equation (3) arises.

In sum, when agents employ a MSE forecast loss their expectations are the expected values of future inflation based on their subjective knowledge and conditional on their information set. Otherwise, expectations are naturally biased and thus deemed inefficient by standard rationality tests.

2.2.2 Rational Expectations

According to Tesfatsion (2019) the i-th agent's expectations are regarded as *rationally weak* if

$$\Pi^h_{i,t} = E_{P_i}\left[\Pi_t|\mathfrak{F}_{i,t-h}\right] = E_P\left[\Pi_t|\mathfrak{F}_{i,t-h}\right] + \mu^h_{t,i} \tag{7}$$

where $\mu^h_{t,i}$ is the *expectation error* at $t-h$, which satisfies $E_P\left[\mu^h_{t,i}|\mathfrak{F}_{i,t-h}\right] = 0$. Furthermore, (7) implies

$$\Pi^h_{i,t} = \Pi_t + \left(E_P\left[\Pi_t|\mathfrak{F}_{i,t-h}\right] - \Pi_t + \mu^h_{t,i}\right) = \Pi_t + \epsilon^h_{t,i} \tag{8}$$

where $E_P\left[\epsilon^h_{t,i}|\mathfrak{F}_{i,t-h}\right] = 0$. From (8) it follows that weak RE are unbiased.

For this RE form definition the existence of an objectively true probability to any inflation outcome is required. This is viewed by many (Tesfatsion (2019) and Pesaran and Weale (2006), for instance) as a strong requirement as it rules out "behavioural (or strategic) uncertainty", under which "agents are uncertain about the behaviour [expectations] of other agents whose actions affect their own actions", Tesfatsion (2019). Furthermore, under behavioural uncertainty of RE heterogeneous agents, the ""infinite regress in expectations" problem ... arise as agents need to forecast the forecasts of others", Pesaran and Weale (2006).

Pesaran and Weale (2006) point out that when agents have heterogeneous information, i.e. the information held by the i-th agent, $\mathfrak{F}_{i,t-h}$ is either public, $\mathfrak{G}_{t-h}$ or completely private $\mathfrak{H}_{i,t-h}$

$$\mathfrak{F}_{i,t-h} = \mathfrak{G}_{t-h} \cup \mathfrak{H}_{i,t-h}$$

for each of the $i = 1, 2, \ldots, N$ agents, "the average expectation error across decision makers"

$$e_t = \Pi_t - \sum_{i=1}^{N} \omega_{i,t} E_P[\Pi_t|\mathfrak{F}_{i,t-h}]$$

is not orthogonal with respect to the decision maker's information sets, where $\omega_{i,t} \geq 0$ are weights such that $\sum_{i=1}^N \omega_{t,i} = 1$ and $\sum_{i=1}^N \omega^2_{t,i} = \mathcal{O}(1/N)$[12]. These authors assume, also,

[12]A function $f(n) = \mathcal{O}(g(n))$ when there exists n_0 and $c \in \mathbb{R}$ such that for all $n > n_0$, $f(n) < c \times g(n)$

that the objective DGP, P yields the future inflation density $f_{\Pi_t|\mathfrak{G}_{t-h}}(\pi_t)$ conditional on public information only.

Since the expectation mixture is a key quantity for strategic decision makers, a weaker form of RE expectations based on it might be "more desirable"[13]. For this, let us denote

$$\overline{f}_{\omega,\Pi_t|\mathfrak{F}_{t-h}}(\pi_t) = \sum_{i=1}^{N} \omega_{i,t} f_{i,\Pi_t|\mathfrak{F}_{i,t-h}}(\pi_t) \tag{9}$$

the expectations mixture density based on individual beliefs and information, where $\mathfrak{F}_{t-h} = \bigcup_{i=1}^{N} \mathfrak{F}_{i,t-h}$ is the set of all available information at $t-h$. Pesaran and Weale (2006) propose that expectations are rational on "average" when the mixture density in Equation (9) equals the objectively true future inflation density conditional on public information

$$\overline{f}_{\omega,\Pi_t|\mathfrak{F}_{t-h}}(\pi_t) = f_{\Pi_t|\mathfrak{G}_{t-h}}(\pi_t) \tag{10}$$

which implies

$$\overline{E}_{\omega}\left[\Pi_t|\mathfrak{F}_{t-h}\right] = \sum_{i=1}^{N} \omega_{i,t} E_{P_i}\left[\Pi_t|\mathfrak{F}_{i,t-h}\right] = E_P\left[\Pi_t|\mathfrak{G}_{t-h}\right] \tag{11}$$

By the same token, the i-th agent's expectations are said to be *strongly rational*, i.e. a la Muth (1961), when

$$f_{i,\Pi_t|\mathfrak{F}_{i,t-h}}(\pi_t) = f_{\Pi_t|\mathfrak{G}_{t-h}}(\pi_t) \qquad\qquad i = 1, 2, \ldots, N \tag{12}$$

where P_i is the subjective knowledge and $\mathfrak{F}_{i,t-h}$ the information set available to agent i. Under Equation (12) the subjective knowledge of agent i equals the objective DGP, $P_i = P$, and the information set available to agent i is the public information available to anyone, $\mathfrak{F}_{i,t-h} = \mathfrak{G}_{t-h}$. As a result, strong RE implies

$$E_{P_i}\left[\Pi_t|\mathfrak{F}_{i,t-h}\right] = E_P\left[\Pi_t|\mathfrak{G}_{t-h}\right] \qquad\qquad i = 1, 2, \ldots, N \tag{13}$$

which is a common form of specifying *strong RE*. Equation (13) has two important implications. On one hand, expectations show no cross section disagreement as they all equal a constant $E_P\left[\Pi_t|\mathfrak{G}_{t-h}\right]$ for $i = 1, 2, \ldots, N$ in virtue of the right hand side of Equation (13). On the other, private information play no role in expectation formation.

[13]Let $X_1, X_2, \ldots, X_n$ a set of random variables with corresponding density functions $f_1(x), f_2(x), \ldots, f_n(x)$. A mixture Z of these random variables with weights $\omega_1, \omega_2, \ldots, \omega_n$ such that $w_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$ is the random variable arising from choosing at random an index $j \in \{1, 2, \ldots, n\}$ employing $\{\omega_1, \omega_2, \ldots, \omega_n\}$ as probabilities, and then drawing from $f_i(x)$ a realisation z. This mixture has density $\sum_{i=1}^{n} \omega_i f_i(x)$

Strong RE imposes agents to have a great amount of information about both, the objectively true DGP and the information set. To exemplify how unbelievably strong this requirement is, Tesfatsion (2019) detailed the information that should be available to the agents for strong RE to hold in a small dynamic equilibrium setting.

2.2.3 Testing for Rationality

Real world sample survey expectations cross section data is at odds with strong RE because of the simple fact that there is cross section expectations disagreement. As a matter of fact, the discussion below Equation (13) led to the conclusion that strong RE are equal for all agents in the cross section. This arises because agents have an excessive amount of information. Therefore, cross section individual disagreement constitutes simple and direct evidence against strong RE, and thus, no further rationality tests are required for strong form RE.

Two different cases arise when testing weak RE. First, under MSE loss standard tests based on the Mincer and Zarnowitz (1969) regression are feasible. And second, under general loss further tests are required.

2.2.3.1 Testing for Weak Rationality Under MSE

Weak RE is suitable for statistical testing when it involves moments rather than the whole density of future inflation. As a matter of fact, statistical tests for the equality of the densities such as those in Equations (10) and (12) are hard to come by as data relating to the densities at the right hand side of these Equations is latent. In contrast, the RE moments conditions in Equations (7) and (11) are feasible to test statistically since the latent right hand side expectation might be decomposed as a function of observable data and an unobserved noise, under the null, as in Equation (8). Furthermore, the second order moment conditions in Equations (5) and (6) are directly testable.

Under MSE, weak RE is usually tested through its efficiency. The simplest form of expectations efficiency is unbiasedness, that is, Equation (7). More specifically, from (8) the Mincer and Zarnowitz (1969) regression

$$\Pi_{i,t}^{h} = \beta_0 + \beta_1 \Pi_t + U_{t,i}^{h} \tag{14}$$

might be estimated. Furthermore, under the weak RE null, individual expectations are unbiased estimates of the expected future inflation, employing the objectively true DGP of the economy and conditional on the particular agent's information set. This condition is satisfied when $(\beta_0, \beta_1) = (0, 1)$. Second moment efficiency, that is, Equations (5) and (6) are tested from the moments conditions they imply on $U^h_{t,i}$, under the null, in Equation (14).

2.2.3.2 Testing for weak Rationality Under General Loss

Testing the traditional RE hypothesis on data coming from agents that optimise their expectations with respect to asymmetric loss functions leads to meaningless results. In fact, under asymmetric loss the coefficients of the Mincer and Zarnowitz (1969) regression are biased, and, therefore, the rejection of the RE null does not necessarily indicate lack of rationality. Furthermore, under loss asymmetry large inefficiencies might not be detected, thus leading to frequent false non rejection of the null. See Elliott et al. (2005) and Elliott et al. (2008), for instance.

To test rationality and probe the shape of the loss function Elliott et al. (2005) propose to employ a "flexible parametric family" of loss functions

$$L(p, \alpha, e_t) = \left[\alpha + (1 - 2\alpha) I_{e_t}(\mathbb{R}^-)\right] |e_t|^p \tag{15}$$

where $e_t = m(\mathfrak{F}_{i,t-h}) - \Pi_t$ is the forecast error associated to a decision function $m \in \mathfrak{B}$, $p > 0$ is a positive integer exponent, $0 < \alpha < 1$ is a symmetry parameter, and $I_x(A)$ is the indicator function of $x \in A$, for x a variable and A a set, that is $I_x(A) = 1$ if $x \in A$ and $I_x(A) = 0$ otherwise. Equation (15) encompasses a number of widely known losses such as the MSE $(p = 2, \alpha = 1/2)$, absolute loss $(p = 1, \alpha = 1/2)$, and their asymmetric counterparts when $\alpha \neq 1/2$, that is the quad-quad loss $(p = 2)$, and lin-lin loss $(p = 1)$. Although this family is not exhaustive, it approximates a wide array of loss shapes of interest.

In Equation (15) $\alpha > 0.5$ implies a higher loss when expectations are bigger than the finally realized inflation, thus preventing agents to elicit high expectations. When $\alpha < 0.5$ a higher loss when expectations are lower than the finally observed inflation arises, thus leading to a dampening of the inflation troughs. Furthermore, when $p = 2$ the familiar square shape arises while $p = 1$ leads to lin-lin losses.

To employ Equation (15) it is worth noticing that rationality tests can be "viewed as moment conditions, which arise from first order conditions of the forecaster's optimisation problem", Elliott et al. (2008). As a result, rationality tests fall naturally into the GMM framework. For instance, Elliott et al. (2005) found that when $p = 2$ the test for rationality is simply an over-identification test of the form

$$J = \frac{1}{T} \left(\sum_{t=\tau}^{T+\tau-1} \boldsymbol{\nu}_{t-h} \left[\hat{\alpha} - I_{e_t}(\mathbb{R}^-) \right] |e_t| \right)^{\top} \hat{\boldsymbol{S}}^{-1} \left(\sum_{t=\tau}^{T+\tau-1} \boldsymbol{\nu}_{t-h} \left[\hat{\alpha} - I_{e_t}(\mathbb{R}^-) \right] |e_t| \right) \tag{16}$$

where $\hat{\boldsymbol{S}}$ is a consistent estimate of

$$\boldsymbol{S} = E \left[\boldsymbol{\nu}_{t-h} \boldsymbol{\nu}_{t-h}^{\top} \left[\alpha_0 - I_{e_t}(\mathbb{R}^-) \right]^2 |e_t|^2 \right]$$

and $\hat{\alpha}$ is a linear instrumental variable estimator of the true parameter value α_0

$$\hat{\alpha} = \frac{\left[\sum_{t=\tau}^{T+\tau-1} \boldsymbol{\nu}_{t-h} |e_t| \right]^{\top} \hat{\boldsymbol{S}}^{-1} \left[\sum_{t=\tau}^{T+\tau-1} \boldsymbol{\nu}_{t-h} |e_t| I_{e_t}(\mathbb{R}^-) \right]}{\left[\sum_{t=\tau}^{T+\tau-1} \boldsymbol{\nu}_{t-h} |e_t| \right]^{\top} \hat{\boldsymbol{S}}^{-1} \left[\sum_{t=\tau}^{T+\tau-1} \boldsymbol{\nu}_{t-h} |e_t| \right]} \tag{17}$$

where $\boldsymbol{\nu}_{t-h}$ is a $d \times 1$ vector of instrumental variables. Under the null the test statistic in Equation (16) is asymptotically distributed as χ^2_{d-1} and rejection occurs for large J values. An advantages of this test is that an estimate of the non-symmetry parameter, Equation (17), is provided, that is, the test is robust to α.

2.3 Near Unit Root Behaviour

Developing small open economies are subject to different forms of structural breaks in the form of foreign exchange and inflation regime changes, foreign exchange crises, spillovers from global or local crises, etc. These breaks induce near unit root behaviour in variables that would otherwise be expected to be stationary such as the inflation and interest rates.

Colombian inflation data does not escape this fact. As a matter of fact, monetary and exchange regimes have had a great impact not only on average inflation, inflation expectations and interest rates, but also have affected the volatility of other key macro variables such as the output gap and real exchange rate. Evidence on the relationship of monetary and exchange regimes with inflation structural breaks in Colombia up to 1995 may be found may be found in Julio-Román (1995). Since the seventies Colombian inflation fluctuated around a 25% mean until 1991. In the early nineties Colombia engaged in a transition towards inflation targeting that had inflation follow a steady downward trend until 1998. The adoption of the free float in 1998 completed the adoption of the IT regime and coincided with a sharp inflation drop to 9.99%, that is below one digit. Then, inflation dropped slowly but steadily below 5% between 2005 and 2006. Finally, after a reversion that pushed inflation up to 7.6% in 2008, inflation fell for the first time inside the 2-4%

band in 2009. As a result, evidence on the effect of discrete and slow transition between regimes is observed in inflation dynamics, thus fuelling a near unit root inflation dynamics. See Julio-Román (2019) also.

This near unit root behaviour add a new layer to standard RE efficiency tests. As a matter of fact, under the RE expectations null, i.e. Equations (7) and (13), a common trend between inflation and expectations arises. The source of this common trend, in this case, are the structural changes and transitions inflation has been subject to. As a result, rejecting co-integration between inflation and expectations implies the rejection of the RE hypothesis.

Co-integration between inflation and expectations does not, however, imply weak RE. For this, testing the null of $(0, 1)$ coefficients in the Mincer and Zarnowitz (1969) regression, with co-integration compatible methods, is still required.

2.4 Data

In this paper we study four different inflation expectations data-sets belonging to BR and FE. BR conducts two different sample surveys; a quarterly survey of economic expectations, ETE, on a sample of firms from six economic sectors, and a monthly survey on a sample of economic analysts, EME. FE, in turn, conducts a monthly short run, SR, expectations sample survey on a growing sample of local and international financial and consulting firms as well as think tanks. FE reports, also, a series of long run, LR, inflation expectations for which no sample information is disclosed. Table A.1 summarises some of the sample features.

On a monthly basis, BR-EME reports sample survey expectations at both, fixed and variable horizons being the shorter horizons sampled for longer as follows:

- Month on Month, MoM, expectations at $h = 1$ month, Figure B.1, between Sep-2003 and Jan-2023.
- Year on Year, YoY, expectations at $h = 12$ months, Figure B.2, between Sep-2003 and Jan-2023.
- YoY expectations at $h = 24$ months, Figure B.3, between Jan-2015 and Jan-2023.
- YoY expectations at the End of the current Year, EoY, variable horizon, Figure B.4, between Oct-2003 and Jan-2023.
- YoY expectations at the End of the next year or at the end of 1 Year from now, Eo1Y, variable horizon, Figure B.5, between Oct-2008 and Jan-2023.

On a quarterly basis, BR-ETE reports sample survey expectations at fixed horizons, with longer samples for shorter horizons as follows:

- YoY expectations at $h = 1, 2, 3, 4$ quarters, Figures B.6 to B.9, between 2000-Q1 and 2022-Q3.
- YoY expectations at $h = 8$ quarters, Figure B.10, between 2015-Q4 and 2022-Q3 .

On a monthly basis, FE-SR reports Short Run expectations at variable horizons exclusively, as follows:

- YoY expectations at EoY variable horizon. Figure B.11, between Sep-1999 and Sep-2022.
- YoY expectations at Eo1Y, Figure B.12, between Sep-1999 and Sep-2022.

On a monthly basis, FE-LR reports Long Run expectations at variable horizons exclusively, as follows:

- Aggregate (no sample survey data) YoY expectations at EoY, Eo1Y, the End of 2 Years from now, Eo2Y, End of 3 Years from now, Eo3Y, End of 4 Years from now, Eo4Y, End of 5 Years from now, Eo5Y, Figures B.13 to B.18, between Sep-1999 and Sep-2022.

Figures B.1 to B.12 depict percentiles $5, 20, 35, 50, 65, 80$ and 95 of the distribution of survey responses for BR-ETE, BR-EME and FE-SR. These percentiles delimit the inter-percentile ranges filled with different intensities of the blue colour. These intensities show the height of the expectation distribution density. Furthermore, the solid while line in the middle is the median expectation, and the solid black line is the observed inflation at the target date. Figures B.13 to B.18, in turn, show in a blue continuous line the aggregate FE-LR expectation, and in a black solid line the observed inflation at December of the target year.

These Figures are built to ease the comparison between the expected and finally observed inflation at each x-axis date of each figure. For fixed horizons, $h = 1, 12, 24$ months and $h = 1, 2, 3, 4, 8$ quarters the x-axis depicts the target date. In this way, the y-axis lines and bands are the percentiles and inter-percentile bands of of expectations and the solid black line is the observed rate of inflation. For variable horizons, that is, at the end of December of j years ahead, for $j = 0, 1, \ldots, 5$, the x-axis date is a composite of the target year and the survey month. In this way, the y-axis lines and bands are the percentiles and inter-percentile bands of of expectations elicited j years before at the month in the axis. In turn, the black solid line is the inflation rate observed at December of the figure date. From these figures the following results arise:

Result 1. ***Median expectation accuracy reduce with the expectation horizon.*** *Expectation accuracy has to do with how close median expectations (the white solid line) is to the finally realized inflation (the black solid line). Theoretically, expectations inaccuracy arises from the fact that under MSE loss, RE errors depend exclusively on the shocks that arise along the expectation horizon, which are unknown to the agent at the survey date. Since longer horizons implies more unaccounted shocks in the expectation, an inverse relationship between accuracy and the expectation horizon arises. This result explains the strong influence of current outcomes on expectations mentioned by Sargent (2023). This fact can clearly be observed for BR-ETE results in Figures B.6 to B.10, where expectation errors increase, on average, with the horizon.*

Result 2. ***Sample survey expectations appear to be precise compared to historical central bank forecast errors.*** *Expectations precision has to do with how different individual expectations are from each other. For instance, the average 5-95% percentile rank for BR-ETE expectations at a 2 year horizon is just 1%, which compares favourably with the predictive density for inflation of historical monetary policy reports, Banco de la República (2022, Fig. 1.1. pp. 9), for instance.*

However, it is worth noticing that precision does not have to do with expectation volatility but expectation disagreement, Mankiw et al. (2004) and Capistrán and Timmermann (2009).

3 Results

3.1 Expectations Display some of the Properties of Formal Forecasts

In this subsection we compare the expectations reported to BR and FE to the behaviour of formal forecasts. For this, We compare specific stylized facts found in expectations data with the properties of expectations derived from a very general but simple econometric set up. This comparison shed some light on the expectation formation process.

3.1.1 Some Expectations Stylized Facts

In this subsection we show that theoretical RE share the following stylized facts with observed expectations regardless of their source (i.e. BR or FE), or frequency (monthly or quarterly):

- The absolute median expectation error increases with the horizon. This can be observed from Figures B.6 to B.9 where the horizon increases from 1 to 4 quarters, respectively. These Figures also show that the shorter the horizon the average absolute expectation error increases. Further evidence on this fact comes from the first column of Table A.6 which show that the median absolute forecast error increases with the horizon within each survey.
- Closely related to the previous fact is that the median expectation error variance increases with the horizon. This fact follows in a simple manner from the previous one. Further evidence may be found in the second column of Table A.6 which show that the 5-95% inter quantile error grows with the horizon within each survey.
- Median expectations smooth out with the horizon. This fact can also be directly observed in Figures B.6 to B.9. This is also a result of the fact that the variances of expectation errors and expectations themselves balance out. Additional evidence regarding this fact comes from the fourth column in Table A.6, which shows that the average inter quantile range reduce with the horizon within each survey.
- Covariation between median expectations and the observed inflation at the survey date decreases with the horizon. This fact is a result of the previous one as correlation falls down when of the parties smooth out. More evidence on this fact may be found in the third column in Table A.6, where correlation between median expectations and the last observed inflation are noticed to reduce with the horizon.

In other words, expectations are copies of the last observed inflation that smooth out with the horizon. Thus, absolute expectation error as well as its variance increases with the horizon. In the next subsection we show this stylized facts also arise in theoretical RE in a simple but general econometric set up.

3.1.2 A Simple Econometric Set up

The dynamics of the economy is governed by its *true* DGP, we denote P. The economy is summarized in a $m \times 1$ vector process $\boldsymbol{Y}_t$, which contains all the information of the economy at any integer period of time $t \in \mathbb{Z}$. This vector may be split into the inflation process $\{\Pi_t\}_{t\in\mathbb{Z}}$ and the remaining economic processes $\{\boldsymbol{X}_t\}_{t\in\mathbb{Z}}$, as follows

$$\boldsymbol{Y}_t = \begin{pmatrix} \Pi_t \\ \boldsymbol{X}_t \end{pmatrix}$$

Let us assume that the true DGP of the economy is causal, ergodic and stationary so that laws of large numbers as well as central limit theorems hold. See Brockwell and Davis (1987), for instance.

Under these circumstances, the economy has a multivariate Wold representation as follows

$$\boldsymbol{Y}_t = \boldsymbol{\kappa}_t + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j \boldsymbol{\varepsilon}_{t-j} \tag{18}$$

where $\boldsymbol{\Psi}_0 = \boldsymbol{I}$, $\sum_{j=0}^{\infty} |\boldsymbol{\Psi}_j| < \infty$, $\boldsymbol{\varepsilon}_t$ is the one period ahead best linear predictor error of $\boldsymbol{Y}_t$, and $\boldsymbol{\kappa}_t$ is a perfectly deterministic process in the sense that it is known without error at any point in time. In this case, $\{\boldsymbol{\varepsilon}_t\}_{t\in\mathbb{Z}}$ is a $(\boldsymbol{0}, \boldsymbol{\Sigma}_\varepsilon)$ multivariate white noise process. Furthermore, for convenience we assume $\boldsymbol{\kappa}_t = 0$, $\forall t \in \mathbb{Z}$.

The economy's Wold representation (18) implies a representation for inflation as follows

$$\Pi_t = \sum_{j=0}^{\infty} \boldsymbol{\Psi}_{j,[1,]} \boldsymbol{\varepsilon}_{t-j} \tag{19}$$

where $\boldsymbol{\Psi}_{j,[1,]}$ is the first row of matrix $\boldsymbol{\Psi}_j$.

Let us denote $f_{\Pi_t}(\pi_t)$ be the unconditional probability density of inflation at time t, whose mean and variance are denoted μ_Π and σ^2_Π, respectively. Let us also remember that $\{\pi_t\}_{t=1}^{T}$ is the realized inflation path, that is the set of inflation figures reported by the national statistical institute.

Let, additionally, $\mathfrak{F}_t = \{\boldsymbol{Y}_\tau | \tau \leq t\}$ be the set of *all* the information regarding the economy up to time t. A standard result establishes that $\mathfrak{F}_t = \{\boldsymbol{\varepsilon}_\tau | \tau \leq t\}$. See Brockwell and Davis (1987).

A crucial assumption, Angrist and Pischke (2009, Thm. 3.43.1. pp 28), is that the Conditional Expected Function, CEF, of inflation with respect to $\mathfrak{F}_{t-1}$ is an affine function of lagged inflation Π_{t-1} and a $(m-1) \times 1$ vector of other determinants $\boldsymbol{T}_t$ as follows

$$E_P[\Pi_t | \mathfrak{E}_t] = \begin{bmatrix} \Pi_{t-1} & \boldsymbol{T}_t^\top \end{bmatrix} \begin{bmatrix} \rho \\ \boldsymbol{\beta} \end{bmatrix} = \rho \Pi_{t-1} + \boldsymbol{T}_t^\top \boldsymbol{\beta} \tag{20}$$

where $\boldsymbol{T}_t$ contains lagged elements of $\boldsymbol{Y}_t$ but $\{\Pi_{t-1}, \Pi_{t-2}, \Pi_{t-3}, \dots\}$. Lagged inflation is set apart in Equation (20) to make explicit the relationship between the expected and finally realised inflation. The parameter $\rho > 0$ may be interpreted as inflation persistence, and we assume $|\rho| < 1$ for stationarity. Furthermore, the quantity in Equation (20) is the expected value of the conditional distribution $f_{\Pi_t|\mathfrak{F}_{t-1}}(\pi_t)$, where the conditional density is inherited from the true DGP of the economy.

The assumption in Equation (20) implies several widely known results. First, it leads to the CEF decomposition

$$\begin{aligned}\Pi_t &= E_P[\Pi_t|\mathfrak{F}_{t-1}] + \varepsilon_t^{\Pi} \\ &= \rho\Pi_{t-1} + \boldsymbol{T}_t^{\top}\boldsymbol{\beta} + \varepsilon_t^{\Pi} \end{aligned} \tag{21}$$

where $\varepsilon_t^{\Pi} = \Pi_t - E_P[\Pi_t|\mathfrak{F}_{t-1}]$ satisfies $E_P[\varepsilon_t^{\Pi}|\mathfrak{F}_{t-1}] = 0$ and $\varepsilon_t^{\Pi} \perp \mathfrak{F}_{t-1}$ in the sense that if $\boldsymbol{Z} \in \mathfrak{F}_{t-1}$ then $\mathrm{Cov}(\boldsymbol{Z}, \varepsilon_t^{\Pi}) = \mathbf{0}$, and $\mathrm{Cov}(\Pi_{t-1}, \varepsilon_t^{\Pi}) = 0$.

Furthermore, from Equation (21) it also follows that ε_t^{Π} is the first element of the Wold representation error $\boldsymbol{\varepsilon}_t$. Second, it is worth noticing the similarity of Equation (20) to structural economic equations such as the Phillips curve. Third, according to Angrist and Pischke (2009, Th. 3.1.2. pp. 25), Equations (20) and (21) make the CEF in Equation (20) the optimal inflation forecast. However, for this result to hold it is required to assume that the forecaster experiences a MSE forecast loss, Granger (1969). And fourth, two results related to the Law of Iterated Expectations arise. On one hand,

$$E_P[\Pi_t] = E[E[\Pi_t|\mathfrak{F}_{t-1}]] \tag{22}$$

and on the other, the Analysis of Variance

$$V_P[\Pi_t] = V[E[\Pi_t|\mathfrak{F}_{t-1}]] + E[V[\Pi_t|\mathfrak{F}_{t-1}]] \tag{23}$$

whose expected values are calculated from distributions inherited from the true DPG of the economy. See Angrist and Pischke (2009, Th. 3.1.3. pp. 26).

Furthermore, we may also find a representation for the remaining determinants in terms of the economy's Wold representation as follows

$$\boldsymbol{T}_t = \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j^{*}\boldsymbol{\varepsilon}_{t-j} \tag{24}$$

where each $\boldsymbol{\Psi}_j^{*}$ is a $(m-1)\times(m-1)$ depends on a finite set set of matrices $\boldsymbol{\Phi}_j$, $\boldsymbol{\Psi}_0^{*} = \mathbf{0}$, and since $\sum_{j=0}^{\infty}|\boldsymbol{\Phi}_j| < \infty$, $\sum_{j=0}^{\infty}|\boldsymbol{\Psi}_j^{*}| < \infty$.

Under the above circumstances, we can calculate a representation for inflation by successive substitution as follows

$$
\begin{aligned}
\Pi_t &= \rho \Pi_{t-1} + \boldsymbol{T}_t^{\top} \boldsymbol{\beta} + \varepsilon_t^{\Pi} = \\
&= \rho \left[\rho \Pi_{t-2} + \boldsymbol{T}_{t-1}^{\top} \boldsymbol{\beta} + \varepsilon_{t-1}^{\Pi} \right] + \boldsymbol{T}_t^{\top} \boldsymbol{\beta} + \varepsilon_t^{\Pi} = \\
&= \qquad \vdots \qquad = \\
&= \rho^h \Pi_{t-h} + \left(\sum_{k=0}^{h-1} \rho^k \boldsymbol{T}_{t-k} \right)^{\top} \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi} \right) = \\
&= \rho^h \Pi_{t-h} + \left(\sum_{k=0}^{h-1} \rho^k \left[\sum_{j=0}^{\infty} \boldsymbol{\Psi}_j^* \boldsymbol{\varepsilon}_{t-k-j} \right] \right)^{\top} \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi} \right) = \\
&= \rho^h \Pi_{t-h} + \left(\sum_{k=0}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k} \right)^{\top} \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi} \right) \qquad (25)
\end{aligned}
$$

after substitution of Equation (24) and factoring the unique elements ε_{t-k} with the following factors

$$
\boldsymbol{\Phi}_k = \begin{cases} \sum_{i=0}^{k} \rho^{k-i} \boldsymbol{\Psi}_i^* & \text{if } 0 \leq k \leq h-1 \\ \sum_{i=0}^{h-1} \rho^{h-1-i} \boldsymbol{\Psi}_{k-i}^* & \text{if } k > h-1 \end{cases}
$$

From the representation in Equation (25) it is easy to derive inflation's RE, that is, the h periods ahead optimal expectation under MSE loss as follows

$$
\begin{aligned}
\Pi_t^h &= E_P[\Pi_t | \mathfrak{F}_{t-h}] = \\
&= E_P \left[\rho^h \Pi_{t-h} + \left(\sum_{k=0}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k} \right)^{\top} \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi} \right) \Bigg| \mathfrak{F}_{t-h} \right] = \\
&= \rho^h \Pi_{t-h} + \left(\sum_{k=h}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k} \right)^{\top} \boldsymbol{\beta} + \cancelto{0}{E_P \left[\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi} | \mathfrak{F}_{t-h} \right]} = \\
&= \rho^h \Pi_{t-h} + \left(\sum_{k=h}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k} \right)^{\top} \boldsymbol{\beta} \qquad (26)
\end{aligned}
$$

which depends on the last observed inflation Π_{t-h} and all the economy's shocks arising since the *the survey date*, $t-h$, up to the *target date*, t, i.e. only on the shocks that hit the economy along the forecast horizon.

Now, from Equations (25) and (26) the RE error becomes

$$
\begin{aligned}
e_t^h &= E_P[\Pi_t|\mathfrak{F}_{t-h}] - \Pi_t = \\
&= \left(\sum_{k=0}^{h-1} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k}\right)^\top \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi}\right) = \\
&= \sum_{k=0}^{h-1} \left[(\boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k})^\top \boldsymbol{\beta} + \rho^k \varepsilon_{t-k}^{\Pi} \right] = \\
&= \sum_{k=0}^{h-1} \left[\tilde{\boldsymbol{\Phi}}_k \boldsymbol{\varepsilon}_{t-k} \right]^\top \tilde{\boldsymbol{\beta}} \qquad (27)
\end{aligned}
$$

since ε_t^{Π} is the first element in $\boldsymbol{\varepsilon}_t$ for all k, where

$$
\tilde{\boldsymbol{\Phi}}_j = \begin{cases} \left[\begin{array}{ccccc} \rho^k & 0 & 0 & \dots & 0 \\ \hdashline & & \boldsymbol{\Phi}_j & & \end{array} \right] & \text{if } j = 0, 1, \dots, h-1 \\ \left[\begin{array}{ccccc} 0 & 0 & 0 & \dots & 0 \\ \hdashline & & \boldsymbol{\Phi}_j & & \end{array} \right] & \text{if } j \geq h \end{cases}
$$

and

$$
\tilde{\boldsymbol{\beta}} = \left[\begin{array}{c} 1 \\ \boldsymbol{\beta} \end{array} \right]
$$

Equation (27) shows that RE errors depend exclusively on the shocks that arise *along the forecast horizon*, so the only way to improve upon RE forecasts is to anticipate these shocks. This can be achieved by employing non sample information, for instance. These facts lead to the following result.

Result 3. ***Theoretical absolute RE errors increase with the horizon.*** *This result follows from the fact that the information loss, i.e. the number of shocks in errors, increase with the horizon. This fact is also found to happen in observed forecast errors. See the first column of Table A.6.*

Furthermore, from Equation (27) we can calculate the RE error variance as follows

$$
\mathrm{Var}_P\left[e_t^h\right] = \mathrm{Var}_P\left[e_t^h|\mathfrak{F}_{t-h}\right] = \tilde{\boldsymbol{\beta}}^\top \sum_{k=0}^{h-1} \left[\tilde{\boldsymbol{\Phi}}_k \boldsymbol{\Sigma}_\varepsilon \tilde{\boldsymbol{\Phi}}_k^\top \right]^\top \tilde{\boldsymbol{\beta}} \qquad (28)
$$

which clearly *increases with the horizon* leading to the following result:

Result 4. ***The variance of the theoretical RE error increases with the horizon.*** *This fact was also noticed in observed expectations errors as mentioned above. See the second column in Table A.6.*

Additionally, Equation (27) gives a explicit value to ε_t^{Π} in Equation (21), $\Pi_t = E_P[\Pi_t|\mathfrak{F}_{t-h}] - e_t^h$, where the two terms to the right are orthogonal, i.e. uncorrelated. These results yield the following analysis of variance

$$\begin{aligned}
\operatorname{Var}\left[\Pi_t\right] &= \operatorname{Var}\left[E_P[\Pi_t|\mathfrak{F}_{t-h}]\right] + \operatorname{Var}\left[e_t^h\right] \Leftrightarrow \\
\sigma_\Pi^2 &= \operatorname{Var}\left[E_P[\Pi_t|\mathfrak{F}_{t-h}]\right] + \tilde{\boldsymbol{\beta}}^\top \sum_{k=0}^{h-1} \left[\tilde{\boldsymbol{\Phi}}_k \boldsymbol{\Sigma}_\varepsilon \tilde{\boldsymbol{\Phi}}_k^\top\right]^\top \tilde{\boldsymbol{\beta}} \Leftrightarrow \\
\operatorname{Var}\left[E_P[\Pi_t|\mathfrak{F}_{t-h}]\right] &= \sigma_\Pi^2 - \left[\tilde{\boldsymbol{\beta}}^\top \sum_{k=0}^{h-1} \tilde{\boldsymbol{\Phi}}_k^\top \boldsymbol{\Sigma}_\varepsilon \tilde{\boldsymbol{\Phi}}_k \tilde{\boldsymbol{\beta}}\right] \qquad (29)
\end{aligned}$$

which shows that *the unconditional RE variance balances out with the unconditional RE error variance.* And since the latter *increases with the expectation horizon*, the former *reduces with the horizon*, leading to the following result:

Result 5. ***The variance of the theoretical RE reduce with the horizon.*** *This a result of the fact that the variances of RE and expectation errors balance out. This fact was also noticed in observed RE as mentioned above. See the fourth column in Table A.6.*

Finally, from Equation (26) we can calculate the covariance of the RE with the inflation rate at the survey time as follows

$$\begin{aligned}
\operatorname{Cov}_P\left[\Pi_{t-h}, \Pi_t^h\right] &= \operatorname{Cov}_P\left[\Pi_{t-h}, \rho^h \Pi_{t-h} + \left(\sum_{k=0}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k}\right)^\top \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi}\right)\right] = \\
&= \rho^h \sigma_\Pi^2 + \operatorname{Cov}_P\left[\Pi_{t-h}, \left(\sum_{k=0}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k}\right)^\top \boldsymbol{\beta} + \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi}\right)\right] = \\
&= \rho^h \sigma_\Pi^2 + \operatorname{Cov}_P\left[\sum_{j=0}^{\infty} \boldsymbol{\Psi}_{j,[1:]} \boldsymbol{\varepsilon}_{t-h-j}, \left(\sum_{k=0}^{\infty} \boldsymbol{\Phi}_k \boldsymbol{\varepsilon}_{t-k}\right)^\top \boldsymbol{\beta}\right] + \\
&\quad + \cancelto{0}{\operatorname{Cov}_P\left[\sum_{j=0}^{\infty} \boldsymbol{\Psi}_{j,[1:]} \boldsymbol{\varepsilon}_{t-h-j}, \left(\sum_{k=0}^{h-1} \rho^k \varepsilon_{t-k}^{\Pi}\right)\right]} =
\end{aligned}$$

$$
\begin{aligned}
&= \rho^h \sigma_\Pi^2 + \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \boldsymbol{\Psi}_{j,[1:]} \mathrm{Cov}_P \left[\boldsymbol{\varepsilon}_{t-h-j}, \boldsymbol{\varepsilon}_{t-k}\right] \boldsymbol{\Phi}_k^\top \boldsymbol{\beta} = \\
&= \rho^h \sigma_\Pi^2 + \sum_{k=h}^{\infty} \boldsymbol{\Psi}_{k-h,[1:]} \boldsymbol{\Sigma}_\varepsilon \boldsymbol{\Phi}_k^\top \boldsymbol{\beta} \qquad (30)
\end{aligned}
$$

since

$$
\mathrm{Cov}_P \left[\boldsymbol{\varepsilon}_{t-h-j}, \boldsymbol{\varepsilon}_{t-k}\right] = \begin{cases} \boldsymbol{\Sigma}_\varepsilon & \text{if } j = k - h \\ \mathbf{0} & \text{Otherwise} \end{cases}
$$

As a result, $\left|\mathrm{Cov}_P\left[\Pi_{t-h}, \Pi_t^h\right]\right|$ *decreases towards zero as the expectation horizon h increases* because the right hand side sum to the right of Equation (30) is absolutely convergent, which leads to the following result.

Result 6. ***The covariance between theoretical RE and the inflation rate at the survey date decreases with the horizon.*** *This is consistent on observed median expectations. See the third column in Table A.6.*

3.2 Disagreement

A direct implication of Equation (12) is that expectations are the same for all respondents at every cross section and horizon, i.e. they do no show cross section disagreement. As a matter of fact, in this Equation strong RE equal the expected future inflation based on the objectively true DGP, conditional on the publicly available information set, which is constant across agents. This fact leads to the following result.

Result 7. ***Cross section disagreement in BR-EME, BR-ETE and FE-SR sample expectations surveys is a simple and direct proof against strong RE in Colombian expectations:*** *Evidence of expectations disagreement may be found in Figures B.1 to B.12 for BR-EME, BR-ETE and FE-SR, where at every cross section these figures show whole distributions of sample responses rather than singe values. It can also be found in Iregui-Bohórquez et al. (2021, Figures 12 and A9.9) who shows the cross section distributions of BR-EME and BR-ETE for selected dates. Further evidence may be found also in Figure B.19 where the cross section distribution of FE-SR EoY and Eo1Y are shown for four selected dates. More specifically, Figure B.11 shows that the* 5-95% *percentile interval of EoY expectations surveyed at Jan-2001 is* $[7.85, 11.25]\%$*, which is consistent with a sizeable* 90% *central percentile range of* 3.4% *in a period when observed inflation was* 7.65% *at the target date Dec-2001. In the same way, Figure B.12 reveals that the* 5-95% *percentile interval of Eo1Y expectations surveyed at Jan-1999 is* $[7.85, 9.25]\%$*, with a* 90%

percentile range of 1.4% *in a period when the observed inflation was* 7.65% *at the target date Dec-2001.*

We follow Mankiw et al. (2004) in the way we measure disagreement. Disagreement is usually measured through the variability of the cross section distribution of expectations survey responses. Several alternatives arise for this task such as the standard deviation, the inter quartile range, the range, etc. In this paper we follow Iregui-Bohórquez et al. (2021) and measure cross sectional disagreement through the 5-95% Inter Quantile Range, IQR.

According to Mankiw et al. (2004), however, disagreement informs not only about expectation formation, but is also a key component in macroeconomic analysis. For this, these authors report that disagreement is time varying and relates to key variables in the economy.

Result 8. ***Disagreement in Colombian expectations sample surveys results is time varying:*** *As a matter of fact, Figures B.11 and B.12 show a rich disagreement dynamics in FE-SR cross section results for EoY and Eo1Y surveys. Furthermore, Figure B.19 show important disagreement changes between the dates the cross section FE-SR distributions are depicted. Evidence about a rich disagreement dynamics may also be found in BR-EME and BR-ETE sample surveys in Iregui-Bohórquez et al. (2021, Figures 12 and A9.9) where important disagreement heterogeneity is observed among the dates reported.*

The existence of disagreement and its variation along the time is a result of agent heterogeneity, and thus it may become an important source of macro economics dynamics. Alternative explanations of disagreement have been offered in the literature. Mankiw and Reis (2002) propose that agent's beliefs update in a staggered rather than immediate manner, which led Mankiw et al. (2004) to formulate a staggered information model that mimics both, observed US inflation dynamics and expectations disagreement. In turn, Cukierman and Wachtel (1979) propose that disagreement as well as inflation volatility are driven by changes in the variance of aggregate demand shocks. In the same way, Brock and Hommes (1997) and Branch (2007) are able to induce expectations disagreement by having agents chose between a set of different models each survey date. Capistrán and Timmermann (2009), finally, propose that agents not only employ arbitrary forecast error losses, but also that these loss functions are heterogeneous. Additionally, these authors include a constant bias term that accounts for the observed US inflation expectations over prediction.

Furthermore, Mankiw et al. (2004) report that disagreement relates to several macro variables. We found a similar results as follows.

Result 9. ***Disagreement relates to inflation in Colombia:*** *As a matter of fact, Figure B.20 show a clear direct relationship between FE-SR EoY disagreement and observed inflation. It shows also a mostly direct relationship between observed inflation and FE-SR Eo1Y disagreement. Furthermore, this may also be detected in Figure B.21 that shows the scatter plot of observed inflation and FE-SR EoY and Eo1Y disagreement, respectively, along with a fitted straight line. Evidence of the relationship of BR-EME and BR-ETE disagreement with observed inflation may also be found in Iregui-Bohórquez et al. (2021, Figures 12, 15, A9.9, A9.11 and A9.13).*

Result 10. ***Disagreement also relates to inflation expectations in Colombia:*** *This result arises because expectations are lagged versions of the inflation observed at the survey date that smooth out with the horizon. Evidence of this relationship for BR-EME, BR-ETE and FE-SR may also be found in Iregui-Bohórquez et al. (2021, Figures 13 and A9.10 and A9.11).*

Result 11. ***Disagreement relates to the observed inflation change,*** $\Pi_{t-h} - \Pi_{t-h-1}$ ***at the survey date in Colombia too:*** *As a matter of fact, Figure B.22 reveals a U shaped relationship between the observed inflation change and FE-SR EoY disagreement, and a closer relationship for with FE-SR Eo1Y disagreement. Evidence of this relationship for BR-EME and BR-ETE may also be found in Iregui-Bohórquez et al. (2021, Figures 14 and A9.12).*

According to Mankiw et al. (2004) the fact that expectation disagreement increases with inflation changes suggests that expectations adjust in a staggered rather than immediate manner.

Result 12. ***Disagreement relates weakly to the output gap:*** *Evidence of this relationship for FE-SR may be found in Figure B.23 where often times output gap peaks and troughs relate inversely to FE-SR EoY and Eo1Y IQR disagreement ones. For BR-EME and BR-ETE more clear evidence of this relationship may be found in Iregui-Bohórquez et al. (2021, Figures 15 and A9.13).*

Summarizing, our analysis above show that

- Disagreement increases with the inflation rate observed at the survey date as well as with inflation expectations.

- Disagreement also increases with the change of the inflation rate observed at the survey date.
- Disagreement relates to the output gap.

Regarding these results, we follow Mankiw et al. (2004) as we do not claim any causality from these relationships. However, these results suggest that expectations follow a staggered flow of information.

3.3 Near Unit Root Behaviour

There are several reasons and strong evidence of regime shifts and slow transitions in recent inflation data in Colombia. As a matter of fact, the slow transition towards the long term inflation target that started in 1991 along with the sudden inflation drop in 1998 added to recent events such as COVID-19 and its aftermath, increase the overall persistence of the inflation process leading to a non stationary or near unit root inflation and expectations behaviour.

There are two ways to deal with the econometric issues that arise from non stationary or near unit root inflation and expectations. On one hand, a common trend between observed and expected inflation may be removed from both series. And on the other, unit root time series econometric procedures may be applied. We follow the latter approach since it is flexible enough to deal with sudden and slow transitions and avoids the possible bias of two steps procedures such as the former one. The unit root tests in Table A.2 where the results of the Kwiatkowski, Phillips, Schmidt, and Shin (1992), KPSS, and Phillips and Perron (1986), PP, unit root tests on observed inflation and expectations at different horizons may be found. These results lead to the following result.

Result 13. ***Observed inflation and expectations show evidence of a unit root:*** *As a matter of fact, the unit root tests results in Table A.2 show strong evidence in favour of a unit root in observed inflation since the KPSS stationarity null is strongly rejected, while the unit root null in PP is rejected but for a very small margin. Provided that KPSS is more powerful than PP, strong evidence in favour of an observed inflation unit root is present. Strong evidence is also found for BR-ET at horizons of* $2Q$*,* $3Q$ *and* $4Q$ *and BR-EME at horizon* $12M$ *in Table A.2 as the KPSS test strongly rejects the stationarity null and PP does not reject a unit root for BR-ET* $2Q$ *and BR-EME* $12M$*. Weak evidence in favour of a unit root may also be found FE-SR at horizons of* $h = 1, 3, \ldots, 8$ *as well as BR-ETE at* $1Q$ *horizon since the PP unit root null is not rejected. However, stationarity is not rejected for the remaining expectation series. Notwithstanding, the evidence in FE-SR,* *FE-LR and BR-ETE at* 8*Q should be viewed with scepticism as the sample sizes are small and some of them do not cover the slow transition thta started in* 1991 *and the sudden fall of* 1998*. As a result, we found evidence that suggests the presence of a unit root*[14].

3.4 Rationality

3.4.1 Testing for Weak Rationality Under MSE Loss

Under MSE loss a co-integration relationship between inflation at the target date and inflation expectations arises. As a matter of fact, under RE the fact that expectations are efficient implies that expectation errors are both zero mean and uncorrelated up to order h, i.e. have time invariant moments. Therefore, under RE Equation (8) the Mincer and Zarnowitz (1969) defines a co-integration equation and the co-integration errors become the expectation errors.

However, co-integration between observed inflation and expectations is not sufficient for weak RE. Indeed, for weak RE it is also required that the long-run intercept and slope be 0 and 1 respectively. When these coefficients differ from the RE requirement, long-term bias arises when the slope is still 1, and full blown weak RE rejection arises otherwise.

A two step Engle and Granger (1987) co-integration test gives rise to the following result:

Result 14. ***A co-integration relationship between expectations and observed inflation at the target date is present:*** *The co-integration relation is defined as the Mincer and Zarnowitz (1969), and We analyse the stationarity of the co-integration errors through the KPSS unit root and PP stationarity tests for the different surveys and horizons. For the majority of cases the KPSS results do not reject co-integration, normal face numbers in the third column of Table A.3. Co-integration is rejected only for BR-ETE* $h = 1, 2, 3, 4$ *quarters ahead and BR-EME* $h = 12$ *months ahead by th KPSS test. However, the unit root null is rejected by the PP test for BR-ETE* $h = 1, 3$*, thus suggesting that co-integration in present in these cases as well. Therefore, we conclude that there is evidence that observed inflation and expectations co-move in the long run, i.e. co-integrate.*

[14]Two errors may be committed; rejecting stationarity when the process is truly stationary, and not rejecting stationary when the process has a unit root. Because of the properties of forecasts and the impulse response function it is worse to conclude stationarity when the process has a unit root than concluding a unit root. If the process is stationary around a deterministic linear trend, differencing takes care of the trend and leads to a stiationary process. However if stationarity is concluded, a linear trend may be removed but the remaining process still has a unit root.

However, as mentioned above, the existence of co-integration is not sufficient to conclude weak RE. For this hypothesis to hold the intercept and slope of the co-integration relationship must be $(\beta_0, \beta_1) = (0, 1)$, and for unbiasedness the co-integration error should have zero mean. As a result, the test of the $(\beta_0, \beta_1) = (0, 1)$, that is the test that these are the co-integration coefficients, reduce to the unit root test on the expectations errors.

The KPSS and PP stationarity and unit root tests results on the expectation error at every horizon of the different surveys in Table A.4 as well as the zero mean expectation error tests results in Table A.5 lead to the following result:

Result 15. ***Weak RE can not be rejected in Colombian Expectations:*** *As a matter of fact, the third column of Table A.4 show no rejections of the null, giving strong support to the statement that $(\beta_0, \beta_1) = (0, 1)$ is a co-integrating vector. Furthermore, the zero mean expectation test results show that under the $(\beta_0, \beta_1) = (0, 1)$ null, the expectation errors are zero mean, that is there is no significant long-run expectation bias. Therefore, there is evidence that Colombian expectations as weakly Rational.*

3.4.2 Testing for Weak Rationality Under General Loss

Under a general loss function, We summarized in Section 2.2.3.2 the test procedure proposed by (Elliott et al., 2005) for the rationality hypothesis, i.e. the efficient use of information. This test is formulated in terms of moment conditions in Equations (16) to (17). In this case, $e_t = \pi_t^h - \pi_t$ is the observed forecast error for expectations at the h horizon, $\boldsymbol{\nu}_t$ is a vector of instrumental variables.

We follow closely (Elliott et al., 2008) in the definition of six different sets of instruments based on a constant, lagged forecast errors, lagged observed inflation at the survey date, lagged expectations, and lagged absolute forecast errors. The lagged value is subtracted from the expectations and last observed inflations to avoid instrument persistence[15].

Under the assumption that $p = 2$, the GMM test for the following moment condition is the rationality test under general loss

$$E\left[\boldsymbol{\nu}_{t-h}\left[\hat{\alpha} - I_{e_t}(\mathbb{R}^-)\right]|e_t|\right] = \mathbf{0} \tag{31}$$

and the estimated α parameter comes as a by-product. See Equation (17). As a result, the over-identification test J-test provides a sense of the validity of the moment conditions, i.e.

[15] Inst1: constant plus lagged errors; Inst2: constant plus lagged errors (two lags); Inst3: constant plus absolute lagged errors; Inst4: constant plus lagged change in actual inflation values; Inst5: constant plus change in expectations; Inst6: constant plus lagged errors, and lagged change in actual inflation values.

the rationality constraints. Tables A.7 and A.8 contain the results of the over-identification J-test and $\hat{\alpha}$, respectively. From the first Table the following result follows

Result 16. ***There is evidence in favour of the employment of asymmetric forecast error losses in Colombian inflation expectations.*** *As a matter of fact, the p-values of the over-specification test for the null of asymmetric error loss, i.e. the rationality constraints, contained in Table A.7, show strong evidence in favour of non symmetry. This evidence corresponds to the fourth set of instruments (a constant plus the lagged change in actual inflation values at the survey date) and the fifth (a constant plus the change of current expectations at the survey date). For the fourth set of instruments thirteen out of eighteen the null is not rejected, and rejections occur at short horizons mostly. For the fifth set of instruments the longest run FE-LR (60 months horizon) is not rejected. Not surprisingly, short horizon expectations do reject the validity of RE restrictions with any set of instruments; BR-ETE at* $1Q$ *horizon, BR-EME at* 1 *month horizon, FE-SR at* 1 *month horizon and FE-LR at* 60 *months horizon.*

Furthermore, from Table A.8 the following result follows

Result 17. ***There is clear evidence of Forecast Error Loss non-symmetry.*** *As a matter of fact, the estimated* $\hat{\alpha}$ *in Table A.8 show a strong trend towards high,* $\hat{\alpha} > 0.5$*, values, thus indicating a higher forecast loss when expectations are above inflation as the horizon increases. This fact is consistent with (or may help explain) the dampening of expectations as the horizon grows in Result 5. For this explanation it is also important to notice that the size and frequency of inflation peaks is higher than inflation troughs, thus inflation dampening is more noticeable for peaks.*

4 Conclusion

In this paper we studied the behaviour of three sample expectations surveys BR-ETE, BR-EME, and FE-SR, as well as a long-run non sample expectations report, FE-LR, whose analyses help us draw the following conclusions.

First, sample survey expectations show disagreement which is at odds with strong form RE. However disagreement has two broad implications. On one hand, cross-section expectations spread measures do not point to expectations uncertainty but to disagreement. And second, disagreement may be key to macroeconomic analysis in Colombia, as it relates with inflation, inflation changes, expectations and the output gap. This result, according to (Mankiw & Reis, 2002) and (Mankiw et al., 2004), indicates the presence of information rigidities.

Second, observed and theoretical RE share the stylized fact that they both *look like* lagged copies of the inflation rate at the survey date that smooth out with the forecast horizon. This fact arises from agents not being able to anticipate the shocks arising along the forecasting horizon. These information loss increases with the horizon as the number of unforeseen shocks accumulate. In this respect, according to (Tesfatsion, 2019), weak RE reduces to the assumption that agents make "optimal use of whatever information they have" at the survey date, while strong RE have agents sharing a greater amount of information. Had agents been able to foresee the shocks along the forecast horizon, they may behave as perfect foresight-ers or perfect foresight RE, the former being able to perfectly foresee the shocks along the forecast horizon, and the latter not being subject to any of such shocks at all.

Third, We stablish the presence of unit roots or near unit root behaviour in inflation and expectations. This is the result of inflation being subject to slow transition shifts between steady states and sudden drops such as the ones that occured during the 1990's inflation fall. As a result, testing for weak RE implies that the (Mincer & Zarnowitz, 1969) regression is a co-integration relationship with co-integration coefficients $(0, 1)$ for long-run expectations unbiasedness, and co-integration error equal to the expectation error, which has to have mean zero. We found that Colombian expectations follow this profile and thus might be weakly rational.

Fourth, We found that Colombian expectations are also weakly rational but arising from agents that employ non-symmetric forecast error losses. These losses impose a heavier penalty to positive deviations (i.e. expectation higher than the finally observed inflation) than negative ones. Furthermore, We found that this asymmetry increases with the horizon. This result arises from two facts. On one hand, inflation shows strong asymmetry as peaks are taller may be longer lasting than troughs. And second, the dampening expectations display as the horizon increases is the result of agents applying a stronger penalties to positive deviations during peaks as the horizon extends. As a result, expectations dampen along the horizon, but dampening is more notorious for peaks than troughs.

Our research suggests that the effect of weakening RE through information rigidities and bounded rationality in the current policy models in Colombia should be assessed, especially when explaining the height and duration of recent peaks and troughs.

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Appendices

A Tables

Table A.1: Sample Features

Survey	**Monthly Survey of Economic Analysts EME**	**Quarterly Survey of Economic Expectations, ETE**	**LatinFocus Consensus Forecast**
Population	Financial institutions and think tanks	Firms from six economic sectors in Bogotá, Medellín, Cali and Barranquilla	Financial and consulting firms as well as think tanks
Organization	Banco de la República	Banco de la República	Focus Economics
Average percentage response	On average 28% of respondents each date for horizons of 12 and 24 months, and 27% of dates for each individual. For a 1 month horizon on average 32% individuals every date and 32% dates per individual	On average 39% of respondents per date, and 38% dates per individual	Between 15% y 20% individuals per date
Sample	Sep-2003 to Jan-2023 for $h = 1, 12$ months. Oct-2003 to Jan-2023 for EoY. Oct-2008 to Jan-2023 for Eo2Y. Jan-2015 to Jan-2023 for $h = 24$ months. Jun-2022 to Jan-2023 for $h = 60$ months	Q1-2000 to Q3-2022 $h = 1, 2, 3, 4$ quarters. Q4-2015 to Q3-2022 $h = 8$ quarters. Q2-2022 to Q3-2022 $h = 20$ quarters	Sep-1999 to Sep-2022 for EoY Eo2Y. Nov-2000 to Sep-2022 for EoY, Eo2Y, ... , Eo5Y.
Frequency	Monthly	Quarterly	Monthly
Horizons	1 month (monthly inflation),12 and 24 months (yearly inflation)	1,2,3 and 4 quarters ahead (quarterly inflation)	end of year and end of next year

Source: Authors' calculations

- EoY = End of current year
- Eo2Y = End of the following year
- Eo3Y = End of two years from now
- Eo4Y = End of three years from now
- Eo5Y = End of four years from now

Table A.2: KPSS Unit Root and Phillips-Perron Stationarity Tests Statistics and 5% Critical Values for Observed Inflation and Inflation Expectations

Survey	h	KPSS Stat	Critical Value KPSS	PP Stat	Critical Value PP
FE-SR	1	0.218	0.463	-2.456	-3.052
	2	0.379	0.463	**-3.217**	-3.011
	3	0.386	0.463	-2.836	-3.011
	4	0.303	0.463	-2.460	-3.003
	5	0.290	0.463	-2.636	-3.011
	6	0.293	0.463	-2.762	-3.011
	7	0.299	0.463	-2.740	-3.011
	8	0.303	0.463	-2.692	-3.011
	9	0.323	0.463	**-3.200**	-3.011
	10	0.354	0.463	**-4.180**	-3.003
	11	0.370	0.463	**-5.905**	-3.003
	12	0.370	0.463	**-6.245**	-3.011
	13	0.383	0.463	**-8.116**	-3.003
	14	0.385	0.463	**-7.431**	-3.011
	15	0.383	0.463	**-8.164**	-3.011
	16	0.384	0.463	**-8.060**	-3.011
	17	0.379	0.463	**-8.583**	-3.019
	18	0.381	0.463	**-8.545**	-3.019
	19	0.382	0.463	**-8.755**	-3.019
	20	0.381	0.463	**-9.213**	-3.019
	21	0.377	0.463	**-10.09**	-3.019
	22	0.384	0.463	**-10.418**	-3.011
	23	0.384	0.463	**-10.893**	-3.011
	24	0.356	0.463	**-6.890**	-3.100
FE-LR	1	0.369	0.463	**-3.017**	-3.003
	12	0.369	0.463	**-6.850**	-3.011
	24	0.374	0.463	**-7.808**	-3.019
	36	0.366	0.463	**-6.347**	-3.029
	48	0.362	0.463	**-8.420**	-3.052
BR-ETE	1Q	0.414	0.463	-1.918	-2.893
	2Q	**0.524**	0.463	-2.460	-2.893
	3Q	**0.603**	0.463	**-3.344**	-2.894
	4Q	**0.652**	0.463	**-3.841**	-2.894
	8Q	0.379	0.463	-0.661	-3.019
BR-EME	1M	0.140	0.463	**-4.913**	-2.874
	12M	**0.983**	0.463	-2.810	-2.874
	24M	0.264	0.463	**-2.934**	-2.901
Inflation		**1.429**	0.463	**-3.059**	-3.052

Source: Authors' calculations

- Boldface shows rejection of the null at 5%
- KPSS null is Stationarity
- Phillips-Perron null is Non Stationarity

Table A.3: KPSS Unit Root and Phillips-Perron Stationarity Tests Statistics and 5% Critical Values for Co-integration Error between Expectations and Observed Inflation at Target Date

Survey	h	KPSS Stat	Critical Value KPSS	PP Stat	Critical Value PP
FE-SR	1	0.334	0.463	**-4.886**	-3.052
	2	0.302	0.463	**-5.427**	-3.011
	3	0.352	0.463	**-5.848**	-3.011
	4	0.442	0.463	**-3.234**	-3.003
	5	0.427	0.463	**-3.357**	-3.011
	6	0.421	0.463	**-3.714**	-3.011
	7	0.409	0.463	**-3.504**	-3.011
	8	0.399	0.463	**-3.140**	-3.011
	9	0.406	0.463	-2.670	-3.011
	10	0.435	0.463	-2.462	-3.003
	11	0.434	0.463	-2.162	-3.003
	12	0.412	0.463	**-3.354**	-3.011
	13	0.430	0.460	-1.700	-3.003
	14	0.405	0.463	-2.785	-3.011
	15	0.404	0.463	-2.673	-3.011
	16	0.405	0.463	-2.673	-3.011
	17	0.395	0.463	**-3.241**	-3.019
	18	0.398	0.463	**-3.208**	-3.019
	19	0.399	0.463	**-3.109**	-3.019
	20	0.397	0.463	-2.975	-3.019
	21	0.392	0.463	-2.879	-3.019
	22	0.411	0.463	-2.798	-3.011
	23	0.412	0.463	**-3.081**	-3.011
	24	0.337	0.463	**-7.244**	-3.100
FE-LR	1	0.402	0.463	**-4.963**	-3.003
	12	0.412	0.463	**-3.273**	-3.011
	24	0.393	0.463	**-4.447**	-3.019
	36	0.380	0.463	**-4.594**	-3.029
	48	0.369	0.463	**-8.458**	-3.052
BR-ETE	1Q	**0.608**	0.463	**-5.021**	-2.893
	2Q	**0.662**	0.463	-2.876	-2.893
	3Q	**0.691**	0.463	**-3.085**	-2.894
	4Q	**0.706**	0.463	-2.727	-2.894
	8Q	0.281	0.463	-1.354	-3.019
BR-EME	1M	0.228	0.463	**-7.883**	-2.874
	12M	**0.992**	0.463	-2.789	-2.874
	24M	0.150	0.463	**-3.251**	-2.901

Source: Authors' calculations

- Boldface shows rejection of the null at 5%
- KPSS null is Stationarity
- Phillips-Perron null is Non Stationarity

Table A.4: KPSS Unit Root and Phillips-Perron Stationarity Tests Statistics and 5% Critical Values for the Expectation Errors, i.e. the Test for the Null that $(\beta_0, \beta_1) = (0, 1)$ is the Co-integrating Vector

Survey	h	KPSS Stat	Critical Value KPSS	PP Stat	Critical Value PP
FE-SR	1	0.317	0.463	**-5.303**	-3.052
	2	0.302	0.463	**-5.427**	-3.011
	3	0.349	0.463	**-5.957**	-3.011
	4	0.462	0.463	**-3.630**	-3.003
	5	0.441	0.463	**-3.684**	-3.011
	6	0.450	0.463	**-3.976**	-3.011
	7	0.449	0.463	**-4.182**	-3.011
	8	0.433	0.463	**-4.362**	-3.011
	9	0.428	0.463	**-3.926**	-3.011
	10	0.418	0.463	-1.777	-3.003
	11	0.420	0.463	-1.444	-3.003
	12	0.404	0.463	-1.485	-3.011
	13	0.428	0.463	-1.337	-3.003
	14	0.398	0.463	**-3.699**	-3.011
	15	0.399	0.463	**-3.708**	-3.011
	16	0.398	0.463	**-3.638**	-3.011
	17	0.393	0.463	**-3.995**	-3.019
	18	0.399	0.468	**-3.807**	-3.019
	19	0.397	0.463	**-3.625**	-3.019
	20	0.391	0.463	**-3.493**	-3.019
	21	0.389	0.463	**-3.385**	-3.019
	22	0.425	0.463	-1.009	-3.011
	23	0.429	0.463	-1.041	-3.011
	24	0.417	0.463	0.259	-3.100
FE-LR	1	0.292	0.463	**-4.514**	-3.003
	12	0.372	0.463	-0.210	-3.011
	24	0.396	0.463	-0.373	-3.019
	36	0.390	0.463	-0.250	-3.029
	48	0.363	0.463	-0.189	-3.052
BR-ETE	1Q	0.357	0.463	**-4.745**	-2.893
	2Q	0.368	0.463	-2.153	-2.893
	3Q	0.368	0.463	-1.576	-2.894
	4Q	0.372	0.463	-1.022	-2.894
	8Q	0.369	0.463	2.487	-3.019
BR-EME	1M	0.236	0.463	**-9.555**	-2.874
	12M	0.128	0.463	-0.877	-2.874
	24M	0.357	0.463	1.129	-2.901

Source: Authors' calculations

- Boldface shows rejection of the null at 5%
- KPSS null is Stationarity
- Phillips-Perron null is Non Stationarity

Table A.5: Unbiasedness Co-integration Errors Test Results for Weak Rationality

Survey	h	P-Value
FE-SR	1	0.766
	2	0.702
	3	0.502
	4	0.580
	5	0.678
	6	0.726
	7	0.774
	8	0.726
	9	0.799
	10	0.940
	11	0.869
	12	0.702
	13	0.822
	14	0.848
	15	0.750
	16	0.848
	17	0.945
	18	0.945
	19	1.00
	20	0.945
	21	0.864
	22	0.949
	23	0.799
	24	0.561
FE-LR	1	0.893
	12	0.774
	24	0.733
	36	0.674
	48	0.579
BR-ETE	1Q	0.413
	2Q	0.432
	3Q	0.534
	4Q	0.499
	8Q	1.00
BR-EME	1M	0.285
	12M	0.107
	24M	**0.0008**

Source: Authors' calculations

Table A.6: Stylized Facts Expectations and Expectations Errors

Survey	h	Median Absolute Forecast Error	Forecast Errors IQR	$\text{Corr}(\Pi_t, \Pi_t^h)$	Median Expectations IQR
BR-ETE	1Q	0.503	0.702	0.991	2.850
	2Q	0.777	0.649	0.978	2.670
	3Q	1.012	0.730	0.961	2.640
	4Q	1.145	0.797	0.940	2.545
	8Q	1.532	0.869	0.970	0.727
BR-EME	1M	0.185	0.104	0.847	0.385
	12M	0.968	0.436	0.855	1.205
	24M	0.805	0.450	0.910	0.400
FE-SR	1	0.466	0.299	0.972	2.955
	6	0.556	0.400	0.945	2.771
	12	0.814	0.387	0.720	1.703
	18	1.208	0.424	0.583	1.725
	24	1.262	0.420	0.487	0.412
FE-LR	1	0.464		0.970	2.761
	12	0.768		0.724	1.742
	24	1.162		0.364	1.397
	36	1.092		0.251	1.466
	48	1.318		0.220	0.900
	60	1.062		0.078	0.300

Source: Authors' calculations

- All columns but Corr correspond to the median of the cross section measures in the column title
- The third column is the correlation along the time of the median expectation and the finally realized inflation
- The IQR is the 5-95% inter quantile range

Table A.7: P-Value for the GMM Over-identification Test for Asymmetric Loss

Survey	h	Inst 1	Inst 2	Inst 3	Inst 4	Inst 5	Inst 6
BR-ETE	1Q	$1.6E{-6}$	$1.7E{-5}$	$6.1E{-7}$	$2.5E{-4}$	$1.7E{-6}$	0.001
	2Q	0.015	**0.061**	$6.8E{-5}$	**0.122**	0.019	**0.068**
	3Q	0.012	0.023	**0.056**	**0.394**	**0.355**	**0.058**
	4Q	0.039	**0.059**	$2.3E{-4}$	**0.359**	**0.331**	**0.132**
	8Q	0.006	0.002	0.037	**0.130**	**0.116**	$7.1E{-4}$
BR-EME	1M	$2.0E{-17}$	$6.8E{-16}$	$1.5E{-14}$	$6.7E{-13}$	$9.5E{-14}$	$6.3E{-16}$
	12M	0.049	**0.102**	0.011	**0.613**	**0.596**	**0.057**
	24M	**0.497**	**0.633**	**0.143**	**0.653**	**0.443**	**0.616**
FE-SR	1	$1.2E{-4}$	$6.1E{-8}$	$2.3E{-5}$	$8.9E{-8}$	$9.0E9$	$1.7E{-7}$
	6	$2.7E{-4}$	0.016	0.003	**0.090**	0.012	0.011
	12	0.001	0.0005	0.005	**0.149**	**0.069**	0.018
	18	$2.5E{-4}$	0.004	$8.5E{-9}$	**0.124**	0.009	0.026
	24	$3.9E{-15}$	$3.4E{-8}$	0.001	**0.085**	$9.8E{-4}$	**0.427**
FE-LR	1	$1.0E{-6}$	$1.0E{-6}$	$8.3E{-6}$	0.002	$4.7E{-7}$	0.002
	12	$2.7E{-4}$	$2.3E{-4}$	0.003	**0.152**	**0.056**	0.010
	24	0.001	0.003	0.006	**0.100**	0.020	0.040
	36	0.008	0.007	0.012	$9.2E{-4}$	0.031	$5.0E{-6}$
	48	0.044	0.076	0.038	**0.094**	**0.306**	**0.155**
	60	$7.3E{-4}$	**0.054**	$5.6E{-5}$	0.0007	**0.399**	$1.8E{-5}$

Source: Authors' calculations

- - Inst1: constant plus lagged errors
- - Inst2: constant plus lagged errors (two lags)
- - Inst3: constant plus absolute lagged errors
- - Inst4: constant plus lagged change in actual inflation values
- - Inst5: constant plus change in expectations
- - Inst6: constant plus lagged errors, and lagged change in actual inflation values

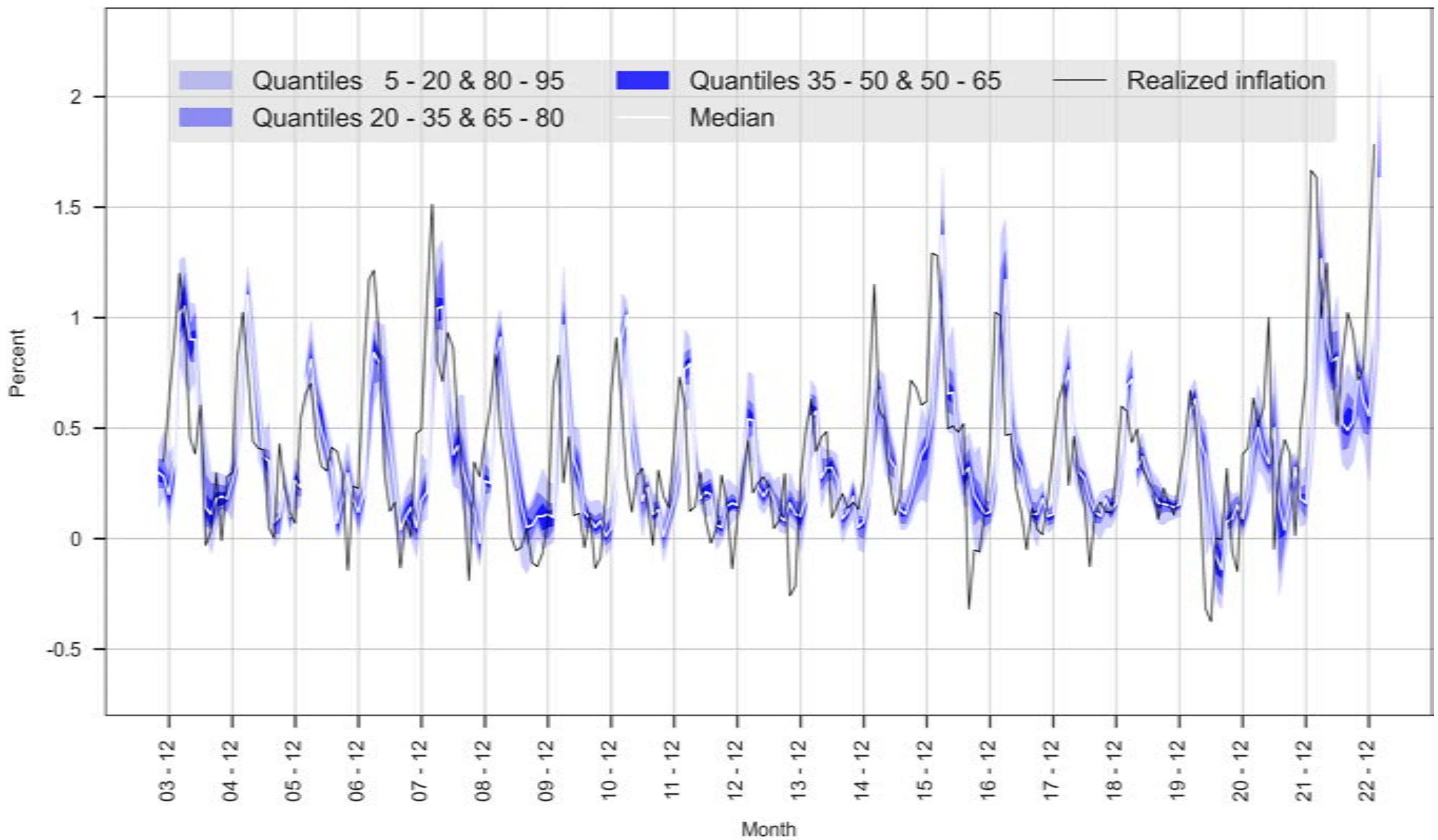
Table A.8: Estimates of the Asymmetry Parameter α in General Forecast Loss Function

Survey	h	Inst 1	Inst 2	Inst 3	Inst 4	Inst 5	Inst 6
BR-ETE	1Q	0.707	0.810	0.464	0.952	0.693	0.965
	2Q	0.801	0.781	0.416	0.868	0.871	0.882
	3Q	0.851	0.857	0.182	0.808	0.721	0.919
	4Q	0.879	0.874	0.654	0.837	0.696	0.927
	8Q	1.000	1.000	1.000	0.994	1.000	1.000
BR-EME	1M	0.519	0.488	0.537	0.431	0.456	0.492
	12M	1.056	1.040	1.139	0.915	0.911	1.028
	24M	1.000	1.000	1.000	1.000	1.000	1.000
FE-SR	1	0.233	0.505	0.358	0.441	0.419	0.437
	6	0.892	0.912	0.895	0.878	0.882	0.929
	12	0.942	0.929	0.954	0.901	0.852	0.946
	18	0.905	0.936	0.871	0.893	0.885	0.942
	24	1.040	1.027	0.970	0.960	0.995	0.967
FE-LR	1	0.569	0.573	0.547	0.931	0.779	0.846
	12	0.927	0.926	0.950	0.900	0.855	0.937
	24	0.945	0.943	0.966	0.937	0.905	0.960
	36	0.935	0.969	0.957	1.000	0.930	1.000
	48	0.874	0.922	0.953	0.965	0.874	0.964
	60	0.840	0.996	1.000	1.000	0.972	1.000

Source: Authors' calculations

B Figures

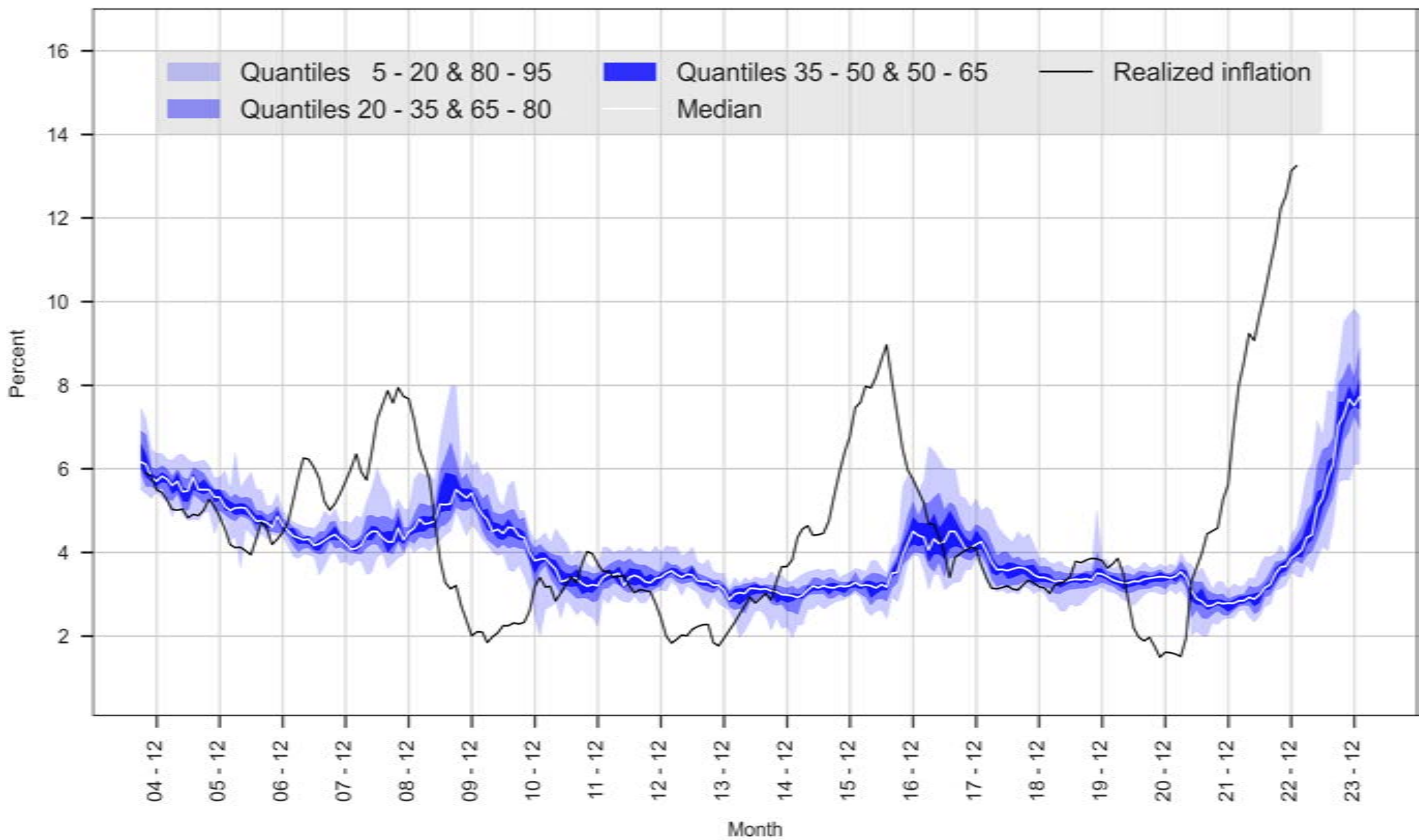
Figure B.1: BR's EME Cross Section Distributions (One Month Ahead MoM Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation target date

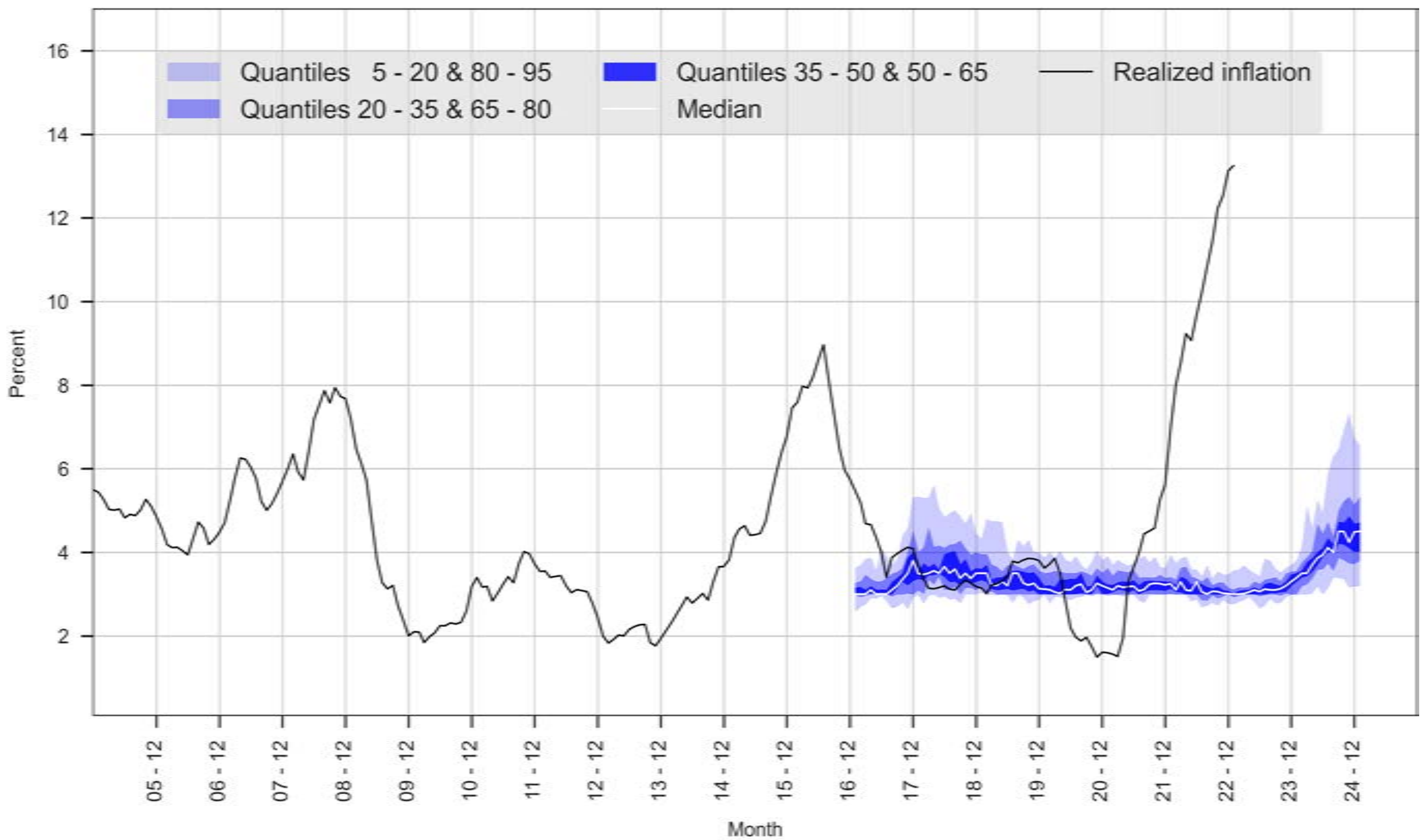
Figure B.2: BR's EME Cross Section Distributions (Twelve Months Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation target date

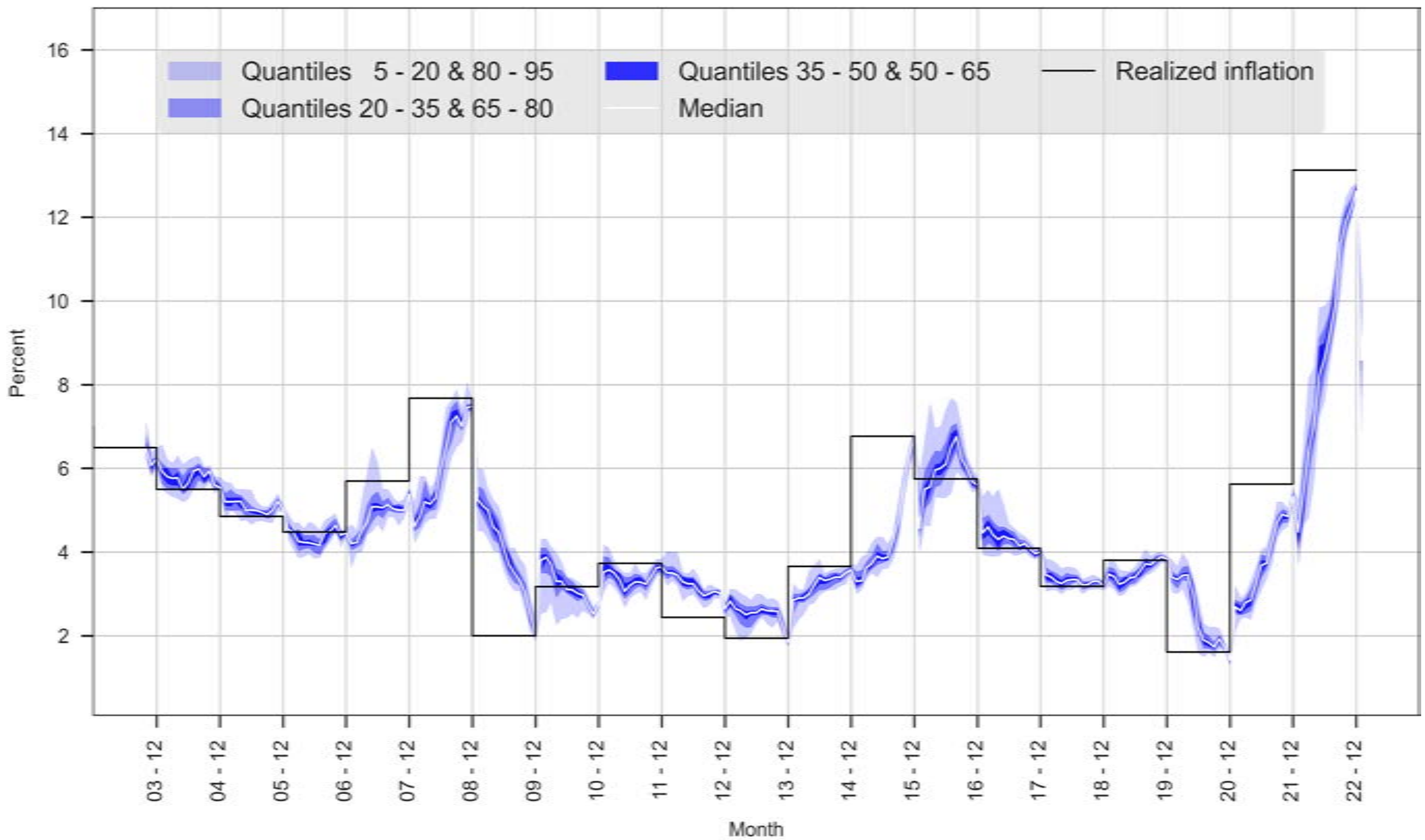
Figure B.3: BR's EME Cross Section Distributions (Twenty Four Months Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation target date

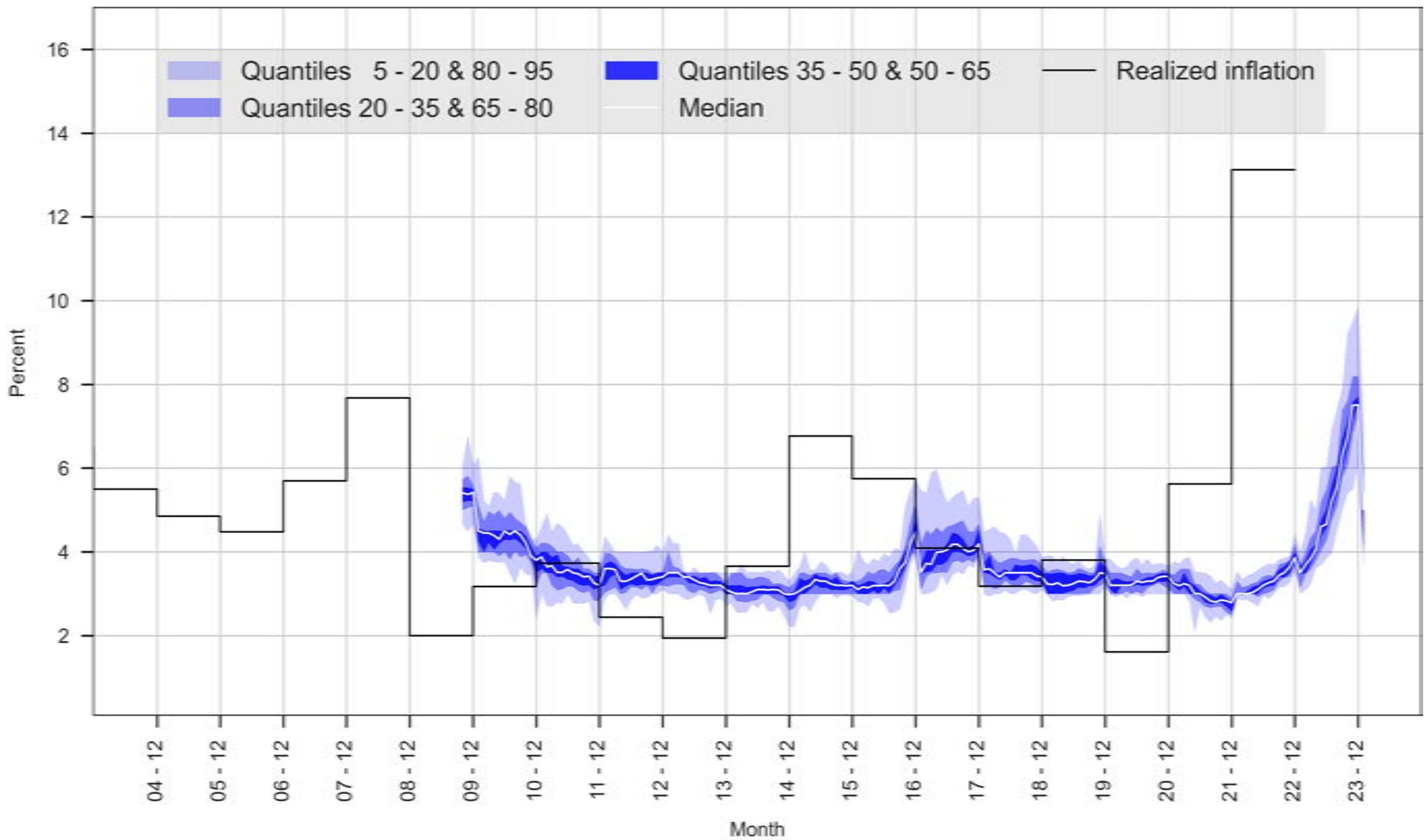
Figure B.4: BR's EME Cross Section Distributions (End of Current Year YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format is the survey date
- Target date is Dec-YY

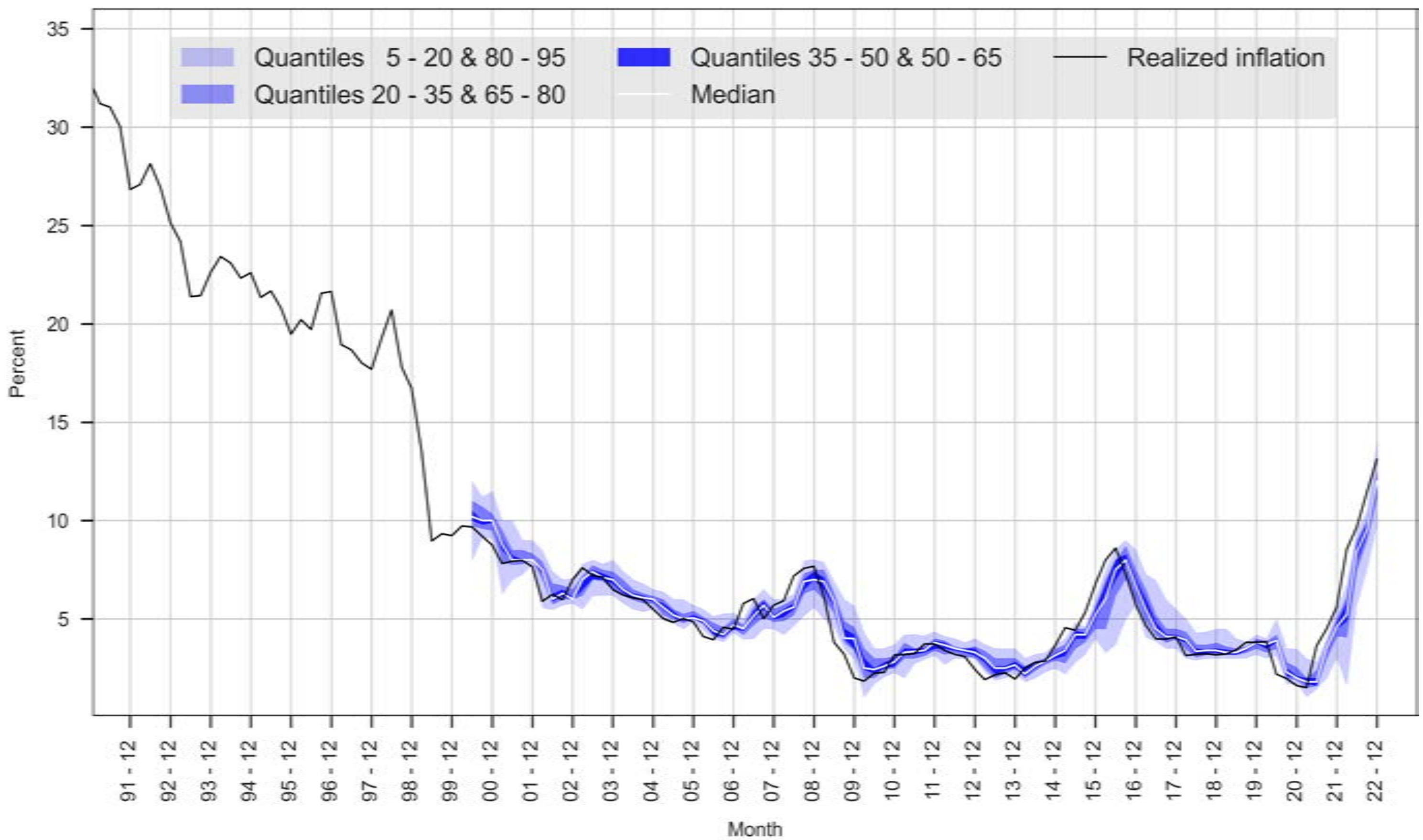
Figure B.5: BR's EME Cross Section Distributions (End of Next Year YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format
- Survey date is month MM of previous year
- Target date is Dec-YY

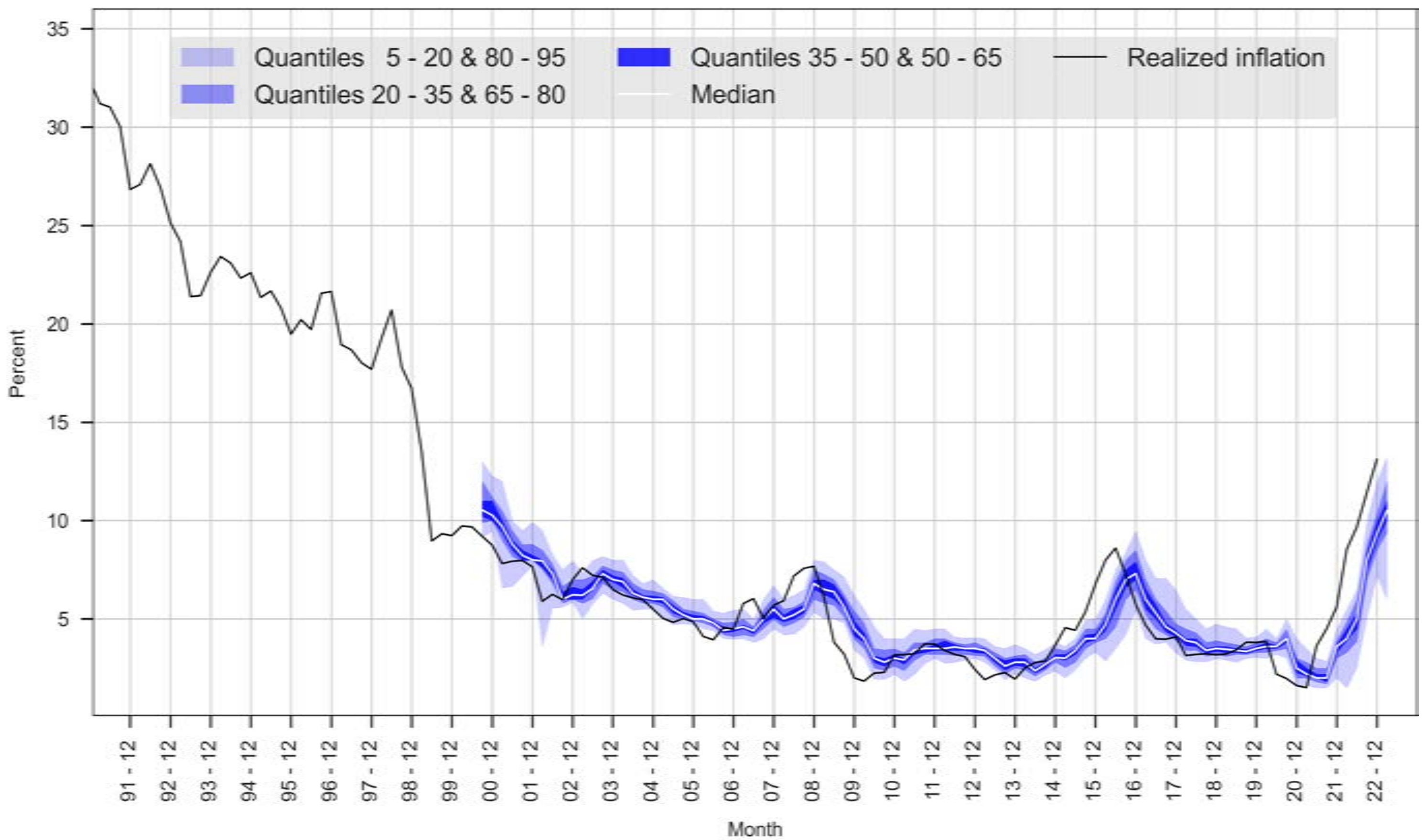
Figure B.6: BR's ETE Cross Section Distributions (One Quarter Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation's target date

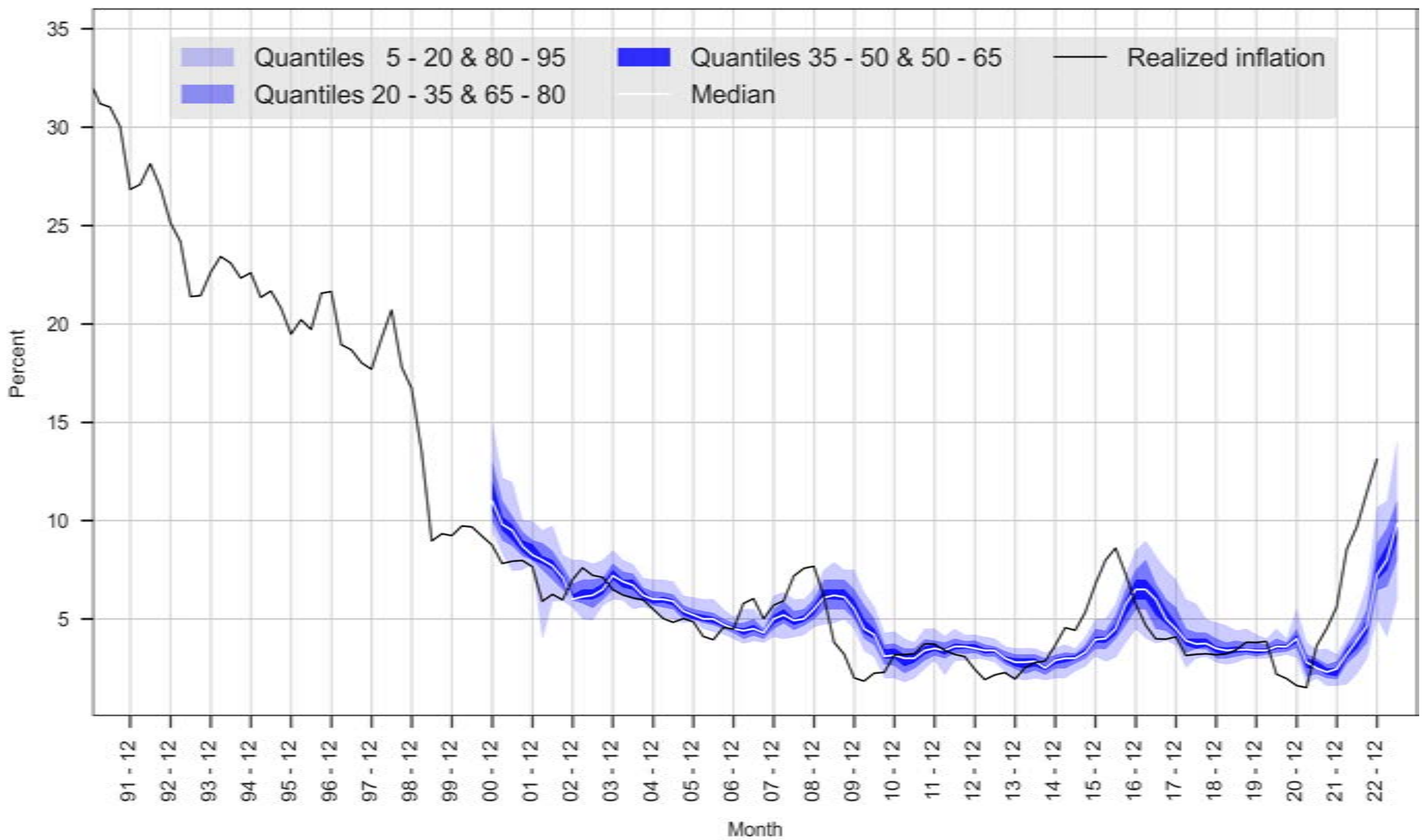
Figure B.7: BR's ETE Cross Section Distributions (Two Quarters Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation's target date

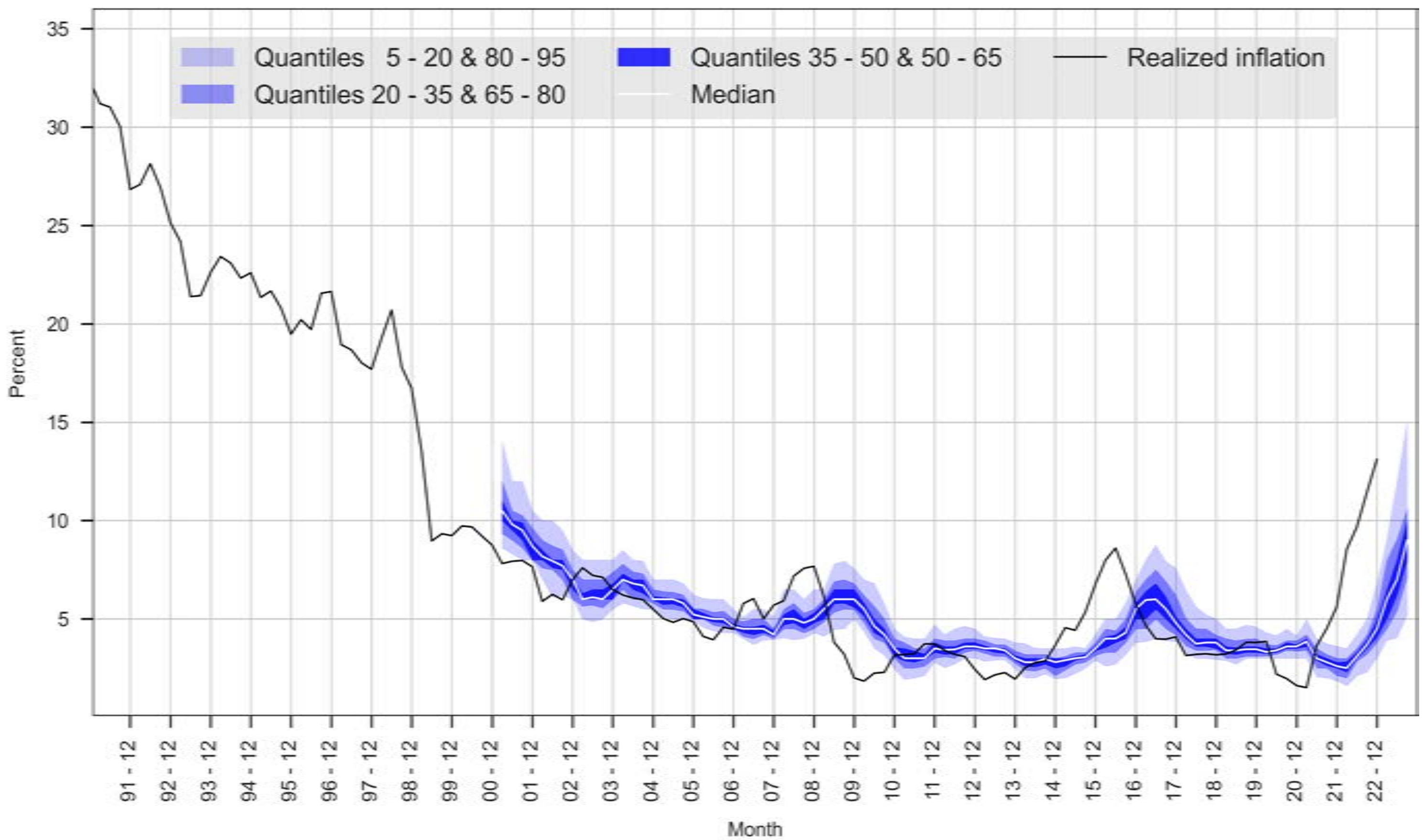
Figure B.8: BR's ETE Cross Section Distributions (Three Quarters Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to the expectation's target date

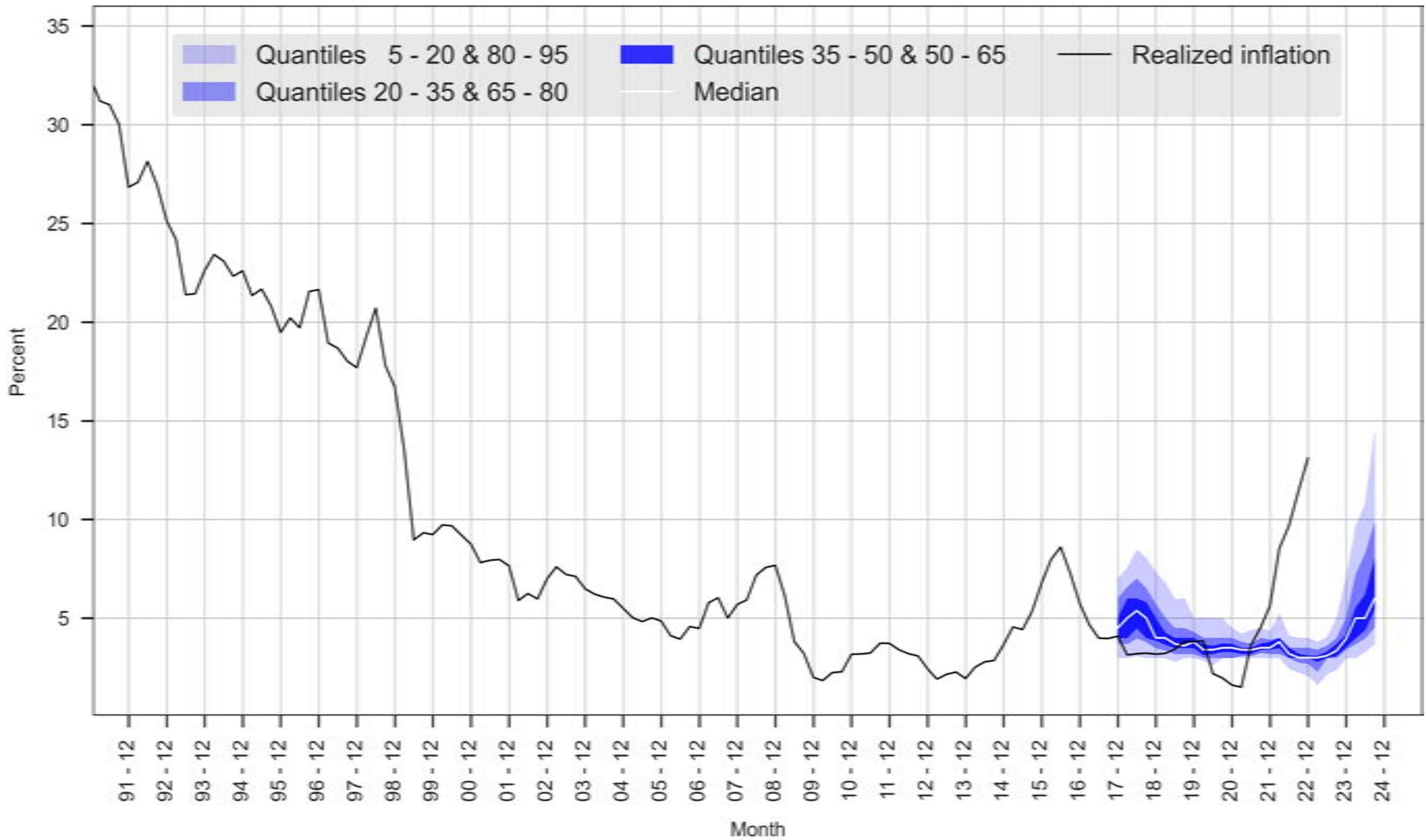
Figure B.9: BR's ETE Cross Section Distributions (Four Quarters Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to the expectation's target date

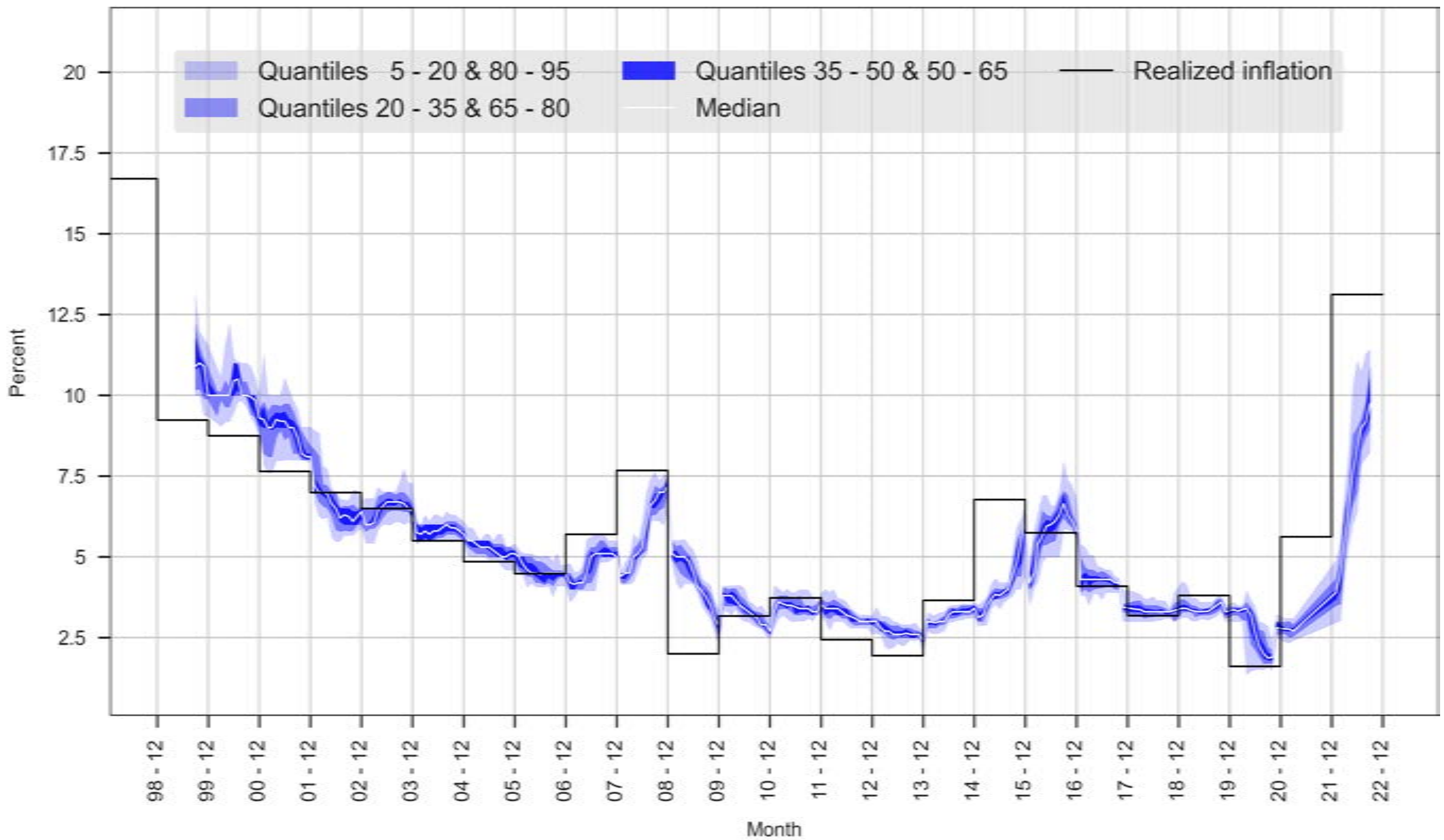
Figure B.10: BR's ETE Cross Section Distributions (Eight Quarters Ahead YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to the expectation's target date
- Realised inflation at YY-MM

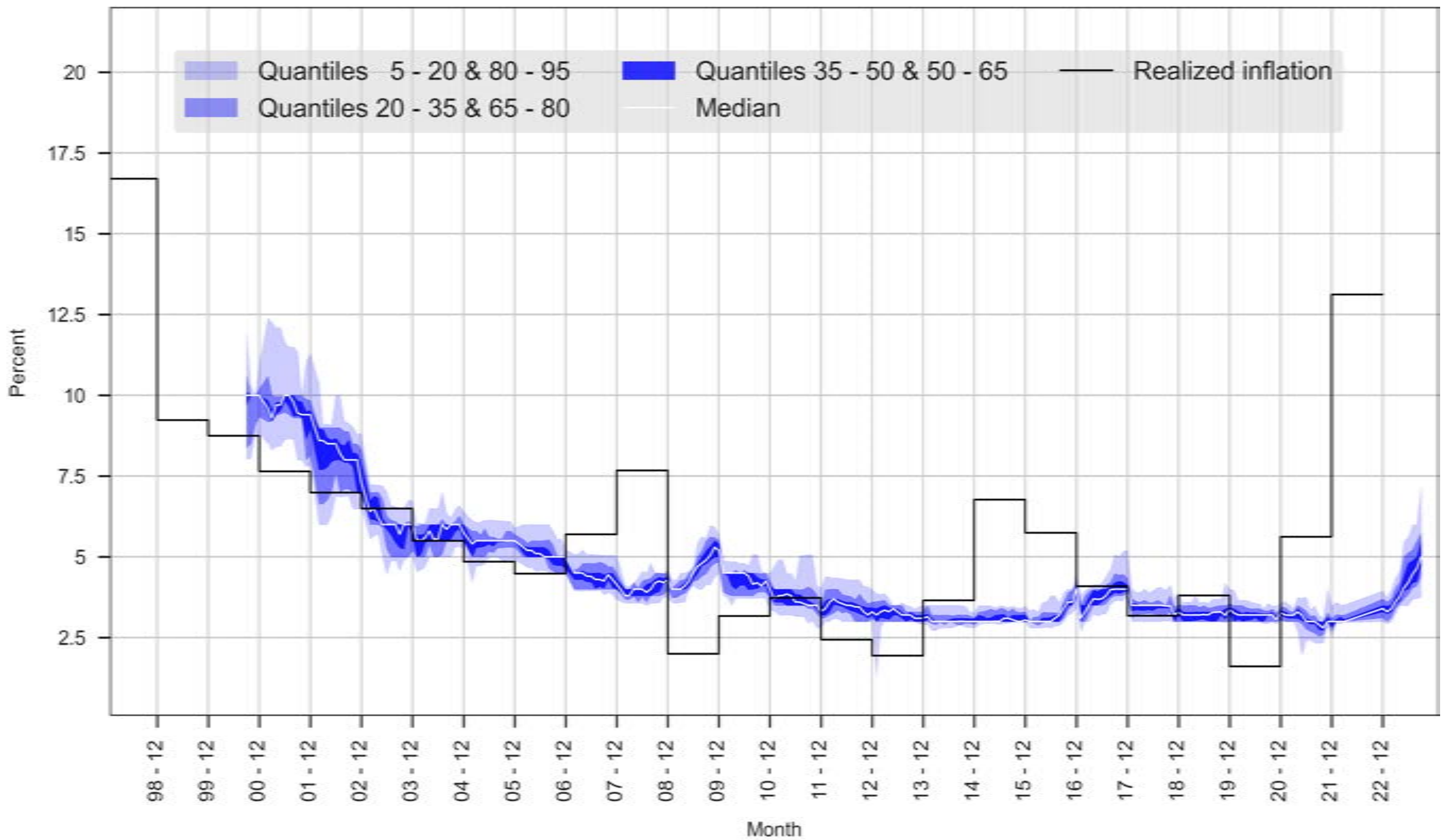
Figure B.11: Cross Section Distributions of Focus Economics Expectation Survey Results (End of the Current Year YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers expectation survey
- Realised inflation at the end of the current year

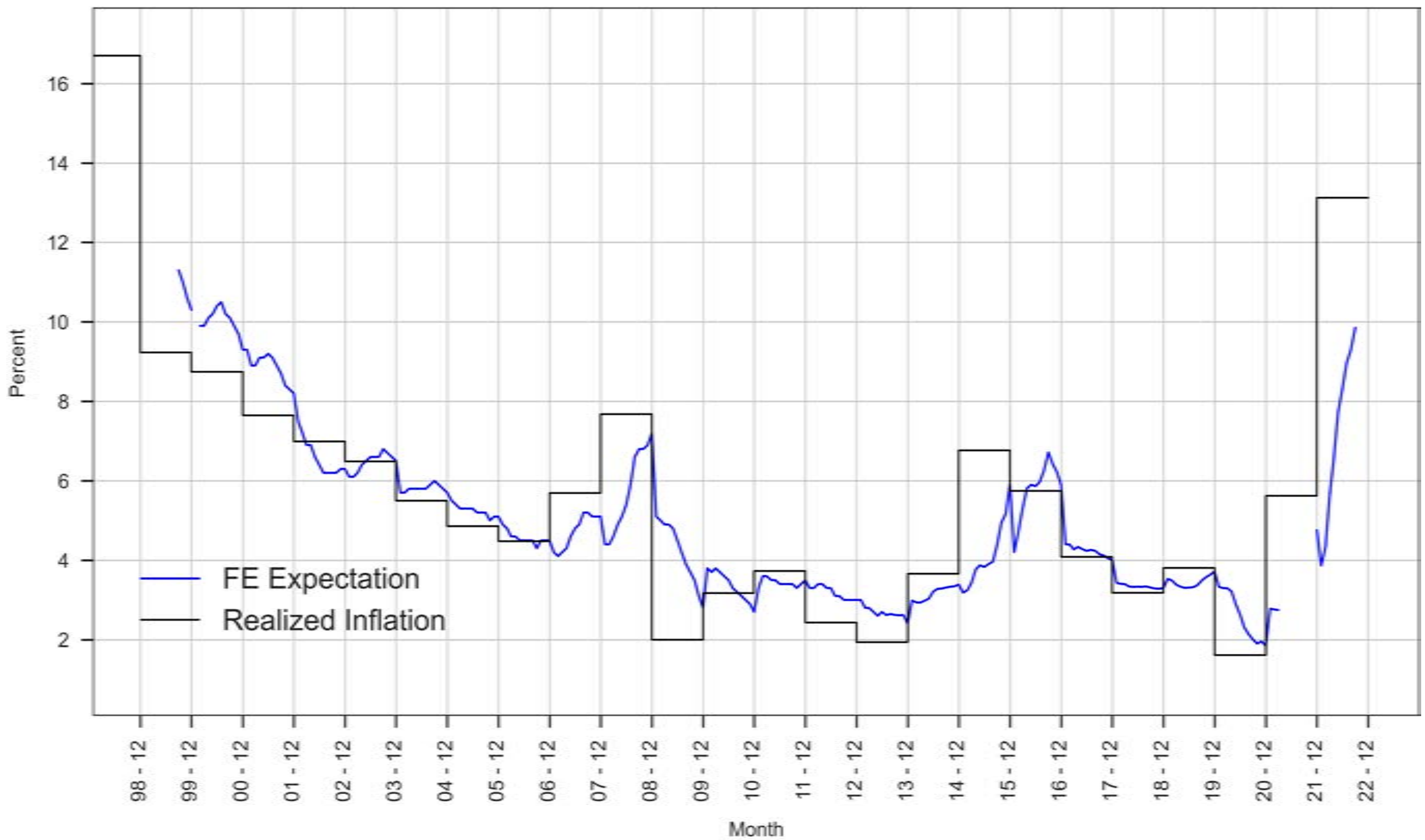
Figure B.12: Cross Section Distributions of Focus Economics Expectation Survey Results (End of the Next Year YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey of the past year
- Realised inflation at the end of the current year

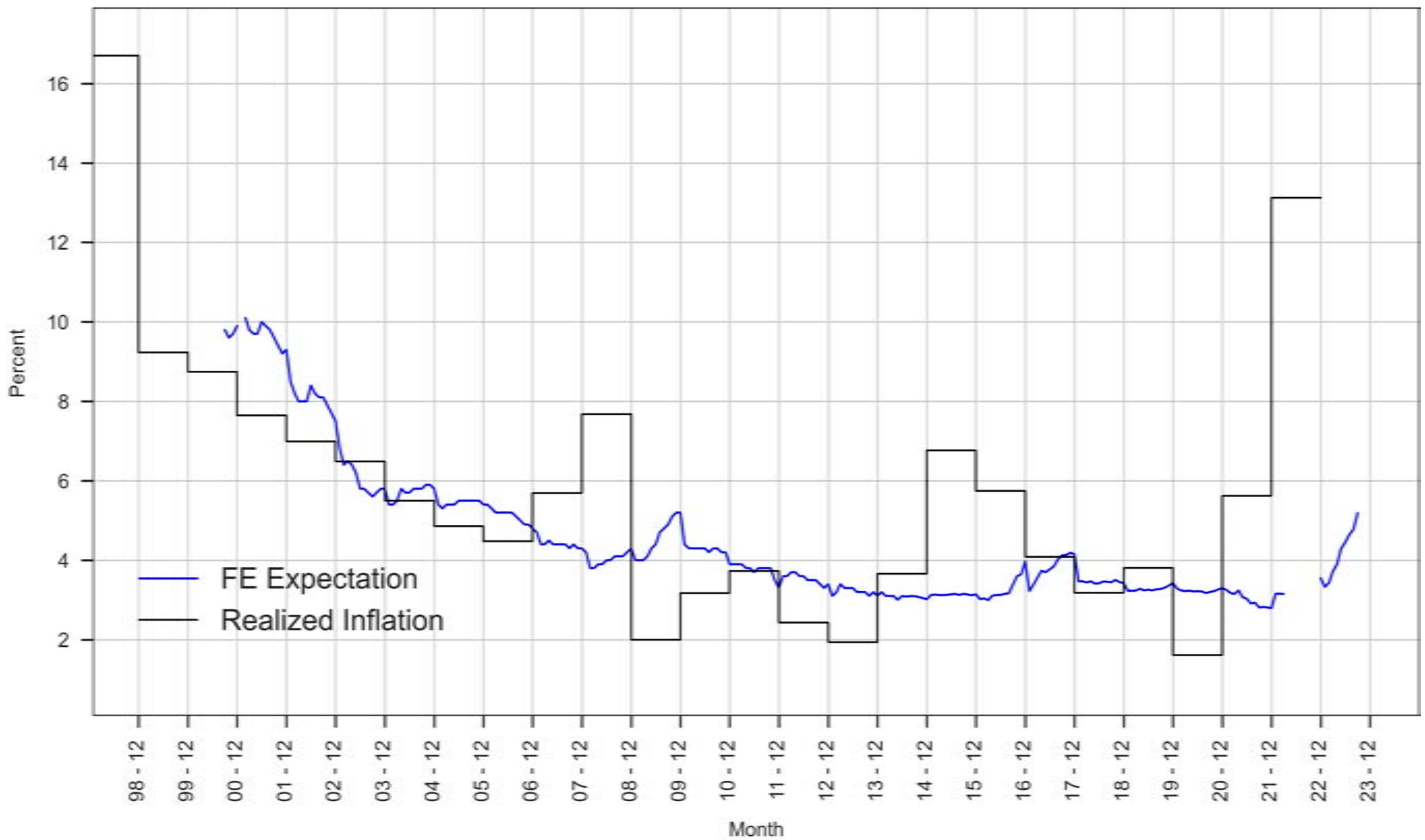
Figure B.13: Focus Economics Long Run Expectation Survey Results (End of the Current Year YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey
- Realised inflation at the end of the current year

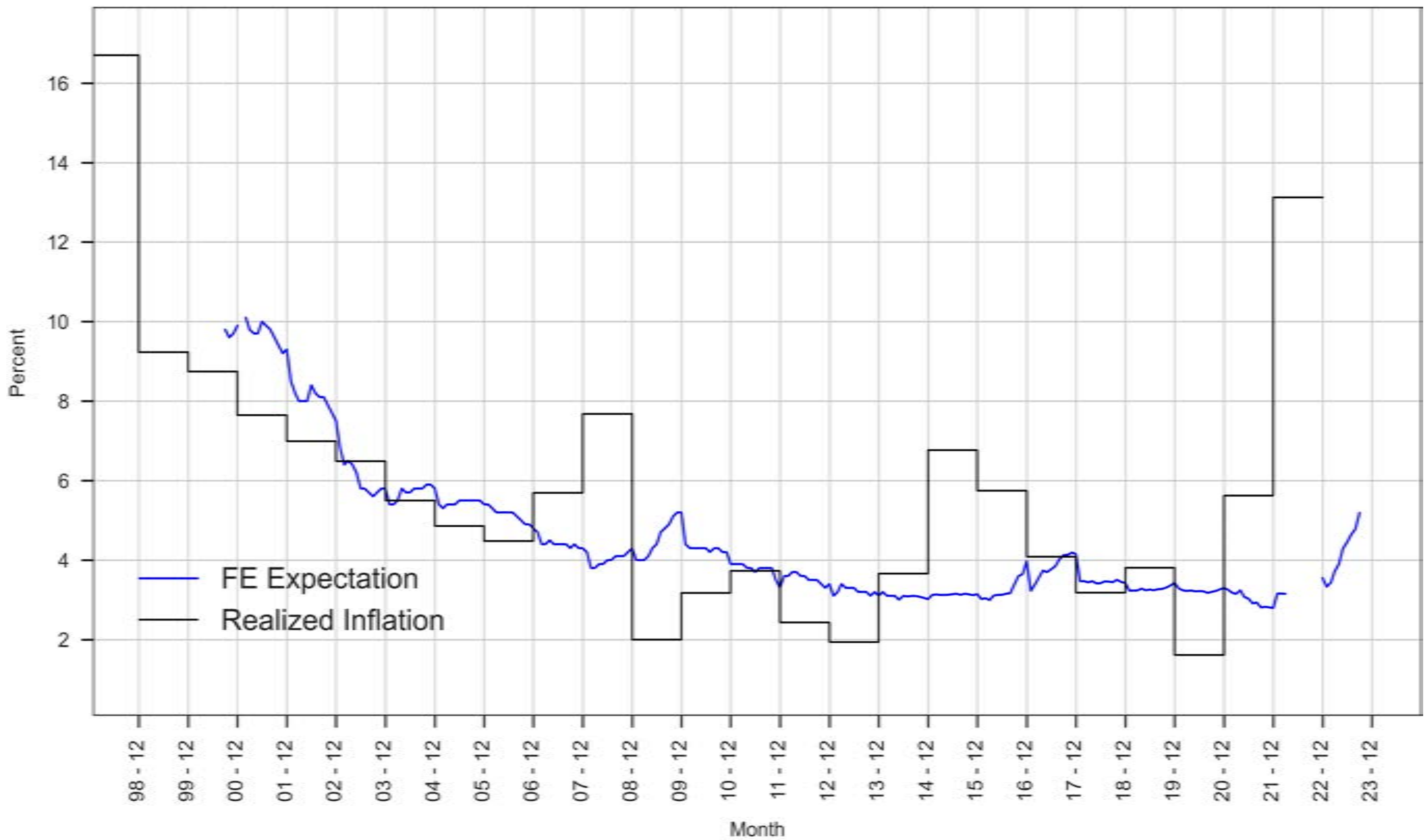
Figure B.14: Focus Economics Long Run Expectation Survey Results (End of the Following Year YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey last year
- Realised inflation at the end of the current year

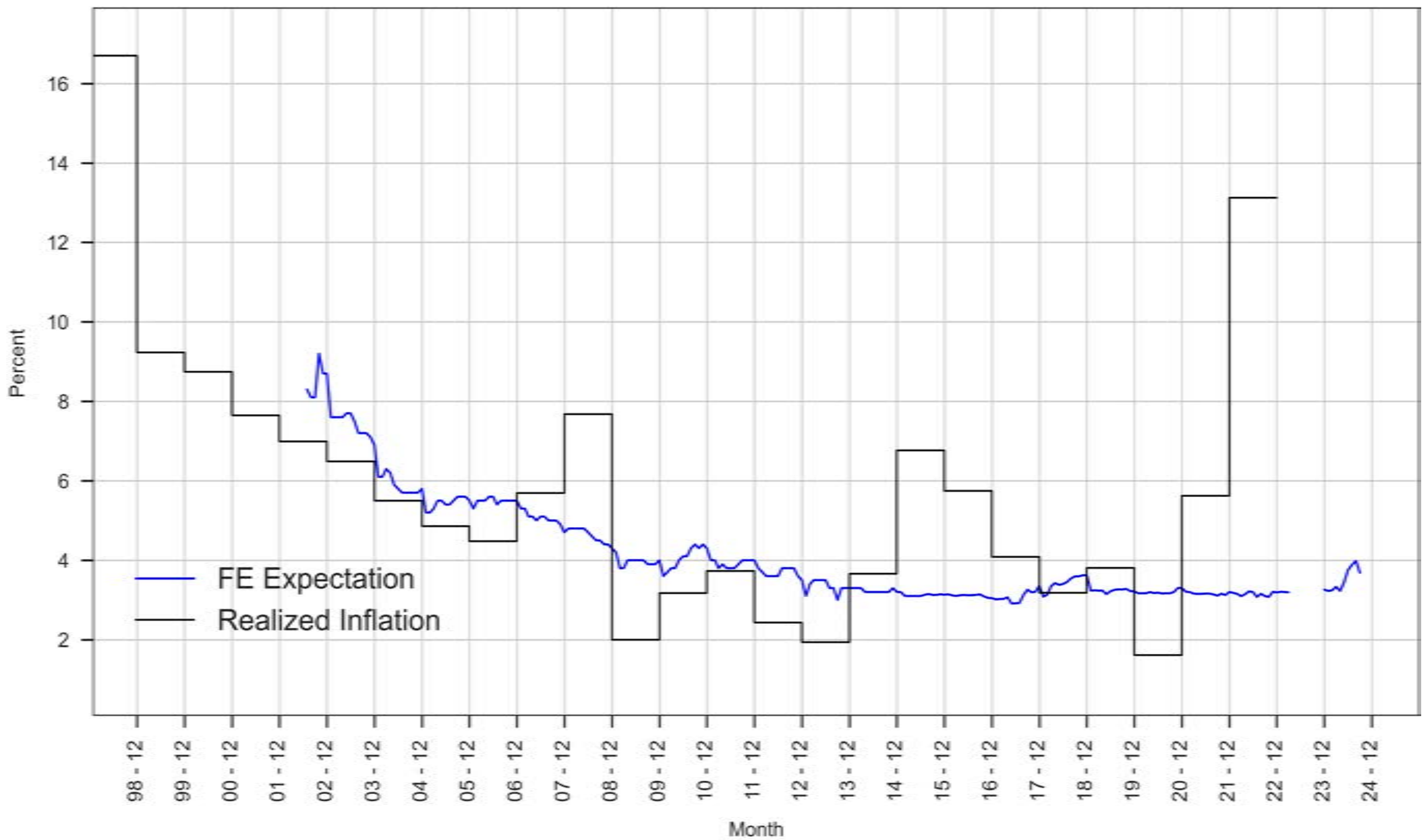
Figure B.15: Focus Economics Long Run Expectation Survey Results (End of the Seciond Year from Now YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey two years ago
- Realised inflation at the end of the current year

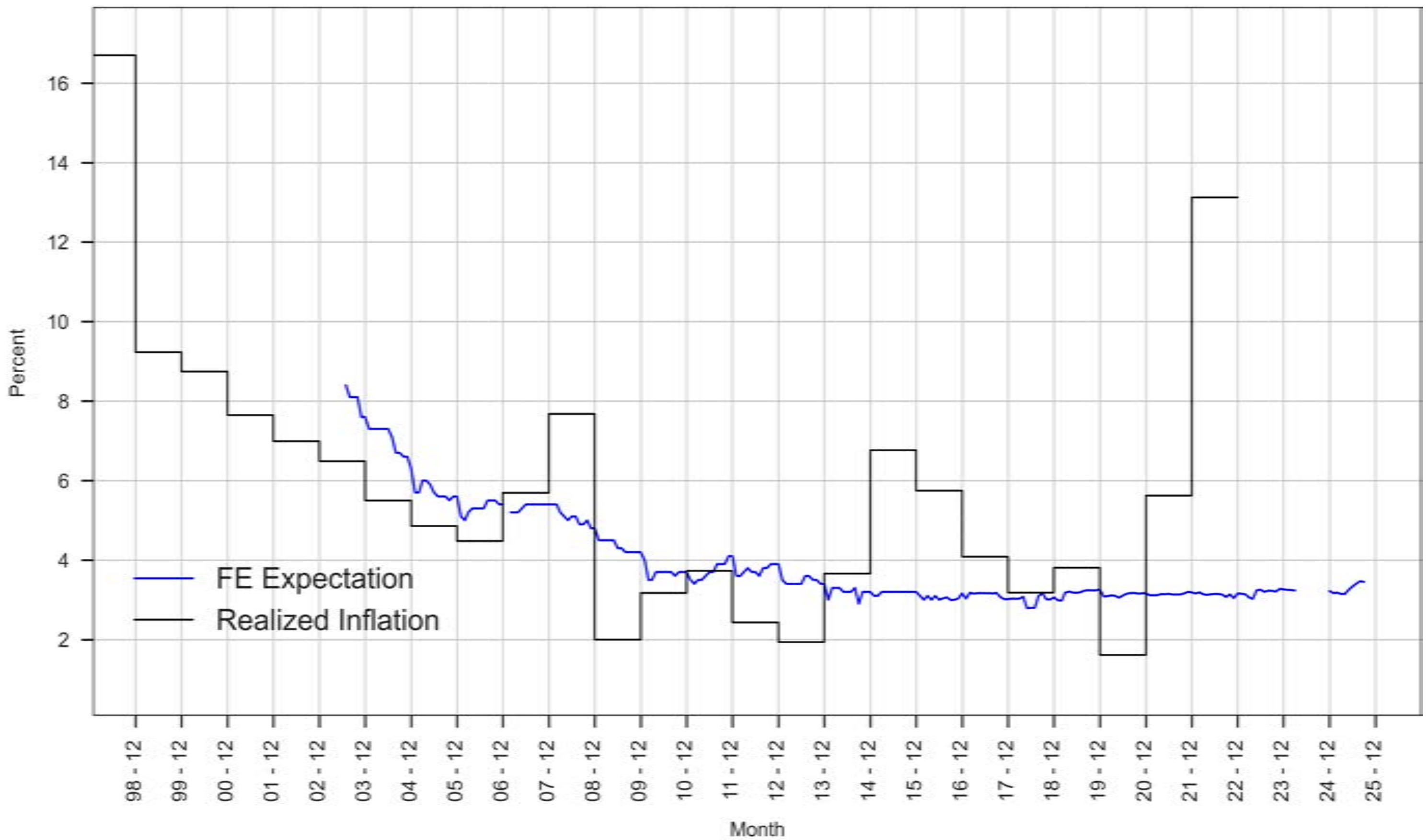
Figure B.16: Focus Economics Long Run Expectation Survey Results (End of the Third Year from Now YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey three years ago
- Realised inflation at the end of the current year

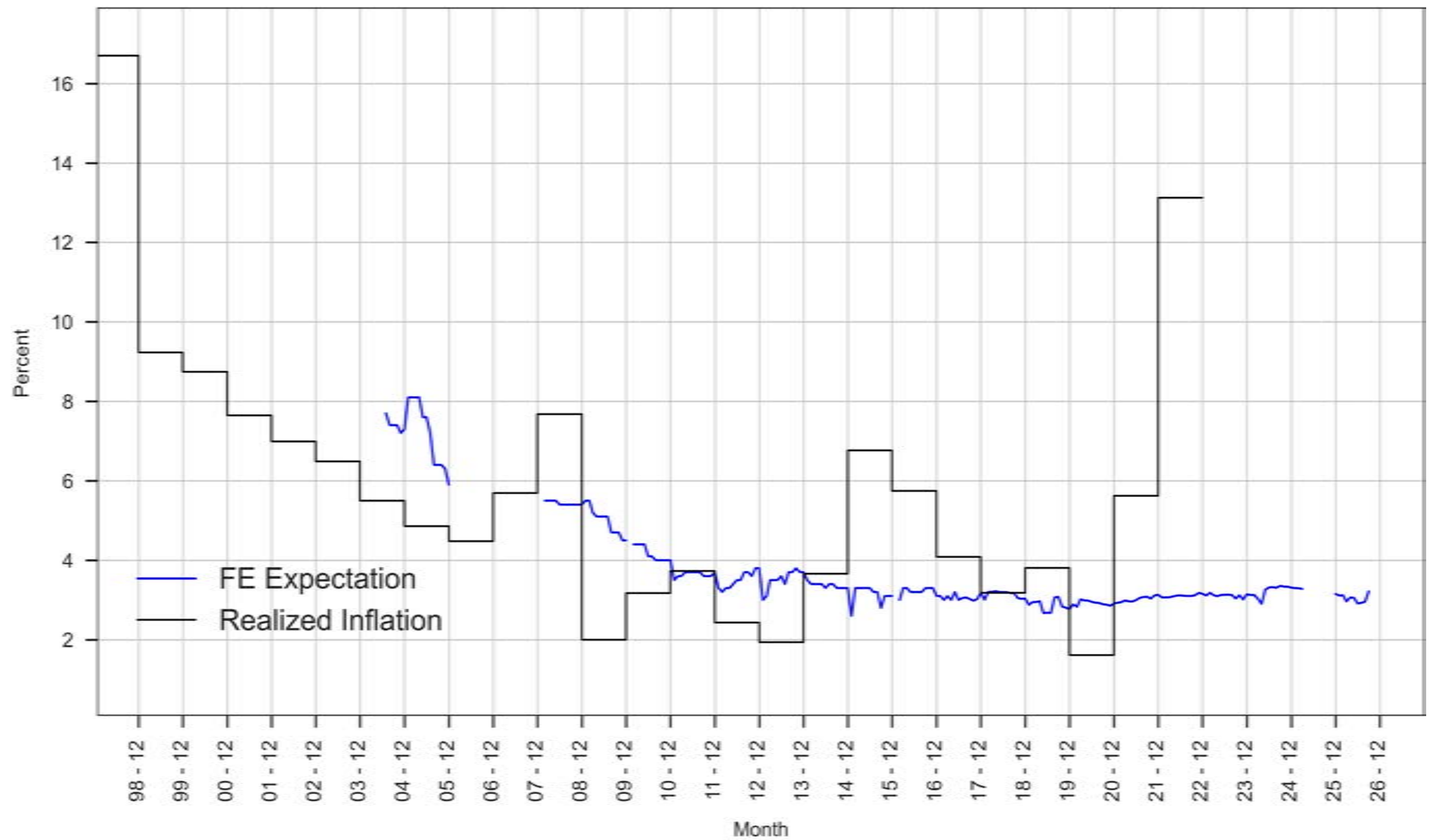
Figure B.17: Focus Economics Long Run Expectation Survey Results (End of the Fourth Year from Now YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey four years ago
- Realised inflation at the end of the current year

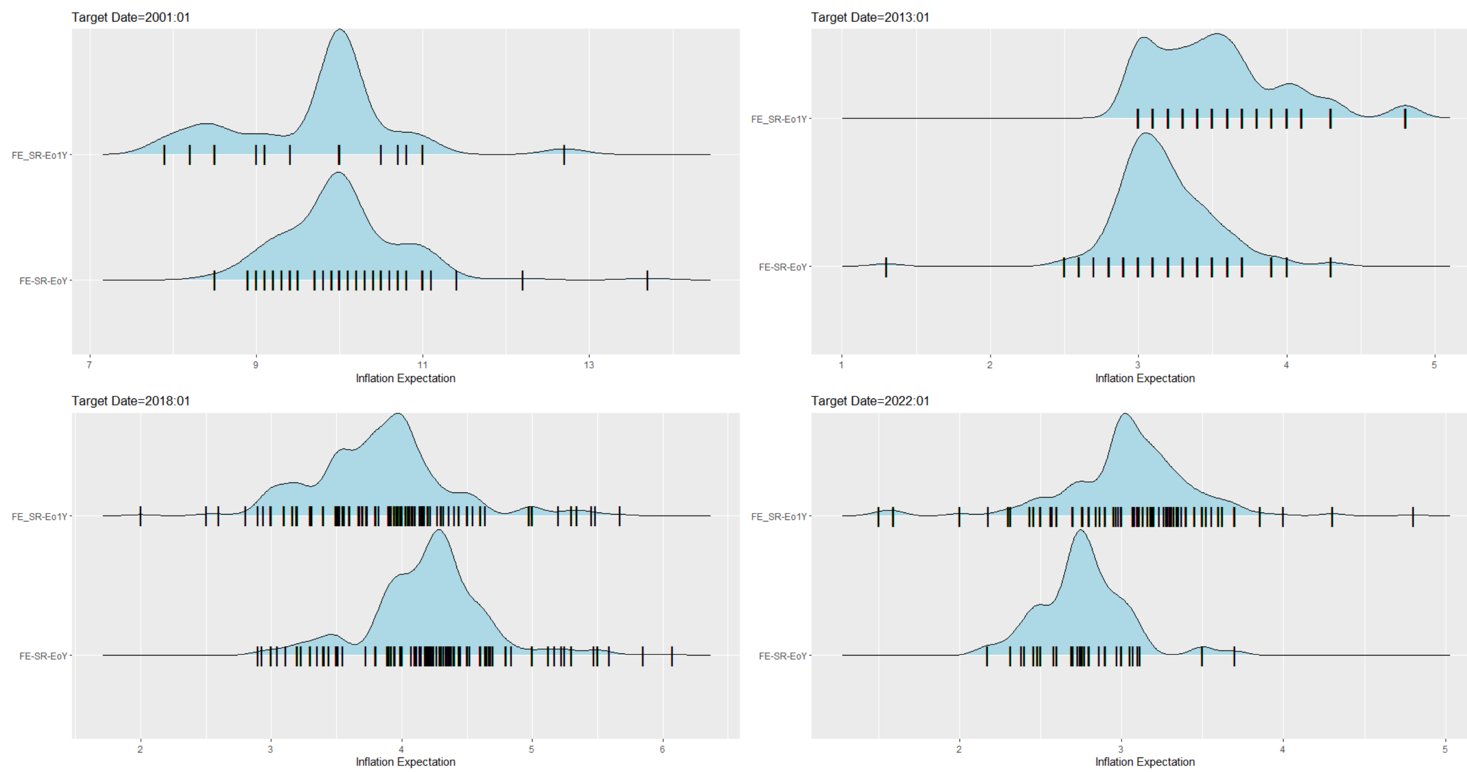
Figure B.18: Focus Economics Long Run Expectation Survey Results (End of the Fifth Year from Now YoY Inflation)



Source: Authors' calculations

- Date in YY-MM format refers to expectation survey four years ago
- Realised inflation at the end of the current year

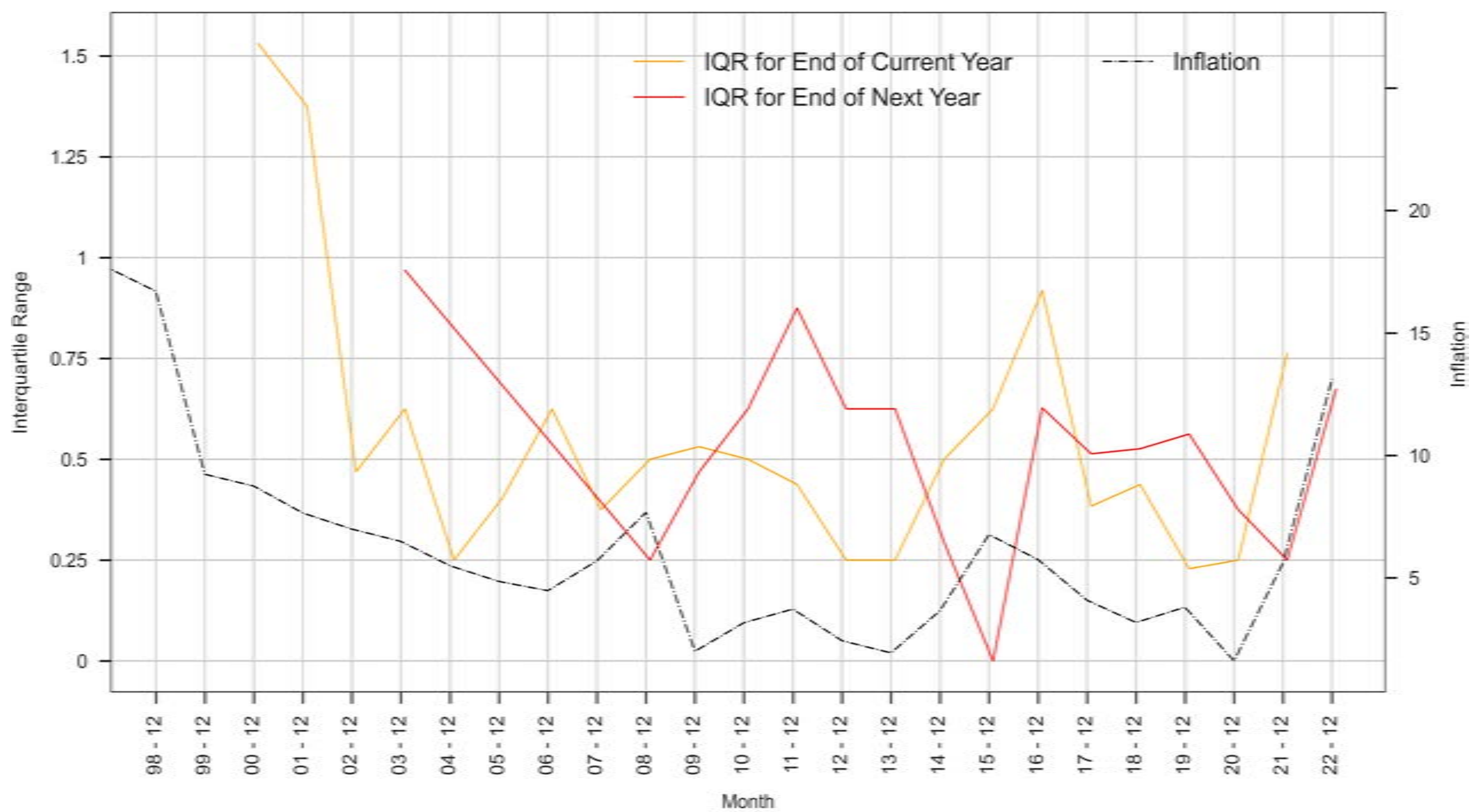
Figure B.19: Focus Economics Short Run Expectations Cross Section Distributions for selected Dates (End of the Current and Next Years YoY Inflation)



Source: Authors' calculations

- Shaded cyan area is the Kernel density estimates of survey responses
- Tick marks crossing the x-axis show the survey responses

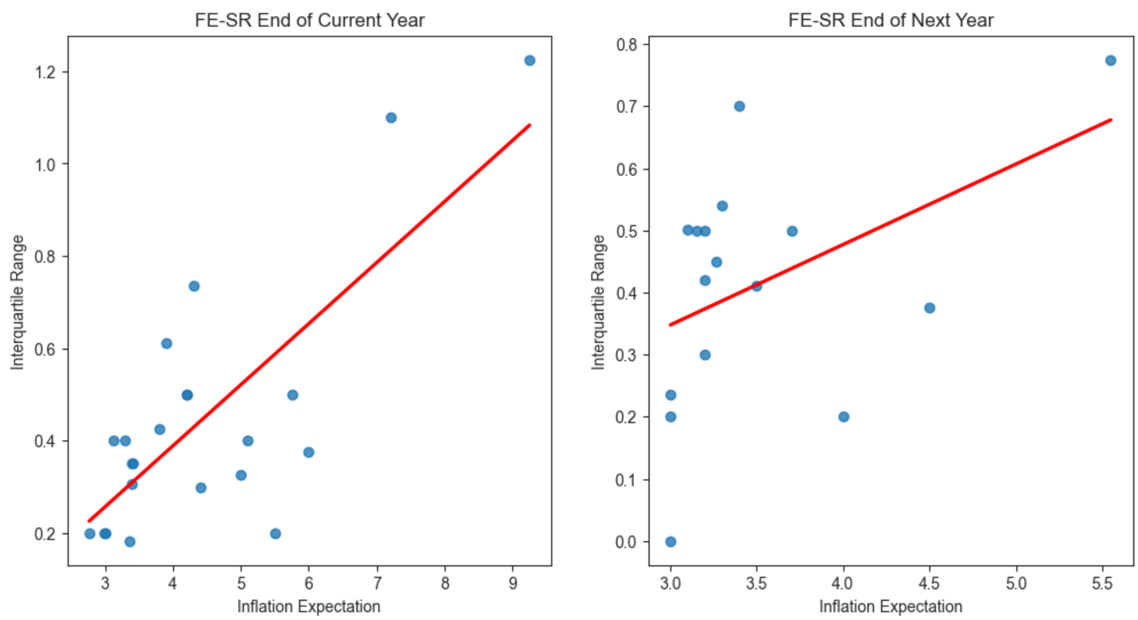
Figure B.20: Focus Economics Short Run Expectations 5-95% Inter Quantile Range (End of the Current and Next Years YoY Inflation) and Observed Inflation



Source: Authors' calculations

- Date in YY-MM format refers to expectation date
- IQR is the 5-95% Inter Quantile Range of survey responses at each date

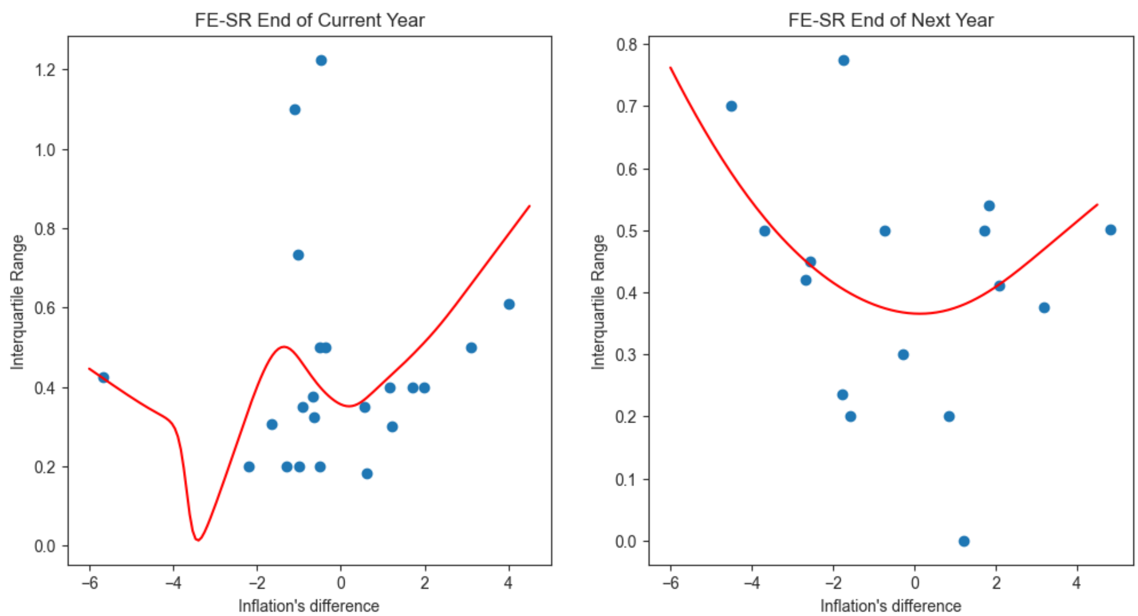
Figure B.21: Focus Economics Short Run Expectations 5-95% Inter Quantile Range (End of the Current and Next Years YoY Inflation) and Observed Inflation



Source: Authors' calculations

- IQR is the 5-95% Inter Quantile Range of survey responses at each date

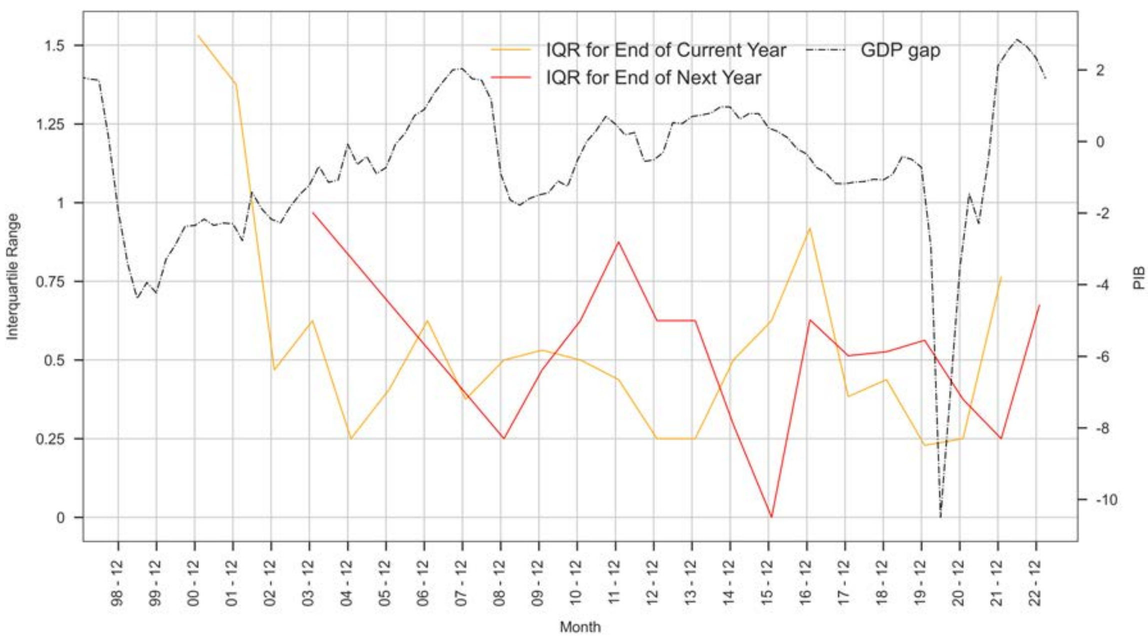
Figure B.22: Focus Economics Short Run Expectations 5-95% Inter Quantile Range (End of the Current and Next Years YoY Inflation) and Observed Inflation Change



Source: Authors' calculations

- IQR is the 5-95% Inter Quantile Range of survey responses at each date

Figure B.23: Focus Economics Short Run Expectations 5-95% Inter Quantile Range (End of the Current and Next Years YoY Inflation) and the Output Gap



Source: Authors' calculations

- IQR is the 5-95% Inter Quantile Range of survey responses at each date