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Abstract

The Equilibrium Real Exchange Rate (ERER) is a latent variable commonly monitored as a potential indicator of external vulnerabilities, yet its estimation is challenged by the non-linear interactions among the underlying fundamentals and by the time-varying nature of their influence. This paper proposes a Neural Network Equilibrium Real Exchange Rate (NN-ERER) model based on a Branch Directed Neural Network (BDNN) architecture that estimates the ERER for the Colombian peso by separating long- and short-run determinants of the real exchange rate (RER). The long-run component is constructed from blocks associated with terms of trade, fiscal policy, productivity, and external debt, while the short-run component is divided into blocks of variables with positive and negative expected effects. Theoretical sign restrictions are encoded into the architecture, constraining the estimation while preserving non-linear flexibility. This structure also allows the RER misalignment to be partially decomposed into short-term determinants and unexplained residuals. The empirical results indicate that the relative contribution of the fundamentals underlying the ERER has evolved over time, with terms of trade and external debt particularly relevant during the period of high commodity prices in the mid-2000s, and fiscal factors gaining weight in the post-COVID-19 period. Misalignments in 2008–2009 and after 2021 are largely attributable to short-run factors, plausibly linked to global volatility, uncertainty, and shifts in interest rate differentials.
Keywords: Equilibrium Real Exchange Rate, Neural Network, Real Exchange Rate Misalignment.
JEL Classification: C45, F31, F32, F41.

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Tasa de Cambio Real de equilibrio basada en Redes neuronales

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Resumen

La Tasa de Cambio Real de Equilibrio (ERER, por sus siglas en inglés) es una variable latente comúnmente monitoreada como indicador de potenciales vulnerabilidades externas, cuya estimación se puede ver afectada por las interacciones no lineales entre los fundamentales subyacentes y por la naturaleza cambiante en el tiempo de su influencia. Este trabajo propone un modelo de redes neuronales (NN-ERER) basado en una arquitectura de Red Neuronal Dirigida por Ramas (BDNN, por sus siglas en inglés) que estima la ERER para Colombia separando los determinantes de largo y corto plazo de la tasa de cambio real (RER). El componente de largo plazo se construye a partir de bloques asociados a términos de intercambio, política fiscal, productividad y deuda externa, mientras que el componente de corto plazo se divide en bloques de variables con efectos positivos y negativos esperados. Las restricciones teóricas de signo se codifican en la arquitectura, restringiendo la estimación mientras se preserva la flexibilidad no lineal. Esta estructura también permite descomponer parcialmente el desalineamiento de la RER en determinantes de corto plazo y residuales no explicados. Los resultados empíricos indican que la contribución relativa de los fundamentales subyacentes a la ERER ha evolucionado en el tiempo, con los términos de intercambio y la deuda externa particularmente relevantes durante el período de altos precios de commodities a mediados de los 2000, y los factores fiscales ganando peso en el período post-COVID-19. Los desalineamientos en 2008–2009 y después de 2021 son atribuibles en gran parte a factores de corto plazo, posiblemente ligados a volatilidad global, incertidumbre, y cambios en los diferenciales de tasas de interés.
Palabras clave: Tasa de cambio real de equilibrio, Red neuronal, Desalineamiento de la Tasa de cambio real de equilibrio.
Clasificación JEL: C45, F31, F32, F41.

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1 Introduction

The Equilibrium Real Exchange Rate (ERER) constitutes a central benchmark for assessing whether observed movements in a country's real exchange rate (RER) reflect transitory disturbances or permanent changes in its macroeconomic fundamentals (Clark & MacDonald, 1998; Edwards, 1988). Deviations in the RER from its estimated equilibrium are routinely monitored as potential indicators of external vulnerabilities. Consistent with this view, a large body of literature has estimated the ERER and the associated misalignment under alternative methodological frameworks and for various country groups (Adler & Grisse, 2017; Jeong et al., 2010; MacDonald, 2000; Roudet et al., 2007).

The conceptual frameworks underlying most of these analyses have been shaped by two complementary traditions: the Fundamental Equilibrium Exchange Rate (FEER) approach, pioneered by Williamson (1983), which defines the equilibrium as the RER consistent with simultaneous internal and external balance; and the Behavioral Equilibrium Exchange Rate (BEER) tradition, popularized by Clark and MacDonald (1998), which posits a long-run relationship between the RER and a set of fundamentals, such as the terms of trade, productivity differentials, government debt, and net foreign asset positions, that the literature has identified as relevant explain the long-run behavior of the RER.

ERER estimates are often highly sensitive to the model specification and the choice of fundamental variables (Adler & Grisse, 2017), and the linear reduced-form structure typically adopted in the BEER tradition imposes restrictive assumptions on how fundamentals interact and on the dynamics of adjustment toward equilibrium. This paper proposes a methodology to estimate the ERER as a latent variable while capturing potentially nonlinear relationships among the long-run determinants of the RER. Specifically, developing a Neural Network Equilibrium Real Exchange Rate (NN-ERER) model based on a Branch Directed Neural Network (BDNN) architecture where each branch is constrained by the theoretical direction of its effect, which accommodates flexible interactions across the underlying fundamentals.

Accounting for such nonlinearities is particularly relevant in today's global environment, characterized by greater capital mobility and increasingly complex international balance sheets, even for emerging markets (Lane & Milesi-Ferretti, 2007). In this setting, the interaction between exchange rates, net foreign asset positions, and cross-border financial flows operates through valuation channels and high-dimensional links that linear specifications struggle to capture (Gourinchas & Rey, 2007; Lane & Shambaugh, 2010), motivating modeling approaches flexible enough to accommodate nonlinear interactions among a broad set of fundamentals (Kilian & Taylor, 2003; Taylor et al., 2001).

The NN-ERER is organized around a set of neural network branches, each associated with a specific category of macroeconomic fundamentals widely recognized as key drivers of the ERER and characterized by theoretically grounded directional effects on the RER. These categories cover terms of trade, fiscal expenditure, productivity, and external debt, in the spirit of the *Thousands of BEERs* approach proposed by Adler and Grisse (2017) and applied to Colombia by Salazar-Díaz et al. (2023). Each long-run branch processes a rich set of related variables, summarizing their potentially nonlinear interactions into a single time-varying latent factor that captures the aggregate influence of the corresponding fundamental category.

More broadly, the framework is related to a growing literature that employs machine learning and neural network models to estimate latent macroeconomic variables and to flexibly characterize their relationships with observed series, including Goulet Coulombe (2025)'s Hemisphere Neural Network for the output gap and inflation expectations, the Deep Dynamic Factor Models of Andreini et al. (2020), the Bayesian neural network approach of Hauzenberger et al. (2025), and the Macroeconomic Random Forest of Goulet Coulombe (2024). The ERER is then constructed as a dynamic combination of these latent factors, with time-varying weights and theoretically consistent signs that reflect the underlying transmission mechanisms. This structure allows the model to adapt to evolving economic conditions, in which the relative importance of different fundamentals may shift over time, while remaining disciplined by economic theory.

Traditional approaches such as the BEER and FEER methodologies have been successful in characterizing long-run relationships between the RER and its fundamentals, the study of its misalignment; defined as the gap between the calculated equilibrium and the observed value, has also been explored. Different studies have characterized this component, one of the main references delivering a decomposition between permanent and transitory effects of its fundamentals (Clark & MacDonald, 2000), and other, analyzing the case of Colombia, evaluates the changes in those variables while calculating each contribution combining them with its estimated impulse-response elasticity (Echavarría et al., 2005). To sharpen the identification of the long-run equilibrium and shed additional light on the drivers of misalignment, the architecture of the model proposed in this paper incorporates two separate short-run branches. These components are motivated by the literature where, beyond the fundamentals, interest rates play a role of control monetary policy effects (Adler & Grisse, 2017), models that recognize the particular role of uncovered interest parity when estimating the ERER (Kiptui & Ndirangu, 2015) and its effects over the exchange rate via Taylor rules (Backus et al., 2010). Factors like stock market volatility, uncertainty and confidence indicators are also included.

Recognizing the interpretability limitations of Neural-Network methodology, to identify the most relevant variables within each fundamental group, we further rely on surrogate models that approximate the behaviour of each network branch in a more interpretable framework (Páez, 2024). Specifically, Random Forest models are fitted to replicate the input–output mapping learned by each branch, allowing us to assess the relative importance of the explanatory variables within the group. This strategy preserves the nonlinear structure captured by the neural network while providing a transparent reading of its internal mechanisms.

The empirical results show that the NN-ERER provides a nonlinear and economically interpretable representation of the ERER, allowing us to assess how the importance of different transmission channels has evolved over time. Among the long-run branches, the terms of trade and external debt emerge as the main drivers of the appreciation observed during the commodity price boom of the mid-2000s, consistent with theory and previous empirical evidence (Erten & Ocampo, 2013). More recently, the fiscal branch has gained relative importance and has become a key factor explaining the depreciation observed in the post-COVID-19 period.

Beyond characterizing the long-run equilibrium, the proposed methodology also identifies the short-run factors associated with deviations of the RER from its estimated equilibrium. Surrogate Random Forest models complement the NN-ERER by identifying the variables that account for most of the variation within each group of fundamentals. The results indicate that remittances, public external debt, the central government balance, the relative ratio of tradable to non-tradable GDP between the United States and Colombia, core inflation, and the Geopolitical Risk Index are the most influential variables across the corresponding groups of fundamentals.

The organization of the paper is as follows. The Section 2 reviews the related literature on the estimation of the equilibrium real exchange rate, discusses the studies developed for the case of Colombia, and introduces and positions the NN-ERER methodology. Section 3 describes the Colombian data that is used in the empirical analysis and includes it in each fundamental group, following the proposed methodology. Section 4 presents the architecture of the Deep Neural Network model, defining its principal components as well as its architecture, along with the surrogate models that allow for a better understanding of the results, given the limitations of direct interpretation of neural network models. Section 5 reports the main empirical findings, which include the results of the estimation and behavior of the RER, some dynamics explained by the fundamental variables, the predicted ERER, the RER misalignment, and the explanatory capacity of the variables as modeled by the surrogate models. Section 6 offers a comparative analysis of ERER calculated by the NN-ERER and the one produced by univariate filters, as benchmark for the methodology. Section 7 concludes the findings of the paper, summarizes the characteristics of the estimation and discusses directions for future research.

2 Literature review

The concept ERER is a cornerstone of international macroeconomics, with its role of adjustment variable, it is a benchmark to detect potential misalignment that could signal macroeconomic instability (Edwards, 1988). Consequently, an extensive body of literature has been devoted to estimating it, resulting in the development of several "workhorse" models. Among the most prominent is the FEER approach, a normative framework developed by Williamson (1983) and Williamson (1994). The FEER is defined as the real exchange rate consistent with both internal balance, full employment with low inflation, and external balance, a sustainable current account position. This approach is inherently prescriptive, as it requires defining a desirable or sustainable current account target.

In contrast, the BEER approach, formally introduced and contrasted with the FEER framework by Clark and MacDonald (1998), offers a robust econometric alternative. It seeks to identify and quantify the long-run relationship between the real exchange rate and a set of economic fundamentals. These fundamentals typically include net foreign assets, productivity differentials, often capturing the Balassa-Samuelson effect, terms of trade, and government consumption. The BEER is then calculated as the value predicted by this long-run relationship, using the long-term or sustainable values of the fundamentals. The estimation usually relies on time-series techniques like cointegration analysis to separate long-run trends from short-term fluctuations.

The BEER approach quickly became the dominant paradigm in research on the ERER, providing the foundation on which much of the subsequent literature has built. A first methodological refinement came from Clark and MacDonald (2000), who developed the Permanent Equilibrium Exchange Rate (PEER) to address a specific shortcoming of the original BEER: the use of Hodrick–Prescott filters to smooth fundamentals. The PEER applies the Gonzalo and Granger (1995) decomposition to separate the variables in a Vector Error Correction Model (VECM) into permanent and transitory components, defining the equilibrium exchange rate as the value implied by the permanent component of the fundamentals and thereby exploiting their cointegrating structure more rigorously. A further innovation is the *Thousands of BEERs* approach of Adler and Grisse (2017), which tackles model uncertainty arising from the well-documented sensitivity of estimated coefficients to the specific combination of explanatory variables. The methodology estimates regressions over all admissible combinations of fundamentals and employs Bayesian Averaging of Classical Estimates (BACE) to weight the results in a statistically optimal manner.

Beyond these refinements within the BEER family, other frameworks have sought to address its limitations from a different angle. The Natural Real Exchange Rate (NATREX), developed by Stein (1998), proposes a richer dynamic model in which equilibrium is defined by stock–flow dynamics: the RER must be consistent with saving and investment decisions that, in turn, support sustainable capital flows and a stable net foreign asset position. Although less parsimonious than BEER-type specifications, NATREX provides an explicit medium-run anchor for equilibrium that links the RER to the underlying intertemporal balance of the economy.

The Colombian case offers a particularly rich setting for ERER analysis. As a developing and open economy heavily dependent on commodity exports, most notably coffee, oil, and coal, and with a growing services sector, Colombia provides a natural laboratory in which terms-of-trade shocks, productivity differentials, and external balances all play a quantitatively important role. The country liberalized its exchange rate regime in 1999 and adopted an inflation-targeting framework as monetary policy in the same year, prompting a sustained research effort to characterize the equilibrium real exchange rate and its deviations. Relevant studies in this line are those of Echavarría et al. (2005) and Echavarría et al. (2007), who studied the ERER for Colombia using cointegration methods, a structural vector error correction (SVEC) approach to characterize the long-run equilibrium and documented extended periods of real overvaluation as well as a sustained depreciation following the liberalization of the exchange rate regime. More recently, Salazar-Díaz et al. (2023) apply the *Thousands of BEERs* methodology to Colombia, estimating the ERER over a wide space of specifications and combinations of fundamentals, characterising the appreciation episode of 2004–2013.

Recently, the field of economic and financial forecasting has been revolutionized by the application of Artificial Intelligence (AI) and Machine Learning (ML) models, allowing for new horizons and methodologies for the estimation of macroeconomic variables. Techniques such as Artificial Neural Networks (ANNs), Support Vector Machines (SVMs), and ensemble methods like Random Forests and Gradient Boosting have shown remarkable success in forecasting volatile time series, including exchange rates (Galeshchuk & Mukherjee, 2017). The primary advantage of these models lies in their ability to capture complex, non-linear patterns and high-dimensional interactions between variables that may be missed by traditional linear econometric models (Anesti et al., 2024; Goulet Coulombe et al., 2022). While a representative sample of the existing literature focuses on using ML for short-term and medium-term forecasting of the nominal or real exchange rate (Bolhuis & Rayner, 2020; Qureshi, 2025), their application to estimating unobserved long-run equilibrium or latent values is still an emerging field where this paper may contribute.

In the field of deep learning, recent evolution has highlighted the utility of a shift from purely data-driven black-box approaches toward the integration of domain-specific knowledge in modeling. A paradigm widely adopted in physics, where integrating physics principles is critical because standard unconstrained neural networks produce results that could be physically inconsistent, violating fundamental constraints such as the conservation laws of mass or energy (Beucler et al., 2021; Farea et al., 2024). This necessity gave rise to the Physics-informed neural networks (PINNs), a specialized type of neural network that integrates physics principles into its learning process (Farea et al., 2024). These networks incorporate principles through hard and soft constraints. Soft constraints modify the penalty of the loss function to guide the optimizer and minimize the residual of the governing equations, although they do not guarantee mathematical accuracy. Hard constraints, on the other hand, are directly incorporated into the neural network architecture through fixed layers or structures that force the model to satisfy analytical laws or equilibrium conditions (Beucler et al., 2021; Lu et al., 2021).

Drawing on the historical relationship between physics and economics, where complex systems are governed by analogous mathematical models (Sornette, 2014), the concept of introducing expert knowledge into neural networks has recently been present in the economics literature. In equilibrium models, Azinovic and Zemlivcka (2023) introduce "Market Clearing Layers", a neural network architecture that enforces market clearing conditions and borrowing constraints to solve high-dimensional dynamic stochastic economic models.

This is achieved through layers that subtract excess demand, divided equally among the agents, and activation functions that ensure that prices and wealth shares are always positive. Wu et al. (2024) present the Informed Equilibrium Neural Network framework to solve partial differential equations focused on economic models in continuous time. The core of the methodology lies in soft constraints that incorporate economic theory through the loss function, adding in a weighted manner the losses from the Hamilton-Jacobi-Bellman (HJB) equations, boundary conditions, inequality constraints, market emptying conditions, and first-order optimality.

Closer to the present work, Goulet Coulombe (2025) employs hard constraints via a branched neural network to study the components of the Phillips Curve, named Hemisphere Neural Network. In this model, each branch represents a group of macroeconomic variables (expectations, real activity, and commodity prices) that prevents unwanted interactions between variables that theory dictates as separate. The outputs of these branches are combined in an additive layer, yielding interpretable latent states for the Phillips Curve.

While traditional econometric models, such as BEER and FEER have been the standard for the ERER estimation in developing countries, such as Colombia, has partially explained the misalignment by the introduction of cointegration relations and short-run deviations from the long-run equilibrium (Clark & MacDonald, 2000; Echavarría et al., 2005), this paper proposes a new way to understand the misalignment by including a couple of short-run branches (as is explained in Section 3) and consider interest rate differentials, implicit risk premiums, uncertainty variables and stock market volatility indexes as the main determinants of deviation from the equilibrium.

The proposed NN-ERER offers several contributions. First, it integrates macroeconomic transmission mechanisms into a deep neural network architecture, allowing for the modeling of nonlinear interactions among fundamentals while remaining disciplined by economic theory. Second, the methodology enables an approximate decomposition of both long-run dynamics and RER misalignment into interpretable blocks of variables. Third, the proposed network architecture enforces theoretical restrictions, avoiding undesired interactions between variables that theory dictates as separate and preserving the direction of the effects implied by the transmission mechanism. This design facilitates an economically interpretable decomposition of the estimated equilibrium RER and its misalignment, providing suggestive evidence on the variables most closely associated with deviations of the RER from its estimated equilibrium and on how their relative importance has evolved over time through the model's time-varying coefficients.

3 Data

To estimate the real equilibrium exchange rate, we use quarterly data spanning from the first quarter of 2000 to the fourth quarter of 2024. The sources are presented in Table (A1). Most of the data come from the Colombian Central Bank, the National Statistics Office of Colombia (DANE, for its acronym in Spanish), the U.S. Bureau of Labor Statistics (BLS), and the Federal Reserve Economic Data (FRED). The output RER, defined as the logarithm of the multilateral real exchange rate index: $q_t = \ln(\text{Real Exchange Rate Index})$. This index, published by the Colombian Central Bank, is constructed using non-traditional trade weights.

These weights correspond to the 12-month moving average of the share in total trade of Colombia's 22 main trading partners (for imports and exports, excluding coffee, oil, coal, ferronickel, emeralds, and gold), with the CPI used as the deflator for all countries. We use a set of fundamental variables to explain the behavior of the RER. Each group corresponds to a branch incorporated into the neural network model. For each group of variables, we describe their expected effect on the RER, based on the transmission mechanisms identified in the BEER literature (Adler & Grisse, 2017; Salazar-Díaz et al., 2023). The groups of fundamentals, each of which summarizes the underlying behavior into a single latent variable incorporated into the neural network model, are the following:

- **Terms of Trade:** this factor represents the price of a country's exports relative to its imports and is a key determinant of external imbalances, especially for commodity-exporting economies. An improvement in terms of trade generates a positive income effect, increasing demand for all goods and typically leading to an appreciation of the RER to restore equilibrium according to the work of Balassa (1964) and Samuelson (1964), and also proved empirically by De Gregorio and Wolf (1994).Also, to capture the income effect not associated with exports of goods, we incorporate a remittance variable.

- **Fiscal:** based on the insights of Balassa (1964) and Samuelson (1964), and similar to Adler and Grisse (2017) an increase in government expenditure increases domestic demand, which in turn puts upward pressure on the prices of non-tradable goods and leads to an appreciation of the RER. This mechanism is also consistent with the general rise in domestic prices that tends to appreciate the RER (Balvers & Bergstrand, 2002). The empirical relevance of this relationship was confirmed by Edwards (1988) for developing countries, establishing government spending as an important determinant in the estimation of the RER estimation.

  Furthermore, the transmission mechanism aligns with the findings of Salazar-Díaz et al. (2023): An expansion in government spending tends to stimulate output and overall demand, increasing the need for labor and capital. This increased factor demand puts upward pressure on wages and production costs, contributing to rising prices. As inflation accelerates, monetary authorities typically respond by tightening the policy, which attracts foreign capital and results in an appreciation of the exchange rate.

- **Productivity:** The first fundamental theoretical explanation for systematic deviations from Purchased Power Parity (PPP) came from the celebrated Balassa-Samuelson effect (Balassa, 1964; Samuelson, 1964). This postulate argues that countries with higher productivity and faster growth, particulary in the tradable goods sector (relative to the non-tradable sector) will experience a trend appreciation of their RER as wages rise across the economy, pushing up the relative price of non-tradable goods (Begun, 2000; Salazar-Díaz et al., 2023).

- **External Debt**: A country's net foreign asset position and its ability to access international capital markets are also critical. Sustained current account deficits lead to an accumulation of external debt, which may require a future depreciation of the RER to generate the trade surpluses needed for repayment (Asonuma, 2014). In addition, and particularly in the case of emerging economies, increased external debt leads to depreciation (Ojeda-Joya & Sarmiento, 2016).

The variables that affect RER dynamics in the short run are divided into two groups: those that have a positive effect and those that have a negative effect. These groups encompass the factors identified in the literature as relevant for explaining short-run RER fluctuations associated with cyclical dynamics and temporary factors. They are summarized as follows:

- **Positive short-run**: in this branch, we include variables related to volatility and geopolitical factors that can increase investment risk in developing countries and influence overall risk appetite in financial markets. Consequently, we incorporate measures of geopolitical risk, volatility indicators, and indices that capture domestic uncertainty conditions. On the one hand, global uncertainty and volatility are highly correlated with important depreciations for developing countries, due to the search for less-riskier assets in advanced economies (Glebocki & Saha, 2024). In addition, domestic uncertain conditions increase the exchange rate, particularly through its nominal component, as accounts Dabrowski (2002), enumerating cases for emerging economies where political instability, domestic military conflicts, low legal security, or constitutional crises may arise risk as sudden stops of capital inflows, promoting several outflows and currency substitution.

- **Negative short-run**: The branch incorporates the variables related to the interest rate differential between the interest rate of Colombia and that of the United States, including the differential that uses the Treasury rate and other measures of external interest rates. The purpose of these variables is to capture the effect of the Uncovered Interest Parity condition (Dornbusch, 1976; Rogoff, 2002). According to this condition, a positive differential, in which the interest rate of Colombia is higher than the external rate, is associated with an appreciation of the real exchange rate. This assumption is consistent with the previous research of Adler and Grisse (2017) and Salazar-Díaz et al. (2023), who support the same premise that a higher domestic interest rate relative to foreign rates increases the returns available to investors, attracts foreign capital, and consequently leads to an appreciated real exchange rate.

A summary of the expected effects on the RER from both the long-run and short-run variables, the group to which each variable belongs, its associated branch in the neural network, and the transmission mechanism identified in the literature are presented in Table (1). Note that the expected signs in the fourth column are defined based on the relationships that the literature has identified between these variables and the RER. This approach is consistent with our objective of obtaining an interpretable estimation of the ERER that aligns with the macroeconomic theory of the real exchange rate.

Most variables included in the model that were not already expressed in percentages are incorporated in logarithmic form to ensure comparable magnitudes. Seasonal adjustment is applied to some variables where seasonality is present. Furthermore, following the approach of Coulombe et al. (2021), we compute the Moving Average Rotation (MARX) transformation to control for implicit priors in the Neural Network model when applied to time series data. This transformation is based on the inclusion of four lags for each variable, along with moving averages of order 2 and 4 for the variable and its lags.

Table 1: Description of fundamental variables

Fundamentals group	Branch	Variables	Expected sign
Terms of trade	$X^{LR_{Terms\ of\ trade}}$	**Terms of trade** • Based on National Accounts. • Based on PPI. **Real Brent Price**. **Remittances (%GDP)**.	Negative, $\delta_{terms\ of\ trade} = -1$: an increase in the terms of trade is associated with higher national income, which could increase demand for non-tradable goods. According to the Balassa-Samuelson model, this suggests a real exchange rate appreciation.
Fiscal	$X^{LR_{Fiscal}}$	**Government consumption** • Public consumption. • Public consumption (% GDP). • Public consumption (% total expenditure). **Government Expenditure** • Central Government Expenditure. • Central Government Expenditure (% GDP). • Non-financial Public sector Expenditure (% GDP). **Government Balance**[1] • Central Government Balance. • Central Government Balance (%GDP).	Negative, $\delta_{Fiscal} = -1$: An increase in public expenditure could increase demand for non-tradable goods, leading to real exchange rate appreciation according to the Balassa-Samuelson model.
Productivity	$X^{LR_{Productivity}}$	**Productivity ratios** • Relative tradable-to-non-tradable GDP ratio (US/Colombia) • Relative tradable-to-non-tradable GDP ratio (US/Colombia), MA four quarters • Per capita relative index US/COL	Positive, $\delta_{Productivity} = 1$: Given that this productivity measure reflects the differential between US and Colombian productivity, an increase suggests that foreign (US) productivity grows faster than domestic (Colombian) productivity. According to the Balassa-Samuelson model, this leads to a depreciation of the real exchange rate.
External debt	$X^{LR_{External\ debt}}$	**Net External Assets** • Net Foreign Assets (NFA) prime real • NFA (%GDP) • Real NFA • NFA PRIME (%GDP) **External Debt** • Total private external debt • Total public external debt • Total external debt	Positive, $\delta_{External\ debt} = 1$: An increase in debt raises risk perception, while a deteriorating NFA position necessitates trade balance improvement; both factors drive real exchange rate depreciation.
Negative Short-run	$X^{SR_{Negative}}$	**Interest Rate Differential** • Real lending rate differential (Colombia vs. USA). • Real deposit rate differential (Colombia vs. USA). • 90-day Time deposit rate differential - LIBOR. • Real Sovereign bond yield differential (TES[2]- U.S. Treasuries). • Real Sovereign bond Yield Differential (TES–U.S. Treasuries), 12-Month Average • Real Monetary Policy Rate Differential (MPR Colombia –Federal Funds Rate). • Real Monetary Policy Rate Differential (MPR Colombia –Federal Funds Rate), 12-Month Average. **Confidence indices** • Industrial Confidence Index. • Retail Confidence Index. **Core inflation.** **Foreign Direct Investment.**	Negative, $\delta_{SR\ Negative} = -1$: Interest rate differentials between Colombia and the United States, covering both monetary policy (MPR vs. Fed Funds Rate) and market rates (sovereign bonds like TES vs. Treasuries) tends to incentivize capital inflows from investors seeking higher yields, thereby putting pressure for a an appreciation of the real exchange rate.
Positive Short-run	$X^{SR_{Positive}}$	**Risk and Uncertainty Metrics** • Uncertainty Index. • Geopolitical Risk Index. • National Financial Conditions Index • CBOE Volatility Index	Positive, $\delta_{SR\ Positive} = 1$: Investor sentiment and risk aversion, at both global and local levels, are key determinants, in this sense variables such as the global financial cycle, the geopolitical risk index, and foreign exchange risk aversion could discourage the investors' appetite for emerging market assets, leading to a depreciation in the real exchange rate.

1. NFA and the government balance are multiplied by -1 to ensure consistency with the sign conventions used for external debt and the fiscal balance, respectively.
2. Títulos de Tesorería (Sovereign debt of Colombia).

4 Model

The neural network is composed of specific subnetwork branches that operate independently, each consisting of input layers, output layers, and sets of hidden layers connected with ReLU activation functions. The results from each independent branch are combined to obtain the model's final prediction. The model is trained using the backpropagation algorithm, employing the Adam optimizer and MSE as the optimization criterion. This training is performed over a predetermined number of epochs, during which the parameters are iteratively updated. The bootstrap resampling scheme generates multiple subsets of estimates necessary to obtain the predictions and variability of each network component.

4.1 The Architecture

The NN-ERER is based on the BDNN architecture, which consists of independent branches that jointly model the long-run and short-run components that aggregate into RER. Also, each branch is constrained by the theoretical direction of its effect. We decompose the observed RER q_t into a permanent long-run component $\overline{q}_t$, interpreted as the ERER, and a transitory short-run component $\tilde{q}_t$, which captures deviations of the RER from its equilibrium fluctuations. This decomposition is formalized by the following identity:

$$q_t = \overline{q}_t + \tilde{q}_t \tag{1}$$

The long-run component is decomposed into separate branches, each associated with a group of fundamental variables described in Table 1. Following Adler and Grisse (2017) and Salazar-Díaz et al. (2023), we consider terms of trade, fiscal expenditure, productivity, and external debt as key determinants of the ERER. Each branch processes a set of proxy variables and produces a single latent factor, denoted $LR_{t,j}$, which captures the non-linear interactions within each group and the long-run dynamics through which fundamentals affect the RER. The short-run component is divided into two branches to handle separately the positive and negative factors described in Table 1, with positive and negative expected signs of their relationship with the RER, each producing a latent factor $SR_{t,i}$ that captures transitory, business-cycle-related movements and explains deviations of the RER from the ERER.

Following Goulet Coulombe (2025), we incorporate a dedicated trend branchs that generates time-varying weights $\theta_{t,j}$ for each long-run and $\omega_{t,i}$ for each short-run blocks, allowing the contribution of each latent factor to evolve over time. In addition, we propose the inclusion of hyperparameters to encode prior information on the expected sign of specific economic relationships between the latent factors and the RER. Concretely, we define δ_j for long-run blocks and δ_i for short-run blocks as hyperparameters that take the value +1 when the expected relationship between the corresponding latent factor and the RER is positive, and −1 when it is negative, conditioning the mapping between fundamentals and the RER according to the transmission mechanisms documented in the literature. In this role, δ_j and δ_i work as hard constraints to promote estimates consistent with theoretical priors and the empirical evidence, while leaving the magnitude and functional form of the effects unrestricted. Finally, we use an aggregation block that combines the latent factors of the branches in a single architecture.

$$\underbrace{q_t}_{RER} = \underbrace{\sum_j \delta_j \theta_{t,j}\, LR_{t,j}}_{\bar{q}_t \text{ (ERER)}} + \underbrace{\sum_i \delta_i \omega_{t,i} SR_{t,i}}_{\tilde{q}_t \text{ (short run)}} + \underbrace{\varepsilon_t}_{\text{Residuals}} \tag{2}$$

Each fundamental branch, therefore, produces two conceptually distinct objects. The latent factor $LR_{t,j}$ is a non-linear index that summarizes the information in the variables of the fundamental group j, and it can be interpreted as the latent state that reflects what the fundamentals are signaling. The trend term $\theta_{t,j}$, by contrast, is a time-varying loading that governs how strongly this latent state maps into the RER at each point in time. Because $\theta_{t,j}$ is generated by a dedicated branch that takes only t as input, it captures the low-frequency, structural evolution of the corresponding transmission channel, without contaminating the non-linear mapping between fundamentals and their latent factor. The same interpretation applies to $SR_{t,i}$ and $\omega_{t,i}$ for the short-run blocks.

In this sense, $\theta_{t,j}$ plays the role of a time-varying semi-elasticity of the RER with respect to the latent state of block j, and constitutes the deep learning counterpart of the time-varying coefficients used in the BEER and VEC traditions (Beckmann et al., 2011). Whereas a standard BEER imposes a constant long-run elasticity for each fundamental (Clark & MacDonald, 1998; MacDonald, 1998), the trend branch allows this elasticity to evolve smoothly over the sample.

The architecture of the model is illustrated in Figure 1, where each branch is associated with a fundamentals group in Table 1 and its trend. The inclusion of explicit short-run branches is crucial for identification, as it allows controlling for temporary fluctuations, thereby isolating the long-run component and ensuring that the estimated ERER is purged of cyclical influences. Additionally, each branch within the long-run group allows us to decompose the contribution of fundamental latent factors in explaining the level of the ERER. Hence, we can derive the RER misalignment relative to its ERER as follows:

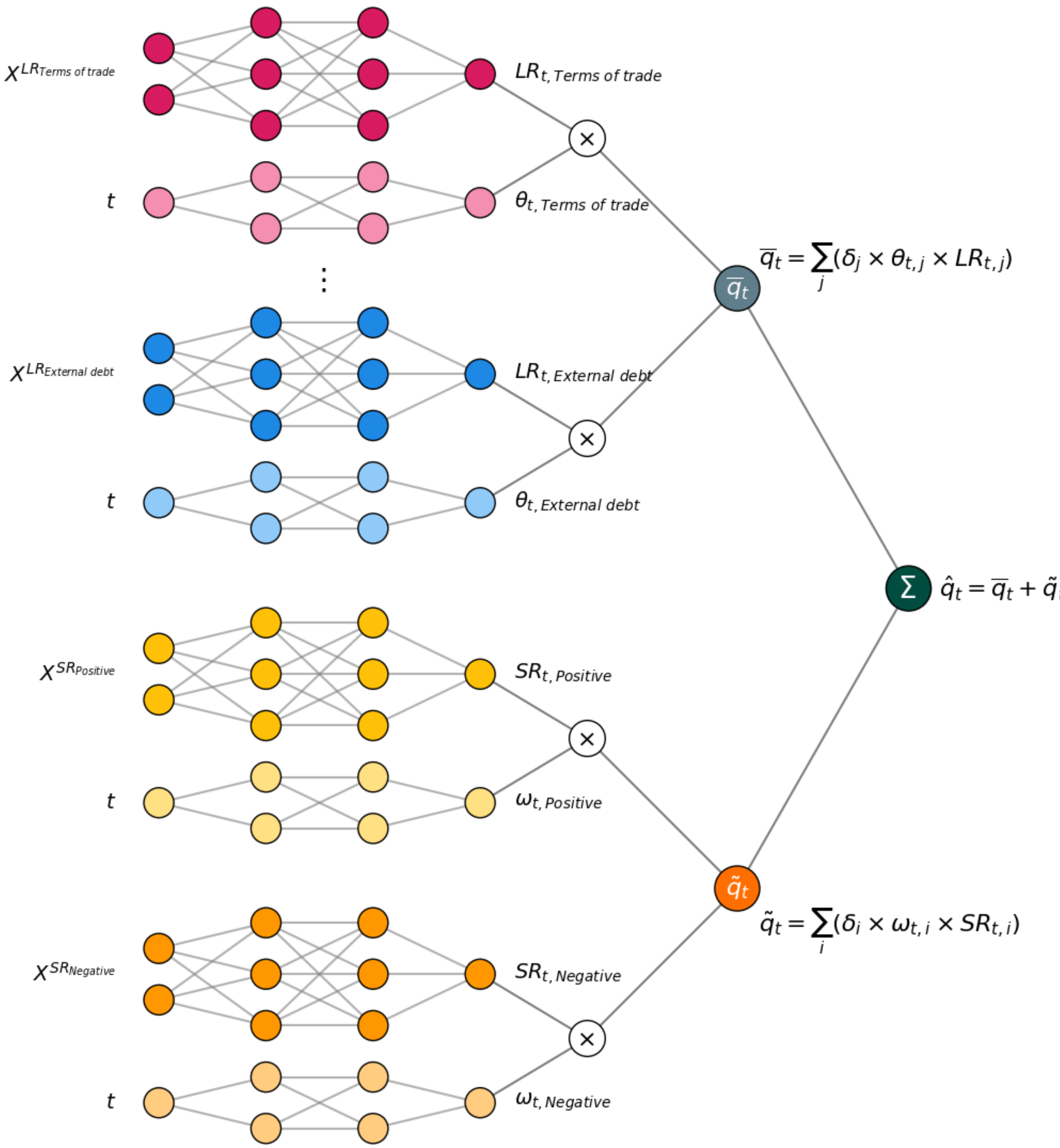

Figure 1: Arquitecture of the NN-ERER

$$\underbrace{q_t - \bar{q}_t}_{\text{RER Misalignment}} = \underbrace{\sum_i \delta_i \omega_{t,i} SR_{t,i}}_{\tilde{q}_t \text{ (short run)}} + \underbrace{\varepsilon_t}_{\text{Residuals}} \tag{3}$$

This allow identification and quantification of short-run determinants underlying deviations of the real exchange rate from its equilibrium level. In particular, the latent factors extracted from the short-run component summarize transitory shocks and cyclical dynamics that generate temporary misalignments of the real exchange rate relative to the ERER. This framework enables a decomposition of observed misalignments into economically interpretable components, providing empirical evidence on the short-run transmission mechanisms operating around the long-run equilibrium and enhancing the understanding of real exchange rate dynamics

4.2 Estimation and Tuning

The architecture of the model is organized into branches that capture the long-run and short-run dynamics that affect the RER. Each branch is implemented as a standard feedforward fully connected neural network with three hidden layers of 400 neurons each. Also, the trend branches that uses a feedforward fully connected networks with hidden three layers of 100 neurons each.

Hyperparameters are common across branches and include a dropout rate of 0.2 and a learning rate of 0.02. Training is conducted for up to 500 epochs with early stopping defined by a patience of 50 epochs and a mean squared error tolerance of 0.001. Since the objective of the model is not to forecast the observable RER but to estimate a non observable variable ERER, all available observations are used for training the model.

All variables incorporated in each branch are standardized to have zero mean and unit variance. In the same spirit of Goulet Coulombe (2025), to control for differences in the number of variables across branches, each standardized variable is divided by the square root of the number of variables in its corresponding branches. This normalization mitigates the risk of overrepresentation of branches with a larger set of explanatory variables.

The estimation is performed using 200 bootstrap replications with a sampling rate of 0.85, which allows us to construct confidence intervals for the estimated coefficients, the latent variables, and the contribution of each group of long-run and short-run fundamentals. Also, as we mention before, for each group of varaibles, we include a hyperparameter that takes the value of one when the literature identifies a positive relationship between the corresponding group of fundamentals and the RER, and minus one when the literature identifies a negative relationship, the details of groups is explaned in section 3.

4.3 Surrogate models

The branches of the NN-ERER are flexible non-linear mappings whose internal structure is not directly interpretable. To open this black box and assess which variables drive each latent factor we rely on a surrogate modeling strategy that provides a post-hoc, model-agnostic approximation of each branch (Páez, 2024). The aim is not to improve the predictive performance of the model, but rather to recover the functional relationships learned by the neural network.

Specifically, we fit one independent surrogate model for each of the six latent factor branches, the four long-run and two short-run blocks. For a given branch, the surrogate is trained to reproduce the mapping from the variables of that block, X^{LR_j} or X^{SR_i}, onto the corresponding latent factor, $LR_{t,j}$ or $SR_{t,i}$, produced by the network. This preserves the modular interpretation of the architecture, where the importance of a variable is assessed only within the block to which it belongs.

We employ Random Forests (Breiman, 2001) as the surrogate model because they offer several advantages for our purpose. First, the Random Forests regressor preserves the non-linear relationships and higher-order interactions captured by each branch, so that the approximation remains faithful to the network. Second, they are robust to the strong multicollinearity that characterises our blocks, where several closely related proxies enter the same branch. Third, they deliver a native measure of variable importance, which allows us to rank the inputs within each block. The adequacy of the approximation is documented in Figure A1, where the latent factors reproduced by the surrogate Random Forests track the original factors generated by the network closely across all branches.

The hyperparameters of each surrogate Random Forest were selected through a grid search procedure combined with time-series cross-validation, which preserves the temporal ordering of the observations while systematically evaluating alternative model configurations. Specifically, different combinations of the number of trees, tree depth, and minimum node sizes were assessed, and the specification yielding the highest cross-validated performance was retained. Variable relevance was subsequently quantified using the impurity-based importance criterion, which measures the normalized reduction in prediction error attributable to each predictor across all splits and trees in the ensemble.

5 Results

5.1 RER Behavior and determinants

As discussed in Section 4, the estimation of the ERER within the NN-ERER model allows us to identify both the long-run and short-run components that shape the behavior of the observable RER. Each component yields a latent factor that captures and summarizes the dynamics of a set of variables, along with a time-varying coefficient. The contribution of each factor to the observable RER is defined as the product of the latent variable and its corresponding coefficient. The behavior of the coefficient for each latent factor and the contribution is presented in Figures 2 and 3.

In particular, the long-run component corresponds to the ERER and is constructed from groups of variables related to terms of trade, fiscal policy, productivity, and external debt. As noted above, these groups are well established in the literature as key determinants of the RER in the long run, and provide a framework for capturing the unobservable dynamics of the ERER through their relationship with underlying fundamentals.

Similarly, the short-run component estimates latent factors, time-varying coefficients, and their associated contributions. This component captures the determinants of short-run dynamics and helps explain the deviations of the RER from its equilibrium level. For both the long-run and short-run branches, the coefficients exhibit time-varying behavior over the sample period, and the sign of each branch aligns with the restrictions imposed, as detailed in Table 1.

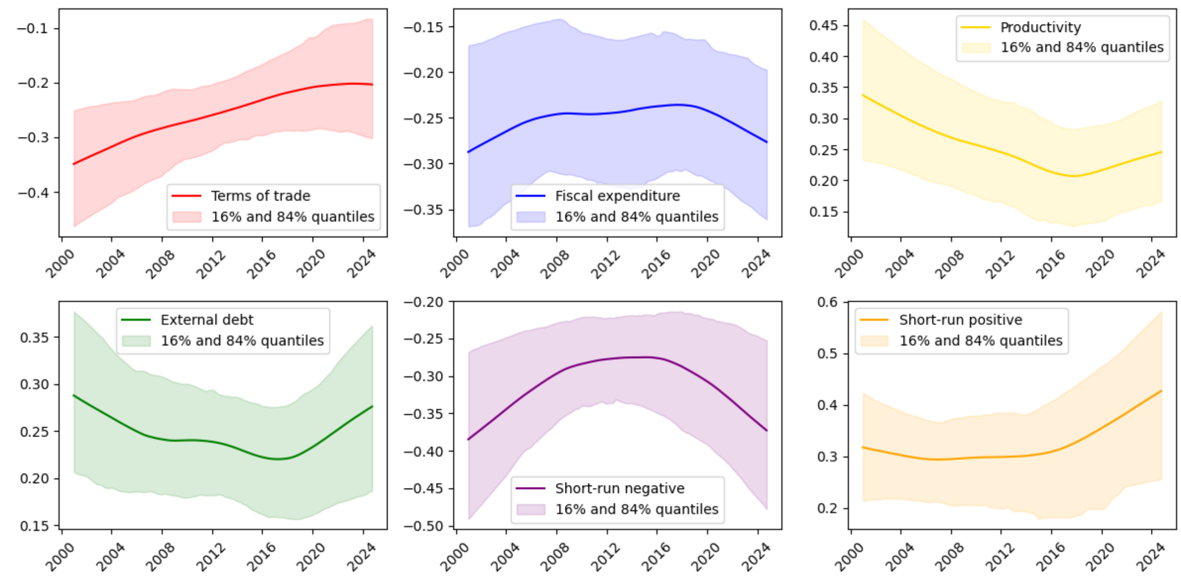

Figure 2: Trend coefficients for the latent variable gaps in the long-run and short-run branches.

In terms of the long-run coefficients, the Terms of trade present an increase compared with the estimation obtained in the beginning of 2000, and recently correspond to the greatest value of this coefficient, which could be associated with a reduction in the magnitude of the negative effect associated with increases in terms of trade. Meanwhile, fiscal expenditure has a concave behavior, showing a coefficient value at the end of the period similar to that estimated for 2000, with some variation throughout the periods, and a greater value of the coefficient after 2016.

For long-run branches with a positive effect,productivity and external debt show a convex behavior. For the productivity coefficient, the values have been lower than those observed at the beginning of the period, while the external debt shows a value similar to that observed at the beginning of the period.

In the short run, the coefficient associated with the latent variables that capture the negative effect on the RER has a concave behavior, with a level similar to that we estimate in the beginning of the sample. However, the coefficient associated with short-run variables that has a negative effect on the RER shows a strong increase in its magnitude, which suggests that short-term factors that negatively affect the real exchange rate have increased in relevance over time.

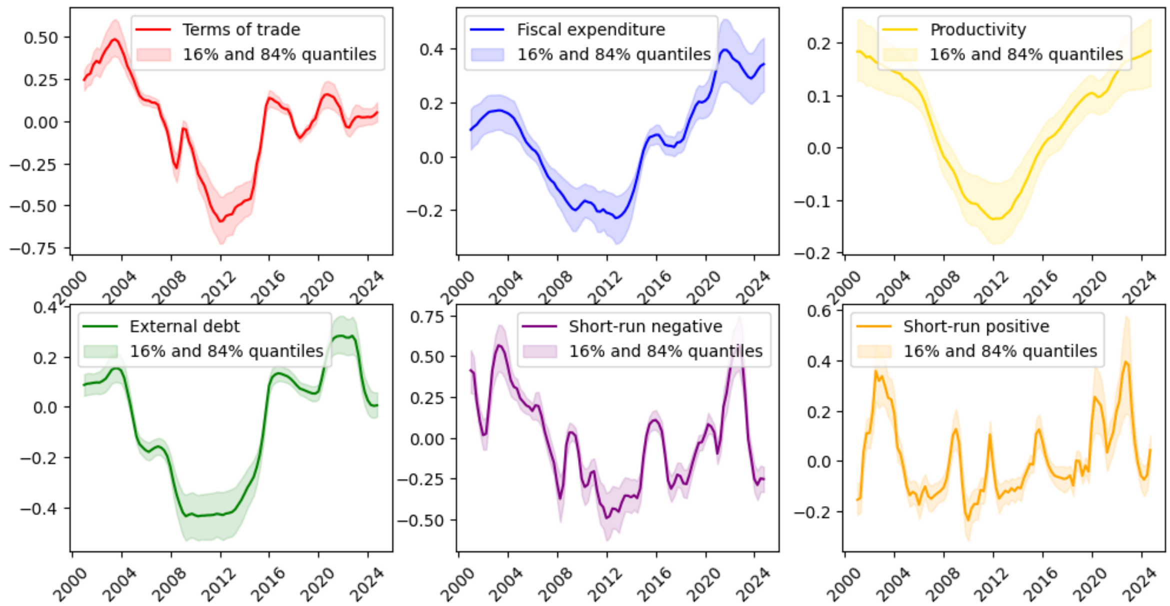

Figure 3: Contributions of the long-run and short-run branches to the RER.

The contribution behavior, understood as the product of the coefficient and the latent factor for each branch, is presented in Figure 3. On the other hand, Figure 4 presents the fitted RER and how each component affects its behavior according to the NN-ERER model. At the period between 2000 and 2008, the contribution of the branches from the long run and short run behavior explains the depreciation of the RER, with a relevant contribution of the Terms of Trade and the short run variables.

The positive contribution of the terms of trade during this period may be associated with low levels of income derived from commodity exports, which partly explained the depreciation observed at the beginning of the 2000s. As commodity prices, particularly oil, began to rise, an appreciating trend in the real exchange rate emerged. At the same time, the short-run component made a positive contribution at the beginning of the sample, likely reflecting heightened geopolitical uncertainty and increases in volatility indices that contributed to the observed depreciation.

This pattern was most evident between 2000 and 2006, when depreciation pressures gradually moderated in line with rising commodity prices and a decline in key indicators of volatility and uncertainty, which shaped the behavior of the short-run component.

Subsequently, a negative contribution is observed from the terms of trade, followed by the external debt component and short-run factors over the period from 2007 to mid-2015. This behavior is partly explained by the upward trend in oil prices, which reached elevated levels in 2008 due to strong global demand driven largely by emerging economies, particularly in Asia. After the decline in prices during the global financial crisis, oil prices recovered along with improvements in the terms of trade.

The increase in oil prices also contributed to a moderation of the country's net foreign asset position, as higher export revenues improved external balances. This explains the significant negative contribution of the external debt component during this period, together with a moderation in total debt as a share of GDP supported by increased oil-related revenues. These developments ocurred in a context of improved relative productivity, which exerted appreciation pressures on the exchange rate, in line with the Balassa-Samuelson effect. In addition, the fiscal consumption component contributed negatively during this period.

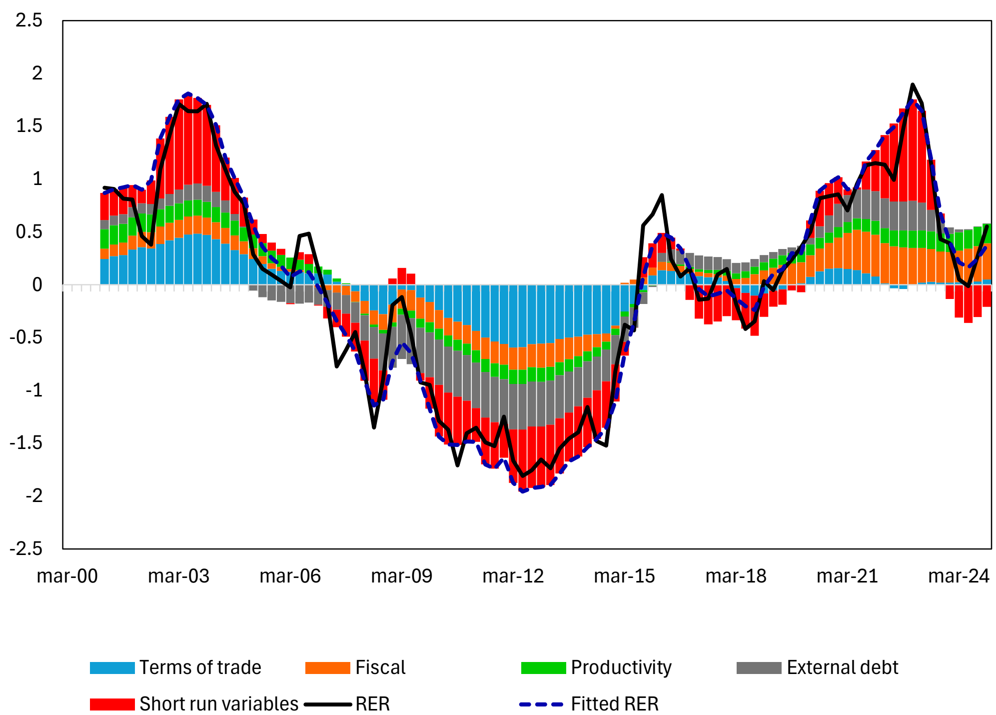

Figure 4: Decomposition of the RER based on the contributions of the long-run and short-run branches.

The short-run component that generated appreciation pressures can be linked to rising inflation and the subsequent increase in interest rates in Colombia, particularly between 2006 and 2008. These dynamics widened interest rate differentials and contributed to the appreciation of the real exchange rate. Relatively high interest rate differentials persisted from around 2010 to mid-2015, consistent with a contractionary monetary policy stance adopted in response to inflationary pressures. Between 2008 and 2014, an improvement in the net foreign asset position contributed to the appreciation of the real exchange rate. This was reinforced by a moderation in central government expenditure as a share of GDP between 2010 and 2011, which further supported the appreciation observed during this period.

By the end of 2015, a depreciation of the real exchange rate is observed, in line with the decline in oil prices. This generated a positive contribution from the terms of trade, explaining part of the depreciation, along with inflationary pressures that increased interest rate differentials and contributed to depreciation through the short-run component. An appreciation followed until 2018, during which the short-run component remained particularly relevant. From 2019 onward, a marked trend of depreciation is observed until the end of 2022, driven by the contributions of fiscal factors, external debt, and short-run dynamics.

This pattern unfolded in the context of the COVID-19 crisis and the subsequent economic recovery after 2020, characterized by a significant increase in household consumption and expansionary fiscal policies aimed at supporting economic activity. These developments, along with rising public debt, help explain the depreciation associated with the external debt component. At the same time, a deterioration in relative productivity generated additional depreciation pressures through the productivity component.

The short-run component during this period was shaped by increasing interest rate differentials driven by a sharp rise in inflation after the pandemic, in the context of economic recovery and supply shocks related to higher food prices linked to the war between Russia and Ukraine. In addition, higher levels of geopolitical uncertainty and volatility further contributed to the short-run dynamics. After 2022, an appreciation of the real exchange rate is observed, mainly driven by short-run factors associated with lower interest rate differentials. Nevertheless fiscal and productivity components continue to exert depreciation pressures.

5.2 ERER Estimation and determinants

Based on the above, the ERER estimation is obtained by excluding the short-run contribution of the RER, as defined in equation 2. The resulting ERER dynamics, incorporating the contributions of the long-run components and compared with the observed RER, is presented in Figure 5.

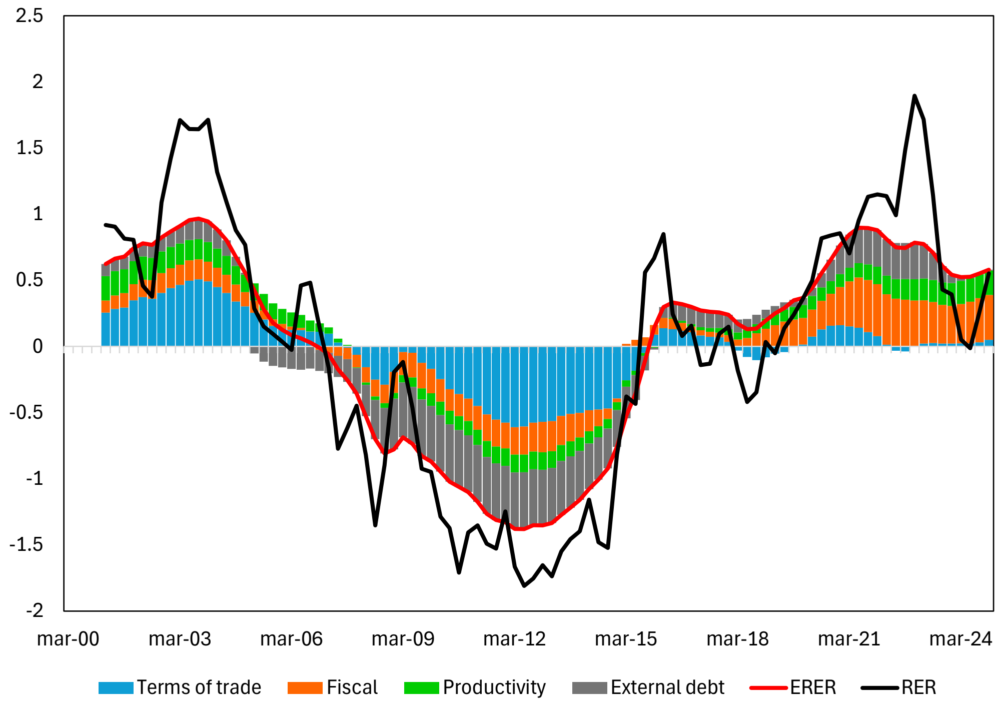

Figure 5: Decomposition of the ERER based on the contributions of the long-run components.

As shown in Figure 5, the ERER is determined by a set of variables identified in the literature as key drivers of the long-run behavior of the RER. Furthermore, the observed RER fluctuates around the ERER, which captures its structural and fundamental dynamics.

Several periods in which the estimated ERER reflects clear trends of appreciation or depreciation were identified in the sample. At the beginning of the sample, around 2000, the ERER exhibits a depreciation, mainly explained by the contribution of the terms of trade in a context of low commodity prices, particularly oil prices. Other groups of variables, such as External Debt, Fiscal conditions, and Productivity, also contribute to this depreciation.

After 2003, the contribution of the terms of trade declines, and around 2005, the external debt begins to contribute to an appreciation of the ERER. This pattern is associated with rising oil prices and an improvement in net foreign assets. From 2006 onward, a marked appreciation of the ERER is observed, driven by the joint behavior of the terms of trade, external debt, productivity, and fiscal variables. This is largely attributed to the increase in oil prices prior to the 2008 crisis and its subsequent effects on these fundamental variables, as discussed above.

Around 2015, the ERER shows a marked shift toward depreciation, which intensifies in the post-COVID-19 period. This trend is largely explained by the contributions of the Fiscal, Productivity, and External Debt variables, consistent with the effects of expansionary monetary and fiscal policies, the deterioration of external debt positions, and a decline in relative productivity measures.

This finding aligns with the implementation of expansionary fiscal policies aimed at mitigating the pandemic's impact on GDP, likely activating a fiscal dominance channel in which rising debt concerns outweigh traditional demand-side appreciation effects. It is also important to highlight the absolute contributions in order to better understand both the relative importance and magnitude of each group of fundamental variables.

This pattern is consistent with the non-linear behavior of the estimated coefficients, which allows the model to capture changes in the relative importance of the underlying determinants of the ERER. It may also suggest structural changes in the economy associated with these fundamental variables. Notably, the Fiscal component has recently increased its relative weight, becoming a dominant factor in explaining ERER behavior compared to historical norms.

5.3 RER misalignment

We define the RER misalignment as the difference between the observed RER and the estimated ERER. These results are presented in Figure 6, where a positive value indicates a depreciation of the RER above the ERER, while a negative value indicates an appreciation below it.

The first strand of the literature explains the real exchange rate misalignment by separating its transitory and persistent sources. Within the behavioral equilibrium exchange rate (BEER) framework, current misalignment is defined as the deviation of the observed real exchange rate from the equilibrium produced by the values of its fundamentals (Clark & MacDonald, 1998). Because these contemporaneous values can themselves depart from sustainable levels, Clark and MacDonald (2000) extend the approach of the BEER by decomposing the fundamentals into permanent and transitory components, using Johansen cointegration and Gonzalo-Granger common factor decomposition within a VECM and identify the equilibrium from the permanent component alone. The gap between the observed rate and the permanent equilibrium exchange rate (PEER) is the misalignment. The permanent–transitory decomposition thus attributes a given misalignment to different volatility sources, among them, the interest rate differentials, a feature that this article includes in the short-run component.

A second, more recent strand could attribute misalignment to policy distortions without estimating an equilibrium real exchange rate in the BEER or FEER sense. In the External Balance Assessment (EBA) framework developed at the IMF (Phillips et al., 2013), the real effective exchange rate is first modeled in a positive (descriptive) panel regression; the normative stage then defines a REER norm as the regression-implied rate that would be obtained if policy variables were set at their desirable medium-term levels, holding the remaining regressors at their observed values. The deviation of the observed rate from this norm; what they call the REER gap, which plays the role of misalignment in this framework, decomposes, analogously to the current-account gap, into identified policy-gap contributions (the estimated effect of each policy's deviation from its benchmark) and a residual capturing unobserved fundamentals, distortions, and regression error. The policy gaps entering the REER regression span foreign-exchange intervention interacted with capital controls, financial policies with private credit, public health spending, and monetary policy as proxy. Because the norm is conditioned on desirable policy settings rather than on fundamentals at sustainable levels, the EBA gap is conceptually distinct from the equilibrium-based misalignment of the BEER and PEER approaches.

For Colombia, the canonical reference is the VECM-based application Echavarría et al. (2005). Using Johansen cointegration during the period 1958–2005, they modeled the RER and, rather than decomposing the misalignment into permanent and transitory or policy components, they explain the major appreciation and depreciation episodes through a contribution accounting that combines the observed change in each fundamental with its estimated impulse-response elasticity. This yields a linear fundamental-based attribution of Colombian real exchange rate movements, which the present study extends through a macroeconomic informed neural network for the estimation of the ERER and the attribution of its misalignment.

The methodology that this paper proposes is meant to understand the misalignment by including a complete branch of determinants that may produce a considerable level of volatility and deviations from the ERER in what is considered short-run. As noted in Table 1, the architecture recognizes two types of short-run variables: negative short-run $X^{SR_{Negative}}$ includes variables that the literature associates with the appreciation of the RER; such as interest rate differentials, through uncovered parity of interest rates; confidence indexes, core inflation, and FDI flows. On the other hand, positive short-run $X^{SR_{Positive}}$ includes variables that the literature associates with the depreciation of the RER, particularly in open small economies; such as volatility and uncertainty indexes. Thus, the aggregation and incorporation of these factors produces the Figure 6.

Under this methodology, it is possible to identify different periods in which the RER deviated considerably from its equilibrium. For example, between 2008 and 2009, as well as the post-COVID-19 years after 2021, are primarily characterized by an overvaluation of the observed RER driven by short-run dynamics. These results help to better understand the determinants of overshooting behavior. In recent periods following the COVID-19 shock, we observed a depreciation of the observed RER that is not fully aligned with the evolution of fundamental variables. This divergence is mainly explained by short-run factors, potentially associated with increased volatility and uncertainty in global markets, particularly at the onset of geopolitical conflicts.

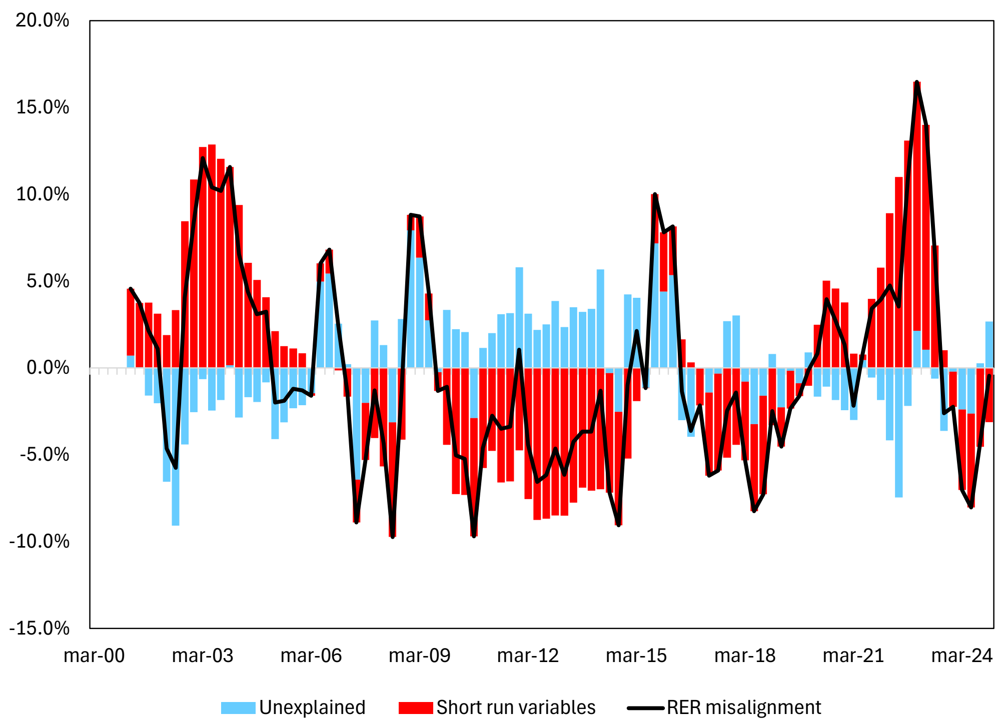

Figure 6: RER misalignment and its decomposition into the short-run $\tilde{q}_t$ and unexplained components.

Following the peak in inflation reductions in Colombia's policy interest rate, exchange rate differentials narrowed, reinforcing short-run depreciation pressures. Subsequently, the moderation of geopolitical risks and market volatility, along with the decline of uncertainty, contributed to an appreciation of the RER, reflected in the short-run component of the decomposition.

5.4 Importances

The surrogate model methodology allows us to measure the explanatory capacity of the variables included in the fundamentals over the variability of the response. In general, the results of this exercise seem to characterize a traditional developing economy in the literature. Additionally, the ability of the surrogate Random Forest models to reproduce the latent factors generated by the neural network is illustrated in Figure A1, where the predicted latent factors closely follow the original ones.

The hyperparameter selection procedure revealed that relatively parsimonious specifications were generally preferred across branches. In most cases, the optimal models required between 10 and 50 trees, while the selected tree depths ranged from shallow structures with three levels to unrestricted depths. Likewise, the minimum node sizes remained close to their default values, with only minor adjustments in a few cases. These results suggest that the latent factors generated by the neural network can be adequately approximated without resorting to overly complex forest structures, thereby reducing the risk of overfitting while preserving the nonlinear relationships embedded in each branch.

Figure 7 shows the main variables by fundamental blocks to represent latent factors. From the first subplot, Remittances (as a share of GDP) constitute the main source of variability within the Terms of Trade group. According to the findings of Carare et al. (2025), remittances are associated with an appreciation of the RER, particularly in countries with flexible exchange rate regimes. Furthermore, shocks to remittances may affect not only the equilibrium level but also the degree of misalignment (Lopez et al., 2007), highlighting their relevance in the dynamics of open-developing economies such as Colombia.

Within the External Debt group, Total public external debt concentrates approximately 50% of the importance, while Total external debt accounts for around 87%, leaving a secondary role for Total private external debt and other related indicators.

Regarding Fiscal Policy, the Balance of the National Government and its share of GDP explain more than 70% of the variability of the latent factor. In contrast, the remaining three groups exhibit variables with similar importance levels, which means that several indicators jointly contribute to explaining variability, with no single variable dominating the explanatory power in the Random Forest models.

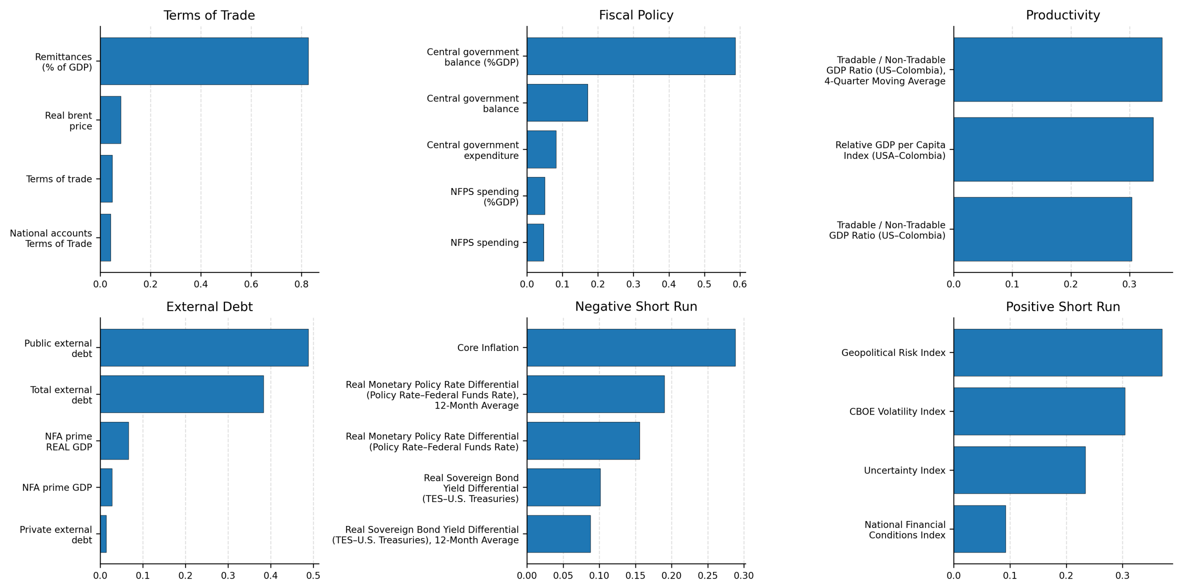

Figure 7: Variables with greatest importance in the representation of latent factors by fundamental blocks

6 Comparative analysis of ERER measures

As expected in the estimation of latent variables, this section compares the results of the methodology with an estimate of the ERER obtained by applying the Hodrick–Prescott (HP) filter to the observed RER (Hodrick & Prescott, 1997). In this approach, the trend component extracted by the HP filter is interpreted as an approximation of the ERER, providing an alternative estimate of this unobservable latent variable (see Figure 8). Compared with the ERER estimated using the NN-ERER methodology, both measures exhibit a common trend, particularly the marked appreciation following the rise in oil prices around 2003. This appreciation was subsequently reversed, leading to a depreciation of the ERER in both measures after 2015, which became more pronounced during the COVID-19 period.

Although both approaches capture a similar overall pattern, NN-ERER methodology shows a stronger increase in the ERER prior to 2015, reaching a higher level in 2016 than the HP-based estimate. These results are consistent with a greater contribution of the terms of trade, external debt, and fiscal fundamentals, which evolved in line with the decline in oil prices. These findings suggest that the proposed methodology provides a useful framework for characterizing the ERER as an unobservable variable, complementing traditional approaches. In particular, it is able to capture specific changes that may alter the equilibrium exchange rate through the nonlinear relationships between macroeconomic fundamentals and the RER. Such nonlinear effects are explicitly accounted for by the NN-ERER methodology, allowing for a richer representation of the determinants of the equilibrium exchange rate.

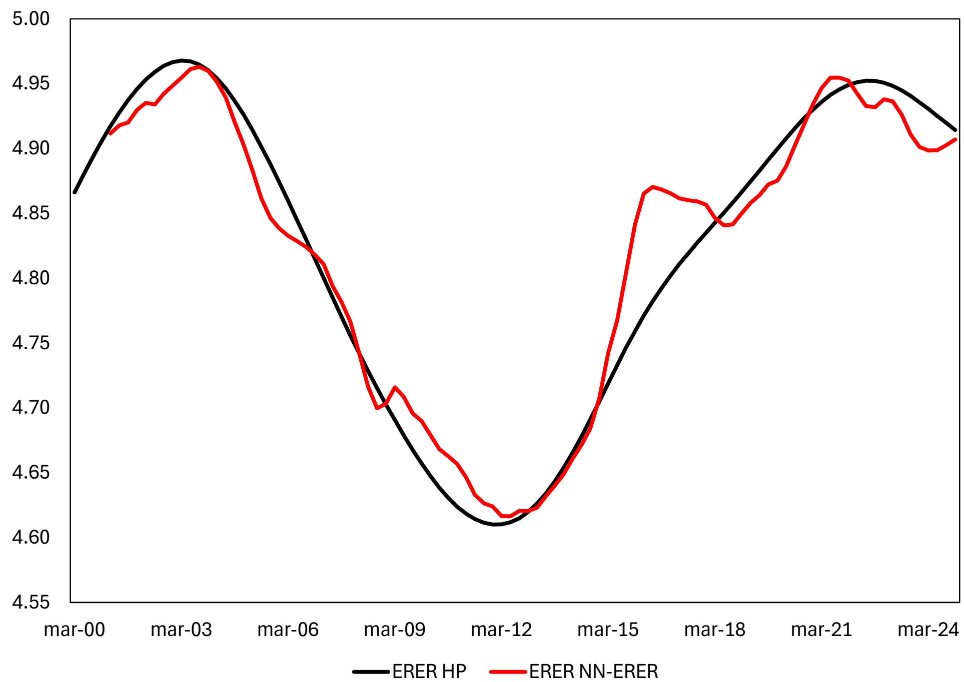

Figure 8: Comparative between NN-ERER and ERER HP. The variable is the logarithm of the multilateral real exchange rate index.

This exercise also compares the misalignment of the ERER obtained using the estimation with the HP filter to the misalignment derived from the NN-ERER methodology (see Figure 9). The results allow to compare the dynamics of both measures and show that they identify several common periods of misalignment. In particular, both approaches indicate that the RER was significantly over-depreciated following the COVID-19 shock and over-appreciated during the period between 2010 and 2015.

However, the estimated degree of misalignment differs after 2015, following the oil price shock. The NN-RER methodology suggests a smaller RER misalignment than the HP-based approach. This result is driven by the stronger depreciation of the ERER estimated by the first methodology relative to the last. Such behavior is consistent with the larger contribution of long-run fundamentals, particularly the terms of trade, external debt, and fiscal factor, which experienced significant adjustments during this period.

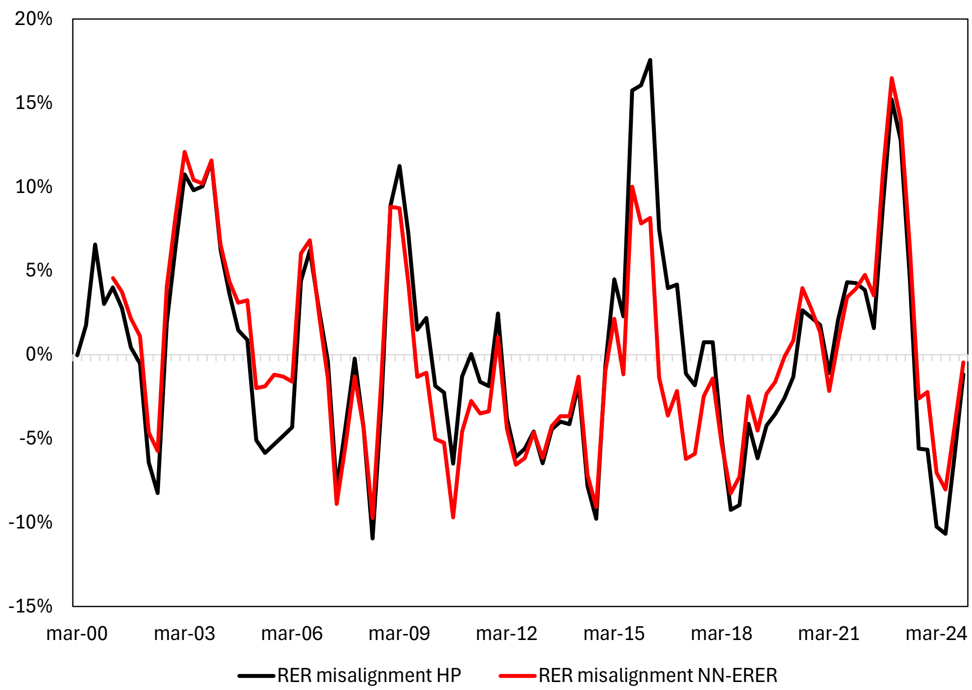

Figure 9: RER misalignment and its decomposition into the short-run $\tilde{q}_t$ and unexplained components.

These findings highlight the ability of the proposed methodology to identify periods of misalignment that are broadly consistent with those obtained using conventional approaches while providing an economically grounded reconstruction based on macroeconomic fundamentals. Moreover, the framework allows to distinguish between long-run equilibrium movements and short-run factors affecting the RER, thereby offering additional insights into the drivers of RER misalignment. In this sense, the NN-ERER methodology complements existing approaches by providing a novel framework for estimating an unobservable latent variable such as RER misalignment while accounting for the nonlinear dynamics linking economic fundamentals to the real exchange rate.

7 Conclusions

This paper proposes a NN-ERER framework for estimating the ERER for Colombia and characterizing its misalignments. The approach decomposes the observed RER into long-run and short-run components, with the long-run component identified as the ERER and constructed from latent factors associated with terms of trade, fiscal policy, productivity, and external debt. The model also integrates theoretical restrictions derived from the transmission channel literature into the network architecture, by encoding the expected sign of each branch. The use of time-varying coefficients allows the model to accommodate changes in the relative importance of these fundamentals over the sample period, providing a flexible framework for capturing the dynamics of an unobserved variable such as the ERER.

The methodology contributes to the literature in three directions. First, it brings together macroeconomic transmission mechanisms and deep learning by embedding theoretically motivated sign restrictions, documented in the BEER literature, directly into the architecture of the network. This design preserves the flexibility of the model in capturing non-linear interactions among fundamentals while keeping its outputs aligned with economic theory. Second, the framework enables a decomposition of both the long-run dynamics of the RER and its misalignment into interpretable blocks of fundamentals, partially addressing a well-documented limitation of BEER and FEER approaches, which deliver misalignments only as residuals from a linear equilibrium. Third, by combining the network with surrogate models, the methodology delivers suggestive evidence on the individual variables most closely associated with the dynamics of the ERER and with deviations of the RER from its estimated equilibrium.

The results indicate that the relative contribution of the fundamentals underlying the ERER has evolved over time. Terms of trade and external debt are particularly relevant for explaining the appreciation observed during the commodity price boom of the mid-2000s, while productivity and Balassa-Samuelson-type effects played a complementary role. The fiscal component, by contrast, has gained relative weight in recent years, becoming a more prominent factor in explaining the depreciation observed in the post-COVID-19 period. This pattern is consistent with the implementation of expansionary fiscal policies during the pandemic and with a deterioration in external debt positions, and may suggest the activation of channels through which fiscal vulnerabilities affect the long-run RER.

The decomposition of misalignments into short-run components and an unexplained residual provides additional insights into episodes of overshooting. Periods such as 2008–2009 and the years following 2021 are characterized by deviations of the observed RER from the estimated ERER that are largely attributable to short-run factors, plausibly linked to global volatility, uncertainty, and shifts in interest rate differentials. The methodology thus allows for a separation between fundamentals-driven dynamics and transitory pressures, which can be informative for the analysis of RER behaviour around episodes of external turbulence. In this sense, the NN-ERER model not only delivers a non-linear and interpretable specification of the estimated ERER, but also offers an alternative that helps identify the short-run factors associated with RER misalignments, filling a gap that the standard BEER methodology does not address. The framework also contributes to the growing literature on the application of machine learning models to the estimation of unobservable variables that are relevant in macroeconomic analysis.

The application of surrogate Random Forest models complements these findings by identifying the variables that account for most of the variability within each group of fundamentals. Remittances emerge as a relevant component of the terms of trade factor, consistent with the literature on the role of remittance flows in shaping the RER of developing open economies (Carare et al., 2025; Lopez et al., 2007). Public external debt accounts for the bulk of the variability within the external debt component, and the central government balance dominates within the fiscal factor. These results help characterize the channels through which the underlying fundamentals are transmitted to the ERER and align with stylized features of a developing economy with significant external linkages.

Several caveats are worth noting. The interpretation of the long-run component as the ERER relies on the identification assumptions imposed on the model, and the estimated misalignments should be read as informative signals rather than as deterministic indicators of disequilibrium. The unexplained residual in the misalignment decomposition reflects the limits of any reduced-form approach to characterizing an unobserved variable, and the time-varying coefficients, while flexible, do not provide a structural interpretation of the mechanisms behind the observed changes in importance. Future research could extend the framework to incorporate alternative identification strategies and additional fundamentals associated with the financial account and cross-border balance sheets.

Overall, the proposed methodology offers a flexible alternative for estimating the ERER and analyzing its misalignments in a setting characterized by complex, potentially non-linear interactions among fundamentals. Estimates of the ERER and the corresponding misalignments constitute a useful input for the monitoring of macroeconomic conditions in Colombia, particularly in light of the structural changes documented in the relative importance of fiscal, productivity, and external debt fundamentals over the past two decades.

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8 Appendix

Table A1: Description of fundamental variables

Fundamental groups	Variables	Variable name	Source
Terms of trade	Terms of trade based on National accounts	ITI_CN	DANE[1].
	Terms of trade based on PPI	TOT_CE	Banrep[2].
	Real Brent Price	brent_real	FRED[3].
	Remittances (% of GDP)	Remesas_PIB	Banrep.
Fiscal	Public consumption	CONSP	DANE.
	Public consumption (% GDP)	CONSP_PIB	DANE.
	Public consumption (% total consumption)	CONSP_CONST_SUM	DANE.
	Central Goverment Expenditure	GNC_Gastos	MHCP[4.].
	Central Government Expenditure (% GDP)	GNC_Gastos_PIB	MHCP.
	Non-financial Public sector (NFPS) Expenditure (% GDP)	SPNF_Gastos_PIB	MHCP.
	Central Government Balance	GNC_Saldo	MHCP.
	Central Government Balance (%GDP)	GNC_Saldo_PIB	MHCP.
Productivity	Relative tradable-to-non-tradable GDP ratio (US/Colombia)	PIB_NT_T_RELATIVO	DANE, FRED.
	Relative tradable-to-non-tradable GDP ratio (US/Colombia), MA four quarters	PIB_NT_T_RELATIVO_PM4T	DANE, FRED.
	GDP per capita relative index US/COL	PER_CAPITA_2	DANE, FRED.
External debt	Net External Assets (NEA) prime real	AEN_PRIME_real	Banrep.
	NEA (%GDP)	AEN_PIB_AG	Banrep.
	Real NEA	AEN_REAL_AG	Banrep.
	NEA PRIME (%GDP)	AEN_PRIME_PIB_AG	Banrep.
	Total private external debt	Dx_privada	Banrep.
	Total public external debt	Dx_publica	Banrep.
	Total external debt	Dx_total	Banrep.
Negative Short-run	Real lending rate differential (Colombia-US)	DIF_LR	Banrep, FRED.
	Real deposit rate differential (Colombia-US)	DIF_DR	Banrep, FRED.
	90-day Time deposit rate differential - LIBOR	DIF_INT_LIBOR	Banrep, FRED.
	Real Sovereign bond yield differential (TES-US Treasuries)	DIF_TES_R	Banrep, FRED.
	Real Sovereign bond Yield Differential (TES–U.S. Treasuries), 12-Month Average	DIF_TES_R_12M	Banrep,FRED.
	Real Monetary Policy Rate Differential (MPR Colombia –Federal Funds Rate)	DIF_TPM_R	Banrep,FRED.
	Real Monetary Policy Rate Differential (MPR Colombia –Federal Funds Rate), 12-Month Average	DIF_TPM_R_12M	Banrep,FRED.
	Industrial Confidence Index	IDX_IND	Banrep
	Retail Confidence Index	IDX_COM	Fedesarrollo[5].
	Core Inflation	IPCB_COL	Banrep,DANE.
	Foreign Direct Investment (FDI) in Colombia	IED_COL	Banrep.
Positive Short-run	Uncertainty Index.	IPEC	Fedesarrollo.
	National Financial Conditions.	NFCI	Fedesarrollo.
	CBOE Volatility Index: VIX	VIX	CBOE[6].
	Geopolitical Risk Index	GPR	GPR[7].

1. Departamento Administrativo Nacional de Estadística (National Statistics Office) `https://www.dane.gov.co`.

2. Banco de la República (Central Bank of Colombia) `https://www.banrep.gov.co/es/estadisticas`.

3. Federal Reserve of St. Louis `https://fred.stlouisfed.org/`.

4. Ministerio de Hacienda y Crédito público (Ministry of Finance and Public Credit).

5. Fundación para la Educación Superior y el Desarrollo.

6. CBOE Global Markets.

7. Geopolitical Risk Index. Data downloaded from `https://www.matteoiacoviello.com/gpr.htm` on december, 2025.

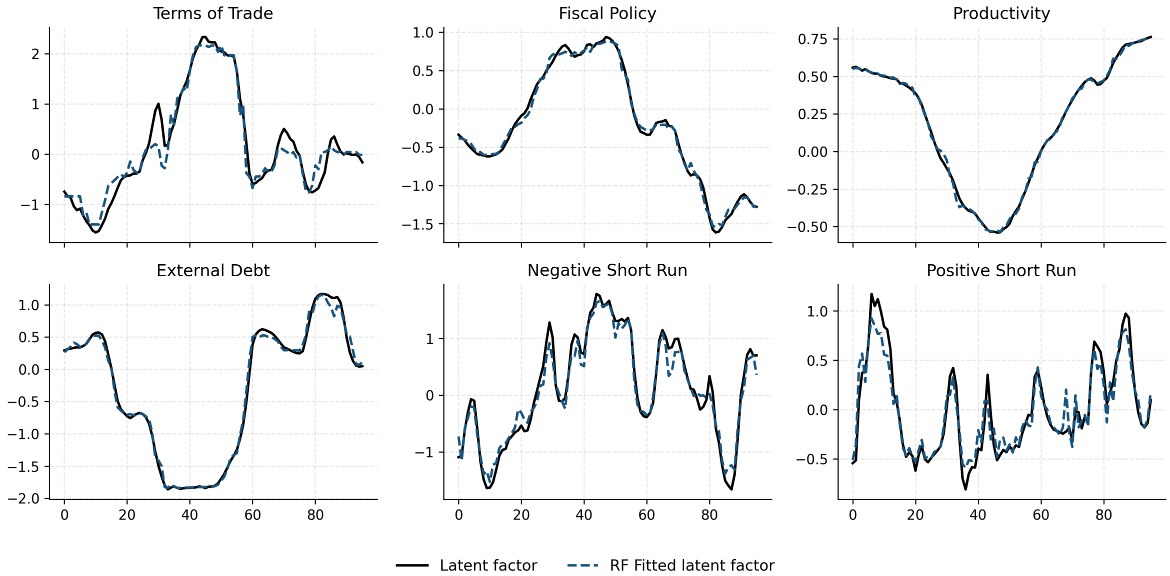

Figure A1: Adjusted latent factors.