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Approach to Estimating Confidence Intervals for a Business Cycle*

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Abstract

This document introduces a novel business-cycle turning-point analysis method that leverages the nonparametric coincident profile tool to construct confidence intervals for turning-point dates. The novel method generalized the coincident profile tool by providing a matrix of coincident relationships among a set of variables. We refer to this object as the coincident matrix. Through a numerical study and two empirical applications: one using economic data from the United States and the other from Colombia, we demonstrate the accuracy of the method in identifying turning points, closely aligning with the reference cycle in each case. In addition, in our analysis of United States economic data, we conduct a pseudo-out-of-sample analysis that further validates the method's superior performance in predicting turning-point dates.

Keywords: Business cycles, Turning points, Non-parametric test, Coincident Profile, Confidence intervals.

JEL Codes: C14, C15, E32, E37

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Un enfoque para estimar intervalos de confianza del ciclo de los negocios*

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Resumen

Este documento presenta un método novedoso para el análisis de los puntos de quiebre del ciclo económico que aprovecha la herramienta no paramétrica de perfil coincidente para construir intervalos de confianza alrededor de las fechas de dichos puntos de quiebre. Este método novedoso generaliza la herramienta del perfil coincidente poniendo en una matriz las relaciones de coincidencia entre un conjunto de variables dadas. A este método lo denominamos matriz de coincidencia. A través de un estudio numérico y dos aplicaciones empíricas: una con datos económicos de Estados Unidos y otra de Colombia, demostramos la precisión del método para identificar puntos de quiebre, que se alinean estrechamente con el ciclo de referencia de cada caso. Además, en nuestro análisis de datos económicos de Estados Unidos, realizamos un análisis pseudo-fuera de muestra que confirma aún más el excelente rendimiento del método para predecir las fechas de estos puntos de quiebre.

Palabras Clave: Ciclos económicos, Puntos de quiebre, Prueba no paramétrica, Perfil coincidente, Intervalos de confianza.

Códigos JEL: C14, C15, E32, E37

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1 Introduction

A traditional problem in economics involves the business cycle, specifically cyclical downturns and their prediction over time. Business cycle analysis relies on the concept of turning point dates, which are closely linked to the economic evolution of a country. These turning points can be seen from switches in the stochastic behavior of time series at random time points, where we can identify local maxima and minima. These local maxima and minima cannot be computed using standard tools of differential calculus because the time series is a realization of a stochastic process, and its dynamics generally do not follow the behavior of a smooth curve. Thus, as mentioned by Burns and Mitchell (1946), these turning points try to describe the periodic behavior of a time series and have been employed to characterize the business cycle, including boom and recession periods typical of the economy.

Given the difficulty of identifying turning points, Bry and Boschan (1971) proposes a dating algorithm to identify them in a monthly time series that incorporates aspects used by the United States National Bureau of Economic Research (NBER). This algorithm is applied to a time series that has been previously seasonally adjusted, and it is based on successively applying a set of rules to identify turning points in a sequence of filtered series, with varying degrees of smoothing. In other words, the filters seek to identify the trend-cycle component of the time series. Typically, a maximum turning point is referred to as a peak, and a minimum turning point as a trough. The temporal distance between a peak and a trough is known as the phase (likewise, trough to peak), and the temporal distance between a peak and a peak is known as the length of the cycle (likewise, trough to trough). Notably, the temporal distance between a peak and a trough is identified as the recession phase of the cycle.

The inference problem over a set of turning point dates is something that, from the seminal work of Burns and Mitchell (1946), has been tackled by considering a set of economic indicators believed to be coincident with the unobserved state of the economy. By employing Bry and Boschan (1971)'s dating algorithm, for instance, it is possible to identify the turning points for each economic indicator, thereby finding a subset of peaks and troughs that align with the peaks and troughs of the state of the economy. This process yields a sequence of temporal groups or zones of peaks and troughs across the set of variables. Thus, we could approximate the business cycle, for example, by taking the mean of the turning points within each group.[1] However, there are often numerous empirical challenges that require a more complex methodology and the consensus of a panel of experts, as discussed by Committee et al. (2010). This procedure enables us to obtain a precise estimate of the business cycle dates.

In this way, it is possible to highlight that to get an adequate estimate of the business cycle, it would be necessary to focus on two key considerations. First, we select the group of time series to be used, which involves examining the lead–lag relationships among a group of macroeconomic time series. Second, we utilize a specific approach to compute the cycle on the selected variables. Thus, once the set of variables has been chosen, the literature traditionally refers to two methods for analyzing the business cycle: "date-then-average" and "average-then-date"; see more details in Stock and Watson (2014).

Regarding the date-then-average method, the literature has proposed several approaches to determine

[1] This approach is referred to as "date then average" as we work with the distribution of the turning points of the disaggregated time series [See Stock and Watson (2014); Camacho et al. (2022)].

possible lead–lag relationships among macroeconomic time series and thus establish the appropriate set of variables for estimating the unobserved business cycle. For instance, Harding and Pagan (2002) developed a measure that assesses the degree of concordance by comparing the cycle of a time series with a given reference cycle. Hardigan and Pagan also provided several simulation exercises to determine the nature of the business cycle once the turning points in the level of economic aggregation have been defined. This approach emphasizes that detrending is not necessary to analyse the business cycle; thus, removing this feature could eliminate one of the most relevant characteristics of the business cycle. In addition, Harding and Pagan (2006) introduced the concept of synchronization and demonstrated how the degree of synchronization in time series can be measured. They developed an algorithm to extract a common cycle, which was applied to identify the reference cycle for the United States, as well as common cycles in stock prices and European industrial production. The authors focused on the turning points in specific series when analyzing the construction of a common cycle. This approach led them to a non-parametric method for extracting the common cycle[2], which is referred to as the reference cycle in the NBER typology. For their method, they used the censoring procedure described in Harding and Pagan (2002). Chauvet and Piger (2008) studied the ability of the algorithm of Harding and Pagan (2006) to identify the NBER-referenced business cycle turning point dates in a real-time scenario.

With respect to the average-then-date method, Stock and Watson (2010) proposed a method to estimate the business cycle based on a set of indicators (270 indicators)[3]. This procedure contributed to the literature by computing the reference business cycles as the means of individual series of turning points ("date then average"). For each indicator, they identified the chronology using Bry and Boschan (1971)'s dating algorithm. Because the chronologies for each indicator do not have the same number of turning-point dates, the panel is unbalanced. With the panel, they used OLS estimators, which allow them to obtain errors and confidence intervals for the turning points. However, this approach considers that the sequence of business cycles is given, which can significantly limit its empirical applicability.

Hamilton (2011) explored efforts to automate the identification of turning points in business cycles. The paper reviewed various methods for dating turning points. All the methods he discussed define recessions or turning points based on a single highly aggregated time series, such as GDP or a monthly index of coincident indicators (average-then-date approach). This approach, which utilizes a few highly aggregated time series to establish the business cycle, was a practice employed by the NBER Business Cycle Dating Committee, as mentioned by Stock and Watson (2014).

Stock and Watson (2014) studied two approaches to estimating turning-point dates, which they referred to as date-then-average and average-then-date. Based on the first approach, the authors define the turning points from a population of disaggregated coincident economic indicators: thus, from a random sample of these economic indicators, they identified some distribution properties, using the mode as an estimator, and therefore they cloud make inference about the estimation of the turning points, such as computing confidence intervals for reference cycle dates estimated from turning points of a random sample of disaggregated series. The distribution's properties are based on the estimation of a kernel for each episode of turning points (peaks or troughs).

[2]See section 6.1 in Harding and Pagan (2006). Also see description of the method in Harding and Pagan (2016).

[3]See Harding and Pagan (2016) section 3.3.2, page 58, for a brief description of the method.

Harding and Pagan (2016) provided a modification to the algorithm they developed in 2006. This modification includes a set of rule-based algorithms and censoring procedures designed to identify dates that minimize a measure of the mean distance between candidates of reference cycle dates and the turning points in disaggregated economic indicators. Their method does not require the concept of a prespecified episode in the reference cycle, as needed in Stock and Watson (2014). They also emphasize the challenges of forecasting turning points because macroeconomic indicators exhibit asynchronous behavior.

Camacho et al. (2022) proposed a methodology to make inferences (confidence intervals for reference cycle dates estimated from turning points) over a sequence of groups of peaks and troughs in the way of Burns and Mitchell (1946). Their approach assumes that the peaks and troughs within these sequences follow a mixture of Gaussian distributions. Additionally, the authors incorporate a Markovian structure and utilize Bayesian estimation to compute both point estimates of the business cycle and corresponding confidence intervals. Although they remark that their methodology does not require a reference cycle as the Stock and Watson (2010) does, they use the reference cycle to establish the priors for the parameters of the mixture of Gaussian distributions in the estimation procedure. This dependence on a reference cycle may limit the empirical applicability of their methodology.

In this document, we propose a methodology for point estimation and confidence interval construction for business cycle dates, following the concepts presented in Burns and Mitchell (1946). Assuming we have a set of economic indicators, we evaluate coincident characteristics using the Coincident Profile (CP) test proposed by Martinez-Rivera et al. (2016). This is a non-parametric tool that allows us to compare coincident characteristics between two time series based on their turning points. In this sense, the CP will enable us to determine whether a set of turning points across a couple of time series is temporally close. Thus, we categorize whether a given economic indicator is coincident with another. Given that the CP provides a coincident measure across a couple of time series, we extend this analysis to the full set of variables by defining a matrix, the **coincident matrix** (CM), that determines the coincident relationships among the variables. Thus, we try to answer the Burns and Mitchell (1946)'s requirement regarding having a set of economic indicators coincident with the reference cycle.

Since the CM operates from a set of turning points, we also find a collection or sequence of peaks and troughs, which are obtained with Bry and Boschan (1971)'s algorithm. Over each group of turning points, we calculate a weighted average to obtain a point estimate of the business cycle dates (the date then average approach, as Stock and Watson (2014)), and we develop a naive Bootstrapping procedure to make inferences for the business cycle dates (confidence intervals).

Thus, this paper contributes to the literature on dating business reference cycles fourfold. First, the nonparametric test method behind the construction of the CM allows the data to speak for themselves when calculating the coincident degree of the economic indicators, thereby avoiding assumptions that can be difficult to determine in advance in real data, such as those related to comparing each economic indicator with the unobserved reference cycle, which is trying to be estimated. Although these methods occasionally rely on expert judgment to assess the alignment of indicators with the reference cycle, the nonparametric approach to computing the coincident degree aligns with Harding and Pagan (2002, 2006). Second, as we use the nonparametric Bry and Boschan (1971) dating algorithm, we avoid making assumptions about the underlying distribution of turning points, which can be difficult to corroborate in empirical applications.

Third, from a set of economic indicators and utilizing the coincident matrix, we identify groups of turning points. Using each of these groups, we not only compute the point estimate of the turning point but also construct confidence intervals for each reference cycle date, estimated using the naive bootstrap. Fourth, our approach can be considered a parsimonious method, as it does not require a parametric model specification during the inference process regarding the turning points, unlike other methods.

The rest of the document is organized as follows. Section 2 describes the methodology proposed to calculate point estimations and confidence intervals for business cycle dates. In Section 3, we examine the performance of our approach through an extensive simulation study. Section 4 presents two empirical applications: one using economic data from the United States and the other from Colombia. We show the performance of our approach on these two datasets. Finally, conclusions are gathered in Section 5.

2 Methodology

To describe the methodology used to compute the estimated confidence intervals for the business cycle dates, we first explain some concepts related to the definition of the Coincident Profile and the Coincident Matrix. In Section 2.3, we present the methodology proposed for estimating confidence intervals for the turning points.

2.1 Coincident Profile (CP)

The Coincident Profile procedure compares two time series[4] through their turning points. This tool provides a measure of time closeness between the two time series, allowing us to determine whether the variables are coincident or one leads the other. Coincidence analysis is achieved by assessing the temporal differences between common turning points. For instance, each maximum from the first time series is compared with the closest maximum from the second time series, and their temporal differences, denoted as d_n, are calculated. Each turning point is used in the comparison only once.

This comparison process generates a set of differences $\{d_1, d_2, ..., d_N\}$, where N represents the number of turning points involved in the comparison. Since these differences can be both positive and negative, a permutation test using the signs of the differences is developed to determine whether the average difference is zero. Furthermore, the procedure adds or subtracts 1, 2, 3,..., l units to the set of differences $\{d_1 \pm l, d_2 \pm l, ..., d_N \pm l\}$ to find the most likely average difference. Thus, in the CP, for each value of l added or subtracted, a permutation test is performed on the newly formed set of differences, which helps determine the most probable average difference between the two time series. The hypotheses to be tested in constructing the main profile can be represented as equation (1).

$$H_0^{(l)} : \sum_{i=1}^{N} (d_i - l) = 0; \quad l = 0, \pm 1, \dots, \pm m \tag{1}$$

where the value of $l > 0$, for which the null hypothesis is not rejected, is considered the leading period. For example, if the p-value obtained from the permutation test, in (1), using the set of differences $\{d_1 -$

[4]Time series in frequency, monthly, or quarterly.

$l, d_2 - l, ..., d_N - l\}$ is the highest among the others with $\pm 1, \pm 2, ..., +l$, we conclude that the most probable average difference is equal to l, $\frac{1}{N}\sum_{n=1}^{N}(d_n - l) = 0$, which means that first time series potentially leads the second one l periods, it is called "*main lag*", see more details in Martinez-Rivera et al. (2016) and Martinez-Rivera (2019).

2.2 Coincident Matrix (CM)

The definition of the Coincident Matrix is grounded in the concept of CP described in the previous subsection 2.1. Then, given a set of K time series denoted by $\{X_{1t}, ..., X_{Kt}\}$, we use the CP to compare in pairs; thus, we obtain a $K \times K$ matrix as follows:

$$CM_{K\times K} = \begin{array}{c c} & \begin{array}{cccccc} X_{1t} & X_{2t} & \cdots & X_{jt} & \cdots & X_{Kt} \end{array} \\ \begin{array}{c} X_{1t} \\ X_{2t} \\ \vdots \\ X_{it} \\ \vdots \\ X_{Kt} \end{array} & \begin{pmatrix} l_{11} & l_{12} & \cdots & l_{1j} & \cdots & l_{1K} \\ l_{21} & l_{22} & \cdots & l_{2j} & \cdots & l_{2K} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ l_{i1} & l_{i2} & \cdots & l_{ij} & \cdots & l_{iK} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ l_{K1} & l_{K2} & \cdots & l_{Kj} & \cdots & l_{KK} \end{pmatrix} \end{array} \tag{2}$$

where l_{ij} is the main lag between X_{it} and X_{jt}. Also, l_{ij} satisfies the conditions $l_{ij} = -l_{ji}$ and $l_{ii} = 0$. A negative value for l_{ij} means that X_{it} leads X_{jt}, and a positive value signifies the opposite. If $l_{ij} = 0$ implies a coincident relationship between X_{it} and X_{jt}. Also, we note that $CM = -CM'$, where CM' is the transpose matrix of CM.

2.3 Algorithm to compute the confidence intervals for the business cycle dates

We will now outline the algorithm for estimating confidence intervals for business cycle dates, using the Coincident Matrix concept described in Subsection 2.2.

Step 1: Let $\{X_{1t}, ..., X_{Kt}\}$ be a set of time series, where each X_{it} can have different sample size, this is $t = t_i, ..., T$ for $i = 1, ..., K$.

a) For $i = 1, 2, ..., K-1$ and $j > i$, join X_{it} and X_{jt} by time ($t = t_i = t_j$): it means that we merge the common information available between a couple of time series X_{it} and X_{jt}. This is necessary for two reasons: First, the set of variables is not mandatory that have the same observed sample period, and second, in the next step, we use the CP to establish a coincident relationship between X_{it} and X_{jt}, whose comparison is done in pairs and the variables do not have missing observations.

b) Compute the CM as Subsection 2.2.

c) For the CM, if $|l_{ij}| \leq 1$, store the turning points from X_{it} and X_{jt} to continue the process; otherwise, ignore that comparison.

d) Repeat steps from (a) to (c) for $i = 1, 2, ..., K-1$.

Step 2: Based on the store, the turning points in step 1.

e) Separating the stored turning points into maximums and minimums determines common zones. For example, suppose that the first couple of minimums come from X_{1t} and X_{2t}. Then, this couple of minimums is compared with the other couples to look for intersections. Cases where this intersection is not empty are classified as a common zone. We repeat this process with every couple of minimums and maximums. This process enables us to identify zones of maximums and minimums based on the number of couples per zone. Thus, we obtain a frequency of couples per zone.

f) Calculate the proportion of couples per zone. In every zone, we can find at most $\binom{N^*}{2}$, where N^* is the total number of variables with available data in the zone[5]. In this sense, the denominator of the proportion is $\binom{N^*}{2}$.

g) Keep the zones where the proportion exceeds a value that defines a minimum proportion admissible per zone, $MinP$.

h) Compute a weighted average. The weighted average is calculated based on the frequency of the variables with available minimums (maximums) in every zone. Every common zone comprises the location[6] of minimums (maximums) and the frequency. Thus, we replicate each location according to its frequency, then calculate the average as a point estimate of the common minimum zone across locations.

i) Compute confidence intervals (1 - $\alpha/2$)100%. We use the same information at step h), then we apply a naive Bootstrap (Efron (1979)) with the replicate locations to calculate confidence intervals (1 - $\alpha/2$)100%. Since the naive Bootstrap procedure uses the turning point dates directly, the confidence intervals that we created are closed. It means the limits of the intervals are included.

j) Guarantee the alternation between the zones of minimums and maximums. We use the standard specifications for cycle and phase durations established by the NBER.

3 Numerical studies

To evaluate the proposed methodology, we simulate a reference cycle y_t as a time series using a class of unobserved component (UC) models (see Harvey (1990) for a complete discussion). Particularly, we use the three additive components: local trend μ_t, stochastic cyclical ψ_t models, and an irregular term ε_t as follows.

$$y_t = \mu_t + \psi_t + \varepsilon_t, \qquad t = 1, ..., T., \tag{3}$$

where the components μ_t and ψ_t are specify in equations (4) and (5), respectively. The term ε_t is a white-noise disturbance normally distributed serially independent $NID(0, \sigma^2_\varepsilon)$. The local trend model is

[5] $N^* \leq N$ due to the availability of information.

[6] In obedience to the dates of the turning points.

defined as

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_t + \eta_t, \quad \text{with} \quad \eta_t \sim NID(0, \sigma^2_\eta), \\ \beta_t &= \beta_{t-1} + \zeta_t, \quad \text{with} \quad \zeta_t \sim NID(0, \sigma^2_\zeta), \end{aligned} \tag{4}$$

where η_t and ζ_t are mutually uncorrelated. The cycle stochastic equation is defined as follows

$$\begin{pmatrix} \psi_t \\ \psi^*_t \end{pmatrix} = \rho \begin{pmatrix} \cos(\lambda) & \sin(\lambda) \\ -\sin(\lambda) & \cos(\lambda) \end{pmatrix} + \begin{pmatrix} \psi_{t-1} \\ \psi^*_{t-1} \end{pmatrix} + \begin{pmatrix} \kappa_t \\ \kappa^*_t \end{pmatrix}, \quad \text{with} \quad \begin{pmatrix} \kappa_t \\ \kappa^*_t \end{pmatrix} \sim NID(0, \sigma^2_\kappa I_2), \tag{5}$$

where ψ_t and ψ^*_t are the cycle and the auxiliary process; the parameters ρ, λ, and σ^2_κ are the dumping factor, the cyclical frequency, and the cycle variance, respectively. The period of the cycle is specified by $2\pi/\lambda$. In addition, if $|\rho| < 1$, and $0 < \lambda < \pi$ the cycle ψ_t and ψ^*_t are stationary process. The processes κ_t and κ^*_t are white-noise disturbances, which must be assumed either to have the same variance or to be uncorrelated; these conditions are necessary for the model to be identifiable.

3.1 Simulation settings

Koopman and Lee (2009) proposed several UC models used to estimate the dynamics of variables such as United Kingdom visits abroad, United States (US) unemployment, and US industrial and dwelling production. Following their estimations, in particular for the US unemployment time series in the period from January 1948 to December 2006, we simulate the reference cycle y_t by setting $\{\sigma^2_\varepsilon, \sigma^2_\eta, \sigma^2_\zeta, \rho, \lambda, \sigma^2_\kappa, \psi_0, \psi^*_0\}$ $= \{0.1, 0, 0.003^2, 0.99, 0.25, 0.005^2, \sqrt{2}/2, \sqrt{2}/2\}$. Under the condition $\sigma^2_\eta = 0$, the trend follows a smooth integrated random walk in the local trend model. The initial values for ψ_0 and ψ^*_0 provide an amplitude 1 and phase $\pi/4$ in the cosine wave model, see details in Harvey (1990), page 39.

We simulate the reference cycle $y_t + 100$, from now on y_t, with a total length of 120 observations, after burning the first 60 data points. Now, we apply the log transformation and identify the cycles and phases, as shown in Figure 1. Figure 1 highlights with gray zones "recessions zones", zones that cover from a peak to a trough. From the reference cycle, we simulate coincident indicators $y_{t,i}(\sigma_s)$ by adding to y_t a Gaussian random perturbation $N(0, \sigma^2_s)$, where σ_s takes values in the set $\{0.05, 0.1, 0.2, 0.3, 0.4, 0.5\}$. For each σ_s, we simulate 100 variables $y_{t,i}(\sigma_s)$, $i = 1, ..., 100$. This type of simulation enables us to obtain a set of variables that coincide with the reference cycle and where the degree of coincidence decays as the value of σ_s increases, as shown in Table 1. Thus, we attempt to mimic real-world applications in which a set of coincident indicators exists, as in Burns and Mitchell (1946), which coincide with an unobserved reference cycle but not necessarily perfectly. In this sense, by sampling from the set of simulated variables, we employ several sampling schemes to evaluate the performance of the proposed methodology in identifying the dates of the true turning points (i.e., the turning points of the reference cycle) and assessing the quality of the confidence intervals.

Before discussing sampling strategies, Table 1 summarizes the comparison between y_t and every $y_{t,i}(\sigma_s)$ using the CP tool. We denote NTP in Table 1 as the number of turning points that match the turning points of the reference cycle. Since the CP tool provides a measure of time-closeness between two time series based on the number of turning points match, thus if $l_{ij} = 0$ the couple of time series y_t and $y_{t,i}(\sigma_s)$

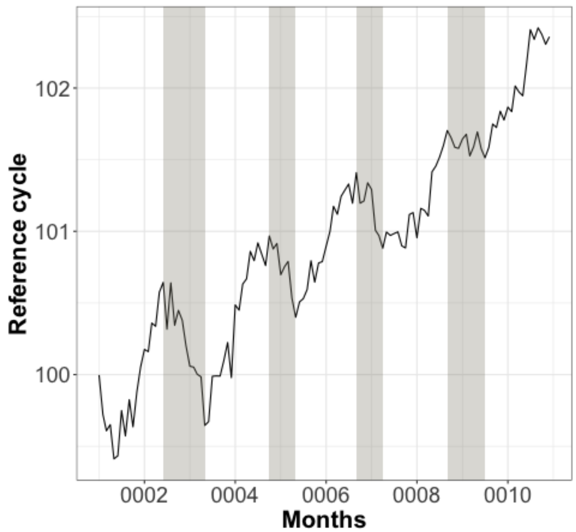

Figure 1: Reference cycle y_t. Assuming a monthly time series from the year 1. Gray zones represent the usual recession phases.

are coincident, if $l_{ij} < 0$ it means that $y_{t,i}(\sigma_s)$ leads y_t, or if $l_{ij} > 0$, $y_{t,i}(\sigma_s)$ is lagged to y_t. For example, using $\sigma_s = 0.05$, from the 100 variables, 38 variables exhibit at least 8 turning points that match the turning points of the reference cycle, and they are coincident, while 5 lead and 26 are lagged to y_t one month, respectively. Particularly, if we emphasize the range $|l_{ij}| \leq 1$ values, we notice that as σ_s increases, the number of variables in this range diminishes. We now describe, in the following Subsection 3.2, the sampling scenarios we developed to evaluate our methodology.

3.2 Sampling scenarios and simulations results

We perform three sampling scenarios to evaluate the quality of the estimation of the turning point dates and their confidence intervals (95%). For the three cases, we select random variables by sequentially increasing the sample size for the selected variables to 10, 20, and 30. With every case, we repeat the procedure 100 times.

Case 1. Using the full collection of 600 variables.

Case 2. Focusing on the subset of variables inside the range $|l_{ij}| \leq 1$.

Case 3. Sampling from the set of variables with $\sigma_s = 0.05$.

In Table 2, we report the results from Case 1. We select the repetitions that fully identify the four zones or the 8 turning points in the reference cycle y_t, $Peaks$, and $Troughs$, respectively. The number of repetitions for sampling 10, 20, and 30 variables is 55, 22, and 1, respectively ($Niter$). Thus, we summarize from these repetitions the weighted average of the estimate peaks and troughs (Peaks and Troughs), the

σ_s	0.05					0.1					0.2				
NTP	4	5	6	7	8	4	5	6	7	8	4	5	6	7	8
l_{ij}															
-3	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
-2	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
-1	0	0	1	0	5	1	0	3	0	7	0	0	8	1	7
0	0	0	8	3	38	0	0	7	6	29	2	1	15	3	14
1	1	0	2	6	26	1	1	9	7	16	1	3	11	5	14
2	0	1	2	3	4	0	0	2	5	3	0	1	4	0	4
3	0	0	0	0	0	0	0	0	2	1	0	0	1	0	0

σ_s	0.3						0.4						0.5					
NTP	3	4	5	6	7	8	3	4	5	6	7	8	3	4	5	6	7	8
l_{ij}																		
-5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
-3	0	0	0	4	0	1	0	0	1	1	0	2	0	0	0	1	0	1
-2	0	0	0	1	0	2	0	0	4	3	1	2	0	3	1	2	1	7
-1	0	1	0	2	2	12	1	3	2	4	3	7	0	1	1	9	3	8
0	0	6	2	11	1	12	1	7	1	9	3	6	0	3	5	6	2	3
1	1	2	3	9	2	9	0	2	3	3	4	4	1	3	3	7	3	3
2	0	0	0	5	2	1	0	1	3	4	1	3	0	1	1	3	3	2
3	1	0	0	0	1	2	0	0	1	4	1	3	0	2	1	1	1	2
4	0	2	0	0	1	0	0	1	0	0	0	0	0	0	1	1	0	1
5	1	0	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0

Table 1: Summary of coincident indicators.

inferior and superior limits of the intervals for the peaks (PLI and PLS), the inferior and superior limits of the intervals for the troughs (TLI and TLS), the duration or the time units between a peak and trough, the number of variables that provide information to estimate peaks and troughs (NvarP and NvarT), the absolute difference between the point estimation peaks and troughs and the true reference cycle's peaks and troughs (DifP and DifT), respectively; and the percentage of coverage of the intervals to the reference cycle's peaks and troughs (CovP and CovT).

From these results in Table 2, we observe that, on average, sampling 10 variables over 55 repetitions, the point estimates of the peaks and troughs differ by 1 and 3 months from the corresponding reference cycle's peaks and troughs. The coverage of the peaks' confidence intervals ranges from 98% to 100%, whereas the coverage of the troughs' confidence intervals ranges from 89% to 100%. The durations are close to the reference cycle's durations: 11, 7, 7, and 10, respectively. The number of variables that provide information to estimate peaks and troughs ranges from 7 to 10. In addition, for the last zone or the last couple of peaks and troughs, although the point estimates differ between 1 and 3 months, the confidence intervals overlap. The difficulty lies in identifying the peak; as we add more variables, the difference between the estimated peak and the true peak increases, and the amplitude of the confidence interval expands. Also, the number of repetitions required to estimate the four zones decreases as the number of variables increases. This occurs mainly because the new 10 variables are not coincident with the previously selected variables, and their variability potentially increases as well.

To study the behavior of the proposed approach, in order to avoid selecting variables that do not

									y_t						
Peaks	Troughs	PLI	PLS	TLI	TLS	Duration	NvarP	NvarT	Peaks	Troughs	DifP	DifT	CovP	CovT	Niter
sampling 10 variables															
18	29	15	22	26	33	11	9	7	18	29	1	1	98	100	55
44	54	40	48	51	57	9	9	8	46	53	2	1	98	100	55
68	79	64	72	75	84	11	9	9	69	76	1	3	100	89	55
96	102	90	106	98	107	6	9	7	93	103	3	1	100	96	55
sampling 20 variables															
18	29	13	22	22	34	11	19	16	18	29	1	1	100	100	22
44	54	39	48	49	58	10	18	15	46	53	2	1	100	100	22
68	79	63	72	73	85	11	18	20	69	76	1	3	100	100	22
98	103	90	111	97	111	5	20	13	93	103	5	1	100	100	22
sampling 30 variables															
19	31	15	23	26	37	12	30	26	18	29	1	2	100	100	1
44	54	39	48	48	59	10	27	30	46	53	2	1	100	100	1
68	80	63	73	73	88	12	26	25	69	76	1	4	100	100	1
98	101	89	114	99	104	3	29	24	93	103	5	2	100	100	1

Table 2: Results Case 1. Peaks and Troughs represent point estimates for peaks and troughs; PLI and PLS provide results of the inferior and superior limits of the intervals for the peak; likewise for troughs, TLI and TLS. Duration is the time unit between a peak and a trough. NvarP and NvarT are the number of variables that provide information to estimate peaks and troughs. DifP and DifT are the absolute difference between the point estimation peaks and troughs and the true reference cycle's peaks and troughs (y_t Peaks and Troughs), respectively. CovP and CovT are the percentage of coverage of the intervals to the reference cycle's peaks and troughs. Finally, Niter denotes the number of repetitions used to sample 10, 20, and 30 variables that identify the four zones.

coincide with the reference cycle, the second case restricts this selection to variables whose coincident results, as shown in Table 1, fall within the range $|l_{ij}| \leq 1$. In Table 3, we summarize these results. Under this approach, the number of repetitions that identify the four zones increases. Thus, sampling 10 variables, we obtain 80 repetitions that estimate the 8 turning points. Increasing the number of variables by 10 yielded 38 repetitions. By sampling 30, we found that 19 repetitions cover the four zones. Following this scenario in Case 2, although the precision of the point estimation does not improve substantially, the precision of the confidence intervals does. However, for the last peak, the confidence interval estimate improves only with 10 variables. The reason it is difficult to estimate a narrower confidence interval for the last peak is that the last zone tends to be flat, as shown in Figure 1.

									y_t						
Peaks	Troughs	PLI	PLS	TLI	TLS	Duration	NvarP	NvarT	Peaks	Troughs	DifP	DifT	CovP	CovT	Niter
sampling 10 variables															
18	29	14	21	26	32	11	10	8	18	29	1	1	99	99	80
44	53	40	48	50	57	9	9	7	46	53	2	1	99	99	80
67	78	63	71	74	83	11	9	10	69	76	2	3	99	96	80
96	102	91	104	98	107	6	9	7	93	103	3	2	99	98	80
sampling 20 variables															
18	29	13	22	25	34	11	20	16	18	29	0	1	100	100	38
44	54	39	48	50	58	10	18	15	46	53	2	1	100	100	38
68	79	62	72	74	85	12	18	20	69	76	1	3	100	97	38
98	102	89	112	98	109	4	20	13	93	103	5	2	100	97	38
sampling 30 variables															
18	29	12	23	21	34	11	29	26	18	29	1	2	100	100	19
44	54	38	48	49	58	10	27	24	46	53	2	1	100	100	19
67	79	62	72	73	85	12	27	29	69	76	2	3	100	95	19
99	103	89	113	98	111	4	30	18	93	103	6	2	100	95	19

Table 3: Results Case 2. Note: Same description as in Table 2.

Now, restricting the selection to Case 3, this scenario ensures the selection of variables with similar characteristics regarding the degree of coincidence and variability properties, see Table 4. Under this scenario, all the repetition samples of 10, 20, and 30 successfully identify the four zones. Additionally, the point estimates and their estimated confidence intervals of the last peak and trough improve compared to cases 1 and 2. Thus, we observe that selecting variables that coincide with the reference cycle reveals their coincidence, allowing us to obtain good estimates of both the turning points and the confidence intervals. Empirically, because we lack a reference cycle, the methodology relies on the classification of the coincident matrix to identify coincident relationships among economic indicators.

									y_t						
Peaks	Troughs	PLI	PLS	TLI	TLS	Duration	NvarP	NvarT	Peaks	Troughs	DifP	DifT	CovP	CovT	Niter
sampling 10 variables															
19	30	17	20	29	30	11	10	8	18	29	1	1	100	100	100
45	53	42	48	53	54	8	10	6	46	53	1	0	100	100	100
69	80	68	71	76	83	11	10	10	69	76	1	4	100	100	100
95	102	93	98	99	104	6	9	8	93	103	2	1	100	100	100
sampling 20 variables															
18	30	16	20	29	30	12	20	16	18	29	0	1	100	100	100
45	53	42	48	52	54	8	20	14	46	53	1	0	100	100	100
69	80	67	72	76	84	11	19	20	69	76	0	4	100	100	100
95	102	92	100	99	104	6	18	17	93	103	2	1	100	100	100
sampling 30 variables															
18	30	16	20	29	30	12	30	23	18	29	0	1	100	100	100
45	53	41	48	52	54	8	29	19	46	53	1	0	100	100	100
69	80	67	72	75	84	11	29	30	69	76	0	4	100	100	100
96	102	92	100	99	104	6	27	26	93	103	3	1	100	100	100

Table 4: Results Case 3. Note: Same description as in Table 2.

4 Empirical applications

This section aims to provide an empirical assessment of the accuracy of the proposed methodology for estimating confidence intervals around the reference business cycle turning points. We illustrate this with two examples using real data: one from the United States and another from Colombia. For this purpose, subsection 4.1 compares our approach with the NBER-referenced business cycle dates for the case of the United States, while subsection 4.2 presents the application of the proposed method to the Colombian dataset.

4.1 Application of the methodology to United States data

In this subsection, we illustrate the proposed methodology for estimating a confidence interval for turning-point dates, using real data from the NBER. This dataset, based on the Business Cycle Dating Committee's 2010 methodology, primarily follows the dynamics of 10 monthly economic indicators. Using these indicators, we evaluated the quality of our approach to identifying the reference cycle dates reported by the NBER and the coverage of the estimated confidence intervals. The list of the 10 indicators are: Macroeconomic Advisers' monthly GDP (x_{1t}), the Stock-Watson index of monthly GDP (x_{2t}), their index of monthly GDI (x_{3t}), an average of their two indexes of monthly GDP and GDI (x_{4t}), Real manufacturing and trade sales (x_{5t}), index of Industrial Production (x_{6t}), real personal income less transfers (x_{7t}), aggregate hours of work in the total economy (x_{8t}), payroll survey employment (x_{9t}), and household survey employment (x_{10t}). This dataset covers a sample from January 1959 to June 2010; however, the number of available observations varies by indicator due to differing starting measurement periods or record capture.

We now follow the proposed methodology, as outlined in Section 2.3, step by step on the dataset mentioned. Since not all variables start at the same time point, particularly the time series x_{1t} and x_{5t} have observations from January 1967 and April 1992, respectively. So, the comparisons with these two variables require following part (a) of step 1. Applying parts (b) to (d) of step 1, we obtain the coincidence matrix in (6), by keeping the results where the condition $|l_{ij}| \leq 1$ is accepted. In this sense, the results from relationships between variables whose absolute values in the coincidence matrix exceed 1 are excluded from subsequent steps.

$$\widehat{CM} = \begin{array}{c} \\ x_{1t} \\ x_{2t} \\ x_{3t} \\ x_{4t} \\ x_{5t} \\ x_{6t} \\ x_{7t} \\ x_{8t} \\ x_{9t} \\ x_{10t} \end{array} \begin{array}{c} \begin{array}{cccccccccc} x_{1t} & x_{2t} & x_{3t} & x_{4t} & x_{5t} & x_{6t} & x_{7t} & x_{8t} & x_{9t} & x_{10t} \end{array} \\ \begin{pmatrix} 0 & 0 & -4 & 4 & 1 & 1 & -2 & -3 & -5 & -4 \\ & 0 & -1 & 0 & 0 & 0 & -2 & -2 & -4 & -3 \\ & & 0 & 0 & 3 & 0 & 0 & -2 & -3 & -2 \\ & & & 0 & -2 & 0 & -1 & -3 & -3 & -3 \\ & & & & 0 & -1 & -2 & -4 & -2 & -3 \\ & & & & & 0 & 0 & -3 & -1 & -2 \\ & & & & & & 0 & -2 & -1 & -1 \\ & & & & & & & 0 & 1 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & 0 \end{pmatrix} \end{array} \qquad (6)$$

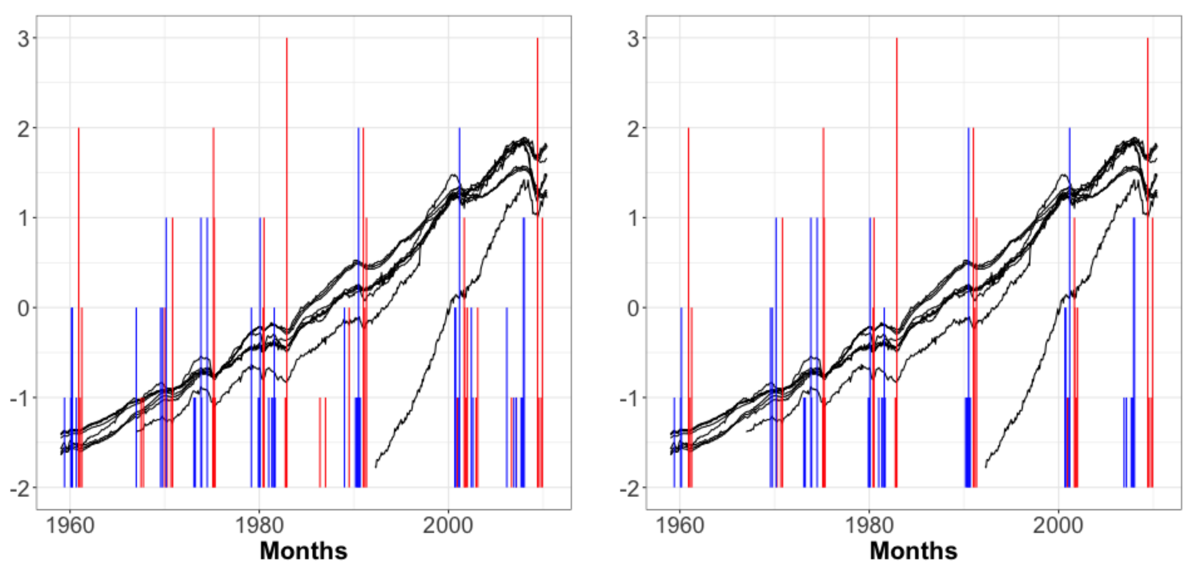

Figure 2: Identification of maximums (blue lines) and minimums (red lines) zones. Following part (e) of step 2, the left panel. Using the value for $MinP$ =0.2, determination of peaks and troughs zones, right panel.

Continuing with step 2 in Figure 2, we summarize the identification of maximum and minimum zones. In this figure, the variables are standardized (each variable is subtracted by its mean and divided by its standard deviation). Figure 2 shows the groups of maximums (blue lines) and the groups of minimums (red lines). The height of the blue and red lines indicates the number of variables that exhibit a maximum or minimum, respectively. For example, the first red line has a height of 4 units, indicating that on this date, 4 variables coincide at the minimum. In Figure 2, the left panel, we find that identifying some zones is challenging because they appear to combine maximums and minimums. Therefore, we use the parameter $MinP$=0.2, which allows us to classify the zones according to their density, i.e., zones that have less than 20% of the variables (two or fewer) that have a maximum or minimum in a given zone are discarded. In addition, we must ensure the alternation of the zones. Once we apply this criterion, the result is shown in the right panel plot in Figure 2. To calculate the value of the $MinP$ parameter, we use the NBER reference cycle as support.

Peaks	Troughs	Duration	NvarP	NvarT	PeaksNBER	TrougsNBER	DifP	DifT	CovP	CovT
1959:12	1961:02	14	4	7	1960:04	1961:02	4	0	0	1
(1959:06, 1960:03)	(1960:12, 1961:04)									
1969:11	1970:10	11	7	5	1969:12	1970:11	1	1	1	1
(1969:08, 1970:03)	(1970:04, 1970:11)									
1973:10	1975:04	18	9	9	1973:11	1975:03	1	-1	1	1
(1973:02, 1974:07)	(1975:02, 1975:05)									
1980:01	1980:07	6	5	5	1980:01	1980:07	0	0	1	1
(1979:12, 1980:02)	(1980:06, 1980:07)									
1981:07	1982:11	16	6	7	1981:07	1982:11	0	0	1	1
(1981:01, 1981:09)	(1982:10, 1982:12)									
1990:06	1991:03	9	9	9	1990:07	1991:03	1	0	1	1
(1990:03, 1990:09)	(1991:01, 1991:05)									
2000:12	2001:10	10	9	9	2001:03	2001:11	3	1	1	1
(2000:09, 2001:03)	(2000:12, 2002:01)									
2007:10	2009:09	23	10	10	2007:12	2009:06	2	-3	1	1
(2006:12, 2008:01)	(2009:06, 2009:12)									

Table 5: Point estimations of the turning point dates (Peaks and Troughs), and their 95% confidence intervals as closed intervals. The duration of a recession (Duration), the number of variables per zone (NvarP -for peaks-, NvarT -for troughs-), peaks and troughs according to NBER reference cycle (PeaksNBER, TroughsNBER), the difference between estimated peaks and the peaks from the reference cycle (DifP), likewise for the troughs (DifT), and indicator CovP= 1 if the estimated interval covers the peak from the NBER (PeaksNBER), and CovP= 0 otherwise (likewise CovT for troughs). Note that, although we use the standard notation for the confidence intervals, these are closed.

Based on the identification of turning point zones, we make an inference about the point estimation and their corresponding confidence intervals, and their characteristics. Table 5 summarizes the results of the estimation of the United States business cycle. We observe that the point estimations for the dates of the turning points (Peaks and Troughs) are close to the respective NBER reference cycle (PeaksNBER and TroughsNBER); we find an average difference of 1.5 months between peaks and 0.75 months between troughs. The number of indicators that exhibit peaks is between 4 and 10, and the number of troughs is between 5 and 10. All the confidence intervals, except for the first peak, cover the peaks and troughs from the NBER. To illustrate these results graphically, Figure 3 shows the 10 indicators standardized as described above, along with the NBER reference cycle (horizontal black lines). Also, in this Figure, the gray-shaded regions indicate the point estimates of the turning points, and the blue and red lines indicate the lengths of the intervals for the peaks and troughs, respectively. Thus, visually, we can see whether the estimated intervals' blue and red tiny lines cover the respective black line, the official business cycle dates.

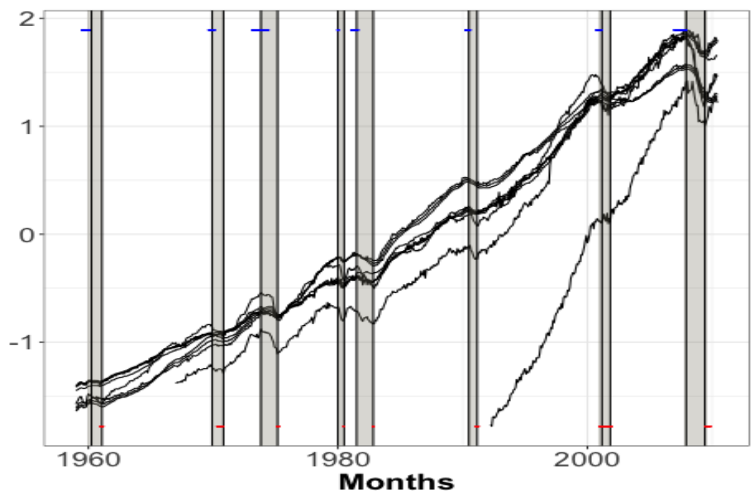

Figure 3: 10 indicators with a standardization transformation are used to illustrate the proposed methodology. The estimated recession zones (gray's zones). The NBER reference cycle is represented with vertical black lines. Tiny blue and red lines represent 95% confidence intervals for peaks and troughs, respectively

4.1.1 Pseudo out-of-sample analysis

Anticipating the dates of the business cycle's turning points is a challenging task and, at the same time, highly interesting (e.g., Hamilton (2011), Camacho et al. (2022)). Based on the NBER reference cycle, using the same set of 10 variables described above (see section 4.1), we calculate the time when our methodology identifies the NBER turning points. To do so, we begin with the fifth U.S. reference cycle[7], adding month by month (an expanding window) until the methodology identifies the target turning point and our confidence interval includes that date.

Camacho et al. (2022) developed a procedure to expand the newly available information month by month for the 10 indicators, shifting from a detection problem to classification. They used two criteria to identify a new cluster of turning points: first, the cluster size increases by three consecutive months; second, the new cluster includes at least one-third of the economic indicators. The authors compared their results with the US reference cycle chronology and with the dates on which the NBER announced the turning points; see Table 6. In this Table, the turning points, peaks, and troughs are listed; the dates

[7]The CP requires at least four turning points to establish the coincident relationship.

of the turning points are announced, and the lag between the turning-point dates and their respective announcement dates is provided.

Taking the NBER turning points as a reference, from the fifth turning point onward, we expand the sample of 10 economic indicators using our methodology. Thus, we determine the time required for the proposed methodology to identify the given NBER turning point. In Table 7, we summarize the estimated chronology, which includes point estimations and confidence intervals (95%). We also report the date on which the methodology identifies the respective turning point (Identify) and the lag relative to the given NBER turning point. Comparing the results of the identify with the announce dates, Tables 6 and 7, our methodology exhibits advantage regarding the relevance of the estimation of the Peaks, mainly for the last three where we obtain a benefit anticipating between 2 and 5 months the identification of the Peaks for the dates of the announce. The announcement of the troughs in general requires more time, between 12 and 25 months, whereas our procedure takes between 6 and 10 months.

Peaks	Announce	Lag	Troughs	Announce	Lag
1960:04	-	-	1961:02	-	-
1969:12	-	-	1970:11	-	-
1973:11	-	-	1975:03	-	-
1980:01	1980:06	5	1980:07	1981:07	12
1981:07	1982:01	6	1982:11	1983:07	20
1990:07	1991:04	9	1991:03	1992:12	21
2001:03	2001:11	8	2001:11	2003:12	25
2007:12	2008:12	12	2009:06	2010:09	15

Table 6: Reference cycle and date NBER announcement turning points, taken from Camacho et al. (2022). Before 1979, there were no formal announcements “-”.

Peaks	Identify	Lag	Troughs	Identify	Lag
1959:12	-	-	1961:02	-	-
(1959:06, 1960:03)			(1960:12, 1961:04)		
1969:12	-	-	1970:09	-	-
(1969:08, 1970:03)			(1970:04, 1970:11)		
1973:08	1974:06	7	1975:03	1975:09	6
(1973:02, 1973:12)			(1975:02, 1975:03)		
1980:01	1980:08	7	1980:07	1981:01	6
(1979:12, 1980:02)			(1980:06, 1980:07)		
1981:07	1982:03	8	1982:11	1983:06	7
(1981:01, 1981:09)			(1982:10, 1982:12)		
1990:06	1991:01	6	1991:03	1991:11	8
(1990:03, 1990:07)			(1991:01, 1991:05)		
2001:01	2001:09	6	2001:10	2002:05	6
(2000:09, 2001:03)			(2001:09, 2001:11)		
2007:10	2008:07	7	2009:08	2010:04	10
(2006:12, 2008:01)			(2009:06, 2009:10)		

Table 7: Pseudo forecast of the NBER reference cycle.

4.2 Application of the methodology to Colombian data

In this application, we apply the proposed procedure to estimate confidence intervals for the turning point dates in the Colombian data. This dataset consists of 41 monthly coincident, seasonally adjusted Colombian economic indicators and spans the period from January 1975 to August 2022; however, the number of available observations varies by indicator due to different starting measurement periods or the capture of records. Some of the indicators used include the total industry employment index, oil production, non-traditional exports, Coffee production, and the total real wage index in the industry, among others.[8] Using these indicators, we aim to evaluate the performance of our approach in identifying the reference cycle dates reported in the Arango et al. (2025). The ESPE document (Arango et al. (2025)) reports a proposed reference cycle dates based on the chronology derived from the cumulative diffusion index (known as the "*Indice de Difusión Acumulado*", IDA index in Spanish).[9] Figure 4 includes the 41 indicators using a standardization transformation (each variable is subtracted by its mean and divided by its standard deviation) jointly with the estimated recession zones (gray zones).

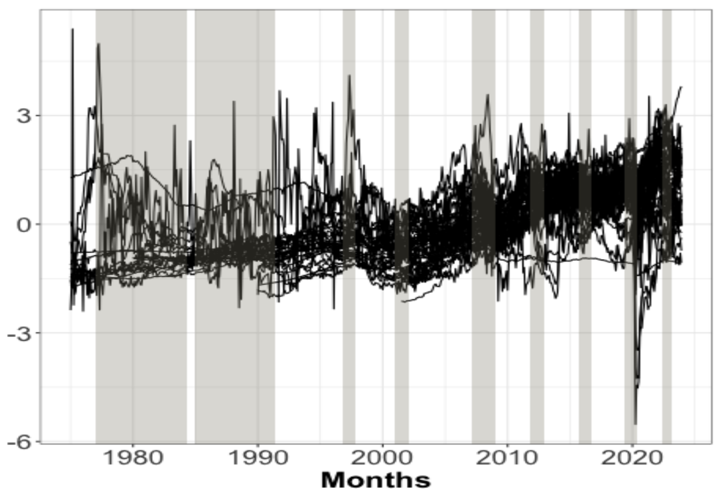

Figure 4: 41 indicators with a standardization transformation are used to illustrate the proposed methodology. The estimated recession zones (gray zones) according to the proposed approach.

Following the proposed methodology outlined in Section 2.3, Figure 5 presents the estimated coincidence matrix represented like a heat map for easier interpretation, particularly given the considerable volume of time series being analyzed. In the heat map, the presence of red and blue squares indicates that the condition $|l_{ij}| \leq 1$ is not satisfied. Thus, the results from relationships between variables whose absolute values in the coincidence matrix exceed 1 are not used in subsequent steps of the proposed methodology.[10]

[8]See Table A.1 of Annex A for more details about the list of 41 indicators used.

[9]See subsection 3.2 in Arango et al. (2025) for more details about IDA index.

[10]Figure B.1 in appendix B shows the estimated coincidence matrix represented like a heat map for the variables that satisfied the condition $|l_{ij}| \leq 1$.

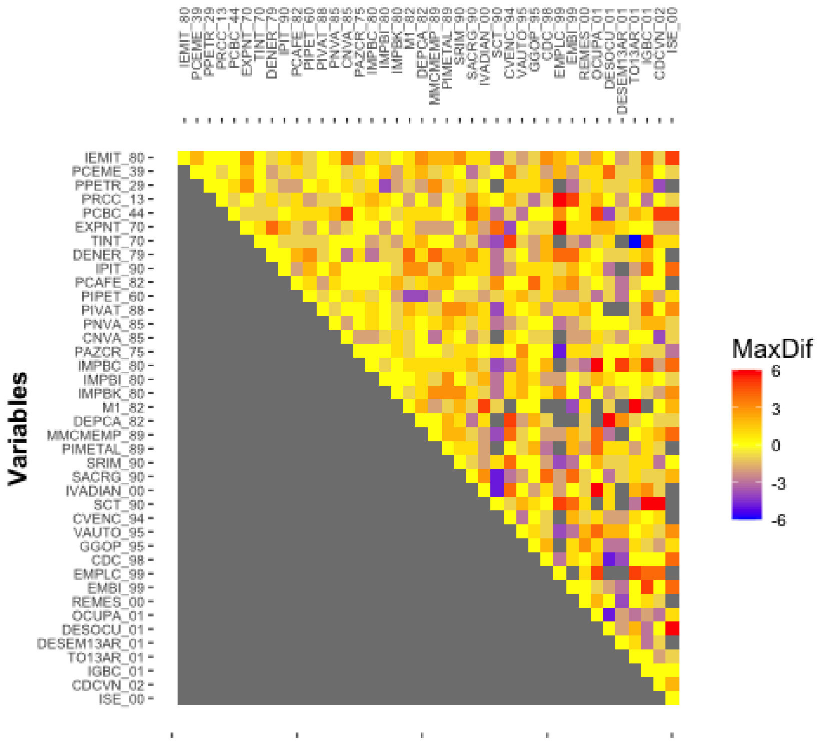

Figure 5: $\widehat{CM}$ matrix that measures the coincident relationship between a variable in the y-axis and the variables in the x-axis. For instance, zero values (yellow color) imply a coincident relationship, whereas negative values indicate a leading relationship between a variable on the y-axis and a variable on the secondary x-axis (positive values mean a lagged relationship). Gray colors along the diagonal indicate no relationship based on the CP analysis. For visualization facility, the colors under the diagonal are gray, but take a look at the properties of the CM matrix in section 2.3.

Based on the identification of turning point zones, we make an inference about the point estimation and their corresponding confidence intervals, and their characteristics. Table 8 presents the results of the estimation of the Colombian business cycle using the proposed method jointly with the IDA reference cycle (see columns *PeaksIDA* and *TrougsIDA*). Thus, according to the IDA reference cycle, Colombia has experienced four complete economic cycles over the last 47 years, with an average duration of 121 months, measured peak-to-peak. The expansions have lasted longer than the contractions. Table 8 indicates that, although our methodology estimates more turning point dates than those established by the IDA chronology, nearly all dates from the reference chronology align with or are part of those estimated by our approach. In most cases, the estimated confidence intervals cover the dates established by the IDA cycle (as shown in columns $CovP$ and $CovT$), and the differences between turning point dates (found in columns DifP and DifT) generally do not exceed five months. Furthermore, the only instance in which the estimated confidence interval does not include the turning-point date from the reference chronology is for the first peak, which occurred in June 1982 (1982:06).

In summary, our approach to estimating confidence intervals for turning-point dates supports the chronology derived from the IDA, as, with one exception, the start or end date of the recession indicated by the IDA chronology falls within the estimated confidence interval. This exception corresponds, as we have just seen, to the peak of the first recessionary phase, which, according to the IDA chronology, occurred in June 1982, while according to our methodology, it occurred sometime between March 1976 and September 1977.

Peaks	Troughs	Duration	NvarP	NvarT	PeaksIDA	TrougsIDA	DifP	DifT	CovP	CovT
1977-01	1984-05	88	6	41	1982:06	1984:03	-	2	0	1
(1976-03, 1977-09)	(1979-08, 1987-10)									
1985-01	1991-05	76	41	41						
(1979-02, 1989-12)	(1989-10, 1993-04)									
1996-11	1997-11	12	41	41	1997:12	1999:08	13	17	1	1
(1994-04, 1998-06)	(1995-08, 2000-04)									
2001-01	2002-02	13	41	29						
(2000-02, 2002-11)	(2001-05, 2002-09)									
2007-03	2009-01	22	40	28	2008:02	2009:03	11	2	1	1
(2006-03, 2008-06)	(2008-05, 2009-10)									
2011-11	2012-12	13	27	24						
(2010-12, 2012-07)	(2012-02, 2013-11)									
2015-10	2016-09	11	41	29						
(2014-10, 2017-06)	(2015-11, 2017-06)									
2019-05	2020-05	12	30	28	2019:10	2020:04	5	1	1	1
(2018-07, 2020-06)	(2019-08, 2020-12)									
2022-05	2023-02	9	28	16	2022:08		3		1	
(2021-11, 2023-02)	(2022-10, 2023-06)									

Table 8: Point estimations of the turning point dates (Peaks and Troughs), and their 95% confidence intervals for Colombia. The duration of a recession (Duration), the number of variables per zone (NvarP -for peaks-, NvarT -for troughs-), peaks and troughs according to IDA reference cycle (PeaksIDA and TroughsIDA), the difference between estimated peaks and the peaks from the reference cycle (DifP), likewise for the troughs (DifT), and indicator CovP= 1 if the estimated interval covers the peak from the chronology of IDA (PeaksIDA), and CovP= 0 otherwise (likewise CovT for troughs). Note that, although we use the standard notation for the confidence intervals, these are closed.

5 Conclusions

In this document, we develop a methodology to estimate the business cycle, yielding point estimates and confidence intervals for the turning-point dates. The methodology utilizes Bry and Boschan (1971)'s dating algorithm to identify turning points across a set of economic indicators. Once these turning points are identified, we use a novel procedure, the coincident matrix, which generalizes the Coincident Profile method proposed in Martinez-Rivera et al. (2016) and enables us to measure the degree of coincidence among economic indicators. Finally, our approach identifies a sequence of turning-point zones, minimums, and maximums that serves as the basis for inference (point estimation and confidence intervals).

We highlight some advantages of the proposed methodology. First, it does not require a specific reference cycle to define the business cycle, as evidenced by the simulation exercises. However, the estimation can be improved by using a tuning parameter calibrated from a reference cycle, as we did in both empirical applications. Second, our approach does not rely on any distributional assumptions about the groups of peaks and troughs, allowing the data to speak for itself. This fact is outlined in the Bry and Boschan (1971)'s algorithm, which does not demand any parametrization.

Through two empirical applications, using United States economic and Colombian economic data, we illustrated the effectiveness of our proposed methodology, providing not only precise point estimates for the turning point dates, which closely align with the reference cycle in each case, but also allowing us to construct confidence intervals around these turning point estimates. Additionally, in our analysis of United States economic data, we conducted a pseudo-out-of-sample exercise that showed our method can anticipate peaks 2 to 5 months in advance and troughs, although these require more time, ranging from 12 to 25 months, and those are detected before the NBER announcements.

In the Colombian case, although it is difficult to clearly identify maximum and minimum zones due to overlap, particularly at the beginning of the sample, once we define the tuning parameter, we can reduce the overlap by retaining the more relevant or coincident relationships. In addition, applying the methodology to a large set of variables is challenging; an alternative approach would be to use a dimension-reduction technique, similar to the United States application. However, we could obtain results similar to those reported in the study by Arango et al. (2025).

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Appendices

A Variables used in the application of Colombian data

Variable	**Acronym**
Total Industry Employment Index	IEMIT_80
Cement Production	PCEME_39
Oil Production	PPETR_29
Representative Price of Colombian Mild Coffee	PRCC_13
Base Purchase Price of Coffee	PCBC_44
Non-Traditional Exports	EXPNT_70
Terms of Trade (IPP Exported/IPP Imported)	TINT_70
Electricity Demand (National Interconnected System)	DENER_79
Industrial Production Index excluding Coffee Processing	IPIT_90
Coffee Production	PCAFE_82
International Oil Price	PIPET_60
Total International Air Passengers	PIVAT_88
Domestic Air Passengers	PNVA_85
Domestic Air Cargo	CNVA_85
Sugar Production	PAZCR_75
Imports of Consumer Goods	IMPBC_80
Imports of Intermediate Goods	IMPBI_80
Imports of Capital Goods	IMPBK_80
M1	M1_82
Savings Account Deposits	DEPCA_82
Total Retail Trade excluding Fuels	MMCMEMP_89
International Metals Price Index	PIMETAL_89
Total Real Wage Index in Industry	SRIM_90
Housing Construction Cost Index	ICCON_90
Livestock Slaughter: Total Cattle and Swine	SACRG_90
Monthly Domestic VAT Revenue	IVADIAN_00
Total Credit Balance	SCT_90
Percentage of Past Due Loans of the total portfolio	CVENC_94
Car sales	VAUTO_95
Total central government expenditures (OPEF)	GGOP_95
Consumer Loans	CDC_98
Employment in the Retail Sector	EMPLC_99
EMBI Spread Colombia	EMBI_99
Remittances from Workers Abroad	REMES_00
Employed Population	OCUPA_01
Unemployed Population	DESOCU_01
Unemployment Rate in Thirteen Areas	DESEM13AR_01
Employment Rate	TO13AR_01
Stock Market Index	IGBC_01
Loans Disbursed for New Home Purchases	CDCVN_02
Economic Tracking Indicator	ISE_00

Table A.1: Variables used in the application of Colombian data. The first column describes the series, while the second column presents the acronym used in this document.

B Heat map of the variables in the Colombian application

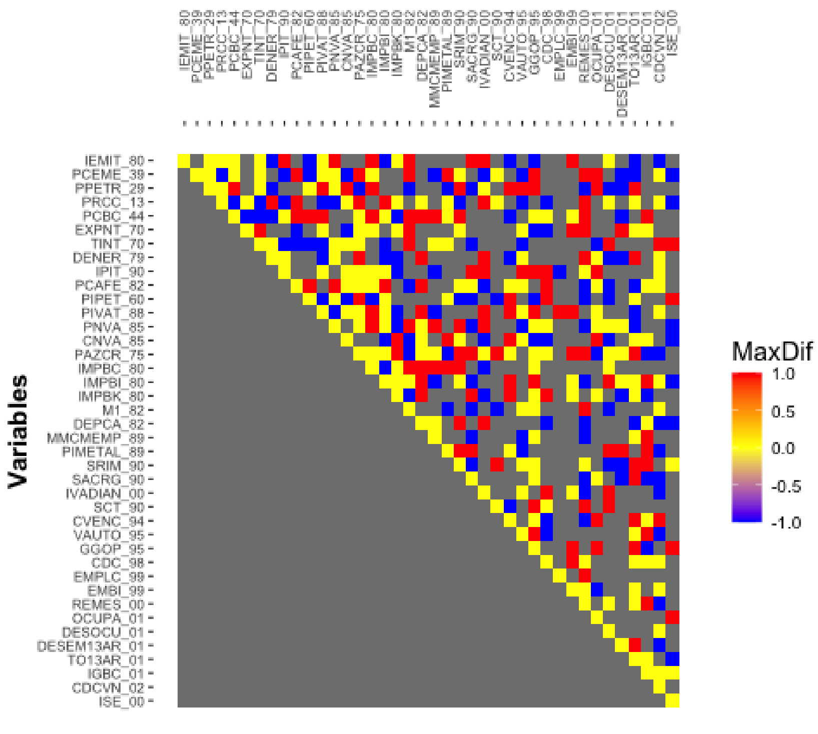

Figure B.1: $\widehat{CM}$ matrix after removing cases without evidence of the coincident relationship, according to the algorithm proposed in section 2.3.