A trend-cycle decomposition with hysteresis

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A trend-cycle decomposition with hysteresis

Javier G. Gómez-Pineda [1] and Julián Roa-Rozo [2] [3] [4]

Abstract

The paper proposes a univariate trend-cycle decomposition of GDP in which trend output exhibits hysteresis when measured in deviation form and business cycles when measured in first difference. Hysteresis is induced as the product of supply and demand shocks that are correlated. We estimate the output gap for 89 economies at a quarterly frequency and 219 economies at an annual frequency. Unlike other existing estimates of trend output, those from trend-cycle decomposition with hysteresis are not smooth, mitigate the problem of the artificial pre-recession boom, and are less sensitive to new data. Volatility and dispersion were low in the Gilded Age and high in the inter-war period, even more so in advanced economies than in emerging-market and developing economies. In the post-war period, however, volatility and dispersion were low in the former and high in the latter.

Keywords: Hysteresis; Business cycles; Business Fluctuations; Univariate model; Trend-cycle decomposition; Trend output; Output gap; Potential output

JEL codes: E32; E50; O47; E58; E37

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Una descomposición tendencia-ciclo con histéresis

Javier G. Gómez Pineda[1] and Julián Roa-Rozo[23]

Resumen

El documento propone una descomposición univariada del PIB entre tendencia y ciclo en la que el producto tendencial presenta histéresis cuando se mide en forma de desviación y presenta ciclos económicos cuando se mide en primera diferencia. La histéresis es inducida como la consecuencia de choques oferta y demanda que están correlacionados. Estimamos la brecha del producto de 89 economías en frecuencia trimestral y de 219 economías en frecuencia anual. A diferencia de otras estimaciones existentes del producto tendencial, las de la descomposición tendencia-ciclo con histéresis no son suaves, mitigan el problema del auge artificial anterior a la recesión y son menos sensibles a los nuevos datos. La volatilidad y la dispersión fueron bajas en la Edad Dorada y altas en el periodo de entreguerras, incluso más en las economías avanzadas que en las economías de mercado emergentes y en desarrollo. En el periodo de posguerra, sin embargo, la volatilidad y la dispersión fueron bajas en las economías primeras y altas en las segundas.

Códigos JEL: E32; E50; O47; E58; E37

Palabras clave: Histéresis; Ciclo de los negocios; Fluctuaciones económicas; Modelo univariado; Descomposición tendencia ciclo; Producto tendencial; Brecha del producto; Producto potencial

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Contents

List of acronyms

AD	Advanced (economies)
DDF	(Trend output) in deviation and difference form
DF	(Trend output) in difference form
EMDE	Emerging-market and developing (economies)
FEVD	Forecast error variance decomposition
FR	Filtering results
HD	Historical decomposition
IRF	Impulse response function
HP	Hodrick-Prescott (filter)
LLT	Local linear trend (filter)
TCDH	Trend-cycle decomposition with hysteresis
WWII	Second world war

1 Introduction

The current range of mainstream models, from univariate filters to structural DSGE models, precludes hysteresis by construction. However, recent work, Cerra et al. (2020), supports the hysteresis view that temporary demand shocks affect trend output. Although Cerra et al. (2020) suggest that, in dealing with hysteresis, the design of economic policy is more important than the design of a statistical filter, the current model toolkit can benefit from a trend-cycle decomposition that explicitly deals with hysteresis.

In this paper, we propose a univariate trend-cycle decomposition with hysteresis (TCDH). The paper fills a gap in the literature by addressing the need for a univariate filter that incorporates hysteresis. In TCDH, hysteresis is imposed by construction, as supply shocks are correlated with and driven by demand shocks. This feature is introduced by a Cholesky decomposition in which the order of causality is from demand shocks to supply shocks.

Working in a univariate setting has advantages and disadvantages. On the advantages side, the simpler setting allows the study of business cycles in a large number of economies and over long sample periods. The cost of the univariate setting is that the model is not as structural as a DSGE model; nevertheless, we venture to call the model semi-structural because of the direction of causality between shocks, referring to shocks that affect the output gap as demand shocks and those that affect the trend as supply shocks.

The main findings of the study can be summarized as follows. First, the estimated trend component is not smooth, which contrasts with the traditional Keynesian view described in Woodford (2009), that the business cycle is caused by shocks that affect aggregate demand while aggregate supply evolves smoothly. It also contrasts with commonly used Hodrick and Prescott (HP) and local linear trend (LLT) filters (see Hodrick and Prescott (1997) and Harvey and Jaeger (1993), respectively). Although this view of an unsmooth trend differs from the traditional view embedded in the HP and LLT filters, it is consistent with some desirable features of a trend. Indeed, according to Harvey (1989), p. 284, "viewed in terms of prediction, the estimated trend is that part of the series which when extrapolated gives the clearest indication of the future long-run movements of the series... The definition makes no mention of smoothness, and it is consistent with the idea of indicating 'general direction'". Second, the model mitigates what we call the problem of the artificial pre-recession boom present in the HP and LLT filters. Third, the estimated output gap or cycle is more stable to new data, so it should be more stable in real-time analysis compared to the HP and LLT filters, where the future strongly influences the past.

The second result is that we estimate the hysteresis elasticity for a large group of economies. The hysteresis elasticity is the percent reduction in the flow of future potential output per percentage-point-year of the present-period output gap (see DeLong and Summers (2012)). We estimate this parameter for 89 economies at the quarterly frequency and 219 economies and territories at the annual frequency. A meta-analysis of these estimates suggests three main findings. First, this parameter averages around 0.28. Second, the hysteresis elasticity is higher in emerging market (EM) economies than in advanced market (AM) economies. Third, there is relevant variation across geographical regions, suggesting that country-specific economic characteristics may play a role in determining the magnitude of the hysteresis elasticity.

The paper makes two contributions to the literature. The first is a trend-cycle decomposition that allows for hysteresis, where trend output is not smooth and undergoes cycles. The second is the estimation of trend output and output gap, the main features of which are analysed in the paper.[5]

The paper draws on several strands of the literature. The first concerns the direction of causality between supply and demand shocks. Two recent papers illustrate opposite directions. The first, Guerrieri et al. (2022), introduces "Keynesian supply shocks", or declines in trend output in one sector that lead to declines in demand in other sectors, i.e. supply leads demand. The second, Furlanetto et al. (2021), extends the supply-demand shock decomposition proposed by Blanchard and Quah (1989) to include demand shocks that can potentially affect output permanently, i.e. demand leads supply.

Our preferred interpretation is the hysteresis view, where, as noted above, temporary demand shocks permanently affect trend output. The order of the shocks in the Cholesky decomposition affects the impulse response functions (IRFs) and forecast error variance decompositions (FEVDs), but not the filtering results (FRs), i.e. the estimated cycle and trend. Then, the estimated trend and cycle can be consistent; on the one hand, with the view of Keynesian supply shocks in Guerrieri et al. (2022), where supply affects demand; and on the other hand, with the view in Furlanetto et al. (2021), where demand affects supply permanently. We use demand shocks first in the Cholesky decomposition for two main reasons.

[5] The paper's web page provides databases with output gap estimates for the 89 and 219 economies and territories at quarterly and annual frequencies respectively; as well as estimates of the hysteresis elasticity for these economies.

First, hysteresis is by definition an effect of demand on supply. Second, the existing hysteresis literature includes the output gap in the trend output equation (among the papers in this literature are DeLong et al. (2012), Jordà et al. (2020b), Ball and Onken (2021) and Kienzler and Schmid (2014)).

The second strand of the literature concerns the magnitude of the hysteresis elasticity. To our knowledge, there are only two studies that provide estimates of this parameter: Jordà et al. (2020a) and Roa Rozo (2024). These papers estimate the hysteresis elasticity for a group of economies rather than for each individual economy. Thus, this paper significantly increases the number of hysteresis elasticity estimates. In terms of the size of the hysteresis elasticity, the results are consistent with the findings of Jordà et al. (2020a) and Roa Rozo (2024).

The paper consists of 12 sections, including this introduction. The model, data, calibration and estimation of the model are presented in Sections 2 to 4. Section 5 discusses the role of demand and supply shocks in the business cycle; in this section it is shown that demand shocks spill over to trend output. Section 6 presents the results of the estimated parameter of the model, the standard deviation of supply shocks. This section also analyses the convergence of the posterior distribution of this estimated parameter. Section 7 presents the results on the hysteresis elasticity and shows that they are similar to those found in the existing literature. Section 8 presents the FRs. This section is divided into three subsections that discuss the sensitivity of the estimated gaps to incoming data, the 2008-2009 financial crisis and pandemic recession, and the business cycle from a historical perspective. Section 9 compares the output gap estimates from the TCDH with those from publicly available databases. This section highlights some of the advantages of the TCDH, e.g. the TCDH mitigates the problem of the artificial pre-recession boom, a feature that is prevalent in commonly used filters. Section 10 discusses the robustness of the results to the calibration of the correlation between demand and supply shocks. We show that the estimated hysteresis elasticity is robust to other, albeit still strong, values of the correlation coefficient. Section 11 discusses the limitations of the TCDH and issues for future research. Section 12 provides some concluding remarks. Online appendixes complement the paper with the model in deviation form, the IRFs at a quarterly frequency, the FRs for the output gap and trend output for each country and sample, the details of the data according to the different data sources, and the model codes at quarterly and annual frequencies.

2 The TCDH

The TCDH is a standard local linear trend (LLT) filter, Harvey and Jaeger (1993) and Durbin and Koopman (2012), extended with correlated shocks.

Let y_t be output; $\hat{y}_t$, the output gap; $\bar{y}_t$, trend output; γ_t, the long-run trend output growth rate; g, the constant average growth rate,[6] $\varepsilon_t^{\hat{y}}$, the demand shock; $\varepsilon_t^{\bar{y}}$, the supply-level shock, and ε_t^{γ} the supply-growth shock.

The decomposition consists of the following equations:[7]

$$\hat{y}_t = \alpha \hat{y}_{t-1} + \varepsilon_t^{\hat{y}}, \tag{1}$$

$$\bar{y}_t = \bar{y}_{t-1} + \frac{1}{4}\gamma_t + \varepsilon_t^{\bar{y}}, \tag{2}$$

$$\gamma_t = \theta \gamma_{t-1} + (1-\theta) g + \varepsilon_t^{\gamma}, \tag{3}$$

$$y_t = \hat{y}_t + \bar{y}_t. \tag{4}$$

Let stack the shocks in vector

$$\varepsilon_t = \left[\varepsilon_t^{\hat{y}}, \varepsilon_t^{\bar{y}}, \varepsilon_t^{\gamma}\right], \tag{5}$$

where

$$\varepsilon_t \sim N(0, \Omega), \tag{6}$$

and

$$\Omega_t = \begin{bmatrix} \sigma_{\varepsilon^{\hat{y}},t} & \rho\sigma_{\varepsilon^{\hat{y}},t}\sigma_{\varepsilon^{\bar{y}},t} & 0 \\ \rho\sigma_{\varepsilon^{\hat{y}},t}\sigma_{\varepsilon^{\bar{y}},t} & \sigma_{\varepsilon^{\bar{y}},t} & 0 \\ 0 & 0 & \sigma_{\varepsilon^{\gamma}} \end{bmatrix}. \tag{7}$$

The output gap equation (1) states that the output gap converges to zero with speed α and is driven by demand shocks $\varepsilon_t^{\hat{y}}$.

The trend output level equation (2) states that trend output has a unit root, grows at the rate γ_t and is driven by supply-level shocks $\varepsilon_t^{\bar{y}}$. Alternatively, the equation states hat trend output growth growth, $4(y_t^{\bar{y}} - y_{t-1}^{\bar{y}})$, fluctuates around the long-run trend output growth rate γ_t, as driven by supply-level shocks $\varepsilon_t^{\bar{y}}$.

Next, the trend output growth equation (3) states that the long-run trend output growth rate, γ_t, converges to the constant average growth rate, g, at the speed θ and is driven by supply-growth shocks ε_t.

In long samples, GDP growth may have a low-frequency component that cannot be captured by equation (3), where the long-run trend output growth rate converges to a constant

[6]This is the constant average, long-run trend output growth rate.

[7]The fraction 1/4 in equation (2) converts quarterly growth into annual terms. In the annual frequency equation (2) becomes $\bar{y}_t = \bar{y}_{t-1} + \gamma_t + \varepsilon_t^{\bar{y}}$. The rest of the equations are unchanged.

average growth rate g. The movements in this low-frequency component of output growth can be ignored in shorter samples but it can hardly be ignored in any country for a long enough sample. For this reason we calibrate the speed of convergence θ either as $\theta < 1$ in samples of about 10 years and also for forecasting purposes and as $\theta \approx 1$ in long samples.

Equation (8) is the non-diagonal matrix covariance matrix of the shocks. Demand and supply-level shocks, $\varepsilon_t^{\hat{y}}$ and $\varepsilon_t^{\bar{y}}$, respectively, are contemporaneously correlated. The parameter ρ is the correlation coefficient. Supply-growth shocks, ε_t, are not correlated with the other shocks.

The matrix Ω_t has a corresponding Cholesky decomposition where we assume that the order of the shocks is demand shocks first and supply-level shocks second. With this order, demand shocks affect both the output gap and trend output in the IRFs and FEVDs, according to equations (1) and (2), respectively. In turn, in this order supply-level shocks affect only trend output in the IRFs and FEVDs, according to equation (2).

We assume that demand shocks come first for three reasons. The first one is the notion of hysteresis as an effect of demand on supply. The second one is the hysteresis literature mentioned above, where the output gap is included in the trend output equation, e.g. DeLong et al. (2012), Jordà et al. (2020b), Ball and Onken (2021) and Kienzler and Schmid (2014). The third one is empirical, namely, an estimation of the effect of demand and supply shocks using output gap data from two publicly available databases, the WEO database and the OECD database. Following Roa Rozo (2024), we use an error correction model for the variables output and trend output. The results are in the form of FEVDs and IRFs. The FEVDs indicate that supply shocks explain less than 5% of the FEVD of the output gap while demand shocks explain 16% of the FEVD of potential output growth. The IRFs, in turn, suggest that a supply shock induces an output gap response that is negative only at the time of the shock, and is followed by a positive output gap thereafter. Thus, the output gap data available in the WEO and OECD databases, and the implicit trend output data, point to the dominance of demand shocks on trend output.[8]

In the Ω_t matrix, the supply-growth shocks ε^{γ} are not contemporaneously correlated with the other two shocks in the model. Thus, the supply-growth shocks ε^{γ} allow counterfactual output to move smoothly.

The correlation structure of the shocks is the only feature of the model that allows for

[8]The results of the error correction model are available upon request.

hysteresis; the second feature of the shock structure in the TCDH is heteroskedasticity.

The shocks were allowed to be heteroskedastic to allow for a treatment of known increases in the variance of the shocks.

The TCDH intersects with other known filters under certain parameter values. When $\rho = 0$ and shocks are homoscedastic, the TCDH becomes the LLT filter in Harvey and Jaeger (1993) and Durbin and Koopman (2012). In turn, if $\alpha = 0$, $\theta = 1$, $\sigma_{\varepsilon_t^{\bar{y}}} = 0$, and also if shocks are homoskedastic, and for an appropriate choice of relative standard deviations of the shocks, the model becomes the Hodrick-Prescott filter.

The responses to demand and supply shocks are shown in Figures 1 and 2.[9] The response of the nonstationary variables, output and trend output are shown in Figure 1. The response of the stationary variables, the output gap and trend output in difference form (DF) is shown in Figure 2.

Given the correlated structure of the shocks, a demand shock generates a response in the output gap and in trend output DF, that is, in demand and supply. The response to a demand shock is shown in Panels A of Figures 1 and 2. Both the output gap and trend output move. The latter reacts due to the order of the shocks in the Cholesky decomposition. Since the demand shock moves output more than trend output on impact, the output gap falls. The demand shock causes a correlated response in both the output gap and trend output DF.

The supply shock moves trend output but not the output gap. The responses to a supply shock are shown in Panels B of Figures 1 and 2.

Note that in the response in Figure 1 a shock to $\varepsilon_t^{\hat{y}}$ has a long-run effect on trend output of

$$\mu_y = \frac{\rho \sigma_{\varepsilon_t^{\bar{y}}}}{\sigma_{\varepsilon_t^{\hat{y}}}}, \tag{8}$$

that is, a demand shock of size $\varepsilon_t^{\hat{y}}$ induces a response of trend output of size $\rho \varepsilon_t^{\bar{y}}$. The ratio normalizes the effect of the shock as per unit shock in $\varepsilon_t^{\hat{y}}$.

Note also that in the response in Figure 2 the shock to $\varepsilon_t^{\hat{y}}$ has a cumulative effect on the output gap of

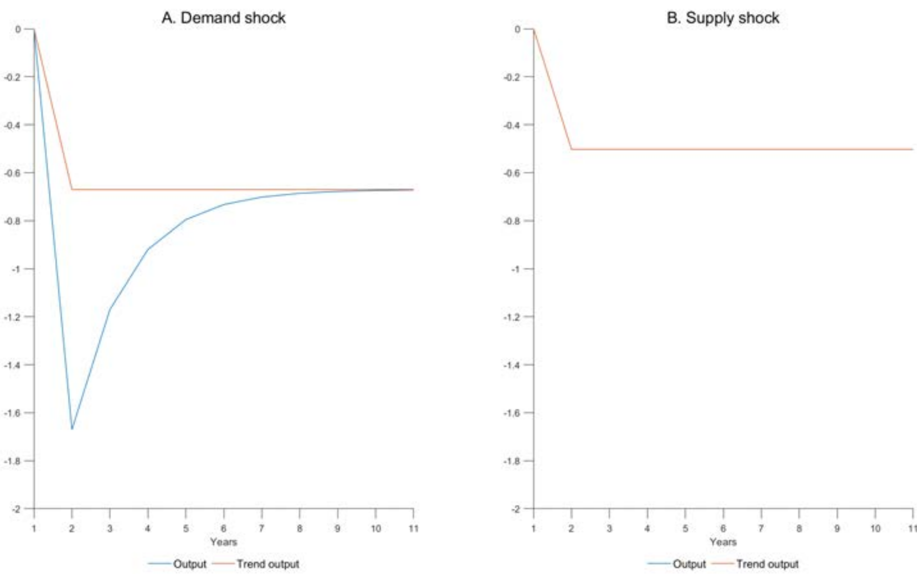
$$\lambda_y = \alpha_y^0 + \alpha_y^1 + \alpha_y^2 + ... = \frac{1}{1 - \alpha_y}, \tag{9}$$

where, locally, parameter α_y takes the sub-index y to denote annual frequency.[10]

[9] For the sake of simplicity, supply-level shocks will be referred to as supply shocks from here on.

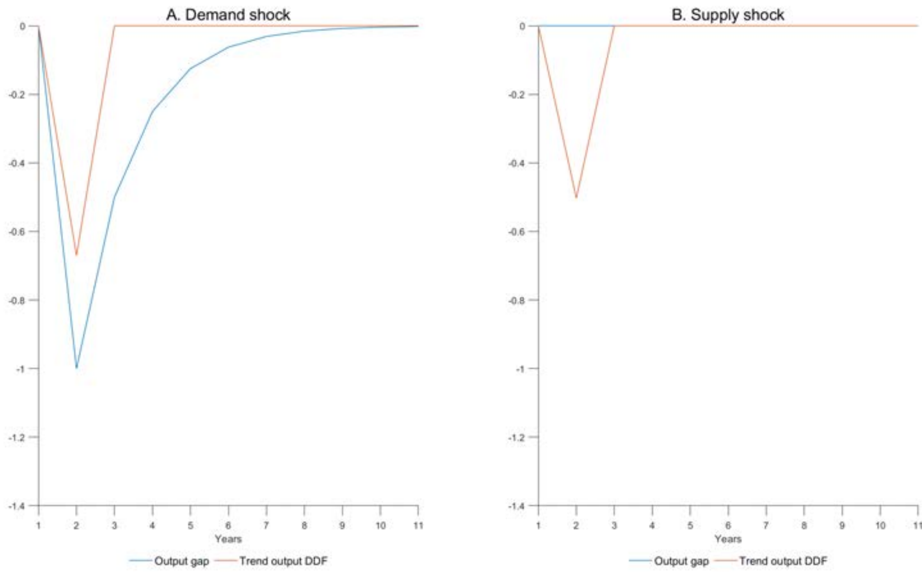
[10] In quarterly frequency equation (9) is $\lambda_q = \frac{1}{4}(\alpha_q^0 + \alpha_q^1 + \alpha_q^2 + ...) = \frac{1}{4(1-\alpha_q)}$, where factor 1/4

Figure 1: The role of demand and supply shocks in the IRFs – The nonstantionary variables



Note: In Panel A, a demand shock moves output and trend output. The output gap is the difference between the blue and the orange line. In Panel B, a supply shock does not induce a response of demand.

Figure 2: The role of demand and supply shocks in the IRFs – The stationary variables



Note: In Panel A, a demand shock moves the output gap. In Panel B, a supply shock does not induce a response of the output gap.

The degree of hysteresis or the hysteresis elasticity, η, is the long-run effect per unit of output gap, and is given by

$$\eta = \frac{\mu_y}{\lambda_y}. \tag{10}$$

While the foregoing completes the exposition of the equations of the TCDH, it is useful to also define trend output in counterfactual terms as well in deviation from the counterfactual.

Trend output is driven by supply shocks, $\varepsilon_t^{\bar{y}}$, and long-run output growth shocks, ε_t^{γ}, according to equations (2) and (3). A counterfactual version of trend output, $\bar{y}_t^C$, growing at the smooth long-run trend output growth rate γ_t, can be obtained from equations (2) and (3) by abstracting from supply shocks $\varepsilon_t^{\bar{y}}$. Alternatively, a measure of trend output in deviation form, $\bar{y}_t^D$, can also be obtained from equations (2) and (3) by abstracting from supply-growth shocks, ε_t^{γ}.

We use trend output in deviation form, $\bar{y}_t^D$, to illustrate hysteresis. In addition, we use trend output DDF, $\bar{y}_t^D - \bar{y}_{t-4}^D$, to illustrate that (in first-difference form) under the assumption of a strongly positive correlation of demand and supply shocks ρ, trend output undergoes

converts the sum of output gaps over four quarters into a percent, annual, representation.

business cycles that are correlated with those of the output gap.[11]

Trend output in deviation form, $\bar{y}_t^D$, gives a notion of hysteresis; that is, a notion of the permanent loss of output due to a given recession. Accordingly, at the onset of a recession, the hypothetical counterfactual trend output continues to grow at the long-run trend output growth rate, γ_t, while actual trend output falls at the rate $4(\bar{y}_t - \bar{y}_{t-1})$, as determined by the correlated supply shocks.[12]

The counterfactual trend output equations are as follows:[13]

$$\bar{y}_t^C = \bar{y}_{t-1}^C + \frac{1}{4}\gamma_t^C, \tag{11}$$

$$\gamma_t^C = \theta\gamma_{t-1}^C + (1-\theta)g + \varepsilon_t^{\gamma}, \tag{12}$$

where γ_t^C is the counterfactual trend output long-run growth rate.[14]

In equation (11) counterfactual trend output is the level of trend output that would have occurred in the absence of correlated supply shocks. By abstracting from supply shocks $\varepsilon_t^{\bar{y}}$, the counterfactual measure of trend output provided by equations (11) and (12) abstracts from the shocks that create a cycle in trend output (in difference form) and provides a measure of counterfactual output that follows the long-run smooth behavior of the trend, driven by the long-run supply shocks ε_t^{γ}.[15]

The equations for trend output in deviation form are as follows:[16]

$$\bar{y}_t^D = \bar{y}_{t-1}^D + \varepsilon_t^{\bar{y}}, \tag{13}$$

$$y_t^D = \hat{y}_t + \bar{y}_t^D. \tag{14}$$

In equation (13), trend output in deviation form is the level of trend output that would have occurred in the absence of trend output growth shocks.

Trend output in deviation form should not be robust to $\sigma_{\varepsilon_t^{\gamma}}$ or θ. We then use trend output in deviation form for illustrative purposes; that is, to show the hysteresis under the given equations and calibrated coefficients. Nevertheless, the trend output DDF, $4(\bar{y}_t^D - \bar{y}_{t-1}^D)$, is stationary and correlated with the cycle in the output gap in a way that is robust to reasonable values around the calibrated correlation coefficient ρ. It can thus be seen as

[11]In annual frequency trend output DDF is $\bar{y}_t^D - \bar{y}_{t-1}^D$

[12]In annual frequency, the rate is $\bar{y}_t - \bar{y}_{t-1}$.

[13]In annual frequency equation (11) becomes $\bar{y}_t^C = \bar{y}_{t-1}^C + \gamma_t^C$.

[14]A measure of counterfactual trend output plus the output gap can be obtained as $y_t^C = \hat{y}_t + \bar{y}_t^C$.

[15]In contrast with this method, a linear extrapolation of past data is inconsistent with the observed data because it assumes that the smooth trend is linear and that the long-run supply shocks are zero.

[16]Note that $\gamma_t^D = 0$.

another manifestation of the business cycle.

The quarterly frequency model was fed with quarterly annualized output growth data, $4(y_t - y_{t-1})$, and the annual frequency model was fed with annual growth data, $y_t - y_{t-1}$.[17]

3 The data

At the quarterly frequency, we use real GDP data from the IMF's International Financial Statistics (IFS) database, 65 countries from seasonally adjusted data, and 30 countries from non-seasonally adjusted data, which we adjust using the X13 ARIMA-SEATS method. Figure 1 shows the sources of the data and the data themselves.

Of the 97 countries in the long (1995Q1-2023Q3) database, the euro area was excluded to avoid double counting and 5 countries were excluded because the available data covered less than 8 years. The number of countries in the final quarterly database is 91, of which 31 are advanced (AD) economies and 50 are emerging-market and developing (EMDE) economies. In addition to the long sample, three 10-year samples (1995Q1-2004Q4, 2005Q1-2014Q4, and 2015Q1-2023Q3) were used to estimate hysteresis at the end of the century, during the 2007-2008 financial crisis and during the pandemic recession, respectively. In these samples, countries with less than 32 quarters of data were excluded. The list of countries in the quarterly databases can be found in Annex 4.

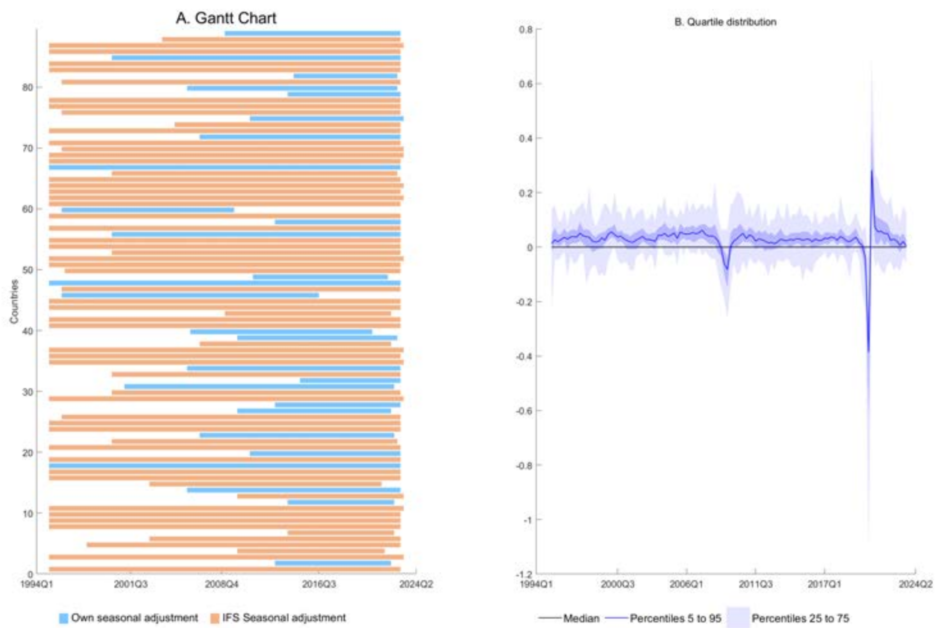
The annual database has the broadest coverage across countries and over time. Four data sources were used: World Bank Development Indicators (WBDI) database, Penn World Tables (PWT) database,[18] [19] the Maddison Project Database (MPD), version 2020, see Bolt and Van Zanden (2020), and the IMF World Economic Outlook database. The sources differ in terms of country and sample coverage. The broadest country coverage is in the WBDI, with data for 217 countries, while the broadest time coverage is in the MAD database, with continuous annual data starting in 1870 for 25 countries and in 1820 for 13 countries.

[17]Log output was defined as $y_t = \log(Y_t)$, where Y_t is real GDP and the log is in base e. Levels were set outside the model by setting trend output and counterfactual trend output equal to 100 in a given base year and multiplying log growth by 100. In turn, trend output in deviation form was set at 0 in the base year. In other words, $\bar{y}_j = \bar{y}_j^C = 100$ and $\bar{y}_j^D = 0$, where j is the base year, defined depending on the database being analyzed.

[18]Of the five growth concepts available in the Penn World Tables database, we used RDGPNA, which is the one recommended by Feenstra, Inkar, and Timmer (2015, pp. 3153, 3154, and 3157) as the one that should be used if the goal is to compare growth over time in each country.

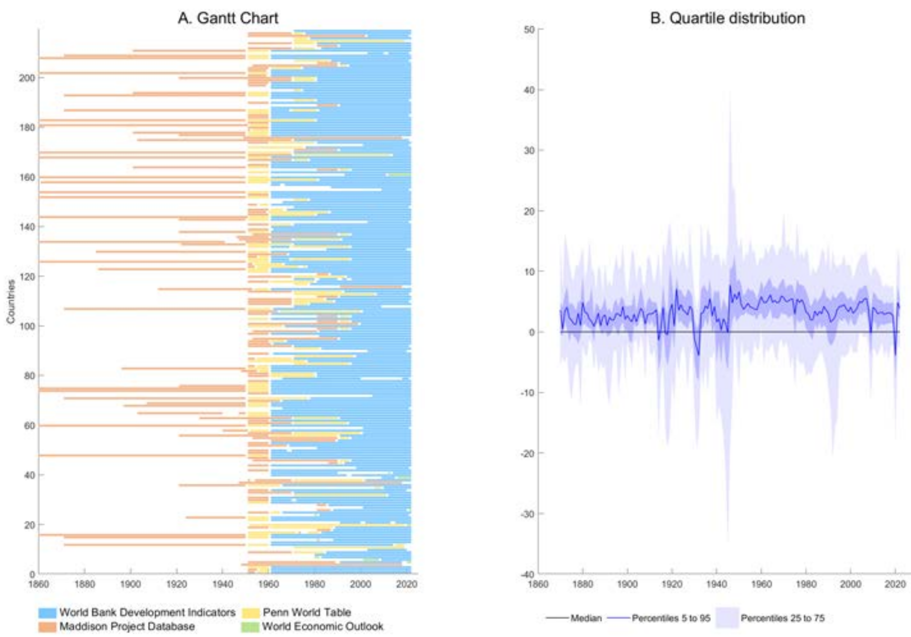
[19]The PWT database also allowed the inclusion of two other countries in the study, Anguilla and the British Virgin Islands.

Figure 3: A summary of the data – The quarterly database



Note: Panel A shows that most of the available data start in 1995 and are mostly available from the IMF IFS in seasonally adjusted form. For most countries the sample ends in 2022Q2 and in a few cases in 2022Q3. Panel B shows the quartile distribution of GDP growth excluding the bottom and top 5 percent.
Source: IMF International Financial Statistics.

Figure 4: A summary of the data – The annual database



Note: Panel A shows that data are available from the MAD database for several countries starting in 1870. The number of countries with available data jumps in 1950 when we use the PWT database. Starting in 1960, data are available from the WBDI, the database with the broadest coverage in terms of number of countries. At the end of the sample, in 2022 in most cases, the database has been supplemented with data and projections from the WEO database. Panel B shows the quartile distribution of GDP growth excluding the bottom and top 5 percent.
Sources: WBDI; PWT; MAD version 2020; and WEO database, April 2023.

An electronic appendix to the annual database shows real GDP growth according to each of the data sources, as well as the source selected in the database for each country and year. As shown in this appendix, GDP growth data are broadly consistent across sources. Nevertheless, it seemed obvious that, when data was available from more than one source, a hierarchy in the choice of source could provide some (although mostly marginal) quality improvements. In adopting this hierarchy, we placed the WBDI first because of its coverage, and the MAD database last because, as a database that is focused on GDP per capita, average GDP growth for some countries and periods did not match that of other databases for some of the time periods where data overlapped. Finally, we used the WEO database mainly to update GDP growth in the last year of the database, in some cases using the WEO projections.

In the MAD database, data are not available for real GDP, but for GDP per capita and population. Therefore, real GDP data were obtained as the product of GDP per capita and population. The calculated data for real GDP therefore depend on the MAD population estimates. For a few countries and for periods covering a limited number of years, the average GDP growth implicit in the MAD database differed from that obtained from other sources. However, the turning points in the growth cycle are consistent between the MAD and other databases.

The data sources and the data themselves for the annual database are shown in Figure 2. The figure shows that the number of economies for which real GDP data are available increases over time. An important jump in the number of countries with available data took place in 1950. Nevertheless, data are available for several AD economies before 1950 and for a few since 1820 (not shown in Figure 2).

Of the 222 countries in the initial annual database, 3 countries were excluded because they had fewer than 8 data points. The number of countries in the final annual database is 219, of which 57 are AD economies and 162 are EMDE economies.

As shown in Figure 4, a decline in long-run output growth occurs in 1979, see and also Kose and Terrones (2015). Then, at the annual frequency, for estimation we used a short and relatively homogeneous sample (1979-2022). An additional long sample (1870-2022) was used to run the model as a calibration.

4 Estimation and calibration

A handful of issues need to be addressed in the calibration and estimation of the model parameters. The parameters to be calibrated and estimated are the correlation coefficient, the standard deviation of the shocks, the speeds of adjustment, and the constant average growth rate. The vector of these coefficients is $\omega = (\rho, \sigma_{\varepsilon_t^{\hat{y}}}, \sigma_{\varepsilon_t^{\bar{y}}}, \sigma_{\varepsilon_t^{\gamma}}, \alpha, \theta, g)$. In addition, other calibration and estimation issues are the sample period, the heteroskedasticity factors, and the number of iterations needed to ensure convergence in the estimation.

One approach to estimating the model is to estimate all the parameters in the vector ω. According to the identification tests in Pfeifer (2013) (not reported), all parameters in the vector ω can be shown to be identified, although the identification strength for each of them is not strong. Therefore we preferred to estimate only one parameter, one that would be relevant for estimating the hysteresis elasticity.

Equations (8) and (10) show that, given a calibrated α, the relevant information for estimating the hysteresis elasticity is the correlation coefficient, ρ, the standard deviation of the supply-level shock, $\sigma_{\varepsilon^{\bar{y}}}$, and the long-run effect of a unit demand shock on trend output, μ_y. For the latter, we used the estimate $\mu_y = 0.67$, estimated by Roa Rozo (2024) with real-time data.[20] Given this calibrated level for the long-run effect μ_y, and noting that $\lambda_y = 1/(1-\alpha_y) = 2$, the prior mean for the hysteresis elasticity is about 0.33 according to equation (10).

At the quarterly frequency, we used a long-run effect of $\mu_q = 0.42$ because it corresponds to the same long-run effect of 0.67 for a series of 4 shocks at the quarterly frequency.[21] In addition, dividing the long-run effect μ_q by the cumulative sum of the output gap in quarterly frequency, $\lambda_q = \frac{1}{4(1-\alpha_q)} = 1.25$, see equation (10), the calibrated hysteresis elasticity in quarterly frequency is also about 0.33.

Regarding ρ and $\sigma_{\varepsilon^{\bar{y}}}$, note that the correlation coefficient, ρ, is by definition less than or equal to one, while the standard deviation of the supply-level shock $\sigma_{\varepsilon^{\bar{y}}}$ is not bounded from above. We then preferred to calibrate the correlation coefficient, ρ, and estimate the standard deviation of supply-level shocks, $\sigma_{\varepsilon_t^{\bar{y}}}$, so that the estimation of the long-run effect

[20]See the hysteresis elasticity on page 3, Table 1, of Roa Rozo (2024) where it appears as the long-run effect in the WEO sample.

[21]Given $1-\alpha_q = 0.2$, a long-run effect of 0.67 over four quarters can be obtained with a row of four shocks of size 1, 0.2 0.2 0.2. Each 0.2 shock offsets the reduction in the impact of the shock explained by the fact that α is 0.8.

Table 1: The calibrated parameters

Parameter	Quarterly frequency	Annual frequency
ρ	0.8	0.8
$\sigma_{\varepsilon_t^{\hat{y}}}$	$0.15[4(y_t - y_{t-1})]$	$0.5(y_t - y_{t-1})$
$\sigma_{\varepsilon_t^{\bar{y}}}$	$\mu_q \sigma_{\varepsilon_t^{\hat{y}}}/\rho$	$\mu_y \sigma_{\varepsilon_t^{\hat{y}}}/\rho$
$\sigma_{\varepsilon_t^{\gamma}}$	$0.25\sigma_{\varepsilon_t^{\hat{y}}}$	$0.25\sigma_{\varepsilon_t^{\hat{y}}}$
μ	0.42	0.67
α	0.8	0.5
θ	0.85 or 0.99	0.55 or 0.99
g	Sample mean	Sample mean
Heterosk. factor 1	2020Q1	Gilded Age
Heterosk. factor 2	2020Q2-2020Q3	Interwar period
Heterosk. factor 3	2020Q4-2021Q4	Pandemic

Note: the parameters are invariant across frequencies.
Source: see text.

μ_y and the hysteresis elasticity η would not be constrained by the ceiling in the estimation of the correlation coefficient, ρ.

Solving equation (8) for the standard deviation of the supply-level shock gives

$$\sigma_{\varepsilon^{\bar{y}}} = \mu_y \sigma_{\varepsilon^{\hat{y}}}/\rho, \tag{15}$$

then, the calibrated prior mean for the standard deviation of the supply-level shock $\sigma_{\varepsilon^{\bar{y}}}$ is the right-hand side of equation 15, as shown in Table 2. By estimating the standard deviation of supply-level shocks, $\sigma_{\varepsilon_t^{\bar{y}}}$, its prior mean is then consistent with the estimate of the hysteresis elasticity in Roa Rozo (2024) and the prior standard deviation allows for a range of posterior estimates of the hysteresis elasticity across economies.

Table 2: The prior distribution

Parameter	Quarterly frequency		Annual frequency	
	Distribution	Interval	Distribution	Interval
$\sigma_{\varepsilon_t^{\bar{y}}}$	Inv-Gamma($\frac{\mu_q}{\rho}\sigma_{\varepsilon_t^{\hat{y}}}$,1.5)	[0 - 10]	Inv-Gamma($\frac{\mu_y}{\rho}\sigma_{\varepsilon_t^{\hat{y}}}$,1.5)	[0 - 10]

Note: the long-run effect $\mu_y = 0.67$ was taken from Roa Rozo (2024). In quarterly frequency, the long-run effect $\mu_q = 0.42$ was set at a value that would give the same hysteresis for a 4-quarter change in the output gap.
Source: see text.

A summary of the calibrated parameters and priors appears in Table 1. The first parameter in the list is the correlation coefficient, ρ. We calibrated ρ at 0.8 so that the assumed correlation would be strongly positive and that the robustness exercise would also include a strongly positive correlation, for the robustness analysis we included ρ equal to 0.7 and 0.9.

In the calibration of the correlation coefficient, it is worth considering the DSGE literature. In a DSGE model, the correlation between trend output and the output gap can be either positive or negative, see for example Vetlow et al. (2011) and Kiley (2013), where demand shocks move output and trend output in the same direction, while permanent technology shocks move trend output and the output gap in the opposite direction. However, existing DSGE models, such as the one reviewed in Vetlow et al. (2011) and the one in Kiley (2013), are not designed to explicitly account for hysteresis and so may not be informative about the sign of the correlation of demand and supply shocks in a model designed to address the issue of hysteresis. Moreover, in DSGE models, "trend output [as opposed to natural or efficient output in DSGE models] most closely resembles the traditional approach to measuring potential output as a persistent, smooth process," (Vetlow et al., 2011), p. 18. For this reason, trend output in DSGE models as illustrated by Kiley (2013), p. 15, is prone to the problem of the artificial pre-recession boom.[22]

The second issue to be addressed with the calibration and estimation of the model is that of the calibrated standard deviations of the shocks, $\sigma_{\varepsilon_t^{\hat{y}}}$, $\sigma_{\varepsilon_t^{\bar{y}}}$ and $\sigma_{\varepsilon_t^{\gamma}}$.

The calibrated standard deviation of the demand shock $\sigma_{\varepsilon_t^{\hat{y}}}$ was set at a fraction of the standard deviation of the observed variable, the output growth rate $y_t^{\Delta} = 4(y_t - y_{t-1})$.[23] This fraction was set as the median standard deviation across countries of the ratio $\sigma_t^{\varepsilon^{\hat{y}}} / \sigma_t^{y^{\Delta}}$ in an estimation of the model shocks with a calibrated model.[24] Using round numbers, the fraction was 0.15 at the quarterly frequency and 0.5 at the annual frequency, as shown in Table 1.

The prior standard deviation of the supply shock $\sigma_{\varepsilon_t^{\bar{y}}}$ was set as a ratio of the standard deviation of the demand shock. This ratio was set to 0.6 at the quarterly frequency and

[22]In sharp contrast with the strong positive correlation, ρ, assumed in this paper Morley et al. (2003) perform an unconstrained estimation of the correlation parameter. They obtain a correlation of -0.9. It is important to note, however, that with this estimated negative correlation, the estimated cycle has a length of about 2 years, which challenges an understanding of this cycle as a traditional business cycle. The length of the estimated cycle with negative correlation suggests the validity of restricting the correlation to positive.

[23]In annual frequency log growth is $y_t^{\Delta} = y_t - y_{t-1}$ and the fraction is denoted as κ_y.

[24]In contrast with a model with estimated standard deviations, in a calibrated model only the relative standard deviations are relevant for the smoothing results.

0.957 at the annual frequency, so that the long-run effect of a unit demand shock is at both frequencies was $\eta = 0.67$,[25] see Table 1.

The calibrated standard deviation of shock $\sigma_{\varepsilon_t^\gamma}$ was set at a ratio of the standard deviation of the demand shock. The ratio was set to 0.25 at both the quarterly and annual frequencies so that the long-run rate of growth, γ_t, would be flexible enough to accommodate long-run changes in growth as well as rigid enough to let trend output DF, $4(\bar{y}_t - \bar{y}_{t-1})$, fluctuate around the long-run trend output growth rate, γ_t, see Table 1.

The third one is the calibration of the speeds of adjustment, α and θ. These parameters were chosen to maintain a certain comparability of the half-life of the shocks across frequencies. Another criterion that was used was to try to keep these intervals in round numbers, for simplicity. For the long sample starting in 1870 theta was calibrated close to one, $\theta \approx 1$, so that the model could capture permanent shifts in the long-run trend output growth rate γ_t.

The fourth issue is that of the constant average growth rate, g. It was calibrated to the historical average of output growth over each 10-year sample considered. Although the calibrated value of g is meaningless in long samples, where we used $\theta \approx 1$, it is relevant to illustrate hysteresis in short samples.[26]

As mentioned above, we estimated the model in different samples. At the quarterly frequency, a long sample (1995Q1-2023Q3) was used to illustrate the business cycle and to estimate the hysteresis elasticity. In addition, three 10-year samples (1995Q1-2004Q4, 2005Q1-2014Q4, and 2015Q1-2023Q3) were used to illustrate the hysteresis in the turn-of-the-century crisis, the 2008-2009 financial crisis, and the pandemic recession.[27] At the annual frequency, a long sample (1870-2022) was used to illustrate the business cycle using a calibrated version of the model. In addition, a shorter sample (1979-2022) was used to estimate the hysteresis elasticity.

Another issue in the estimation is that of heteroskedasticity. The heteroskedasticity factors were set, for each country and episode, as a multiple of the standard deviation of output growth in the periods of lower volatility. These episodes were selected according to the median rolling

[25] At the quarterly frequency, the fraction was set using $\eta = 0.67$ and so as to obtain a one percent increase in the output gap for a series of 4 quarterly shocks.

[26] Regarding the calibration of g in short samples, a more granular approach to the estimation of the model in each country and period could also include judgment. We use the sample average as a way to cope with the number of countries in the study.

[27] The results of the 10-year samples are presented in an electronic appendix available on the paper's website.

standard deviation of log growth.

Finally to ensure the convergence of the estimated standard deviation of supply-level shocks, the number of iterations was first calibrated using the Raftery and Lewis (1992) test. As a cross-check we then applied the Geweke (1992) convergence test.[28]

5 The role of demand and supply shocks in the business cycle

When shocks are not contemporaneously correlated, the output gap and trend output are explained by orthogonal shocks or independent causes. In contrast, the correlated shocks provide an insight into the source or cause of the cycles, with demand shocks explaining movements in trend output, as is the case when there is hysteresis.

In the TCDH, demand shocks lead and supply shocks follow. This role of the demand and supply shocks can be seen in the IRFs and FEVDs. The IRFs, in Figures 1 and 2, show that a demand shock has a temporary effect on the output gap and a permanent effect on trend output. The cycles in the output gap and trend output DF are caused by the demand shock.

The FEVDs in Figures 5 and 6 show that the forecast errors of the output gap are explained by demand shocks alone while those of the trend output DDF are explained by both supply and demand shocks. Thus, there is causality from the demand shocks to the behavior of the trend. With the calibrated $\rho = 0.8$, demand shocks account for a larger share of the FEVD of trend output DDF.[29]

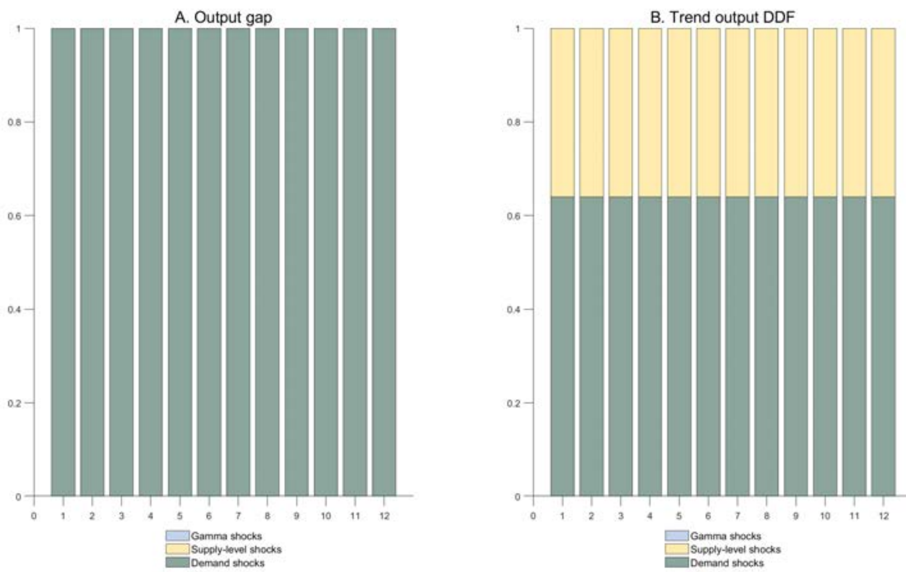
The IRFs of a supply shock in the reverse order of the shocks (not reported) would have the same shape as in Panel A of Figure 1. In this case, a supply shock would have a permanent effect on the trend and a transitory effect on the output gap. This could be said to be a univariate version of the type of supply-demand interaction considered in Guerrieri et al. (2022). However, the causality would run from the supply shock to the output gap, which is not consistent with the definition of hysteresis.

The FEVDs with the reverse order of the shocks (not reported) would indicate that the forecast errors of the output gap would be explained by both demand and supply shocks while

[28]The model was estimated separately for each economy in each sample period so as to speed up the estimation.

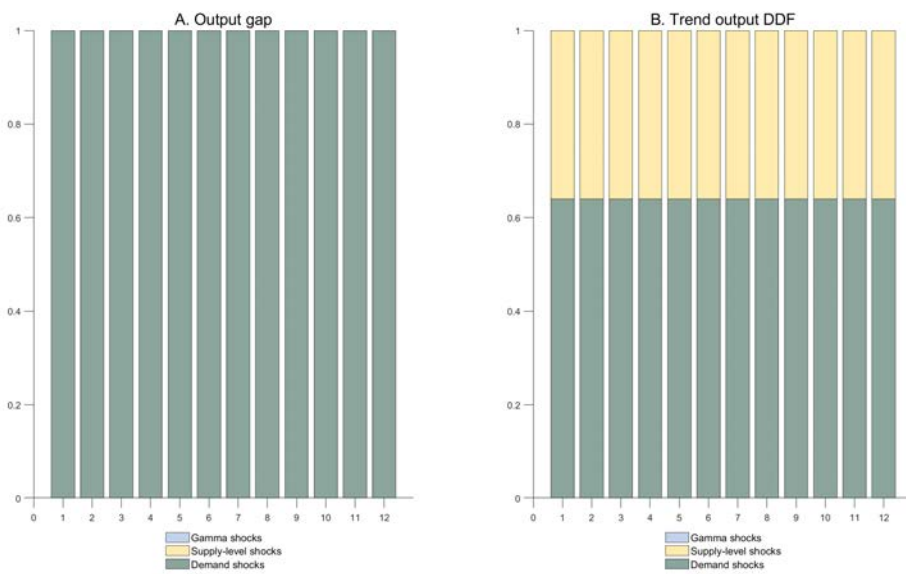
[29]For ρ equal to 0.7, 0.8, and 0.9 the shares are 0.51, 0.64, and 0.81, respectively.

Figure 5: The role of demand and supply shocks in the FEVD:
the United States



Note: according to the FEVD, demand shocks cause the cycle in the output gap while both demand and supply shocks cause the cycle in trend output DDF. Data are at the quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

Figure 6: The role of demand and supply shocks in the FEVD:
the euro area



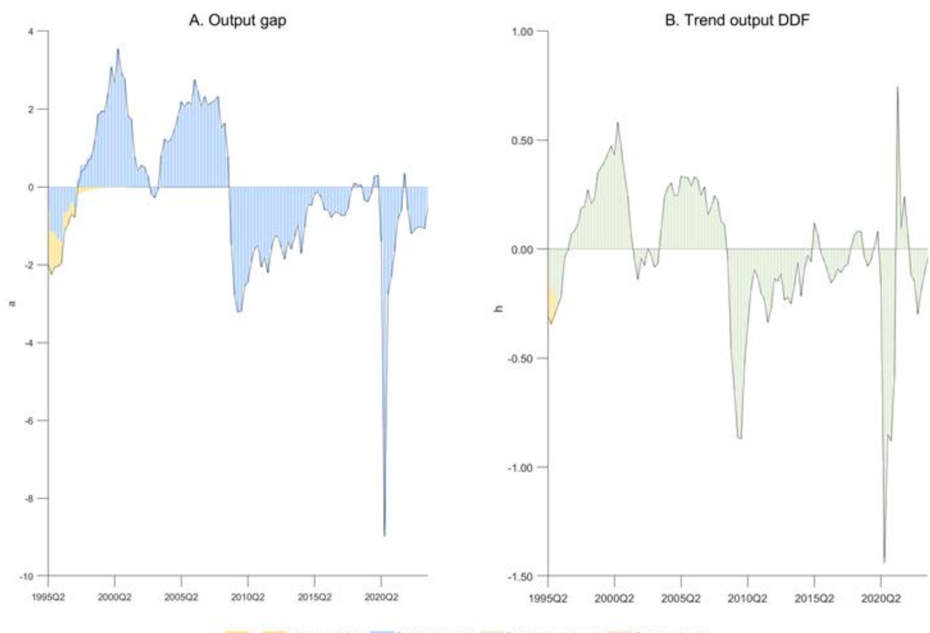
Note: according to the FEVD demand shocks cause the cycle in the output gap while both demand and supply shocks cause the cycle in trend output DDF. Data are at the quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

those of trend output would be explained by supply shocks alone. The spillover from supply shocks to the output gap would imply causality from the supply shocks to the output gap, which is not consistent with the definition of hysteresis.

The role of the demand and supply shocks and the causality mechanism that appear in the IRFs and FEVDs are not present in the FRs and HDs. According to the FRs, the role of the demand and supply shocks and the causality mechanism do not depend on the order of the shocks. According to the HDs, Figures 7 and 8, the role of the demand and supply shocks and the causality mechanism have the output gap explained entirely by demand shocks and trend output DF explained entirely by supply shocks, as in the accounts of the business cycles in the traditional Keynesian view.[30]

Bogot[a3]

Figure 7: The role of demand and supply shocks in the HD:
the United States

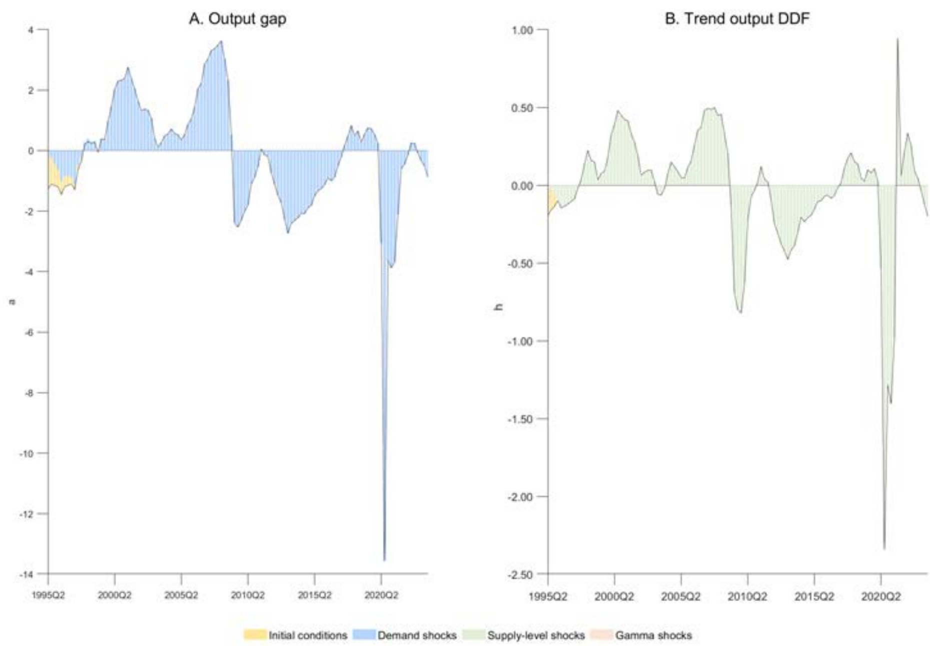


Note: According to the HD, demand and supply shocks explain the demand and supply components of the business cycle, respectively. Data are at the quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

In sum, the correlation of the shocks and their order in the Cholesky decomposition, allowed us to incorporate a transmission mechanism, from demand shocks to trend output, in a simple, univariate model. With this transmission mechanism the univariate model can be

[30]In Figures 7 and 8, trend output in deviation form enters the HD in 4-quarter differences.

Figure 8: The role of demand and supply shocks in the HD:
the euro area



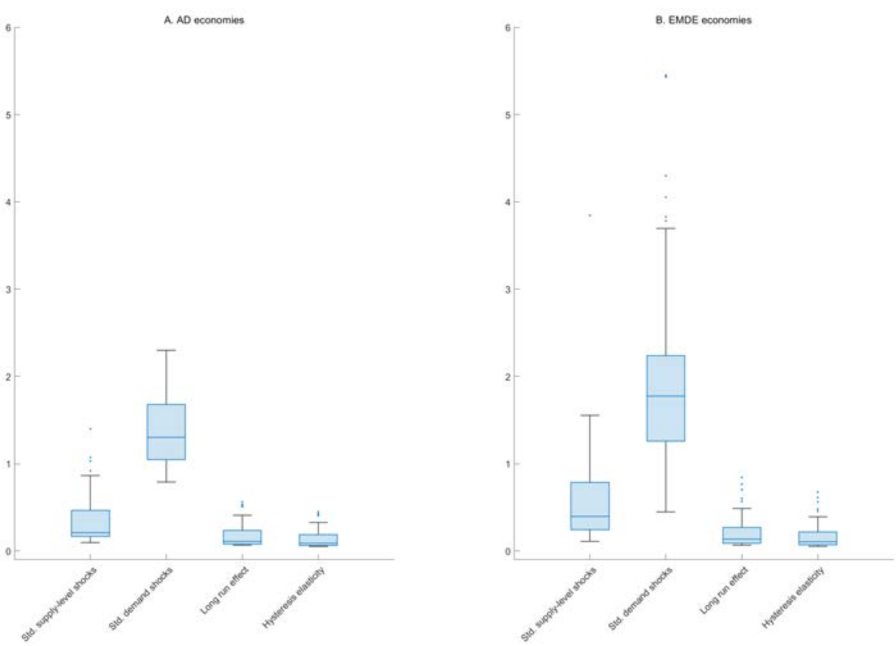
Note: According to the HD, demand and supply shocks explain the demand and supply components of the business cycle, respectively. Data are at the quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

said to be semi-structural.

6 Results on the standard deviation of supply-level shocks

The estimated standard deviation of supply-level shocks, $\sigma_{\varepsilon^{\bar{y}}}$, for the quarterly and annual databases is shown in Figures 9 and 10, respectively. The figures show that the estimated standard deviation of supply-level shocks is larger and more spread out in EMDE economies.

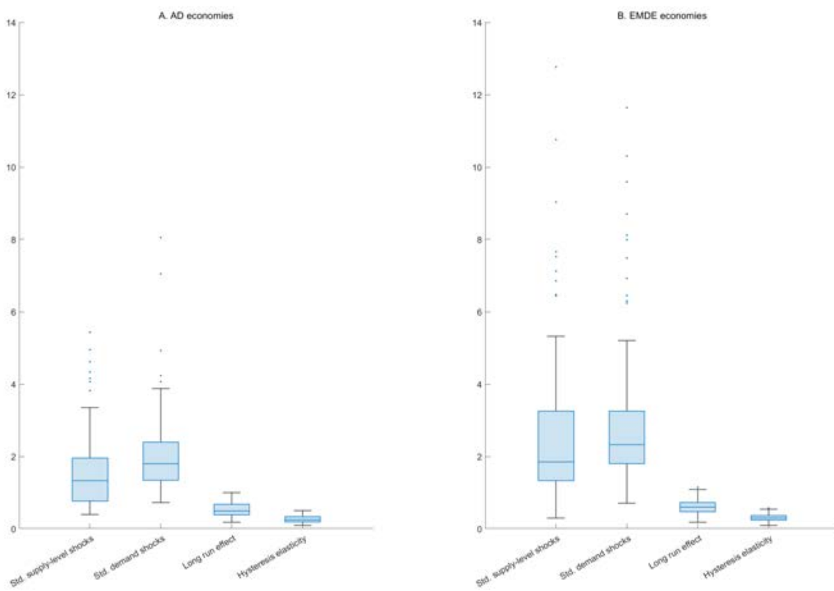
Figure 9: The estimated standard deviation of supply-level shocks – The quarterly database



Note: The estimated standard deviation of supply-level shocks tends to be larger and more widely spread in EMDE economies.
Source: Authors' estimates and the quarterly database.

As mentioned above, to ensure the convergence of the posterior distribution to an invariant distribution, the Raftery and Lewis (1992) test was performed for each country (see also Pfeifer (2013)). At a quarterly frequency, the number of iterations ranged from 11,035 to 15,634 across countries. At an annual frequency, the number of iterations ranged from 15,510 to 10,852. These numbers were applied to the alternative estimations with ρ equal to 0.7 and 0.9. In the baseline case of $\rho = 0.8$ to assure the convergence of the estimation the number of

Figure 10: The estimated standard deviation of supply-level shocks – The annual database

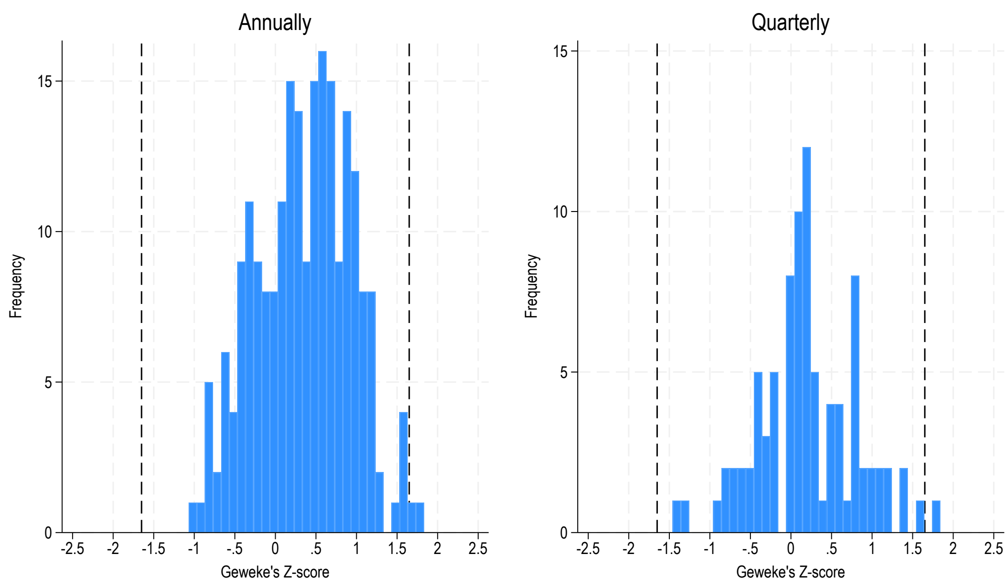


Note: The estimated standard deviation of supply-level shocks tends to be larger and more widely spread in EMDE economies.
Source: Authors' estimates and the annual database.

iterations was increased to 50,000.

A cross-check on the convergence of the posterior distribution, the Geweke (1992) convergence test was performed. The test consists of comparing the mean of initial batches of the parameter with that of batches at the end of the iteration. In the case of the yearly database, the null that the mean of the early and late batches is not different was not rejected. This result holds for all 219 economies studied. This allows us to conclude that the posterior distribution of the standard deviation $\sigma_{\varepsilon^{\bar{y}}}$ is stationary, and therefore inference from the posterior distribution is robust (see Figure **??**). A similar result was found using the quarterly database, although in 4 out of 90 economies the null hypothesis that the parameter is not different was rejected. The number of rejections is close to what would be expected by chance alone.

Figure 11: Geweke convergence test



Note: The graph is a histogram of the Geweke Z-score statistic. The left panel shows the annual database, while the right panel shows the quarterly database. In the annual database, only two economies have a z-score greater than 1.65 in absolute terms, so for most of the economies examined, the equality of the means at the beginning and at the end of the sample and at the end of the sample was not rejected at a the 10% significance level. Similarly, in the quarterly database, only one economy rejects the null hypothesis at the 10% significance level. The number of rejections is lower than would be expected by chance alone.
Source: Authors' estimates and the quarterly database.

7 Results on the hysteresis elasticity

Given the estimates of the standard deviation of supply-level shocks, $\sigma_{\varepsilon_t^{\bar{y}}}$, the hysteresis elasticity was obtained combining equations (8) and (10) as[31]

$$\eta = (1 - \alpha_y)\frac{\rho\sigma_{\varepsilon_t^{\bar{y}}}}{\sigma_{\varepsilon_t^{\hat{y}}}}. \tag{16}$$

As shown in Figures 9 and 10, the estimated hysteresis elasticities are in the range (0.06, 0.7) at the quarterly frequency and (0.1, 0.6) at the annual frequency, with their distribution skewed to the left at the quarterly frequency, and more symmetrically distributed at the annual frequency. Comparing AD and EM economies, the hysteresis elasticity is broadly similar, although only slightly larger and more widely spread in EMDE economies. This similarity is due to the fact that both the numerator and the denominator of equation (16), i.e., both, the estimated standard deviation of supply-level shocks, $\sigma_{\varepsilon_t^{\bar{y}}}$ and that of the estimated demand shocks, $\sigma_{\varepsilon_t^{\hat{y}}}$ are larger and more widely spread in EMDE economies.

The broad coverage of the annual database allowed us to examine whether there are differences in the hysteresis elasticity estimates across regions. Figure 13 shows the average hysteresis elasticity by region and income level.[32] [33] Europe and Central Asia have the highest hysteresis elasticity at 0.36. In contrast, Latin America and the Caribbean and South Asia have relatively low hysteresis elasticities of about 0.23. Overall, the estimated hysteresis elasticity is about 0.28, an estimate slightly smaller than that found in Roa Rozo (2024)

Concerning income level, the hysteresis elasticity tends to be smaller in high-income economies than in low-income economies. For example, it is about 0.27 in AD economies and about 0.31 in EMDE economies. The difference is statistically significant. Using a more granular classification, the null of equality of the estimates is not rejected at the 10% significance level. The difference is not significant but qualitatively remains, the hysteresis elasticity seems to be negatively related to the income level.

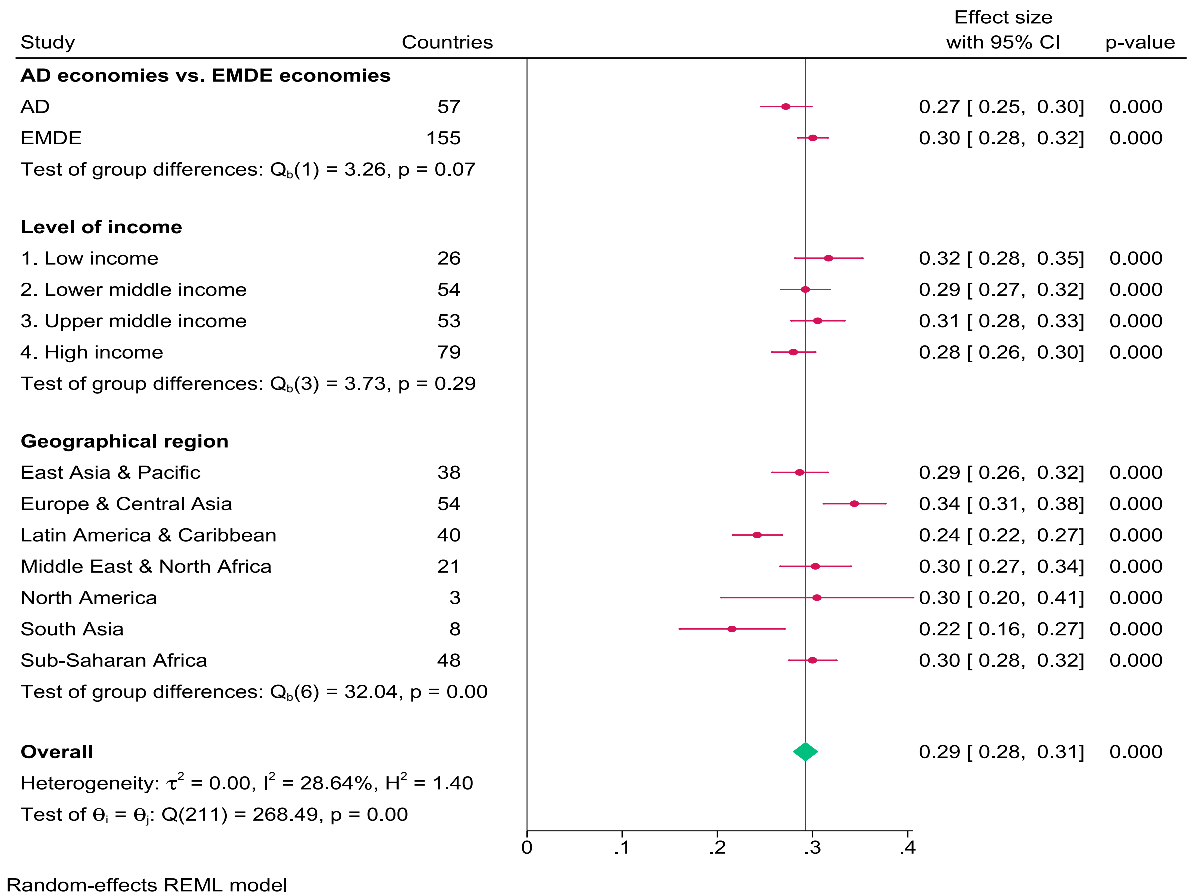
To see how the results on hysteresis relate to the existing literature, it may be useful to recall the definition of hysteresis elasticity. DeLong et al. (2012) defines the hysteresis elasticity,

[31] In quarterly frequency equation (16) is $\eta = 4(1 - \alpha_q)\frac{\rho\sigma_{\varepsilon_t^{\bar{y}}}}{\sigma_{\varepsilon_t^{\hat{y}}}}$

[32] The hysteresis elasticity estimates were assumed to follow a normal distribution centered on the posterior mean and with a standard deviation equal to the standard deviation of the posterior distribution. This simplification does not take into account that in some countries the posterior distribution of the hysteresis elasticity may be asymmetric.

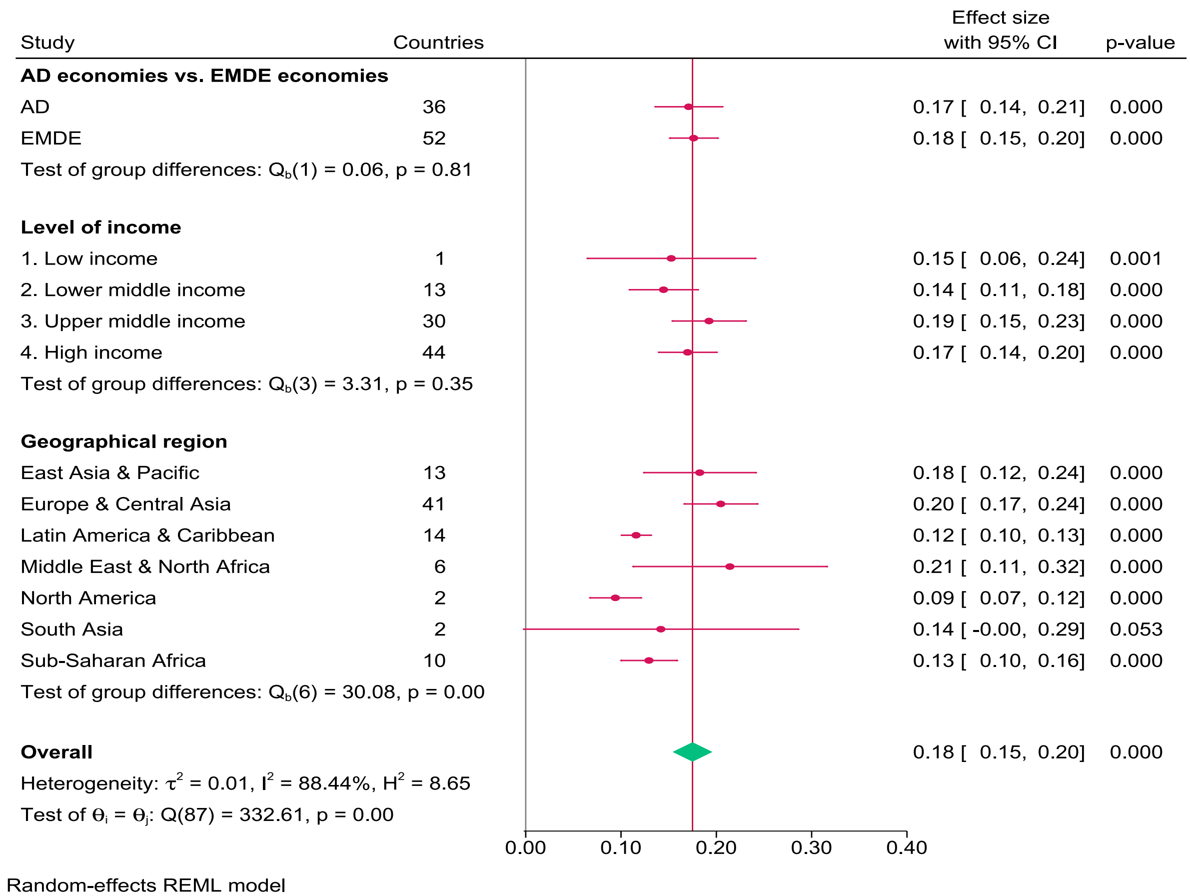
[33] Regions are defined according to the World Bank classification.

Figure 12: The estimated hysteresis elasticity – Annual database
AD and EMDE economies and regions



Note: The figure shows a subgroup meta-analysis of the hysteresis elasticity (η).

Figure 13: The estimated hysteresis elasticity – Quarterly database
AD and EMDE economies and regions



Note: The figure shows a subgroup meta-analysis of the hysteresis elasticity (η).

η, as the percentage reduction in the flow of future potential output, $\bar{y}_t$, per percentage point year of the current period output gap, $\hat{y}_t$. This definition suggests the following equation:

$$\bar{y}_t = \bar{y}_{t-1} + \eta \hat{y}_{t-1} + \varepsilon_t^{\bar{y}}, \tag{17}$$

which is a mechanism included in the survey by Cerra et al. (2020). In turn, Ball and Onken (2021) estimate the hysteresis elasticity, η, using equation (8), which we rewrite here for simplicity as $\eta = \mu_y/\lambda_y$, where μ_y is the long-run effect on GDP from a demand shock, and λ_y is the cumulative sum of the output gap.

Therefore, the results of the paper allow for a connection with the literature in terms of μ_y and η. Estimates of the latter are rather scarce in the literature. To our knowledge, there are only two studies that provide estimates of the hysteresis elasticity η: Jordà et al. (2020a) and Roa Rozo (2024).

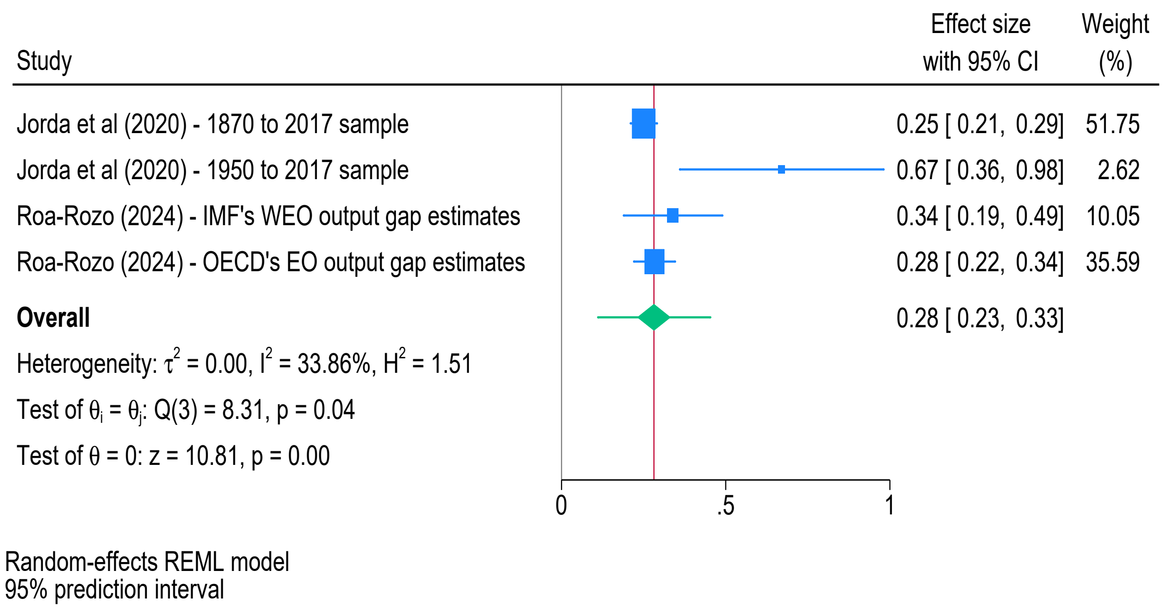
Jordà et al. (2020a) estimated the hysteresis elasticity for a group of developed economies. They used a two-step classical minimum distance approach. In the first step, they estimated the IRFs of utilization-adjusted TFP and a measure of the cyclical gap to a monetary policy shock. In the second step, they estimated the hysteresis elasticity as the ratio of the long-run effect on utilization-adjusted TFP over the cumulative effect on the cyclical gap. Using a sample from 1870 to 2017, they obtained $\eta = 0.25$, with a 95 percent confidence interval (CI) from 0.21 to 0.3. For their post-war sample, they obtained $\eta = 0.67$ with a 95 percent CI from 0.34 to 0.99.

For his part, Roa Rozo (2024) used an error correction model to study the dynamics of GDP and potential GDP data for the developed OECD economies. Using the WEO data, he found $\eta = 0.28$ with a standard error of 0.032. Using the OECD data, he found $\eta = 0.34$ with a standard error of 0.085. The meta-analysis suggests that $\eta = 0.28$ (see Figure 14).

The hysteresis elasticity estimates of this paper contribute to the existing literature. On the one hand, they are quite consistent with previous findings. The meta-analysis of all our estimates, suggests that $\eta = 0.297$ with a prediction interval of 0.14 to 0.46. On the other hand, they cover more than 200 economies, in contrast to previous studies that refer to group averages of high-income economies.

The estimates also compare favorably with previous guesses of the size of the hysteresis elasticity. Some calibrations and well-informed back-of-the-envelope calculations are worth

Figure 14: Meta-analysis of the hysteresis elasticity – Prior literature



Source: Author's calculations with estimates from Jordà et al. (2020a) and Roa Rozo (2024).

mentioning. DeLong and Summers (2012) collected micro and macroeconomic evidence for the United States and suggested that $0 \leq \eta \leq 0.25$. Łukasz Rawdanowicz et al. (2014) provide back-of-the-envelope calculations of the hysteresis elasticity in the OECD countries.[34] They report that the hysteresis elasticity could be as low as 0.08 and as high as 1.2. The average of their reported calibrations of the hysteresis elasticity is 0.5 with a standard error of 0.05. Fatás and Summers (2018) suggest that, if they interpret their results at face value, the hysteresis effect is much larger than any of the previous guesses that were found to be above one. Thus, the hysteresis elasticity results in this paper are also in the range of prior guesses.

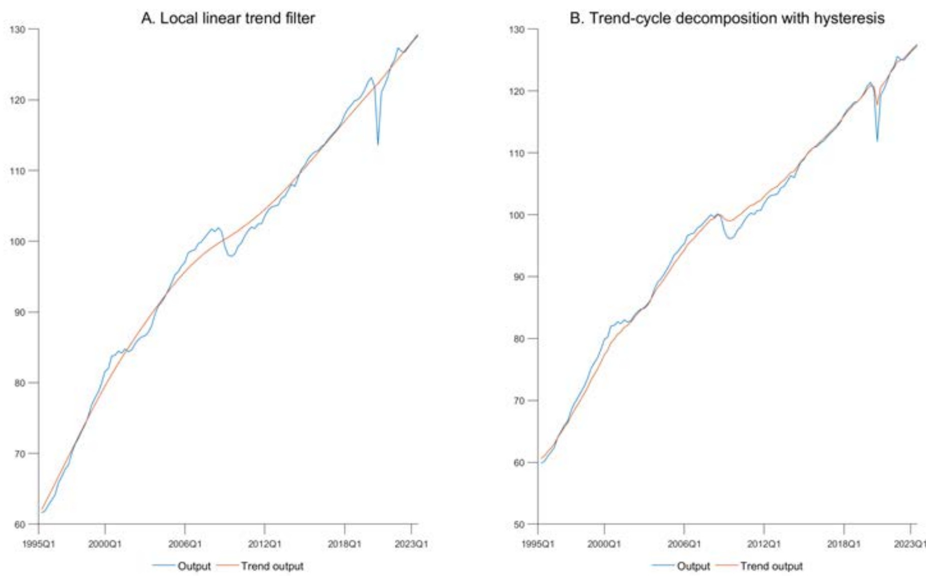
8 The FRs

For illustrative purposes, we first consider the FRs for the United States and the euro area at the quarterly frequency starting from 1995Q1. Figures 15 and 16 compare the TCDH with the LLT filter.[35]

[34] See Table 1 of Łukasz Rawdanowicz et al. (2014)

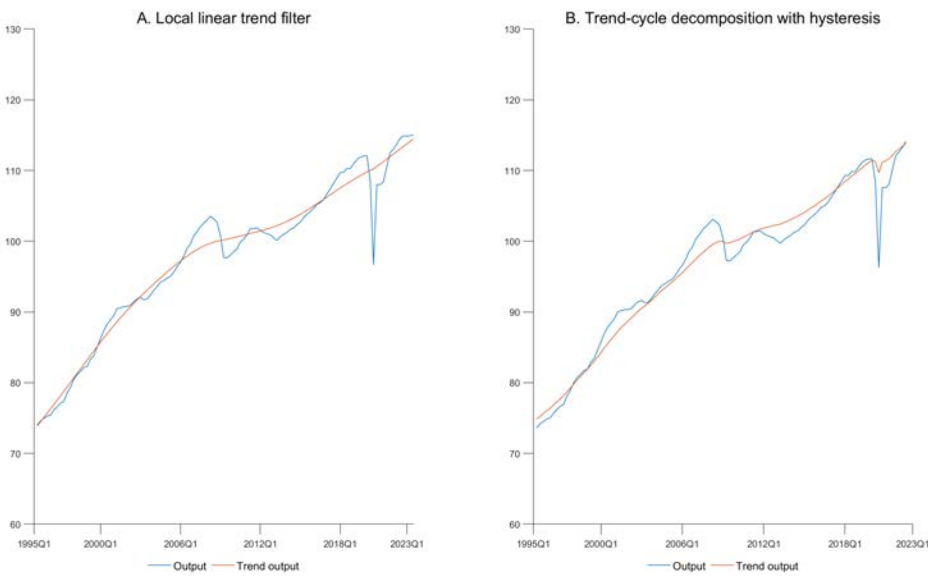
[35] In the LLT filter the standard deviation of demand shocks was calibrated as $\sigma_{\varepsilon_t^{\hat{y}}} = 19$ and the standard deviation of supply shocks was calibrated as $\sigma_{\varepsilon_t^{\bar{y}}} = 0.09$ and $\sigma_{\varepsilon_t^{\gamma}} = 0.21$ so that the sum of

Figure 15: Comparison of the LLT filter
and the TCDH – The United States



Note: Using the TCDH, trend output is not necessarily smooth and does not create the problem of the artificial pre-recession boom. Data are at the quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

Figure 16: Comparison of the LLT filter
and the TCDH – The euro area



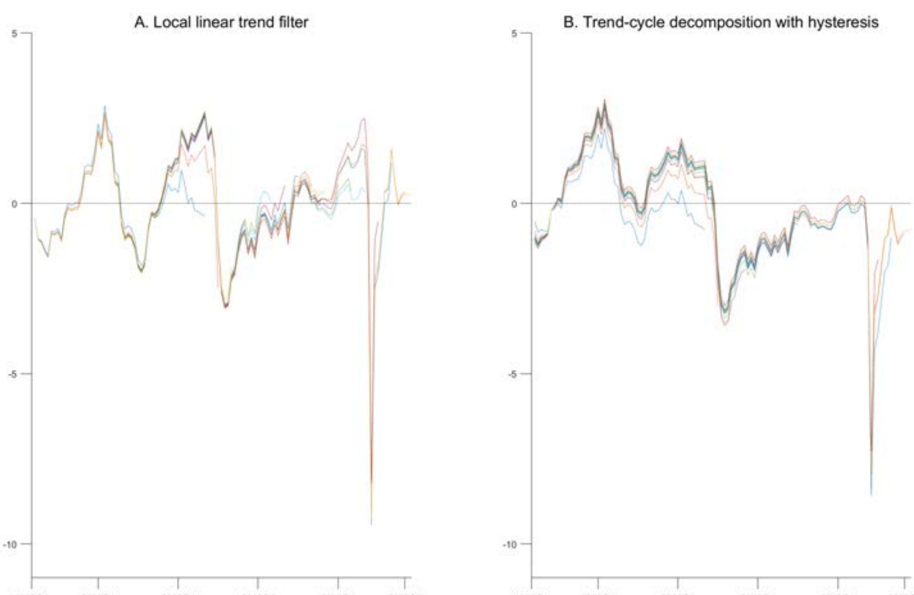
Note: In the TCDH trend output is not necessarily smooth and does not create the problem of the artificial pre-recession boom. Data are at the quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

Using the LLT filter, in Panels A of Figures 15 and 16, trend output evolves smoothly and the output gap, represented by the difference between output and trend output, goes through a large boom before the 2008-2009 financial crisis as well as before the pandemic recession. In contrast, using the TCDH, in Panels B of Figures 15 and 16, trend output falls during the recessions and the output gap mitigates the problem of the artificial pre-recession boom.

8.1 Sensitivity to new data

Another feature of the TCDH is that the output gap is less sensitive to new data than the LLT filter. Figures 17 and 18 show the estimated output gap for the United States and the euro area over progressively longer estimation periods. When the TCDH is used, the output gap revisions are smaller.

Figure 17: Output gap revisions in the LLT filter
and the TCDH – The United States



Note: When the TCDH is used, revisions to the output gap are smaller. The figure shows the output gap estimates over progressively longer estimation periods. The first estimation period is 1995Q1-2007Q4; thereafter, the estimation period includes 4 additional quarters until the estimation runs over the full available sample. Data are at the quarterly frequency for the period 1995Q1-2022Q3.
Source: Authors' estimates and the quarterly database.

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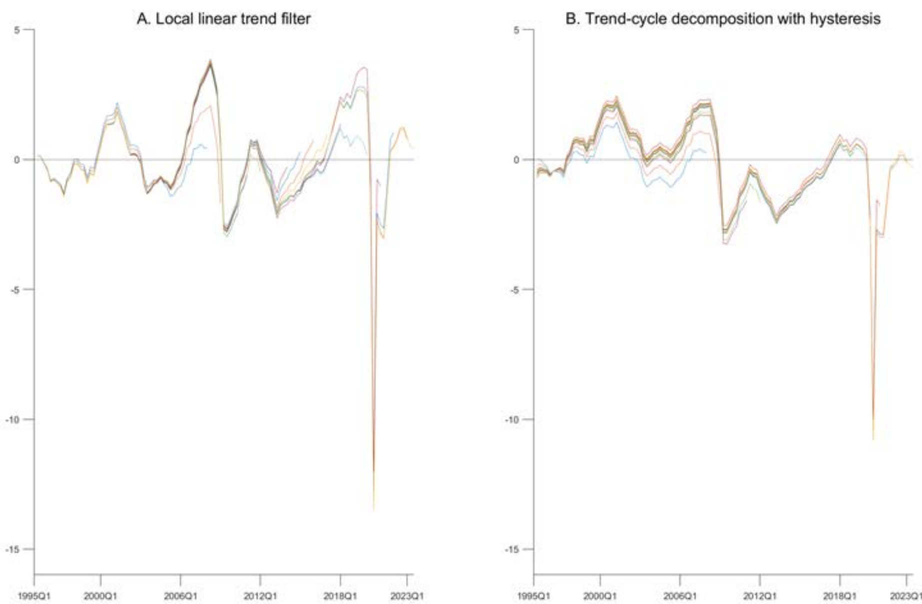
the standard deviations of the supply shocks adds up to 0.3. The standard deviation of the shocks in the TCDH is reported in Table 1.

Figure 18: Output gap revisions in the LLT filter
and TCDH – The euro area



Note: When using the TCDH, revisions to the output gap are smaller. The figure shows the output gap estimates over progressively longer estimation periods. The first estimation period is 1995Q1-2007Q4; thereafter, the estimation period includes 4 additional quarters until the estimation runs over the full available sample. Data are at the quarterly frequency for the period 1995Q1-2022Q3.
Source: Authors' estimates and the quarterly database.

Turning to counterfactual trend output, it grows at the long-run trend output growth rate, γ_t, thus providing a model projection that serves as a benchmark to illustrate hysteresis. Counterfactual trend output, in Panels A of Figures 19 and 20, is equal to trend output before 2008Q3; that is, before the sequence of large negative realizations of correlated shocks during the 2008-2009 financial crisis. Thereafter, counterfactual trend output grows at the long-run trend output growth rate, γ_t, the rate that would have occurred in the absence of a series of negative realizations of correlated shocks.

For trend output in deviation form, the variable we use to illustrate hysteresis, the 2008-2009 financial crisis and the pandemic recession had important effects on hysteresis, as shown in Panels B of Figures 19 and 20. Conditional on the calibrated parameters θ and $\sigma_{\varepsilon_t^\gamma}$, the hysteresis effect of the 2008-2009 financial crisis was around 5 percent in the United States. In the euro area, the hysteresis effect of both the 2008-2009 financial crisis and the euro area crisis was about half of that of the United States, the difference being accounted for by the posterior mean of the standard deviation of supply-level shocks $\sigma_{\varepsilon_t^{\bar{y}}}$. In both the United States and the euro area, the pandemic recession appears to have added a further 2 percentage points to the hysteresis.

8.2 The 2008-2009 financial crisis and the pandemic recession

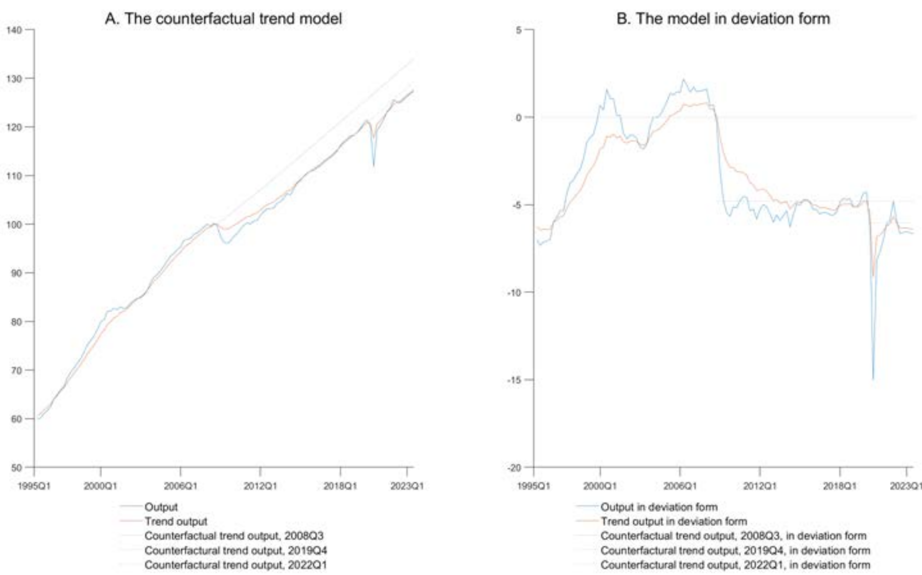
A more general view of the 2008-2009 financial crisis and the pandemic recession can be examined using the quartile distribution of the cycle across countries. Figures 21 to 24 show the interquartile range and the median of the non-stationary variables, output and trend output, and of the stationary variables, the output gap and trend output DDF.[36]

Figure 21 shows the 2008-2009 financial crisis in advanced economies. Panel A shows the non-stationary variables, output and trend output, in deviation form. Output goes through a double-dip recession, while trend output shows hysteresis. In Panel B, the stationary variables, output gap and trend output DDF, go through a full cycle. The sluggish recovery of trend output DDF in Panel B explains the hysteresis in trend output in deviation form in Panel A.

Figure 22 shows the 2008-2009 financial crisis in the EMDE economies. In contrast to the case of the AD economies, trend output in deviation form does not seem to show much

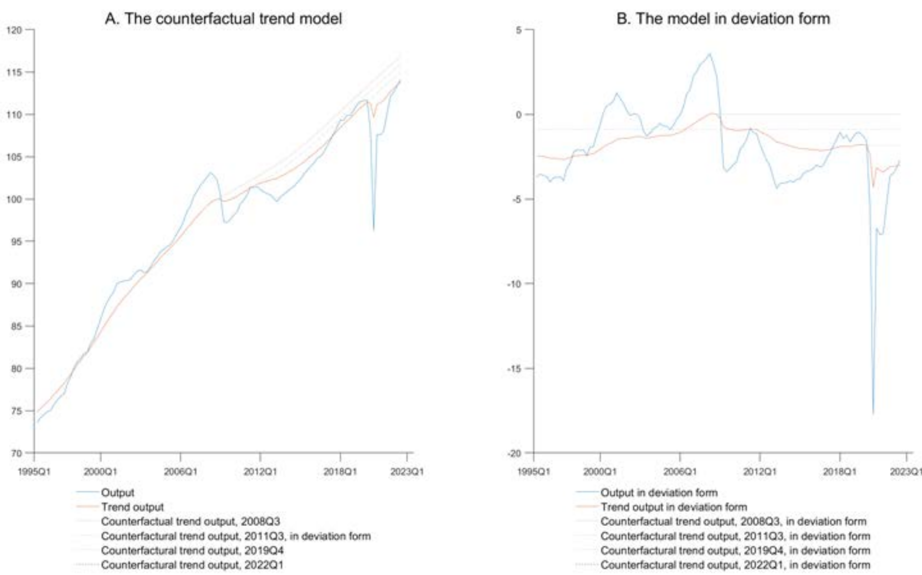
[36] As noted above, the non-stationary variables in Panel A may not be robust to the choice of the prior standard deviation of trend output growth shocks, $\sigma_{\varepsilon_t^\gamma}$, as well as the corresponding posterior estimates, so they are reported for illustrative purposes. In contrast, the stationary variables in Panel B are robust to the choice of the prior for this standard deviation, $\sigma_{\varepsilon_t^\gamma}$, and the posterior estimates.

Figure 19: The model for counterfactual trend output and the model in deviation form – The United States



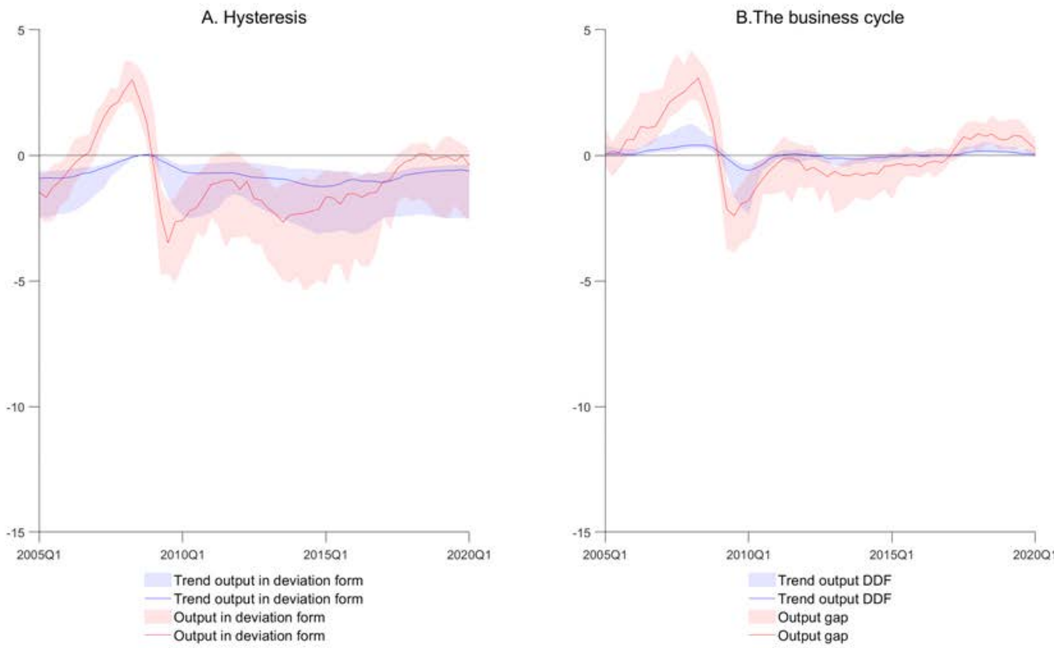
Note: both output and trend output are upward sloping; without sloping, trend output in deviation form serves as a measure of hysteresis. Data are at quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

Figure 20: The model for counterfactual trend output and the model in deviation form – The euro area



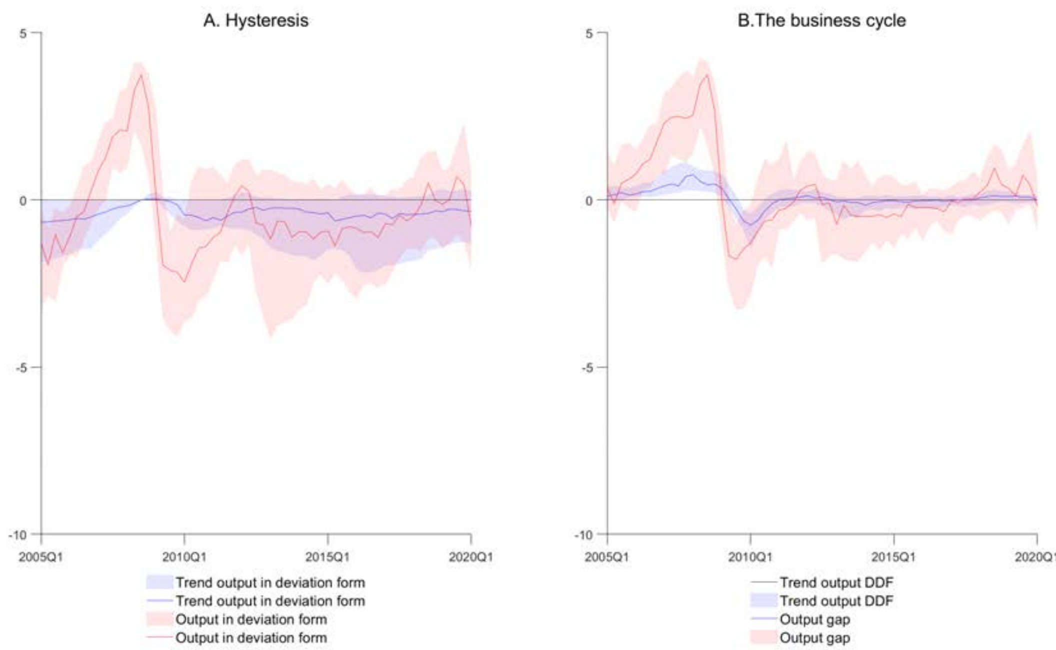
Note: Both output and trend output are upward sloping; without sloping, trend output in deviation form serves as a measure of hysteresis. Data are at quarterly frequency over 1995Q1-2022Q2.
Source: Authors' estimates and the quarterly database.

Figure 21: The 2008-2009 financial crisis – AD economies



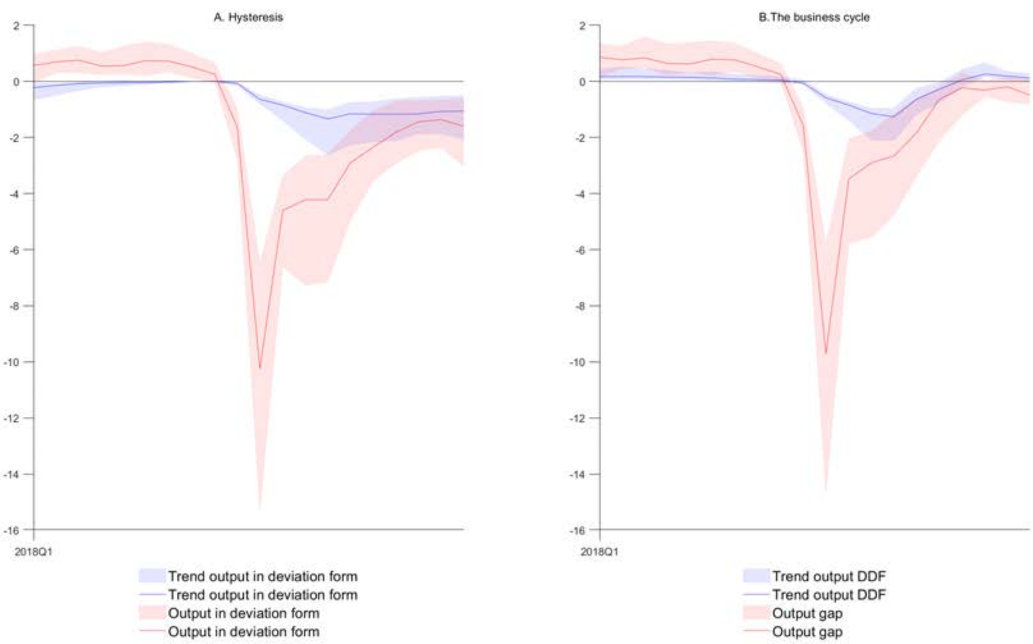
Note: Output and trend output are in deviation form. Shades show the interquartile range, lines show the median. Trend output in deviation form is normalised to zero in 2008Q3. The 2008-2009 financial crisis led to hysteresis in the AD economies.
Source: Authors' estimates and the quarterly database.

Figure 22: The 2008-2009 financial crisis – EMDE economies



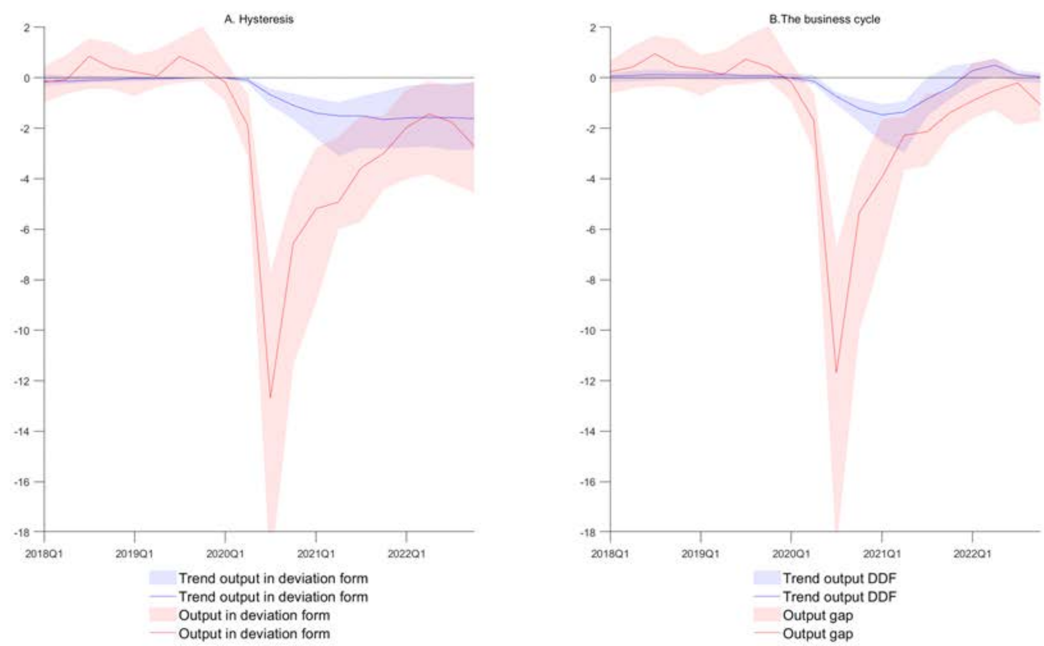
Note: Shades show interquartile range, lines show median. The business cycle is the cycle in both the output gap and trend output DDF. The 2008-2009 financial crisis did not lead to hysteresis in the EMDE economies. Differences are over 4 quarters.
Source: Authors' estimates and the quarterly database.

Figure 23: The pandemic recession – AD economies



Note: Shades show the interquartile range, lines show the median. Output and trend output are in detrended form. Trend output in deviation form is normalised to zero in 2019Q4. The pandemic recession led to hysteresis in the AD economies.
Source: Authors' estimates and the quarterly database.

Figure 24: The pandemic recession – EMDE economies



Note: Output and trend output are in detrended terms. Shades show the interquartile range, lines show the median. Trend output in deviation form is normalised to zero in 2019Q4. The pandemic recession led to hysteresis in the EMDE economies.
Source: Authors' estimates and the quarterly database.

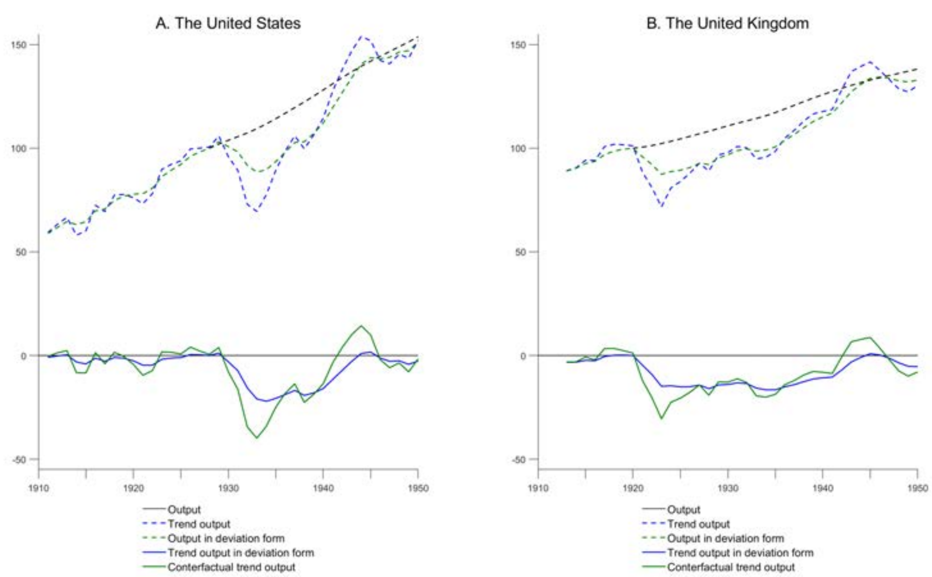
evidence of hysteresis. Moreover, the output gap closes quickly; in other words, output returns to trend quickly. In Panel B, the output gap and trend output DDF also show a complete cycle; however, in contrast to the AD economies, the rapid recovery of trend output DDF in Panel B explains the absence of hysteresis in trend output in deviation form in Panel A.

Figures 23 and 24 show the pandemic recession in the AD and EMDE economies respectively. In both the AD and EMDE economies, the behaviour of trend output suggests significant hysteresis.

8.3 The business cycle in historical perspective

The TCDH can help to shed light on hysteresis in some notable historical episodes. Figure 25 shows extreme hysteresis events in the United States during the Great Depression and in the United Kingdom during the depression of the 1920s.

Figure 25: The TCDH during depression and war



Note: The figure shows the Great Depression in the United States, the depression of the twenties in the United Kingdom, and the recovery during WWII in both countries. Trend output in deviation form is zero in the United States in 1928 and in the United Kingdom in 1918. Data are in annual frequency.
Source: Authors' estimates based on data from the Maddison Project database, version 2020.

In the United States, trend output dropped relentlessly during the Great Depression in response to a series of correlated shocks associated with the financial crisis, as shown in Figure 25. A recovery in the hysteresis indicator, trend output in deviation form, took place in the 1940s in both the United States and the United Kingdom as a result of war-related government spending.

In the United Kingdom, trend output collapsed in events associated with deflation and a return to the gold standard with an overvalued exchange rate, as documented by Eichengreen (2008). In addition, as shown by Crafts (2018), trend output also fell as a result of reduced international trade. Crafts (2018) estimated that the fall in output was around 11.3 percent, due to deflation, reduced trade and an increased burden of public debt. This figure is comparable to the fall in trend output in deviation form shown in Figure 25.

The annual database also allows a more general view of the business cycle to be examined by including the full distribution of both the output gap and trend output DDF. The long historical sample allows comparisons over three time periods: the Gilded Age, 1870-1910; the war and inter-war period, 1910-1950; and the post-war period, 1950-2022. Business cycles are examined in Figures 26 and 27 using the median, interquartile range and the range of the output gap and trend output DDF as measures of volatility and dispersion.

The Gilded Age period was calm in terms of volatility and dispersion in both AD and EMDE economies. Dispersion and volatility, as measured by the interquartile range and the range, were the lowest, see Figures 26 and 27.

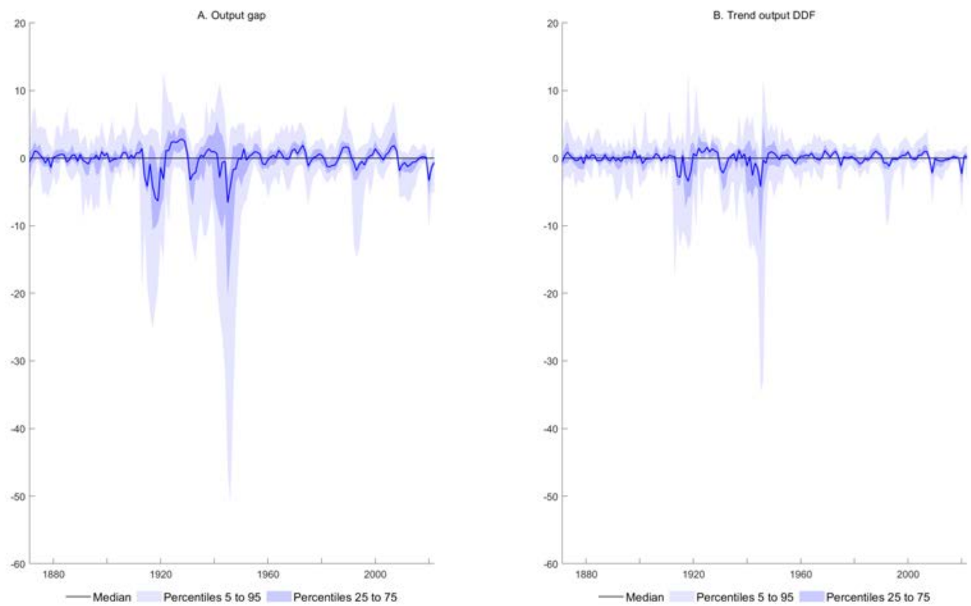
The inter-war period was particularly turbulent for the AD economies. Volatility and dispersion are the highest in all periods, as indicated by the wider interquartile range and range, as shown in Figures 26 and 27.

The post WWII period was relatively calm in the AD economies and turbulent in the EMDE economies. With the exception of the 2008-2009 financial crisis and the pandemic recession, volatility and dispersion are remarkably low in the AD economies (Figure 26). In sharp contrast, volatility and dispersion in the EMDE economies have risen sharply, indicating that several economies have experienced strong fluctuations, as can be seen from the marked increase in the interquartile range and the range in Figure 27.

Driven by supply shocks, trend output DF $\bar{y}_t - \bar{y}_{t-4}$ fluctuates around the long-run trend output growth rate, γ_t. In this sense the long-run trend output growth rate, γ_t, is a kind of core growth rate.

The long-run trend output growth rate, γ_t, evolves smoothly over time, shifting around

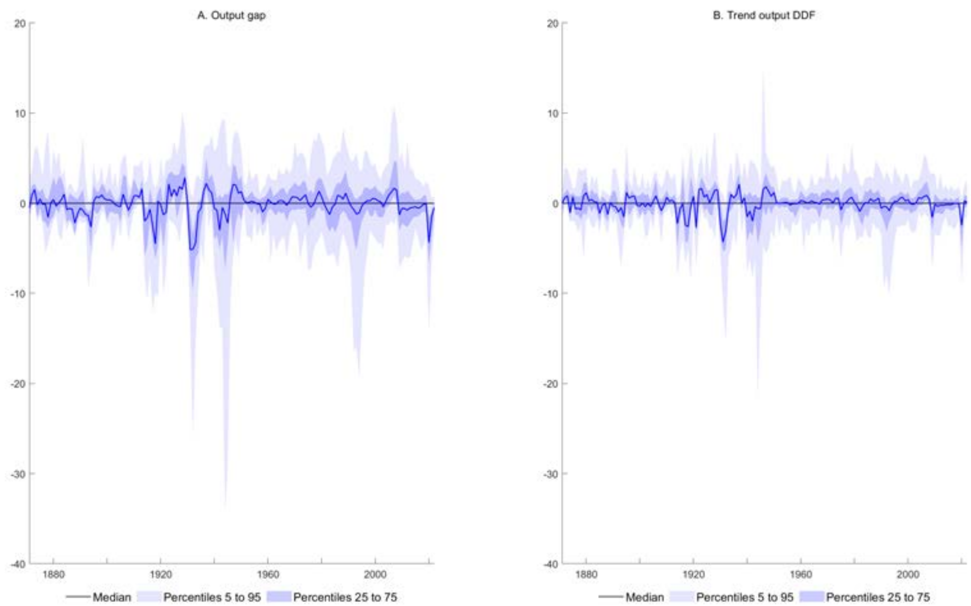
Figure 26: A historical perspective on the business cycle – AD economies



Note: The figure shows the central tendency, dispersion and volatility of the output gap and trend output DDF.

Source: Authors' estimates and the annual database.

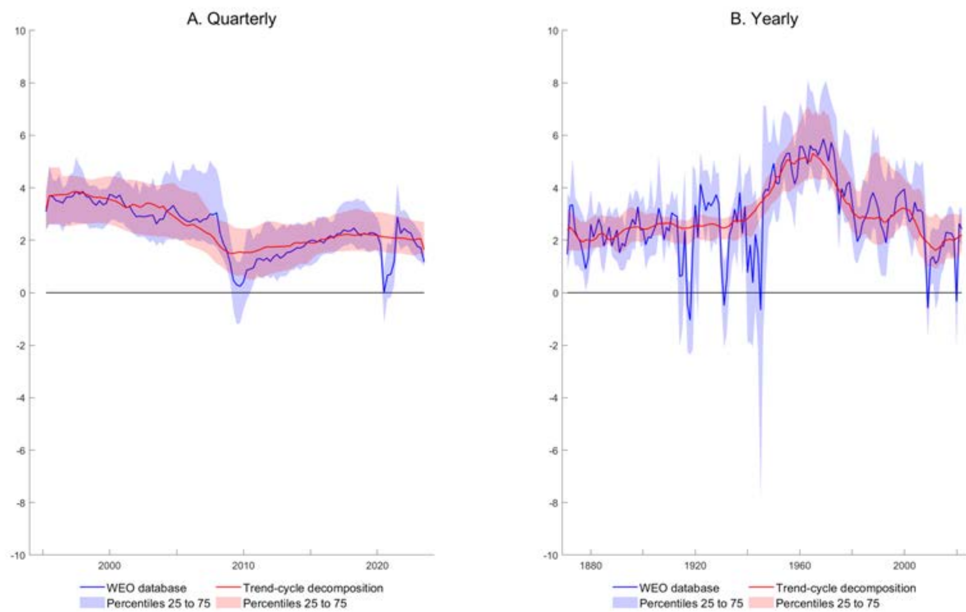
Figure 27: A historical perspective on the business cycle – EMDE economies



Note: The figure shows the central tendency, dispersion and volatility of the output gap and trend output DDF.
Source: Authors' estimates and annual database.

periods of higher and lower long-run growth. In both the AD and the EMDE economies, the long-run trend output growth rate increased markedly during the post-war economic expansion (Figures 28 and 29). From the 1970s onwards, the long-run trend output growth rate declined with the productivity slowdown. Figure 29 also shows that the long-run trend output growth rate has been consistently higher in the EMDE economies.

Figure 28: Short- and long-run trend output growth rates – AD economies

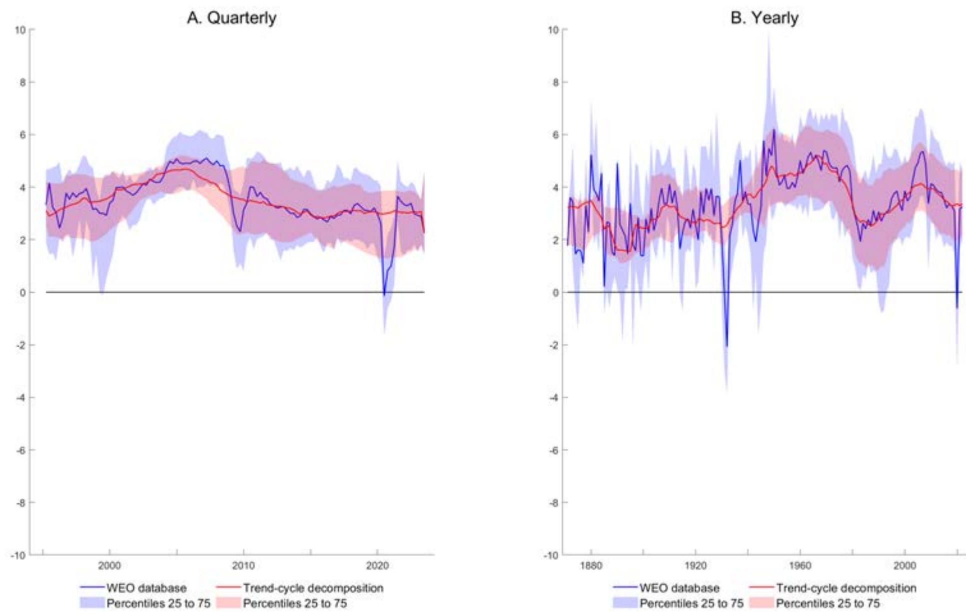


Note: The figure shows the interquartile range, in shading, and the median, in line, of the growth rate $y_t^{\bar{y}} - y_{t-4}^{\bar{y}}$ and g_t. The former growth rate fluctuates around the latter.
Source: Authors' estimates and the quarterly and annual databases.

9 Discussion: Comparison of output gap and trend output

As we have estimated output gaps for a large number of countries, the question arises as to how these estimated output gaps compare with other estimates. Publicly available output gap estimates are currently available from three sources: the IMF's World Economic Outlook (WEO) database, the Macroeconomic Database of the European Commission's Directorate General for Economic and Financial Affairs, and the OECD's Economic Outlook

Figure 29: Short- and long-run trend output growth rates – EMDE economies



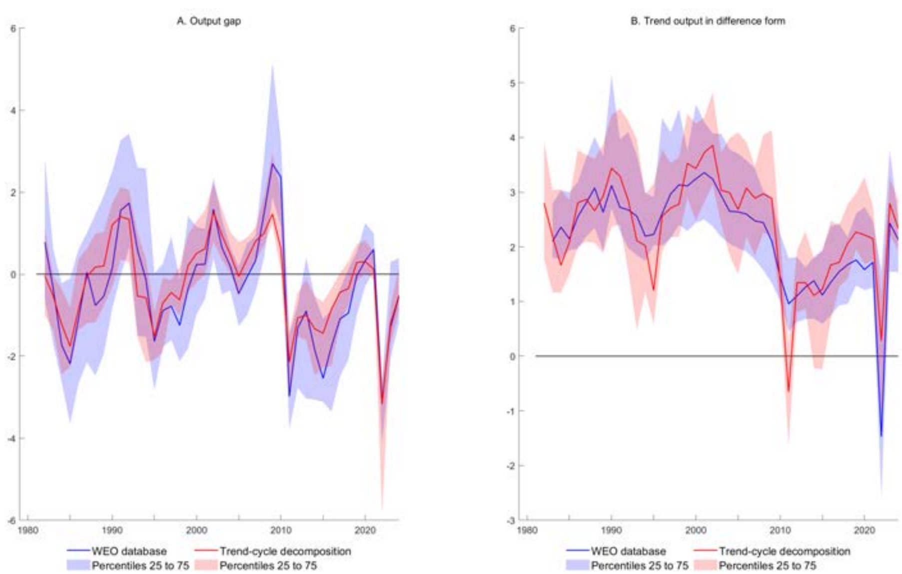
Note: The figure shows the interquartile range, in shading, and the median, in line, of the short term trend output growth rate $y_t^{\bar{y}} - y_{t-4}^{\bar{y}}$ and the long-run trend output growth rate γ_t. The former growth rate fluctuates around the latter. The median of γ_t can change discretely due to a change in the median country.
Source: Authors' estimates and the quarterly and annual databases.

Online Database. All currently available estimates are at the annual frequency.

The first difference is that the TCDH allows for quarterly estimates for a large number of countries. No publicly available database currently provides quarterly estimates of the output gap. Quarterly estimates incorporate more information and thus allow for a more thorough estimation of the output gap and trend output for the periods for which data are available. At the annual frequency, the TCDH allows the output gap and trend output to be estimated with greater coverage and a broader historical perspective.

In any case, when comparing output gaps, it is important to bear in mind that they are estimated using different methodologies, concepts of latent output and also with different analytical purposes. Bearing this caveat in mind, and with the aim of illustrating the characteristics of the TCDH, a comparison of estimated output gaps with other publicly available estimates may be appropriate.

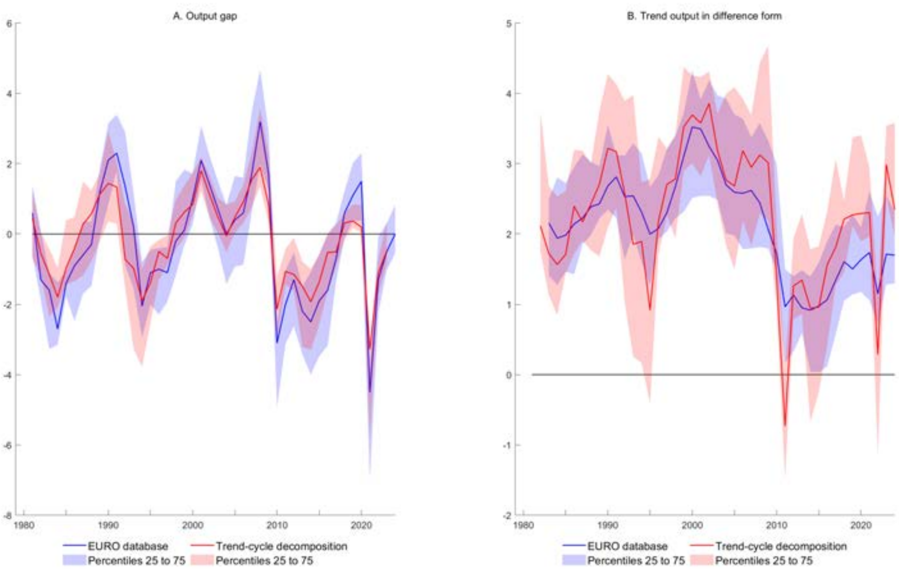
Figure 30: Comparison of estimates from the WEO database and the TCDH



Note: In the TCDH, trend output is not smooth, the output gap booms that precede recessions are smaller and the standard deviation of the output gap is about one-fifth smaller.
Source: Authors' estimates, the annual database and the WEO April 2022 database.

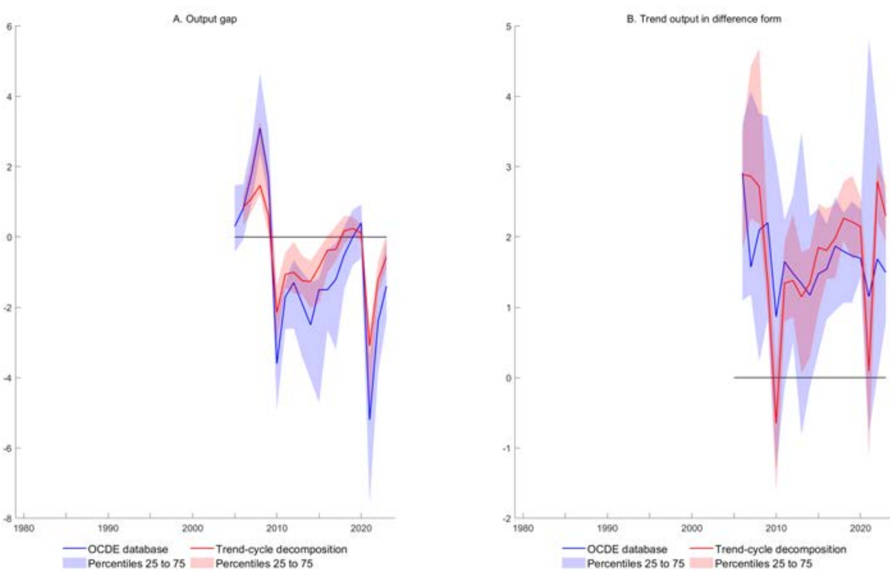
Figures 30, 31 and 32 compare the output gap estimates from each of the currently publicly available databases with the output gap estimates from the TCDH. The figures also compare

Figure 31: Comparison of estimates from the European Commission
and the TCDH



Note: In the TCDH, trend output is not smooth, the output gap booms that precede recessions are smaller and the standard deviation of the output gap is about one-fifth smaller.
Source: Authors' estimates, annual database and macroeconomic database of the European Commission's Directorate-General for Economic and Financial Affairs.

Figure 32: Comparison of estimates from the OECD
and the TCDH



Note: In the TCDH, trend output is not smooth, the output gap booms that precede recessions are smaller and the standard deviation of the output gap is about one-fifth smaller.
Source: Authors' estimates, the annual database and the OECD Economic Outlook Online Database.

the trend output growth implicit in the output gap estimates.[37] The comparison highlights the advantages of the TCDH, namely that trend output does not evolve smoothly and large booms do not necessarily precede recessions; as a result, output gap revisions are smaller. Another difference is that, because trend output does not evolve smoothly, the standard deviation of the estimated output gaps is somewhat smaller than that calculated using output gap estimates available from other sources.

10 Robustness

The paper proposes a trend-cycle decomposition with three main features: trend output is not smooth, trend output in difference-form is correlated with the output gap, and trend output in levels has hysteresis. One robustness question is whether these features hold under different assumptions about the correlation coefficient ρ. A strongly positive ρ is a defining feature of the TCDH. We have assumed this feature pretty much as the HP or LLT filters assume a smooth trend. At the extreme, $\rho = 0$, the TCDH becomes the LLT filter, which does not have the aforementioned defining characteristics of the TCDH. Thus, we approached this robustness question from the angle of sensitivity analysis of the hysteresis elasticity, η, for alternative values of the correlation coefficient—under the condition that these alternative values still imply a strong positive correlation. We used ρ equal to 0.7 and 0.9 as alternative values.

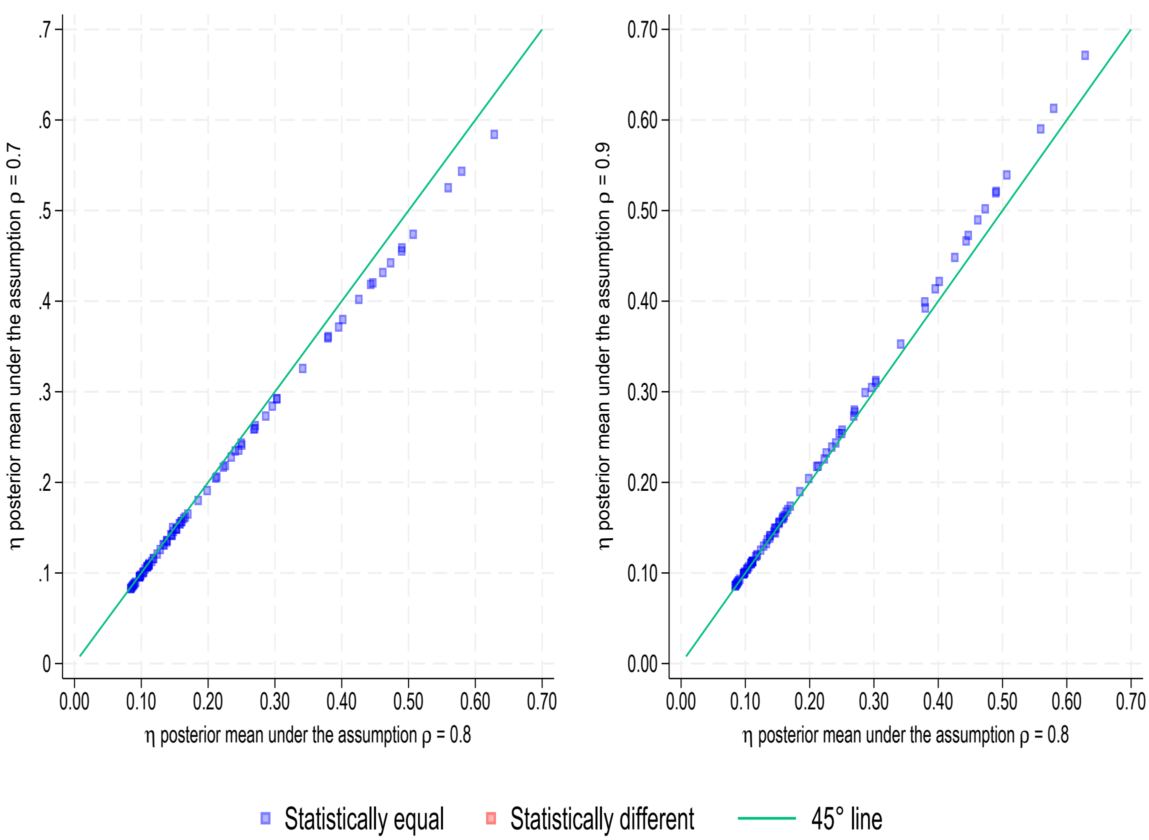
We tested whether the η estimates obtained under the alternative assumptions for ρ were statistically different from those obtained under the baseline $\rho = 0.8$. The test had two conditions. First, the estimate of η under the alternative $\rho = 0.7$ is within the credible interval for the estimate of η under the baseline $\rho = 0.8$. Second, the estimate of η under the baseline $\rho = 0.8$ is within the credible interval for η under the alternative $\rho = 0.7$. If either of these conditions is not met, the estimate of the hysteresis elasticity is considered statistically different and therefore not robust to the assumed ρ. A similar test was performed for the case of high vs. baseline correlations, 0.9 vs. 0.8, respectively.

We present the results in Figures 33 and 34. In each graph, the baseline assumption,

[37] Each comparison covers the set of countries for which output gap estimates are available in each of the publicly available databases. The WEO database has output gap estimates for 27 AD economies, in most cases from 1980 onwards. The European Commission has estimates for 30 economies, 25 AD and 5 EMDE, with some estimates starting in 1965 and most starting in 1995. The OECD database, on the other hand, has estimates for 38 economies, 31 AD and 7 EMDE, starting from 2004.

$\rho = 0.8$, is plotted on the x-axis. The low-correlation alternative, $\rho = 0.7$, is also plotted on the y-axis in Panel A. The high-correlation alternative, $\rho = 0.9$, is also plotted on the y-axis in Panel B. The cases where one of the conditions was not met and the estimated hysteresis elasticity was then considered to be statistically different would have been shown in red if that had been the case. As shown in Figures 33 and 34, the hysteresis elasticity is not statistically different in any country. Furthermore, the estimated hysteresis elasticities are close to the 45-degree line, even more so for countries where the hysteresis elasticity is lower, implying that the estimated hysteresis elasticity is not only statistically but also numerically robust.

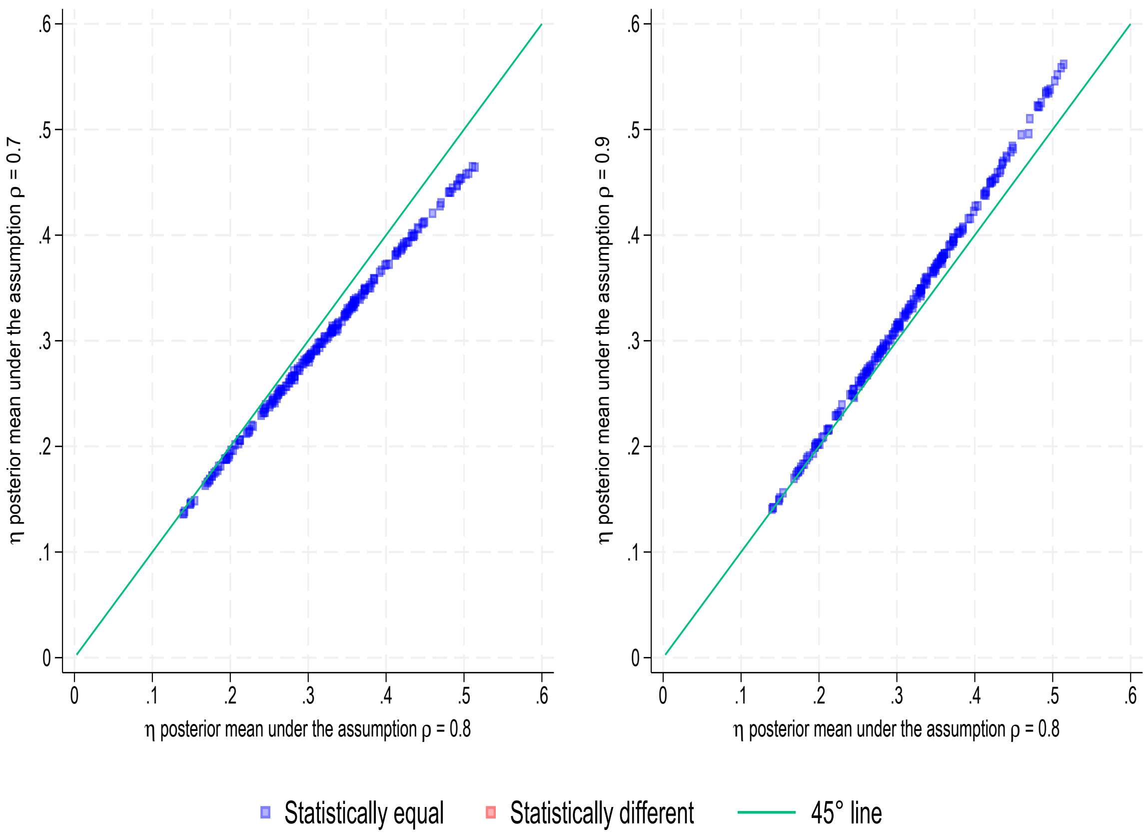
Figure 33: The robustness of η estimate to varying degrees of ρ – Quarterly frequency



Note: The posterior mean of the hysteresis elasticity η is shown under the baseline $\rho = 0.8$ and alternative scenarios, $\rho = 0.7$ and $\rho = 0.9$. Blue squares indicate estimates are not statistically different. Red squares would have indicated statistically different estimates if they had been found.

Another robustness question is the assumed value of the long-run effect. We used $\mu = 0.67$, which is the value found by Roa Rozo (2024) using the IMF sample. However, using the OECD sample, he finds $\mu = 1.21$. We do not use the latter estimate because, as mentioned above,

Figure 34: The robustness of η estimate to varying degrees of ρ – Annual frequency



Note: The posterior mean of the hysteresis elasticity η is shown under the baseline $\rho = 0.8$ and alternative scenarios, $\rho = 0.7$ and $\rho = 0.9$. Blue squares indicate that the estimates are not statistically different. Red squares would have indicated statistically different estimates if they had been found.

the former estimates are comparable to those of other papers in the literature. Moreover, the preferred choice leads to more conservative results.

11 Limitations

Some of the limitations of the paper may be topics for future research; however, they were outside the scope of this paper. One is the symmetric treatment of expansions and recessions. Demand shocks could plausibly have larger hysteresis effects for negative shocks than for positive shocks. An asymmetric treatment of the shocks was also beyond the scope of the paper.

Another is the lack of granularity in some aspects of the model; mainly, in the timing of the heteroskedasticity factors, in the prior for the parameter $\varepsilon_t^{\bar{y}}$, and in the calibrated coefficients. An idiosyncratic treatment of these features of the model remains an interesting research topic at the country or episode level but is naturally beyond the scope of this paper.

Finally, in the longer historical sample, in most cases from 1870 onwards, we have kept most parameters invariant over time. Time-varying coefficients are also an interesting possible issue, especially for long samples, although this topic was also beyond the scope of this paper. Nevertheless, the approach of maintaining uniform and constant coefficients is analogous to that of others such as DeLong et al. (2012), Jordà et al. (2020b), Ball and Onken (2021), it is also common practice with the HP filter, where the λ coefficient of the HP filter is usually used uniform across economies and constant over time.[38]

12 Conclusions

The paper proposes a univariate decomposition of output into trend and cycle. In the decomposition demand shocks lead to hysteresis in trend output. The hysteresis was incorporated with correlated shocks. The correlated shocks allowed for a transmission mechanism from demand to supply; in this sense, the model was considered semi-structural.

As a result of the correlated shocks, the estimated trend output is not smooth, in contrast with the traditional business cycle literature. In addition, compared with filters and public

[38] Other possible features that can be explored are the serial correlation of the shocks, if desired, and the lag structure of the persistence terms.

databases where trend output is smooth, the TCDH helps mitigate the problem of the artificial pre-recession boom and is less sensitive to incoming data.

The results of the paper refer to the estimation of the hysteresis elasticity and the estimated trend output and output gaps. Regarding the hysteresis elasticity, we showed that it tends to be negatively related to income, smaller in AD economies and larger in EMDE economies.

As to the FRs, we showed that during the 2008-2009 financial crisis and the pandemic recession, the business cycle was highly synchronized, which had not been the case before. We also showed that during the 2008-2009 financial crisis hysteresis occurred in AD economies but not in EMDE economies, and that, in contrast, during the pandemic recession, hysteresis occurred in both the AD and EMDE economies.

For the historical sample starting in 1870, three historical periods were identified based on the volatility and dispersion of the business cycle: the Gilded Age, the inter-war period, and the post-war period. The first period was relatively calm; the second was turbulent, especially in the AD economies; the third one was calm in the AD economies and turbulent in several EMDE economies.

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