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Financial Intermediation and Monetary Policy in a Small Open Economy

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Abstract

This paper analyses the role of a costly financial system in the transmission of monetary policy. The new-keynesian model for a small open economy is extended with a simple financial system based in Hamann and Oviedo (2006). The presence of the financial intermediation naturally allows the introduction of standard policy instruments: the repo interest rate and the compulsory requirement of reserves. The model is calibrated to match key steady-state ratios of Colombia and is used to evaluate the alternative policy instruments. The financial system plays an important role in the transmission mechanism of the monetary policy, and determines the final effects on aggregated demand and inflation rates of exogenous modifications of the policy instruments. The monetary policy conducted through the repo interest rate has the standard effects predicted by the new-keynesian framework. But changes in the compulsory reserve requirement rate may generate, under different scenarios, totally different reactions on economic activity, and little quantitative effects on inflation rates and aggregate demand. Therefore this last policy instrument appears to be uneffective and unreliable.

JEL Classification Numbers: E32, E44, E52, F41

Keywords: financial intermediation, small open economy, dynamic stochastic general equilibrium model, monetary policy, Colombia.

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1. Introduction

The financial system is a supervision, monitoring and control tool for information asymmetries, and is the mechanism of intermediation between savings and funding requirements in the economy. Both roles are relevant for aggregated macroeconomic activity.

Banks can be thought as having advantages over other agents in risky projects evaluation, private information management about those projects and knowledge about the payment capacity of the economic agents. The presence of banks in a small open economy reduces these agency/transaction costs and allows the development of more investment projects, and eases the financing of productive activities across the economy.

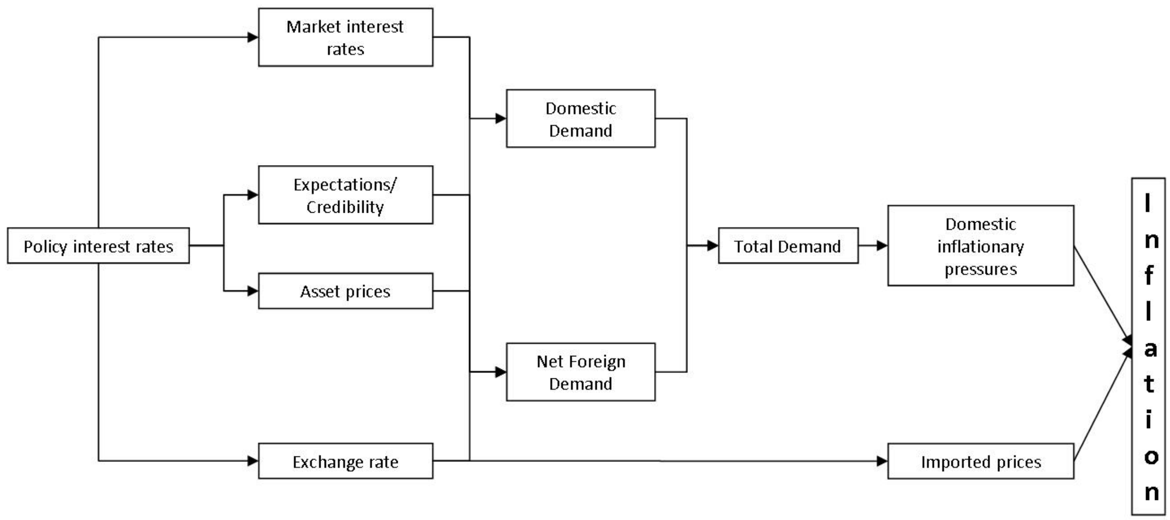
A key characteristic of banks is their dual role of managing deposits and loaning credits. The absence of separated specialized entities for each activity is explained by scope economies (where both activities are seen as financial services production at which banks have some comparative advantages). Also, the intermediation between private resources (deposits) and productive credit (loans) relates the aggregate saving with the investment and is important in the determination of the level of production and consumption, and can be fundamental for the growth rate.

These microeconomic characteristics are reflected in the aggregate activity, because the optimal decisions of banks affects the production of financial services and the market interest rates, thus affecting the optimal choices of other agents in the economy. The relationship between the financial intermediation and the real economy plays a key role for economic policy, since the transmission and effectivity of monetary policy is influenced by the decisions of the banks to exogenous policy shocks. The monetary policy cannot be analized without taking into account these relationships: for instance, an independent effect over aggregate activity by a credit channel can exist if there are not substitutes to bank credit in financing firms and if there are not substitutes to deposits in financing banks (Walsh (2003)).

The transmission mechanism of monetary policy through the financial system is summarized in Figure 1. Policy interventions affect the optimal decisions of the financial intermediators, disturbing the production of financial services and therefore changing the market interest rates. The effect of a policy shock on the savings interest rate can have a direct effect on the aggregate demand, and the effect on the lending rate may affect the marginal costs of the firms. Also, a policy announcement and its implementation has effects on the expectations and on the asset's prices, and may affect the exchange rate. All these variables have effects on domestic and net-foreign demand, affecting the total demand and generating pressures on the inflation rate (see Jalil (2006)). This channel can be relevant in developing countries, where banking credit remains as the main funding source and where the deposits are an important saving mechanism. As argued by Claus (2005), the small open economies tend to have more small bank-dependent firms that are strongly affected by the credit market conditions.

The financial system is also very important for small open economies because it takes care

Figure 1: Monetary policy channel



Source: Jalil (2006).

of the intermediation of foreign resources into the domestic economy, and it can amplificate and propagate the fluctuations in foreign capital flows. Empirically it is known the relation between the domestic real cycle and the domestic and foreign financial cycles. Uribe and Yue (2003) find that real cycles are correlated with the interest rate faced by countries in the foreign market. Also, as reviewed by Oviedo (2005), international interest rate is acyclical in developed countries, countercyclical in developing economies, and leads (by cross correlations analysis) the economic cycles in emerging countries. Some stylized facts reflecting the relation between financial and real cycles for Colombia can be found in Prada (2007).

Some works have focused on the role of the foreign capital market on the domestic business cycle. Calvo and Talvi (2005) suggest that the financial crisis suffered by the emerging countries is due to *sudden stops*, where the financial system plays a key role as amplifier of foreign shocks. With this approach, Villar, Salamanca, and Murcia (2005) analyze the links between the domestic credit, foreign capital flows and financial regulation for Colombia, and recognize the sudden stop as the most important cause in the 1999 crisis.

The monetary policy and its relation with the financial system is key to understand the effects of foreign and domestic shocks on the aggregate economic activity. Gertler, Gilchrist, and Natalucci (2007) explore the connection between the exchange rate regime and financial distress for small open economies, and find that the policy regime and the financial system are both important in explaining the amplification of foreign shocks. The Colombian case is studied in López, Prada, and Rodríguez (2008): balance-sheet effects play an important role in explaining recent Colombian business cycles, and a fixed exchange rate regime could have exacerbated the financial distress in the economy between 1998-1999.

However, despite the clear relevance of the financial intermediation in small open economies, most of the dynamic stochastic general equilibrium models used for policy analysis and forecast ignore the interactions between the financial markets and the real economy. Since in most of modern economies the monetary policy is carried through instruments that often affects directly the financial system, as repo interest rates, compulsory reserve requirements and foreign exchange operations, it is imperative to incorporate explicitly the intermediation sector in those models.

The new-keynesian model for a small open economy is extended with a simple financial system based in Hamann and Oviedo (2006). The banks take deposits from the households and lend it to the firms to finance their productive activities. Deposits are held by households because it is a way to reduce the transaction costs associated with consumption and investment. Firms also hold deposits due to a working capital constraint. Finally, banks face costs that allows the introduction of an operative margin in the market interest rates.

The presence of the financial intermediation naturally allows the introduction of standard monetary policy instruments: the repo interest rate and the rate of compulsory requirement of reserves. The model is calibrated to match key steady-state ratios of Colombia and is used to evaluate the alternative policy instruments.

The monetary policy conducted through the repo rate has the standard effects predicted by the new-keynesian framework, and this result is invariant under different structures of the financial system. But changes in the compulsory-reserve-requirement rate may generate, under different scenarios, totally different reactions on the economic activity, and little quantitative effects on the inflation rates and the aggregate demand. Therefore the reserve rate policy instrument appears to be uneffective and unreliable.

The next section presents the extended new-keynesian model. Section three shows the calibration and properties of the model. Section four presents qualitative/quantitative results. The last section concludes.

2. The Model

This is a small open economy with a unique final good, inhabited by households, firms, banks, government and the central bank.

The total population N_t follows an stochastic-trend process

$$\log(N_t) = n + \log(N_{t-1}) + \epsilon_t^N$$

and the labour augmenting productivity follows the process

$$\log(A_t) = a + \log(A_{t-1}) + \epsilon_t^A$$

where ϵ_t^A and ϵ_t^N are white noise variables.

There is another exogenous process that determines the participation rate and the employment rate. The people that effectively participates in labour market is

$$L_t = (1 - TD_t)\, TBP_t N_t$$

where TD_t is the unemployment rate and TBP_t is the gross participation rate in labour market.

2.1. Households

The economy is inhabited by a continuum of infinitely-lived households indexed by h, that differ in the variety of labour that are able to supply. Each household supplies a different kind of labour to the firms, and has preferences on per-capita consumption of the final good and per-capita leisure time. Preferences are represented through a classic instantaneous utility function $u\left(c_t^{pc}(h), c_{t-1}^{pc}(h), l_t(h)\right)$, where $c_t^{pc}(h)$ is the per-capita consumption bundle and $l_t(h)$ is the per-capita leisure time for the household h. Each period the households have $\bar{l} > 0$ per-capita units of time used only to work for firms and for leisure purposes. The average leisure satisfies $l_t(h) = \bar{l} - (1 - TD_t)\, TBP_t \mathcal{N}_t(h)$ where $\mathcal{N}_t(h)$ are total hours worked by the average inhabitant of household h.

The consumption bundle $c_t(h)$ is composed by domestically produced goods $c_t^d(h)$ and imported consumption goods $c_t^m(h)$:

$$c_t(h) = \left[\gamma^{\frac{1}{\omega}} \left(c_t^d(h)\right)^{\frac{\omega-1}{\omega}} + (1-\gamma)^{\frac{1}{\omega}} \left(c_t^m(h)\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{1}$$

At a first stage the household chooses the optimal combination between domestic and imported consumption to minimize the total expenditure in consumption. The demands for each kind of consumption are:

$$c_t^d(h) = \gamma \left(\frac{p_t^{qd}}{p_t^c}\right)^{-\omega} c_t(h) \tag{2}$$

$$c_t^m(h) = (1-\gamma) \left(\frac{p_t^m}{p_t^c}\right)^{-\omega} c_t(h) \tag{3}$$

and the inflation rate is

$$(1+\pi_t^c) = \left[\gamma \left(1+\pi_t^{qd}\right)^{1-\omega} \left(\frac{p_{t-1}^{qd}}{p_{t-1}^c}\right)^{1-\omega} + (1-\gamma)\left(1+\pi_t^m\right)^{1-\omega} \left(\frac{p_{t-1}^m}{p_{t-1}^c}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} \tag{4}$$

where p_t^{qd} is the unit price of the domestic gross product, p_t^m is the unit price of imported consumption and π_t^{qd} and π_t^m are the corresponding inflation rates.

At a second stage the household h chooses the aggregate level of consumption, investment, capital accumulation and utilization, credit and deposits to maximize the discounted sum of the lifetime expected utility.

Following Hamann and Oviedo (2006) it is assumed that the exchange process requieres the utilization of real resources. This transaction cost depends positively on the ratio between the value of private absorption and household's deposits

$$v_t(h) = \frac{c_t(h) + \frac{p_t^x}{p_t^c} x_t(h)}{d_{t-1}^h(h)} \frac{A_t N_t}{A_{t-1} N_{t-1}} \tag{5}$$

where $x_t(h)$ is the investment in physical capital made by household h, $v_t(h)$ is the velocity of deposits and $d_{t-1}^h(h)$ are the deposits held by the household h in the banks. In order to sustain a higher level of absorption the household must undertake a larger number of costly transactions. The deposits reduce the transaction costs by generating financial services to the households. The velocity of deposits is then a measure of the transactions per unit of financial services.

The transaction cost for absorption unit is given by $\vartheta(v_t(h))$, a positive, increasing, convex twice continuously differentiable function. That is $\vartheta(\cdot) \geq 0$, $\vartheta(0) = 0$, $\vartheta'(\cdot) > 0$ and $\vartheta''(\cdot) > 0$. In particular it is assumed that

$$\vartheta(v_t) = \vartheta_0 v_t^{\vartheta_1} \tag{6}$$

with ϑ_0, $\vartheta_1 > 0$.

The households own the firms and the banks, and receive their profits each period. They also pay lump-sum taxes τ_t to the government. Because they own the productive factors, households get the capital and labour remunerations. They also finance their activities through loans $z_{t-1}^h(h)$ from banks. Finally they make investment in physical capital $k_{t-1}(h)$, which depreciates at a rate that increases with capital utilization, $\delta(u_t(h)) \geq 0$, $\delta'(\cdot) > 0$.

Following Mendoza (1991) and Schmitt-Grohé and Uribe (2003) there are adjustment costs in investment: a higher variation in the capital stock has higher adjustment costs. This costs are given by a non negative function $\Phi(k_t(h) - k_{t-1}(h))$, with $\Phi(0) = 0$ and $\Phi'(0) = 0$[1]. This adjustment of new capital is a service provided by the producers of investment and is given by the quadratic cost function

$$\Phi(k_t(h) - k_{t-1}(h)) = \frac{\psi^X}{2} (k_t(h) - k_{t-1}(h))^2$$

with $\psi^X > 0$.

[1]Note that the presence of this costs will affect the marginal decisions of the households and the dynamic response of the capital and investment. Th conditions $\Phi(0) = 0$ and $\Phi'(0) = 0$ assure that the adjustment costs do not affect the level or the marginal returns of capital in the steady state.

The capital accumulation is given by

$$k_t(h) = x_t(h) + (1 - \delta(u_t(h)))\, k_{t-1}(h) \frac{A_{t-1}N_{t-1}}{A_t N_t} \tag{7}$$

where the depreciation rate is

$$\delta(u_t(h)) = \overline{\delta} + \frac{b}{1+\Upsilon} (u_t(h))^{1+\Upsilon} \tag{8}$$

with $b > 0$, $\Upsilon > 0$. The depreciation rate is an increasing convex function of the utilization intensity of capital $u_t(h)$: a more intense utilization generates a faster depreciation of the capital and this increases the depreciation rate.

Each household is subject to idiosyncratic shocks, due to the ability of setting a non-competitive wage, because of the differentiated labour varieties they supply. Following Erceg, Henderson, and Levin (1999) and Fernández-Villaverde and Rubio-Ramírez (2006), the households are able to exchange state-contingent Arrow-Debreu assets (indexed by household h and by time period t). The asset $a_t(h)$ pays one unit of consumption if the event $w_t(h)$ (certain wage level) occurs, and it is acquired at a real cost of $p_t^a(h)$. These assets allow the households to cover themselves from idyosincratic shocks in such a way that all the non-labour decisions made by all the households are the same. This eases the aggregation.

The household decisions must satisfy the budget constraint

$$\begin{aligned}
c_t(h) + \frac{p_t^x}{p_t^c} x_t(h) + \Phi(k_t(h) - k_{t-1}(h)) \qquad & r_t^k u_t(h) k_{t-1}(h) \frac{A_{t-1}N_{t-1}}{A_t N_t} + w_t(h)(1 - TD_t)\, TBP_t n_t(h) \\
+ \int p_t^a(h) a_t(h)\, dw_t(h) + \tau_t + d_t^h(h) \quad \leq \quad & + \frac{s_t p_t^\star}{p_t^c} tr_t + \Pi_t + \left(\frac{1 + i_{t-1}^d}{1 + \pi_t^c}\right) d_{t-1}^h(h) \frac{A_{t-1}N_{t-1}}{A_t N_t} \\
+ \left(c_t(h) + \frac{p_t^x}{p_t^c} x_t(h)\right) \vartheta(v_t(h)) \qquad & + z_t^h(h) - \left(\frac{1 + i_{t-1}^z}{1 + \pi_t^c}\right) z_{t-1}^h(h) \frac{A_{t-1}N_{t-1}}{A_t N_t}
\end{aligned}$$

The household h finances its expenditures, given by consumption $c_t(h)$, investment $x_t(h)$, adjustment costs, accumulation of Arrow-Debreu assets $a_t(h)$, lump-sum taxes τ_t, accumulation of per-capita deposits $d_t^h(h)$, transaction costs and interest payments i_{t-1}^z on loans $z_{t-1}^h(h)$, with the income given by the remuneration of capital effectively used $k_{t-1}(h) u_t(h)$ and labour time $n_t(h)$, real net foreign transfers tr_t valued in foreign currency, total profits of firms and banks Π_t, the bank interests i_{t-1}^d paid on deposits and credit accumulation $z_t^h(h)$. Here $p_t^\star$ is the consumer price index of the rest of the world, $1 + \pi_t^c = \frac{p_t^c}{p_{t-1}^c}$, and s_t is the nominal exchange rate . Note that $\frac{s_t p_t^\star}{p_t^c}$ is the real exchange rate.

The problem of the household h is to maximize the discounted value of the stream of instantaneous utility subject to the budget constraint. The utility function is separable in consumption and labour

$$u(\cdot) = \frac{\chi_t^u}{1-\sigma} \left[c_t^{pc}(h) - \phi \frac{A_t}{A_{t-1}} c_{t-1}^{pc}(h)\right]^{1-\sigma} - \frac{\chi_t^h}{1+\varsigma} \overline{l}^{-\sigma-\varsigma} A_t^{1-\sigma} \left((1 - TD_t)\, TBP_t n_t^{pc}(h)\right)^{1+\varsigma}$$

where $\sigma > 0$, $\varsigma > 0$ and $\phi \geq 0$. Note that $\phi > 0$ allows the presence of internal habit formation. The exogenous scale factor χ_t^u is a shock to the marginal utility of consumption, and the exogenous scale factor χ_t^h is a shock to the marginal utility of leisure. These shocks follow the process

$$\ln\left(\chi_t^j\right) = (1-\rho_j)\ln\left(\overline{\chi}^j\right) + \rho_j \ln\left(\chi_{t-1}^j\right) + \epsilon_t^j$$

where $\overline{\chi}^j$ is the expected value of the scale factor, $\rho_j \in (0,1)$ and ϵ^j is a white noise variable with null expected value and constant variance σ_j^2, with $j \in \{u, h\}$.

The necessary first order conditions of this problem can be found in Appendix A.

It can be shown in equilibrium that the optimal credit demand of each household will be

$$z_t^h(h) = 0 \tag{9}$$

because banks will charge a interest rate i_t^z that is always higher than the benefit of an additional unit of credit (see equation (28)).

2.2. Wage setting

Households offer differentiated labour services in a monopolistic-competitive market. Each household h is able to set the wage of its labour variety, and knows the labour demand coming from the labour aggregating firm. Also, wages are sticky à la Calvo.

The labour aggregating firm buys the labour varieties supplied by the different households, aggregate them through a CES production and sell the aggregate labour to final-good producer firms.

The aggregator minimize the production costs subject to the CES technological constraint, and the conditional demand for each variety of labour is

$$n_t(h) = \left(\frac{w_t(h)}{w_t}\right)^{-\theta_t^w} n_t^d$$

where $w_t \equiv \left[\int_0^1 w_t(h)^{1-\theta_t^w}\right]^{\frac{1}{1-\theta_t^w}}$ is the price of the aggregate labour bundle and $\theta_t^w > 0$ is the elasticity of substitution between varieties. This elasticity follows an exogenous process.

The wage setting problem follows Erceg, Henderson, and Levin (1999). The households are able to set the wage to be paid for each labour variety. But this wage can be set according to a stochastic signal that follows a Poisson process and arrives each period with probability $(1-\epsilon^w)$. If a household does not receive the signal, the wage is updated according to the non-optimal rule

$$w_t^{rule\,pc}(h) = w_{t-1}^{pc}(h)\frac{A_t}{A_{t-1}}\left(\frac{1+\pi_{t-1}^c}{1+\pi_t^c}\right)$$

If the household receives the signal to optimize in period t, the following problem is solved:

$$\begin{aligned} \max_{w_t(h)} \quad & E_t \sum_{i=0}^{\infty} (\beta\epsilon^w)^i \frac{N_{t+i}}{N_t} u\left(c_{t+i}(h), 1-(1-TD_{t+i}) TBP_{t+i} n_{t+i}(h)\right) \\ \text{s.t.} \quad & n_{t+i}(h) = n_{t+i}^d \left(\frac{w_{t+i}(h)}{w_{t+i}}\right)^{-\theta_{t+i}^w} \\ & w_{t+i}^{pc}(h) = w_t^{pc}(h) \frac{A_{t+i}}{A_t} \prod_{k=1}^{i} \left(\frac{1+\pi_{t+k-1}^c}{1+\pi_{t+k}^c}\right) \end{aligned}$$

and subject to the budget constraint.

The optimal wage of household h satisfies:

$$w_t^{opt}(h) = \frac{E_t \sum_{i=0}^{\infty} (\beta\epsilon^w)^i \frac{N_{t+i}}{N_t} \left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma} \chi_{t+i}^h \left((1-TD_{t+i}) TBP_{t+i} n_{t+i}^d \left(\frac{w_t^{opt}(h)}{w_{t+i}} \pi_{t,i}^c\right)^{-\theta_{t+i}^w}\right)^{1+\varsigma} \theta_{t+i}^w}{E_t \sum_{i=0}^{\infty} (\beta\epsilon^w)^i \frac{N_{t+i}}{N_t} \left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma} \lambda_{t+i} \left(\theta_{t+i}^w - 1\right) \pi_{t,i}^c \left((1-TD_{t+i}) TBP_{t+i} n_{t+i}^d \left(\frac{w_t^{opt}(h)}{w_{t+i}} \pi_{t,i}^c\right)^{-\theta_{t+i}^w}\right)}$$

where $\pi_{t,i}^c = \prod_{k=1}^{i} \left(\frac{1+\pi_{t+k-1}^c}{1+\pi_{t+k}^c}\right)$ and $\lambda_t(h)$ is the Lagrange multiplier associated to the budget constraint of household h. The index h can be dropped from w_t^{opt} because the asset market is complete and all the households choose the same optimal wage.

Since the probability of wage setting is independent across households and time, a fraction $1-\epsilon^w$ of the households set the wage optimally and a fraction ϵ^w follows the non-optimal rule. Consecuently the wage index is:

$$w_t = \left[\epsilon^w \left(w_{t-1} \left(\frac{1+\pi_{t-1}^c}{1+\pi_t^c}\right)\right)^{1-\theta_t^w} + (1-\epsilon^w) \left(w_t^{opt}\right)^{1-\theta_t^w}\right]^{\frac{1}{1-\theta_t^w}} \tag{10}$$

2.3. Final good producer firm

There is a representative producer firm that produces the final good q_t^s using the nested CES production function

$$q_t^s = \chi_t^{qs} \left[\alpha^{\frac{1}{\rho}} \left(v_t^q\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} \chi_t^z \left(z_{t-1}^f \frac{A_{t-1}}{A_t} \frac{N_{t-1}}{N_t}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{11}$$

$$v_t^q = \chi_t^v \left[\alpha_v^{\frac{1}{\rho_v}} \left(k_t^s\right)^{\frac{\rho_v-1}{\rho_v}} + (1-\alpha_v)^{\frac{1}{\rho_v}} \left(A_t (1-TD_t) TBP_t n_t\right)^{\frac{\rho_v-1}{\rho_v}}\right]^{\frac{\rho_v}{\rho_v-1}} \tag{12}$$

where $k_t^s = \int k_{t-1}(h) u_t(h)\, dh$ is the capital stock used by the firm, n_t is the aggregate labour used by the firm and z_{t-1}^f is the amount of financial capital (credit) required to produce. Note that the substitution elasticities are constant and given by $(\rho)^{-1}$ and $(\rho_v)^{-1}$.

χ_t^{qs}, χ_t^z and χ_t^v are exogenous technological scale factors that follow the process

$$\ln\left(\chi_t^j\right) = (1-\rho_j)\ln\left(\overline{\chi}^j\right) + \rho_j \ln\left(\chi_{t-1}^j\right) + \epsilon_t^j$$

where $\overline{\chi}^j$ is the expected value of the scale factor, $\rho_j \in (0,1)$ and ϵ^j is a white noise variable with null expected value and constant variance σ_j^2, with $j \in \{qs, z, v\}$.

This firm operates under perfect competition and sells the total production q_t^s to the retailers at a price p_t^{qs}.

Following Uribe and Yue (2003), the firm faces a *working capital constraint*. To pay a fraction of the wage bill the firm is required to hold deposits in the banks in order to satisfy the CIA constraint

$$d_{t-1}^f \frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} \geq \varphi w_t \left(1 - TD_t\right) TBP_t n_t \tag{13}$$

where d_{t-1}^f are total deposits held by the firm in the banks and $\varphi \geq 0$.

The resource constraint of the firm in units of effective standarized labour is:

$$\begin{aligned}\Pi_t^{qs} \quad &= \quad \frac{p_t^{qs}}{p_t^c} q_t^s + z_t^f + \left(\frac{1+i_{t-1}^d}{1+\pi_t^c}\right) d_{t-1}^f \frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} \\ &\quad -w_t\left(1-TD_t\right)TBP_t n_t - r_t^k k_t^s - \left(\frac{1+i_{t-1}^z}{1+\pi_t^c}\right) z_{t-1}^f \frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} - d_t^f\end{aligned} \tag{14}$$

The profits are the difference between incomes and expenditures. The incomes are given by the value of the output, the credit and the interests received from the deposits of the firms. These resources are used to pay the production factors capital and labour, pay the interests of the credit and to accumulate deposits.

Since the households own the producer firm, they can just ask for credit (or put deposits) where the opportunity cost is minimum (as credit/deposits of the households or as credit/deposits of the firm). Consecuently the interest rates for the firm and for the households are the same in this model.

The firm maximizes the discounted sum of real per-effective-standarized-labour profits transferred to the households, using $\Lambda_{t+i,t}^f = \beta^i \left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma} \frac{N_{t+i}}{N_t}\frac{\lambda_{t+i}}{\lambda_t}$ as the appropiate discount factor.

The first order conditions are:

$$r_t^k = \frac{p_t^{qs}}{p_t^c}\chi_t^{qs}\chi_t^v\left(\frac{\alpha q_t^s}{\chi_t^q v_t^q}\right)^{\frac{1}{\rho}}\left(\frac{\alpha_v v_t^q}{\chi_t^v k_t^s}\right)^{\frac{1}{\rho_v}} \tag{15}$$

$$w_t\left(1+\varphi\Gamma_t\right) = \frac{p_t^{qs}}{p_t^c}\chi_t^{qs}\chi_t^v\left(\frac{\alpha q_t^s}{\chi_t^q v_t^q}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-\alpha_v\right)v_t^q}{\chi_t^v\left(1-TD_t\right)TBP_t n_t}\right)^{\frac{1}{\rho_v}} \tag{16}$$

$$\begin{aligned}\lambda_t &= \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\frac{1+i_t^z}{1+\pi_{t+1}^c}\right) \\ &\quad -\beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\frac{p_{t+1}^{qs}}{p_{t+1}^c}\chi_{t+1}^q\chi_{t+1}^z\left(\frac{\left(1-\alpha\right)q_{t+1}^s}{\chi_{t+1}^q z_t^f}\frac{A_{t+1}}{A_t}\frac{N_{t+1}}{N_t}\right)^{\frac{1}{\rho}}\end{aligned} \tag{17}$$

$$\lambda_t = \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\left(\frac{1+i_t^d}{1+\pi_{t+1}^c}\right)+\Gamma_{t+1}\right) \tag{18}$$

where Γ_t is the Lagrange multiplier associated to the working capital constraint. It is assumed that $\Gamma_t > 0$, so that the constraint is always binding. These are standard marginal conditions: the marginal cost of an additional unit of productive factor must be equal to the marginal profit that it generates.

2.4. Retailers

There is a continuum of retailers, indexed by $j \in (0,1)$, that buy a fraction of the good q_t^s at a price p_t^{qs} and transform it without cost in a differentiated good $q_t(j)$. The differentiated varieties of the good are then aggregate to be sold to the households, banks, investment producers and government. Since their output is differentiated, retailers have the monopolistic power to set prices of these final goods. This allows the introduction of price stickiness in the production side of the model.

Each retailer firm faces a downward-sloping demand curve

$$q_t(j) = \left(\frac{\chi_t^{qd}p_t^q(j)}{p_t^{qd}}\right)^{-\theta_t^q} q_t^d$$

where $q_t^d = \chi_t^{qd}\left[\int_0^1 \left(q_t(j)\right)^{\frac{\theta_t^q-1}{\theta_t^q}}dj\right]^{\frac{\theta_t^q}{\theta_t^q-1}}$ and $p_t^q = \left(\chi_t^{qd}\right)^{-1}\left[\int_0^1 \left(p_t^q(j)\right)^{1-\theta_t^q}dj\right]^{\frac{1}{1-\theta_t^q}}$. Here χ_t^{qd} is an exogenous technological factor, p_t^q is the price of the aggregate bundle q_t^d and θ_t^q is the exogenous elasticity of substitution between varieties of the good.

There is a price stickiness à la Calvo (1983). With probability $(1-\epsilon^q)$ the retailer firm j receives a stochastic signal that allows the optimal price setting. Otherwise, they do not optimize and follow the non-optimal price rule

$$p_t^{qrule}(j) = p_{t-1}^q(j)\prod_{k=1}^{n}\left(1+\pi_{t-k}^{qd}\right)^{\gamma_{qk}}(1+\overline{\pi})^{1-\sum_{m=1}^{n}\gamma_{qm}}$$

where $n \in \mathbb{N}$ is the indexation horizon, $\gamma_k \geq 0$ is the weight assigned to the inflation rate of the k-lag in the rule and $1-\sum_{m=1}^{n}\gamma_{qm} \geq 0$ is the weight assigned to the steady state inflation rate $\overline{\pi}$.

The problem faced by firm j is to choose $p_t^q(j)$ to maximize the discounted value of the profits when it is allowed to set the optimal price once:

$$\begin{aligned}
\max_{p_t^q(j)} \quad & E_t\sum_{i=0}^{\infty}(\epsilon^q)^i\Lambda_{t+i,t}\left[\frac{p_{t+i}^q(j)\,q_{t+i}(j)}{p_{t+i}^c}-\frac{p_{t+i}^q}{p_{t+i}^c}q_{t+i}(j)\right]\\
\text{s.t.} \quad & \\
& q_{t+i}(j) = \left(\frac{\chi_{t+i}^{qd}p_{t+i}^q(j)}{p_{t+i}^{qd}}\right)^{-\theta_{t+i}^q}q_{t+i}^d\\
& p_{t+i}^q(j) = p_t^q(j)\,\pi_{t,i,\{\gamma_{qk}\}_{k=1}^n}
\end{aligned}$$

where $\Lambda_{t+i,t} = \beta^i\left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma}\frac{N_{t+i}}{N_t}\frac{\lambda_{t+i}}{\lambda_t}$ is the discount factor and

$$\pi_{t,i,\{\gamma_{qk}\}_{k=1}^n} = \prod_{l=1}^{i}\left\{\prod_{k=1}^{n}\left(1+\pi_{t-k+l}^{qd}\right)^{\gamma_{qk}}\right\}(1+\overline{\pi})^{i\left(1-\sum_{m=1}^{n}\gamma_{qm}\right)}$$

The optimal price is characterized by

$$\frac{p_t^{qopt}(j)}{p_t^{qd}} = \frac{E_t\sum_{i=0}^{\infty}(\epsilon^q)^i\Lambda_{t+i,t}\left[\theta_{t+i}^q\frac{p_{t+i}^{qs}}{p_{t+i}^c}\left(\pi_{t,i,\{\gamma_{qk}\}_{k=1}^n}\right)^{-\theta_{t+i}^q}\left(\frac{p_{t+i}^{qd}}{p_t^{qd}}\right)^{\theta_{t+i}^{qd}}q_{t+i}^d\left(\chi_{t+i}^{qd}\right)^{\theta_{t+i}^q}\right]}{E_t\sum_{i=0}^{\infty}(\epsilon^q)^i\Lambda_{t+i,t}\left[\left(\theta_{t+i}^q-1\right)\frac{p_{t+i}^{qd}}{p_{t+i}^c}\left(\pi_{t,i,\{\gamma_{qk}\}_{k=1}^n}\right)^{1-\theta_{t+i}^j}\left(\frac{p_{t+i}^{jm}}{p_t^{jm}}\right)^{\theta_{t+i}^j-1}q_{t+i}^d\left(\chi_{t+i}^{qd}\right)^{\theta_{t+i}^q}\right]}$$

Since the probability of price setting is independent across firms and time, a fraction $1-\epsilon^q$ of retailers set the price optimally and a fraction ϵ^q follows the non-optimal rule. Consecuently the inflation is given by the new-keynesian hybrid Phillips curve

$$\left(1+\pi_t^{qd}\right)^{1-\theta_t^q} = (1-\epsilon^q)\left(\frac{p_t^{qopt}}{p_t^{qd}}\right)^{1-\theta_t^q}\left(1+\pi_t^{qd}\right)^{1-\theta_t^q}+\epsilon^q\left[\prod_{k=1}^{n}\left(1+\pi_{t-k}^{qd}\right)^{\gamma_{qk}}(1+\overline{\pi})^{1-\sum_{m=1}^{n}\gamma_{qm}}\right]^{1-\theta_t^q} \tag{19}$$

The final good is used for domestic consumption, domestic investment x_t^d, consumption of the government g_t, exports e_t and as input for the financial system $\xi_t\eta(z_t, d_t)$:

$$q_t^d = c_t^d + x_t^d + g_t + e_t + \xi_t\eta(z_t, d_t) \tag{20}$$

2.5. Financial System

The financial system is composed by a continuum of banks, indexed by j^b. In this model the banks are financial firms specialized in the production of financial services (credit and deposits). It is not explained the reason of existence of the banks, and the financial sector is just given in the economy.

Since the economic agents need deposits and credit, the banks produce the financial services through a production technology that uses real resources from the economy as input. The production technology for the bank j^b is given by the cost function $\xi_t \eta \left(z_t \left(j^b\right), d_t \left(j^b\right)\right)$ that represents the minimum quantity of real resources (final output) needed to produce financial services (deposits $d_t \left(j^b\right)$ and credit $z_t \left(j^b\right)$). The presence of this function is justified because the financial intermediation is costly: the management of deposits and credit, evaluation of customers, monitoring of loans, the rent paid for buildings, the maintenance cost of ATMs, etc.

Following Edwards and Végh (1997), $\xi_t \eta \left(z_t \left(j^b\right), d_t \left(j^b\right)\right)$ is positive for $z_t \left(j^b\right), d_t \left(j^b\right) > 0$, convex, continuously differentiable, increasing in all arguments and homogeneous of degree one[2]. Also the credit and deposits are Edgeworth complements: the cost of providing an additional unit of credit is reduced by a higher level of deposits. It can be thought as if more deposits allow the banks to get a better knowledge of the agents, generating a reduction in the monitoring costs for credit.

ξ_t represents an inverse measure of the total productivity of the financial intermediation sector, and it is a cost scale factor exclusive of the financial sector, and equal for all the banks. The real costs are subject to exogenous shocks. The logarithm of the cost scale factor follows an autoregressive stochastic process

$$\ln \left(\xi_t\right) = \left(1 - \rho_\xi\right) \ln \left(\overline{\xi}\right) + \rho_\xi \ln \left(\xi_{t-1}\right) + \epsilon_t^\xi$$

where $\overline{\xi}$ is the expected value of the cost scale factor, $\rho_\xi \in (0, 1)$ and ϵ^ξ is a white noise variable with null expected value and constant variance σ_ξ^2.

The monetary policy and the financial system are closely related, because the financial system is the bridge between the monetary authorities and the private sector. In particular, the compulsory requirement of reserves and the repo market are the mechanisms through which this policy may operate.

There exists a compulsory requirement of reserves as a fixed proportion $\tau_t^d > 0$ of the total

[2]This minimal cost function is a "*black box*", just as any production function: it does not say how to transform the real resources into financial intermediation, just as the production function does not say how to transform capital, labour and other inputs into output. The homogeneity of degree one implies that to produce $j \geq 0$ times the current credit and deposits quantities the banks need j times the current real resources used as input. Finally a convex cost function may be associated to a concave production function.

deposits, so the bank reserves rb_t satisfy the constraint

$$rb_t\left(j^b\right) \geq \tau_t^d d_t\left(j^b\right) \tag{21}$$

This reserves do not pay interest to the banks and its presence generates an opportunity cost by the foregone earnings of holding an amount that otherwise could have been invested in foreign assets. That is the reason why the reserves constraint holds as equality in the optimum[3]. This reserve requirement may be used as a monetary policy instrument by the central bank, and τ_t^d is assumed as exogenously managed by the monetary authority.

Banks can borrow from the central bank at a nominal rate i_t^{cb}. The net debt of a private bank with the central bank is $b_t\left(j^b\right)$ and this is one source of funding for the financial system. The intervention of the central bank in this bank's debt market is the way in which the central bank manages the monetary base, and this is one important instrument for monetary policy in most of small open economies.

The banks also finance themselves through foreign debt f_t, and they pay the interest rate i_t^f set in the foreign market. It is assumed that the banks are the only private agents that have access to foreign resources. This stresses the importance of the financial system in a small open economy, as intermediator between the foreign funds and the domestic needs for financing.

The representative bank j^b seeks the maximization of the value of assets, the discounted sum of profits $\Pi_t^b\left(j^b\right)$ that will be transferred to the households. The resource constraint of the bank j^b is given by:

$$\begin{aligned}
\left(\frac{1+i_{t-1}^z\left(j^b\right)}{1+\pi_t^c}\right) z_{t-1}\left(j^b\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} & & \left(\frac{1+i_{t-1}^d\left(j^b\right)}{1+\pi_t^c}\right) d_{t-1}\left(j^b\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} + z_t\left(j^b\right) \\
+\frac{s_t p_t^\star}{p_t^c} f_t\left(j^b\right) + d_t\left(j^b\right) & \geq & +\left(\frac{1+i_{t-1}^f}{1+\pi_t^\star}\right)\frac{s_t p_t^\star}{p_t^c} f_{t-1}\left(j^b\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} + \Pi_t^b\left(j^b\right) \\
+b_t\left(j^b\right) + rb_{t-1}\left(j^b\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} & & +\left(\frac{1+i_{t-1}^{cb}}{1+\pi_t^c}\right) b_{t-1}\left(j^b\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} + rb_t\left(j^b\right) \\
& & +\frac{p_t^{qd}}{p_t^c}\xi_t\eta\left(z_t\left(j^b\right), d_t\left(j^b\right)\right)
\end{aligned} \tag{22}$$

The income of the banks, given by credit interest payments at a nominal rate $i_{t-1}^z\left(j^b\right)$, foreign debt accumulation $f_t\left(j^b\right)$, deposits accumulation $d_t\left(j^b\right)$, accumulation of debt with the central bank $b_t\left(j^b\right)$ and the returned reserve from the central bank $rb_{t-1}\left(j^b\right)$, is used to pay for deposits at an interest rate $i_t^d\left(j^b\right)$, to accumulate credit $z_t\left(j^b\right)$, to pay the foreign debt interest i_t^f, to make profit transfers to households $\Pi_t^b\left(j^b\right)$, to pay the interest i_t^{cb} to the central

[3]It is easy to incorporate the fact that the central bank may pay a small interest rate for the reserves, and as long as this rate is less than the foreign interest rate (the opportunity cost of the domestic economy) the constraint will hold with equality. Then the inclusion of a small interest rate paid for the reserves does not change the conclusions derived from the model.

bank, to accumulate new reserves and to pay the real costs of the financial intermediation.

The production technology of the financial services is represented with the cost function

$$\eta\left(z_t\left(j^b\right), d_t\left(j^b\right)\right) = \left[\nu_z\left(z_t\left(j^b\right)\right)^{\nu} + \nu_d\left(d_t\left(j^b\right)\right)^{\nu}\right]^{\frac{1}{\nu}} \tag{23}$$

where $\nu > 1$, $\nu_z, \nu_d > 0$.

The first order conditions for domestic and foreign debt accumulation are:

$$\lambda_t = \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\frac{1+i_t^{cb}}{1+\pi_{t+1}^c}\right) \tag{24}$$

$$\lambda_t\frac{s_t p_t^{\star}}{p_t^c} = \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\frac{s_{t+1}p_{t+1}^{\star}}{p_{t+1}^c}\left(\frac{1+i_t^f}{1+\pi_{t+1}^{\star}}\right) \tag{25}$$

These are standard Euler conditions regarding the holding of substitute assets, and together constitute the uncovered interest rate parity condition of this model.

Following Gerali, Neri, Sessa, and Signoretti (2008), it can be assumed that there is some degree of collusion in the deposits. The collusion consists in the aggregation of the deposits of all the banks through the aggregation function

$$d_t = \left[\int\left(d_t\left(j^b\right)\right)^{\frac{\theta^d-1}{\theta^d}} dj^b\right]^{\frac{\theta^d}{\theta^d-1}}$$

that implies that the demand for the deposits produced by bank j^b is

$$d_t\left(j^b\right) = \left(\frac{1+i_t^d\left(j^b\right)}{1+i_t^d}\right)^{\theta^d} d_t$$

In a symmetric equilibrium in which all banks behave in the same way, the first order condition with respect to deposits is

$$\beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\left(\frac{1+\theta^d}{\theta^d}\right)\left(\frac{1+i_t^d}{1+\pi_{t+1}^c}\right)-\tau_t^d\right) = \lambda_t\left(\left(1-\tau_t^d\right)-\frac{p_t^{qd}}{p_t^c}\xi_t\nu_d\left[\nu_z\left(z_t\right)^{\nu}+\nu_d\left(d_t\right)^{\nu}\right]^{\frac{1}{\nu}-1}\left(d_t\right)^{\nu-1}\right) \tag{26}$$

and the equilibrium condition for the deposits market is

$$d_t = d_t^h + d_t^f \tag{27}$$

Similarly, there is also some degree of collusion in the credit supply. The collusion consists in the aggregation of the credit of all the banks through the aggregation function

$$z_t = \left[\int \left(z_t \left(j^b \right) \right)^{\frac{\theta^z - 1}{\theta^z}} dj^b \right]^{\frac{\theta^z}{\theta^z - 1}}$$

that implies that the demand for the credit produced by bank j^b is:

$$z_t \left(j^b \right) = \left(\frac{1 + i_t^z \left(j^b \right)}{1 + i_t^z} \right)^{-\theta^z} z_t$$

In a symmetric equilibrium in which all banks behave in the same way, the first order condition with respect to credit is

$$\beta E_t \left(\frac{A_{t+1}}{A_t} \right)^{-\sigma} \lambda_{t+1} \left(\frac{1 + i_t^z}{1 + \pi_{t+1}^c} \right) = \lambda_t \frac{\theta^z}{\theta^z - 1} \left(1 + \frac{p_t^{qd}}{p_t^c} \xi_t \nu_z \left[\nu_z \left(z_t \right)^\nu + \nu_d \left(d_t \right)^\nu \right]^{\frac{1}{\nu} - 1} \left(z_t \right)^{\nu - 1} \right) \quad (28)$$

and the equilibrium condition for credit is

$$z_t = z_t^f \quad (29)$$

2.6. Exports and Imports

The exported good e_t is sold at a price that satisfies the PPP

$$\frac{p_t^{qd}}{p_t^c} = \frac{s_t p_t^\star}{p_t^c} \frac{\left(p_t^{exp} \right)^\star}{p_t^\star} \quad (30)$$

where $\frac{\left(p_t^{exp} \right)^\star}{p_t^\star}$ is the relative price of the exports in the foreign market in foreign currency.

The foreign demand for the domestic good is

$$e_t = \left(\frac{\left(p_t^{exp} \right)^\star}{p_t^\star} \right)^{-\mu} c_t^\star \quad (31)$$

The imports are more complex. In order to introduce price stickiness in the imports, it is assumed that there is a continuum of importer firms indexed by $j^m \in (0, 1)$. Each firm buys the foreign good m_t^{dock} at dock price p_t^{mdock} and transforms it costlessly in a differentiated good $m_t \left(j^m \right)$.

Each importer firm j^m faces a downward-sloping demand curve

$$m_t \left(j^m \right) = \left(\frac{p_t^m \left(j^m \right)}{p_t^m} \right)^{-\theta_t^m} m_t$$

where $m_t = \left[\int_0^1 \left(m_t \left(j^m \right) \right)^{\frac{\theta_t^m - 1}{\theta_t^m}} dj^m \right]^{\frac{\theta_t^m}{\theta_t^m - 1}}$ is the aggregate imported good adapted to domestic use and p_t^m is the final price of the imports bundle.

Following the new open economy model literature, there is price stickiness as in Calvo (1983). With a probability $(1 - \epsilon^m)$ the importer receives a stochastic signal that allows him/her to optimally set the price. If this signal does not arrive, the importer follows the price rule

$$p_t^{mrule} \left(j^m \right) = p_{t-1}^m \left(j^m \right) \prod_{k=1}^{n} \left(1 + \pi_{t-k}^m \right)^{\gamma_{mk}} \left(1 + \overline{\pi} \right)^{1 - \sum_{j=1}^{n} \gamma_{mj}}$$

The problem of the importer j^m is to optimally set $p_t^m \left(j^m \right)$ to maximize the discounted sum of expected profits when the price can be adjusted once, subject to the demand and the price rule. The optimal price is characterized by

$$\frac{p_t^{mopt}}{p_t^m} = \frac{E_t \sum_{i=0}^{\infty} \left(\epsilon^m \right)^i \Lambda_{t+i,t} \left[\theta_{t+i}^m \frac{p_{t+i}^{mdock}}{p_{t+i}^c} \left(\pi_{t,i,\{\gamma_{mk}\}_{k=1}^n} \right)^{-\theta_{t+i}^m} \left(\frac{p_{t+i}^m}{p_t^m} \right)^{\theta_{t+i}^m} m_{t+i} \right]}{E_t \sum_{i=0}^{\infty} \left(\epsilon^m \right)^i \Lambda_{t+i,t} \left[\left(\theta_{t+i}^m - 1 \right) \frac{p_{t+i}^m}{p_{t+i}^c} \left(\pi_{t,i,\{\gamma_{mk}\}_{k=1}^n} \right)^{1-\theta_{t+i}^m} \left(\frac{p_{t+i}^m}{p_t^m} \right)^{\theta_{t+i}^m - 1} m_{t+i} \right]}$$

where $\pi_{t,i,\{\gamma_{mk}\}_{k=1}^n} = \prod_{l=1}^{i} \left\{ \prod_{k=1}^{n} \left(1 + \pi_{t-k+l}^m \right)^{\gamma_{mk}} \right\} \left(1 + \overline{\pi} \right)^{i \left(1 - \sum_{j=1}^{n} \gamma_{mj} \right)}$.

Since the probability of price setting is independent across firms, each period a fraction $1-\epsilon^m$ re-optimizes and a fraction ϵ^m follows the non-optimal rule. Then, given the non-optimal price setting rule, the inflation of the imported good is:

$$\left(1 + \pi_t^m \right)^{1-\theta_t^m} = \left(1 - \epsilon^m \right) \left(\frac{p_t^{mopt}}{p_t^m} \right)^{1-\theta_t^m} \left(1 + \pi_t^m \right)^{1-\theta_t^m} + \epsilon^m \left[\prod_{k=1}^{n} \left(1 + \pi_{t-k}^m \right)^{\gamma_{mk}} \left(1 + \overline{\pi} \right)^{1 - \sum_{j=1}^{n} \gamma_{mj}} \right]^{1-\theta_t^m} \tag{32}$$

The PPP holds for imported goods at dock:

$$\frac{p_t^{mdock}}{p_t^c} = \frac{s_t p_t^{\star}}{p_t^c} \frac{\left(p_t^m \right)^{\star}}{p_t^{\star}} \tag{33}$$

where $\frac{\left(p_t^m \right)^{\star}}{p_t^{\star}}$ is the exogenous relative price of imports in foreign currency.

Finally we have the equilibrium condition for imports:

$$m_t = c_t^m + x_t^m \tag{34}$$

where x_t^m is the imported component of investment.

2.7. Investment

Total investment x_t is composed of imported investment x_t^m and domestically produced investment x_t^d:

$$x_t = \chi_t^x \left[(\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^d \right)^{\frac{\omega^x - 1}{\omega^x}} + (1-\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^m \right)^{\frac{\omega^x - 1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x - 1}} \tag{35}$$

The investment producer firms are in perfect competition and seek the maximization of the profits, solving the problem:

$$\begin{aligned} \max_{\{x_t^d, x_t^m\}} \quad & \frac{p_t^x}{p_t^c} x_t - \frac{p_t^{qd}}{p_t^c} x_t^d - \frac{p_t^m}{p_t^c} x_t^m + \Phi\left(k_t - k_{t-1}\right) \\ \text{s.t} \quad & x_t \leq \chi_t^x \left[(\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^d \right)^{\frac{\omega^x - 1}{\omega^x}} + (1-\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^m \right)^{\frac{\omega^x - 1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x - 1}} \end{aligned}$$

Note that the income of the producer of investment is the value of the investment good and the value paid for adjustment costs of capital. The adjustment services are costless to the investment firm.

The demands for x_t^d and x_t^m are given by

$$x_t^d = (\gamma^x) \left(\frac{p_t^{qd}/p_t^c}{\chi_t^x p_t^x / p_t^c} \right)^{-\omega^x} \frac{x_t}{\chi_t^x} \tag{36}$$

$$x_t^m = (1-\gamma^x) \left(\frac{p_t^{m}/p_t^c}{\chi_t^x p_t^x / p_t^c} \right)^{-\omega^x} \frac{x_t}{\chi_t^x} \tag{37}$$

It follows that the inflation rate is

$$(1+\pi_t^x) = (\chi_t^x)^{-1} \left[(\gamma^x) \left(1+\pi_t^q\right)^{1-\omega^x} \left(\frac{p_{t-1}^q}{p_{t-1}^x} \right)^{1-\omega^x} + (1-\gamma^x)\left(1+\pi_t^m\right)^{1-\omega^x} \left(\frac{p_{t-1}^m}{p_{t-1}^x} \right)^{1-\omega^x} \right]^{\frac{1}{1-\omega^x}} \tag{38}$$

2.8. Fiscal and monetary authorities

The monetary authority sets the nominal interest rate of the bank's debt market following a Taylor-type rule:

$$(1+i_t) = (1+i_{t-1})^{\rho_i} \left(\left(1+\bar{i}\right) \left(\frac{1+\pi_t}{1+\overline{\pi}} \right)^{\rho_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\rho_y} \right)^{1-\rho_i} \epsilon_t^i \tag{39}$$

and through this instrument controls the cost of funding of the banks.

The resorce constraint of the central bank is given by

$$\left(\frac{1+i^f_{t-1}}{1+\pi^\star_t}\right)\frac{s_t p^\star_t}{p^c_t}a^{cb}_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+\left(\frac{1+i^{cb}_{t-1}}{1+\pi^c_t}\right)b_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+rb_t=\frac{s_t p^\star_t}{p^c_t}a^{cb}_t+rb_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+b_t+\Pi^{cb}_t \tag{40}$$

where a^{cb}_t is the exogenous stock of foreign net assets held in foreign currency by the central bank[4] and Π^{cb}_t are transfers to the government.

The central bank receives foreign returns at a rate i^f_{t-1} for the net foreign assets a^{cb}_{t-1}, the returns of the loans to the private banks at a rate i^{cb}_{t-1} and the bank's reserves rb_t. This income is used to accumulate new net foreign assets, return the reserves to the banks, emit new loans b_t to the banks and as transfers to the government.

As an institutional restriction it is assumed that the central bank only can transfer to the government the net returns for the net foreign assets and the real return for the loans to the commercial banks:

$$\Pi^{cb}_t=\left(\frac{1+i^f_{t-1}}{1+\pi^\star_t}\frac{s_t p^\star_t}{p^c_t}a^{cb}_{t-1}+\left(\frac{1+i^{cb}_{t-1}}{1+\pi^c_t}-1\right)b_{t-1}\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}-\frac{s_t p^\star_t}{p^c_t}a^{cb}_t \tag{41}$$

The government obtains resources from the lump-sum taxes τ_t, net transfers from the central bank, the transaction costs[5] and foreign debt f^g_t, and uses them to finance the foreign debt interest payment at a rate i^g_t and an exogenous unproductive consumption g_t that follows the process

$$\ln(g_t)=(1-\rho_g)\ln(\overline{g})+\rho_g\ln(g_{t-1})+\epsilon^g_t$$

where $\overline{g}$ is the expected value of the government expenditure, $\rho_g\in(0,1)$ and ϵ^g is a white noise variable with null expected value and constant variance σ^2_g.

Thus the resourse constraint of the government is:

$$\frac{p^{qd}_t}{p^c_t}g_t=\left(c_t+\frac{p^x_t}{p^c_t}x_t\right)\vartheta(v_t)+\tau_t+\frac{s_t p^\star_t}{p^c_t}f^g_t-\left(\frac{1+i^g_{t-1}}{1+\pi^\star_t}\right)\frac{s_t p^\star_t}{p^c_t}f^g_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+\Pi^{cb}_t \tag{42}$$

For political reasons, the government seeks to maximize the discounted sum of resources that is able to transfer to households by choosing the optimal foreign debt path. Under the calibration of this model, the government charges taxes instead of making transfers to the households. Then the problem is equivalent to minimize the discounted sum of taxes.

Formally, the problem faced by the government is

$$\max_{\{f^g_t\}}\quad E_t\sum_{i=0}^{\infty}\Lambda_{t+i,t}\left(\left(c_t+\frac{p^x_t}{p^c_t}x_t\right)\vartheta(v_t)+\frac{s_t p^\star_t}{p^c_t}f^g_t-\left(\frac{1+i^g_{t-1}}{1+\pi^\star_t}\right)\frac{s_t p^\star_t}{p^c_t}f^g_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+\Pi^{cb}_t-\frac{p^{qd}_t}{p^c_t}g_t\right)$$

[4]The exogenous net foreign assets of the central bank a^{cb}_t can be, for instance, international reserves in foreign currency.

[5]It is assumed that the government receives the transaction costs in order to avoid wealth effects associated to those costs in the aggregate constraint.

and the first order condition is

$$\lambda_t \frac{s_t p_t^\star}{p_t} = \beta E_t \left(\frac{A_{t+1}}{A_t} \right)^{-\sigma} \lambda_{t+1} \frac{s_{t+1} p_{t+1}^\star}{p_{t+1}} \left(\frac{1 + i_t^g}{1 + \pi_{t+1}^\star} \right) \tag{43}$$

This equation implies $1+i_t^g = 1+i_t^f$. However, the foreign agent that finances the government is assumed to be different than the foreign agent that finances the banks.

2.9. Foreign Sector

The foreign sector provides resources to the domestic economy. In order to simplify the model, it is assumed that there are two lenders in the foreign economy: the financial system's lender and the government's lender. This allows to introduce the foreign debt of the government.

The interest rate charged by the financial system lender includes a risk-premium if the ratio of per-capita net foreign liabilities to GDP rises above some steady state level $\overline{fs}$. This is a way to induce stationarity in the model, following Schmitt-Grohé and Uribe (2003).

The net foreign liabilities of the financial system are given by the net foreign debt of the private banks and the negative of the net foreign assets of the central bank. Then $fs_t = f_t - a_t^{cb}$ are net foreign liabilities of the financial system in foreign currency. The lender's rule is

$$i_t^f = i_t^s + \psi^b \left[\exp\left(\frac{s_t p_t^\star}{p_t^c} \frac{fs_t}{gdp_t} - \overline{fs} \right) - 1 \right] \tag{44}$$

where i_t^s is the risk-free foreign interest rate and $\psi^b > 0$.

The government's lender follows a similar rule

$$i_t^g = i_t^s + \psi^g \left[\exp\left(\frac{s_t p_t^\star}{p_t^c} \frac{f_t^g}{gdp_t} - \overline{fg} \right) - 1 \right] \tag{45}$$

where $\overline{fg}$ is the ratio of government debt to GDP of steady state and $\psi^g > 0$.

The risk-free foreign interest rate is exogenous and subject to random shocks. It is assumed that i_t^s follows an autoregressive process

$$i_{t+1}^s = (1 - \rho_s) i^s + \rho_s i_t^s + \epsilon_{t+1}^s$$

where i^s is the expected value of the risk-free foreign interest rate and ϵ^s is white noise with null expected value and constant variance σ_s^2.

2.10. National accounts

The real GDP is the final domestic income of the households

$$gdp_t = w_t \left(1 - TD_t\right) TBP_t n_t + r_t^k k_t^s + z_t^h - \left(\frac{1 + i_{t-1}^z}{1 + \pi_t^c}\right) z_{t-1}^h - d_t^h + \left(\frac{1 + i_{t-1}^d}{1 + \pi_t^c}\right) d_{t-1}^h + \Pi_t^{dom}$$

the income from renting the productive factors, the net financial income and the domestic component of the profits.

With this definition the standard macroeconomic identity is obtained

$$gdp_t = c_t + \frac{p_t^x}{p_t^c} x_t + \frac{p_t^{qd}}{p_t^c} g_t + \frac{p_t^{qd}}{p_t^c} e_t - \frac{p_t^{mdock}}{p_t^c} m_t^{dock} \tag{46}$$

From the aggregate budget constraint and after a few substitutions it can be obtained the balance of payments

$$\begin{aligned}
\frac{p_t^{qd}}{p_t^c} e_t \quad = \quad & \frac{p_t^{mdock}}{p_t^c} m_t^{dock} - \frac{s_t p_t^\star}{p_t^c} f_t^g + \left(\frac{1 + i_{t-1}^g}{1 + \pi_t^\star}\right) \frac{s_t p_t^\star}{p_t^c} f_{t-1}^g \frac{A_{t-1}}{A_t} \frac{N_{t-1}}{N_t} \\
& - \left(\frac{1 + i_{t-1}^f}{1 + \pi_t^\star}\right) \frac{s_t p_t^\star}{p_t^c} a_{t-1}^{cb} \frac{A_{t-1}}{A_t} \frac{N_{t-1}}{N_t} + \frac{s_t p_t^\star}{p_t^c} a_t^{cb} \\
& - \frac{s_t p_t^\star}{p_t^c} tr_t - \frac{s_t p_t^\star}{p_t^c} f_t + \left(\frac{1 + i_{t-1}^f}{1 + \pi_t^\star}\right) \frac{s_t p_t^\star}{p_t^c} f_{t-1} \frac{A_{t-1}}{A_t} \frac{N_{t-1}}{N_t}
\end{aligned}$$

Then the model satisfies the national accounts, and this fact can be used for calibration purposes.

3. Calibration

A period in the model corresponds to one quarter. The model is calibrated to match key steady-state ratios of Colombia (see Table 1). The calibrated parameters and additional short-run parameters are summarized in Tables 2 and 3.

A standard value of $\sigma = 2$ is used as the constant relative risk aversion coefficient. This value is common in the literature and is used by Arias (2000). The average annual rate of growth of the total population in Colombia, according to DANE is 1.22%. The value of n is set to match this growth rate. The parameter a is calibrated to obtain an annual rate of growth of the labour-augmenting productivity of 1.5%.

Finally the value of the inverse of the Frisch elasticity, the parameter ς is set at 1.50.

Table 1: Steady state ratios

Variable	Interpretation	Value
gdp	Real GDP level	0.982
w	Real wage level	3.616
h	Labour supply's share of time	0.298
$\frac{c}{y}$	Ratio consumption to GDP	0.640
$\frac{p^x x}{p^c gdp}$	Ratio investment to GDP	0.215
$\frac{p^{qd} g}{p^c gdp}$	Ratio government expenditures to GDP	0.178
$\frac{p^{qd} e}{p^c gdp}$	Ratio exports to GDP	0.163
$\frac{p^{mdock} m^{dock}}{p^c gdp}$	Ratio dock imports to GDP	0.196
$\frac{p^m c^m}{p^c c}$	Ratio imported consumption to total cons.	0.200
$\frac{p^m x^m}{p^x x}$	Ratio imported investment to total investment	0.500
$\frac{sp^\star tr}{p^c gdp}$	Ratio net foreign transfers to GDP	0.035
$\frac{sp^\star (f^g + fs)}{p^c gdp}$	Ratio total net foreign liabilities to GDP	1.20
$\frac{sp^\star f^g}{p^c gdp}$	Ratio government's net foreign debt to GDP	0.653
$\frac{sp^\star a^{cb}}{p^c gdp}$	Ratio central bank's foreign assets to GDP	0.454
$\frac{sp^\star}{p^c}$	Real exchange rate	1.191
$\left(\frac{1+i^d}{1+\pi^c}\right)^4 - 1$	Annualized real bank rate on deposits (per cent)	2.01%
$\left(\frac{1+i^z}{1+\pi^c}\right)^4 - 1$	Annualized real bank rate on credit (per cent)	7.92%
$\frac{d}{gdp}$	Ratio of deposits to GDP	1.123
$\frac{z}{gdp}$	Ratio of credit to GDP	2.100
v	Velocity of the deposits	0.983
$\vartheta_0 v^{\vartheta_1}$	Transaction cost (per cent)	9.77%

3.1. Long-run parameters

The discount factor β is set at 0.999 in order to obtain an annualized foreign steady-state real interest rate of 3.42%, equal to the average of the implicit rate of Colombian foreign debt computed in Mahadeva and Parra (2008). This value is close to the standard value of 4% used by Mendoza (1991) and Claus (2005).

From the National Survey of Households (DANE), the average hours worked weekly in seven metropolitan areas for $2004:1-2007:1$ is 49.40. Because the week has 168 hours, the model is calibrated to produce a steady state value of $n = 0.294$, the share of time dedicated to the labour market. This implies a value of $\overline{\chi}^h = 128.78$.

The exogenous parameters of unemployment and labour participation are set to match the corresponding averages of the Colombian economy: $TD_t = \frac{D_t}{PEA_t}$ is the unemployment rate as reported by DANE and $TBP_t = \frac{PEA_t}{N_t}$ is the gross participation rate in labour market (DANE), where D_t is the number of unemployed people, PEA_t is the economically active population and N_t is total population. The average values during the period $2006:1-2007:4$ of these rates are $TD = 0.134$ and $TBP = 0.537$.

The depreciation parameter b is set at a value of 0.0228 to obtain a steady-state depreciation rate of 0.014, which is close to the value of 0.013 proposed by Mahadeva and Parra (2008). This depreciation rate implies a steady-state ratio of investment to GDP of 0.215 (DANE), equal to the average ratio of gross investment and durable goods consumption to GDP in the period $2006:1-2007:4$.

The scale factor of the foreign demand for domestic exports $c^{\star}$ is set at 0.1078 to obtain a steady-state real exchange rate value of 1.191. This is the average value of the ratio $\frac{sp^{\star}}{p^{c}}$ in the period $2006:1-2007:4$, where $p^{\star}$ is a world price index taken from Mahadeva and Parra (2008).

The relative price $\frac{(p^{m})^{\star}}{p^{\star}}$ is set at 0.843 to match its average value in the period $2006:1-2007:4$.

The steady-state compulsory reserve requirement τ^{d} is set at 0.062, equal to the average ratio Reserves / Liabilities subject to deposit requirements and reserves (Banco de la República) for $2006:1-2007:4$.

From the working capital constraint $d^{f} = \varphi w\,(1-TD)\,TBPn$, the average of the ratio of total deposits of the private sector (private societies, financial accounts - Banco de la República) to the wage bill (National Accounts - DANE) is $\varphi = 0.4801$ for $1996-2004$.

Following Claus (2005) the value for ν is set at 2. The average annualized real lending rate (Weighted lending rates - Banco de la República) is 7.92% and the average annualized real deposit rate (DTF - Banco de la República) is 2.01% and it is assumed that these values hold in the steady state. The parameters ν_{d} and ν_{z} are calibrated at $\nu_{d} = 7.315\times10^{-5}$ and $\nu_{z} = 1.443\times10^{-4}$ to obtain these values of the real bank interest rates.

The total productivity scale parameter $\overline{\chi}^{qs}$ is calibrated at 0.493 to obtain the steady-state level of the GDP of 0.982, equal to the average of the level of GDP expressed in units of effective standarized labour (Mahadeva and Parra (2008)).

The exogenous public expenditure parameter g is calibrated to obtain a steady-state ratio of government expenditure to GDP of 0.183, equal to the average of that ratio in the period $2006:1-2007:4$.

The average value of the ratio of public foreign debt to GDP is 0.653 (foreign liablilites of the central national government, medium-run and long-run, Ministerio de Hacienda) and the parameter $\overline{fg}$ is set to match that value. Following Mahadeva and Parra (2008) the value of total foreign net assets to GDP is set to 1.20, and this implies a value of 0.547 for the parameter $\overline{fs}$.

The average ratio of net foreign assets of the central bank to GDP (net foreign assets, monetary sectorization - Banco de la República) is 0.454, and the parameter $\overline{a}^{cb}$ is set to match this ratio.

The parameters γ and γ^{x} are set in order to obtain the steady-state ratios of imported consumption to total consumption (0.20) and imported investment to total investment (0.50).

These ratios imply that the ratio of dock imports to GDP is 0.196 and the ratio of exports to GDP is 0.163.

Finally the parameters ϑ_0 and ϑ_1 are calibrated to match the value of the average real wage rate $w = 3.616$ and the value of the average ratio of deposits which generate costs to the banks to GDP of 1.123.

Table 2: Calibrated parameters

Parameter	Value	Parameter	Value
β	0.999	b	0.023
a	0.004	$\overline{\delta}$	0.003
n	0.003	ν_d	7.315×10^{-5}
$1+\overline{\pi}$	$(1.03)^{\frac{1}{4}}$	ν_z	1.443×10^{-4}
$1+\overline{\pi}^{\star}$	$(1.0221)^{\frac{1}{4}}$	τ^d	0.062
γ	0.791	γ^x	0.491
$\frac{(p^m)^{\star}}{p^{\star}}$	0.843	ϑ_0	0.098
tbp	0.537	ϑ_1	0.036
td	0.134	$c^{\star}$	0.108
φ	0.48	g	0.183
$\overline{\chi}^{qs}$	0.493	$\overline{\chi}^h$	128.78
$\overline{a}^{cb}$	0.375	$\overline{fg}$	0.653

3.2. Short-run and additional parameters

The values of several short-run parameters are taken from González, Mahadeva, Prada, and Rodríguez (2008). The short-run calibration is summarized in Table 3.

In order to obtain a quadratic endogenous depreciation cost function the parameter Υ is set at 1.

Following Arango, Gracia, Hernández, and Ramírez (1998) the mark-up on production marginal cost is set at 25%, and this implies a value of $\theta^q = 5$. The same mark-up is assumed for the wage setting process. Finally the parameter θ^m is set at 6, and this implies a mark-up of imports prices of 20%.

The Calvo parameters that measure the degree of price stickiness are selected in such a way that, on average, the final good price is adjusted once each year ($\epsilon^q = 0.75$), the imported good price is adjusted once each two quarters ($\epsilon^m = 0.50$) and the wage rate is adjusted once each four months ($\epsilon^w = 0.25$). This configuration is used in González, Mahadeva, Prada, and Rodríguez (2008).

The parameter α is calibrated to get the minimum possible ratio of credit to GDP. The value obtained is 0.931.

When the model presented in the previous section is considered without financial intermediation costs (that is, $\xi_t \eta(\cdot) \equiv 0$), the parameters θ^d and θ^z provide a measure of the mark-down and mark-up of the deposits and the lending rate on the repo rate. Therefore these parameters can be calibrated in the absence of financial costs in order to match the values of the lending and deposit rates. In that case $\theta^d = 343.57$ and $\theta^z = 94.45$. When the model is considered with financial costs it is assumed that there is no monopolistic competition, because this assumption is not needed to explain the spread between interest rates. Then $\theta^d \to \infty$ and $\theta^z \to \infty$.

Table 3: Short-run parameters

Parameter	Value	Parameter	Value
α	0.931	α_v	0.525
ρ	0.75	ρ_v	0.838
ω	1.247	ω^x	1.158
σ	2	ς	1.5
ϵ^q	0.75	θ^q	5
ϵ^m	0.50	θ^m	6
ϵ^w	0.25	θ^w	5
ψ^X	8	Υ	1
ϕ	0	ν	2
ψ^b	0.001	ψ^g	0.001
μ	2	ρ_i	0.40
ρ_π	1.50	ρ_y	0.50

4. Transmission mechanisms

The dynamics of the log-linearized model are studied using impulse responses. In order to assess the role of the costly financial intermediation on transmission of shocks in the economy it is considered the standard repo rate shock, the reserve-requirement rate shock and a pure financial distress shock.

Two alternate configurations of the model presented in section 2 are considered. The "cost" configuration includes the fact that the financial intermediation is a costly process. Therefore in this model it is assumed the presence of the real cost bank function of Edwards and Végh (1997). An alternate "no-cost" configuration, where this real costs are missing, is also considered. In the "no-cost" case monopolistic competition in the financial sector is included in order to account for the observed mark-down of deposit rates and mark-up of lending rates with respect to repo rates. This is done to assess the role of the structure of the financial firms (banks) in the transmission mechanisms in this economy.

For the "no-cost" configuration some first order conditions and budget constraints are altered. Since there are no real cost of the financial system, the production is only used for domestic consumption, domestic investment, public expenditure and exports, changing equation (20):

$$q_t^d = c_t^d + x_t^d + g_t + e_t$$

The profits of banks (equation (22)) change in the obvious way, and the first order conditions of banks (equations (26) and (28)) are simplified to

$$\begin{aligned}
\beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma} \lambda_{t+1} \left(\left(\frac{1+\theta^d}{\theta^d}\right)\left(\frac{1+i_t^d}{1+\pi_{t+1}^c}\right) - \tau_t^d\right) &= \lambda_t \left(1-\tau_t^d\right) \\
\beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma} \lambda_{t+1} \left(\frac{1+i_t^z}{1+\pi_{t+1}^c}\right) &= \lambda_t \frac{\theta^z}{\theta^z - 1}
\end{aligned}$$

4.1. Repo rate shock

A rise in the repo rate increases the funding cost of the banks. By the nature of the shock the financial intermediation as a whole is reduced and the economy has less financial services. The banks search for more resources and set a higher deposit rate. At the same time, they reduce the credit and charge a higher lending rate. Hence, the spread increases. This optimal response of the financial system does not depend on the form of the real banking costs.

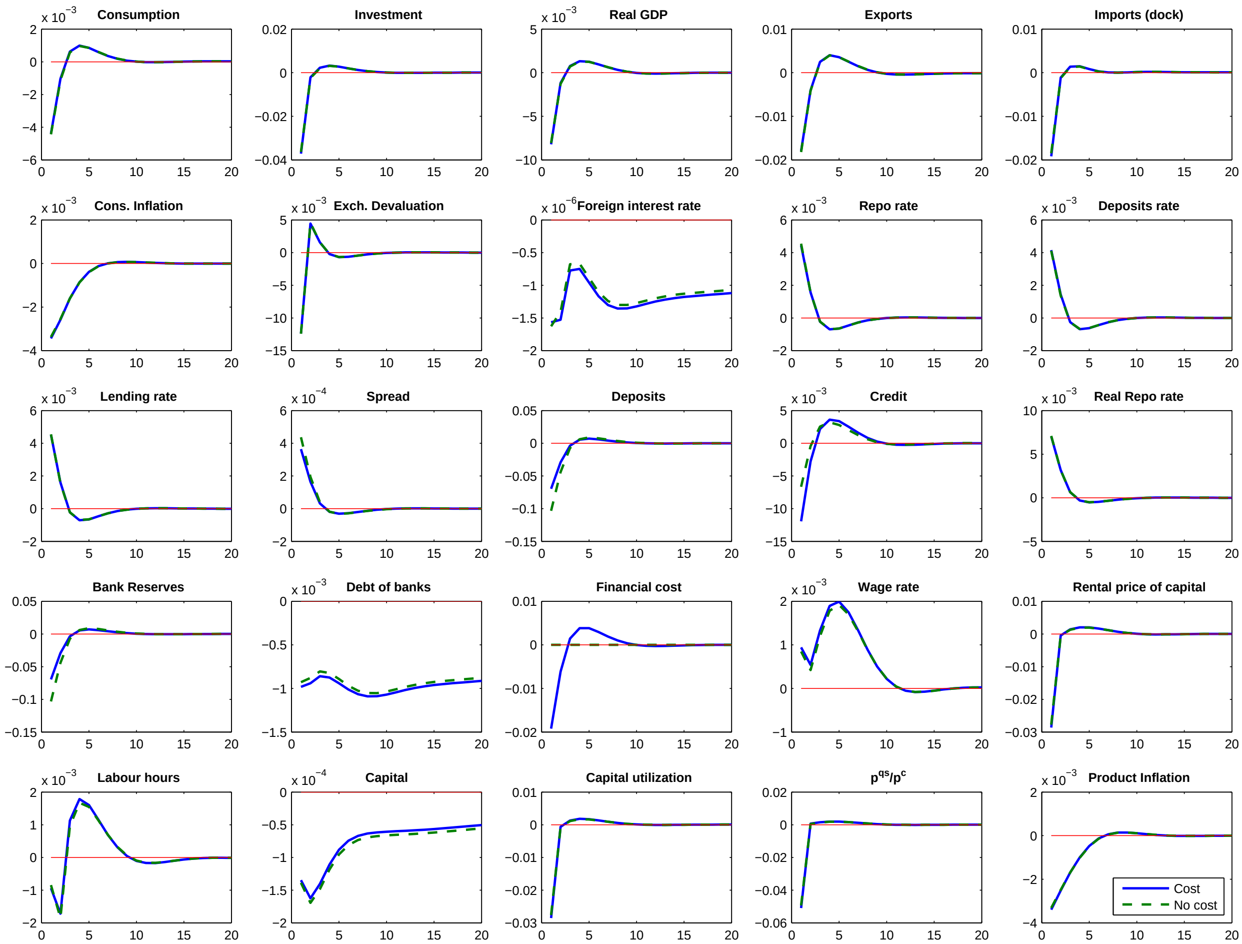
Since there are less deposits the transaction costs are higher, and the rise in deposit rate incentives the households to save. This reduces aggregate demand. The fall in consumption and the rise in the transaction costs makes the household to supply less labour. On the other side, less credit means higher production costs, and the fall in aggregate demand is accompanied by a fall in the demand for productive factors. The rental price of capital falls, and this generates a reduction in the real marginal cost and in the inflation rate.

Then the monetary policy conducted through the repo rate has the standard effects predicted by the new-keynesian framework: a rise in the repo rate induces higher deposits and lending rates, a reduction in financial intermediation, a fall in inflation and a drop of the aggregate demand.

The response of the economy to the repo shock is invariant to the alternate structures of the financial system considered in the model. The presence or absence of the real banking costs has no significative effect on the qualititative/quantitative reactions of the endogenous variables of the economy. This is because the effects on lending and deposits are similar in both scenarios.

The effects of a shock of 1% in the Taylor rule for the repo interest rate are summarized in the Figure 2. The quantitative responses of aggregate demand, GDP and inflation are significative, and according to the model this instrument is an effective policy tool.

Figure 2: Repo rate shock



4.2. Compulsory reserve-requirement rate shock

An exogenous increase in the reserve-requirement rate acts like a tax on the deposits received by the banks. This makes more expensive the bank's funding through deposits and the financial intermediatior optimally chooses to demand less deposits, thus lowering the deposit rate. In the case of a costly financial intermediation à la Edwards and Végh (1997), because credit and deposits are Edgeworth complements, the banks also reduce the quantity of credit. In the absence of these intermediation costs, there are no incentives to reduce the quantity of credit and therefore z_t does not fall. This mechanism also implies that without costs the deposits fall more because of the pure balance-sheet effect. Summarizing: with real costs the supply of credit falls more and the deposits fall less, and this induces higher lending rate and higher deposit rate that in the case of no real costs[6]. In both models the spread raises.

[6]Note that in the economy with no real costs the lending rate equals a constant mark-up over the repo interest rate.

The fall in deposits increases the transaction costs, generates incentives to reduce the household absorption and affects the capacity of payments of the firms because of the working capital constraint. To face this situation the monetary authority reacts by lowering the repo interest rate. In the model without banking costs the lending rate behaves exactly as the repo rate. But when there are real banking costs, the lending rate reacts less. Because of the real banking costs the monetary authority has to make an additional effort to impulse the economy: the repo rate falls more than in the economy without costs.

Now the reaction of the economy depends simultaneously on two financial factors: higher reserve rate and lower repo interest rate. In the economy where the lending interest rate falls more a higher expansive effect is introduced: when there are no real banking costs all real variables are stabilized in such a way that the contractionary effect is almost eliminated. The inflation rate increases in a very small amount.

When there are real financial costs the consumption does not fall because deposits fall less than without costs. This affects the labour market increasing the labour supply. The behaviour of the lending rate generates a small substitution effect towards labour and capital, increasing the wage rate and the capital utilization. But the contractionary effects are still present and the inflation rate falls in this case.

The results for a shock that increases the reserve-requirement rate from 6% to 10% are summarized in Figure 3. Note that the quantitative effects on real variables and inflation due to this shock are small.

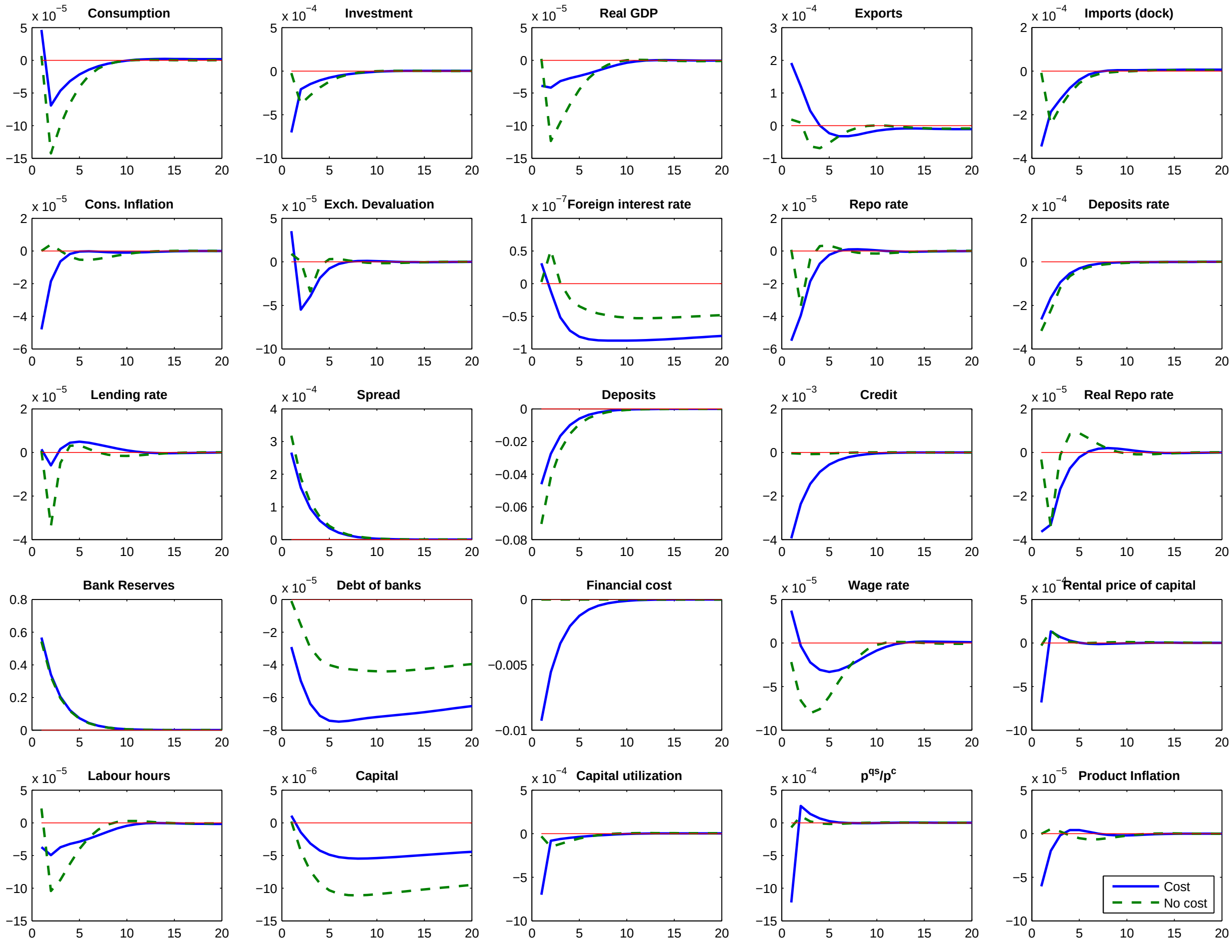
The Taylor rule followed by the central bank generates endogenous responses of the repo rate to exogenous changes in the reserve-requirement rate. In order to understand the pure effect of the shock, an impulse response keeping fixed the repo interest rate is computed. The results are summarized in Figure 4.

The shock makes the deposits less productive for banks, and they respond by lowering their demand for deposits. The deposit interest rate falls and this generates incentives toward consumption and investment. When the economy has real cost of financial intermediation this effect dominates and the consumption and investment slightly rises. This is because the fall in deposits is not as hard as in the case of the economy without banking costs, since credit and deposits are Edgeworth complements and credit is still needed for production. In the absence of costs, however, deposits falls more, and this increases the transaction costs, preventing a recovery in consumption and investment.

Since the firms are subject to a working capital constraint, a strong contraction in deposits also generates a contraction in the demand of labour, because the lack of resources to finance the wage bill. This is specially important in the model without real financial costs, where the wage rate and the labour hours decrease.

If the credit supplied by the banks falls (i.e., when there are banking costs) then the financial productive factor is scarcer and the firms substitute toward value added. This slightly rises the demand for capital and labour, and the prices of these factors increase. But if the lending

Figure 3: Reserve-requirement rate shock

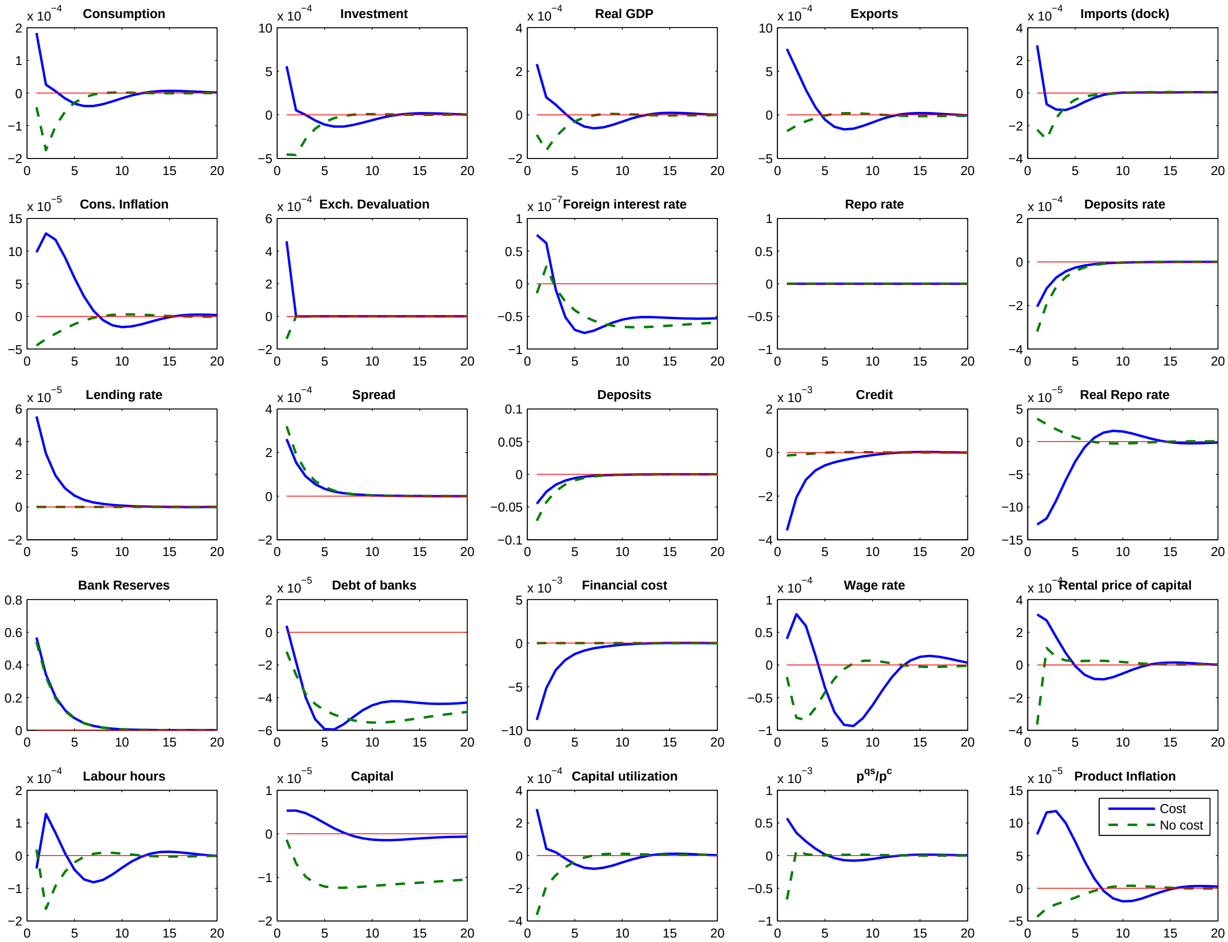


rate do not change and the credit is not contracted (i.e., when there are no banking costs) then there is not an increased demand for the productive factors owned by the households. This implies that the income of the household is not increased in the case of no banking costs, because the demand for productive factors do not grow as much as in the case of real banking costs. This, together with high transaction costs, generates a fall in the aggregate demand.

With financial costs the economy face inflationary pressures, reflected in a lower real repo rate that makes less expensive the financial intermediation process and generates an exchange rate depreciation. This further stimulates aggregate demand and generates inflation in consumption.

Without financial costs the economy is slightly contracted because the effect of a contraction in deposits on the balance-sheet of all the agents. The lack of banking costs reduces the capacity of the financial system to absorb the negative shock and the contractionary effects of the reserve-requirement rate are transmmited through the economy. The prices of productive factors are stable or falling (in the case of rental price of capital) and this lowers the inflation rate. Then the real repo rate rises and this induces an exchange rate appreciation, disminishing exports.

Figure 4: Reserve-requirement rate shock: fixed nominal repo rate



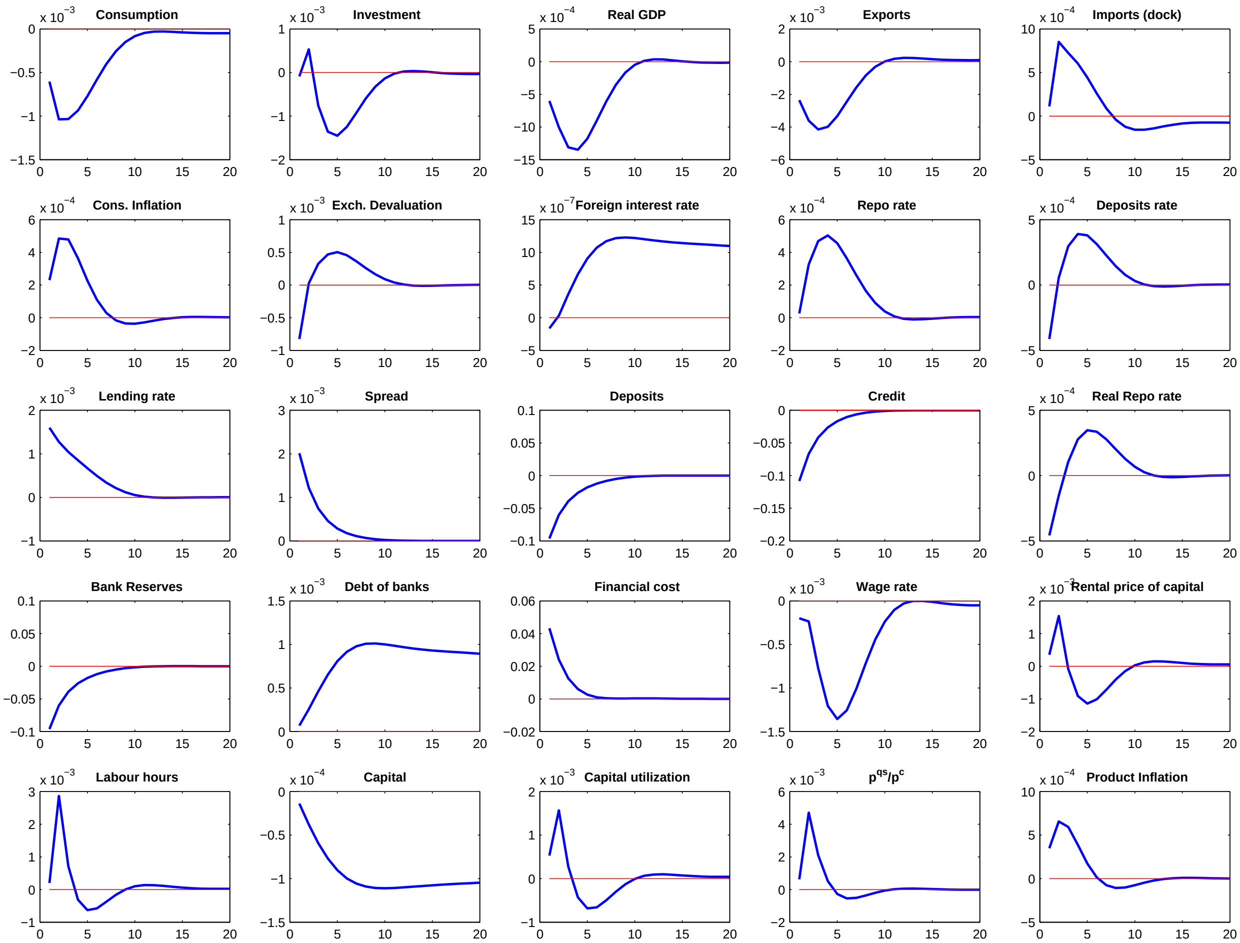
Therefore, the structure of the financial system and the endogenous reaction of other policy instruments are key factors in the response of the economy to a reserve-requirement rate shock.

Finally, in both scenarios the effects of the shock on the aggregate economic activity are rather small. An increase of near 4% in the reserve-requirement rate at most generates an effect of 0.01268% on quarterly consumption inflation and at most a change of 0.02324% in real GDP. This is a negligible effect on real variables and prices. However this shock introduces distress in the financial system: deposits may fall 7.1% due to the shock.

4.3. Banking cost shock

The scale factor of the banking cost can be associated with an efficiency factor: positive (negative) shocks to cost can be seen as negative (positive) shocks to efficiency, given by changes in regulation, technology of the financial system, management problems, excessive risk exposition, macroeconomic instability, intermediation capital loss etc[7].

Figure 5: Banking cost shock



The specific shock to banking cost affects the aggregate activity of the economy. A positive shock in banking cost makes more expensive the financial services. The lending rate increases and the deposit rate decreases. Then the economy has a larger spread because a contraction in the supply of funds. In order to finance their activities, the banks increase the net foreign debt.

The shock causes a reduction in the financial intermediation: the deposits of households and firms and the credit fall. Because the working capital constraint, the reduction in deposits make difficult to pay for the wage bill, and because the firms use credit as input, the final output falls. This reduces the wage rate and the demand for all productive factors falls.

The reduction in the deposits rises the transaction costs, and this makes the consumption

[7] The approach of intermediation capital, introduced by Bernanke (1983), says that a reduction in the information shared by clients to the banks reduces the total productivity of the financial system.

and investment fall. The lower deposit rates generate a negative income effect on households, and the agents choose to work more time and to use more intensely the available capital stock. The rental price of capital increases because the reduction of capital relative supply.

Since the shock affects output by the supply side, it generates inflation through an increase in the real marginal cost, due mainly to the higher lending rate and rental price of capital. The monetary authority responds increasing the repo rate (following the standard Taylor rule) and this generates appreciation in the exchange rate.

This pure financial distress shock generates a contraction of the economy and an increase in inflation. However all the quantitative effects are rather small. The figure 5 summarizes the responses of the economy to a positive shock of 15% in the scale factor of banking costs. This huge shock makes credit and deposit fall in near 10%. But the effect on the real economy is much smaller. The greatest effect on real GDP is of -0.135%. The highest inflation rate is 0.048% above the steady state value (an increase in annual inflation from 3% to 3.2%).

5. Concluding remarks

The new-keynesian model for a small open economy is extended with a simple financial system based in Hamann and Oviedo (2006). The presence of the financial intermediation naturally allows the introduction of standard monetary policy instruments: the repo interest rate and the rate of compulsory requirement of reserves. The model is calibrated to match key steady-state ratios of Colombia and is used to evaluate the alternative policy instruments.

The monetary policy conducted through the repo rate has the standard effects predicted by the new-keynesian framework: a rise in the repo rate induces higher deposits and lending rates, a fall in inflation and a drop of the aggregate demand, with significative quantitative effects, and this result is invariant under different structures of the financial system.

It is shown that a positive shock in the requirement-reserve rate may have different effects on the aggregate economy depending on both the structure of the financial firm and the reaction of the repo interest rates. If the monetary authority has uncertainty about the real structure of the financial system and on the role of other policy instruments, then there is no reasonable way to predict the response of the economy to changes in the reserve rate. The model also predicts little quantitative effects on the inflation rates and on aggregate demand. Therefore the reserve rate policy instrument appears to be uneffective and unreliable.

This result appears to be in line with the findings of previous literature. For instance, in a static general equilibrium model, Brock (1989) shows that the reserve-requirement ratio may be used to maximize the seignorage, by increasing the monetary base and the inflation rate. In the case of Colombia (and forty more countries), there existed a positive correlation between reserve ratio and inflation rates during 1960-1984. Brock (1989) argues that this could suggest that the reserve-requirement ratio is used as an inflationary tax and not as a contractive monetary policy. Romer (1985) arrives to similar conclusions, and in a static framework shows that changing the reserve-requirement ratio does not affect the inflation rate, but adjusts market rates and banks assets, being this requirement rate policy uneffective for the control of the inflation. Finally, Edwards and Végh (1997) show that a counter-cyclical reserve rate may help to isolate the economy from foreing shocks, but Villar and Salamanca (2005) argue that for the Colombian economy this policy may have not had the desired effects.

Then it is not clear the role of the reserve-requirement ratio as policy instrument.

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A. Model's equations

Consumption

$$c_t = \left[\gamma^{\frac{1}{\omega}} \left(c_t^d\right)^{\frac{\omega-1}{\omega}} + (1-\gamma)^{\frac{1}{\omega}} \left(c_t^m\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

$$c_t^d = \gamma \left(\frac{p_t^{qd}}{p_t^c}\right)^{-\omega} c_t$$

$$c_t^m = (1-\gamma) \left(\frac{p_t^m}{p_t^c}\right)^{-\omega} c_t$$

$$(1+\pi_t^c) = \left[\gamma \left(1+\pi_t^{qd}\right)^{1-\omega} \left(\frac{p_{t-1}^{qd}}{p_{t-1}^c}\right)^{1-\omega} + (1-\gamma)\left(1+\pi_t^m\right)^{1-\omega} \left(\frac{p_{t-1}^m}{p_{t-1}^c}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

Household's constraints

$$v_t = \frac{c_t + \frac{p_t^x}{p_t^c} x_t}{d_{t-1}^h} \frac{A_t N_t}{A_{t-1} N_{t-1}}$$

$$\vartheta(v_t) = \vartheta_0 v_t^{\vartheta_1}$$

$$\Phi(k_t - k_{t-1}) = \frac{\psi^X}{2} (k_t - k_{t-1})^2$$

$$k_t(h) = x_t(h) + (1 - \delta(u_t(h)))\, k_{t-1}(h) \frac{A_{t-1} N_{t-1}}{A_t N_t}$$

$$\delta(u_t(h)) = \overline{\delta} + \frac{b}{1+\Upsilon} (u_t(h))^{1+\Upsilon}$$

$$\begin{aligned} c_t + \frac{p_t^x}{p_t^c} x_t + \Phi(k_t - k_{t-1}) \qquad & \qquad r_t^k u_t k_{t-1} \frac{A_{t-1} N_{t-1}}{A_t N_t} + w_t (1 - TD_t)\, TBP_t n_t \\ + \tau_t + d_t^h \quad = & \quad + \frac{s_t p_t^{\star}}{p_t^c} tr_t + \Pi_t + \left(\frac{1+i_{t-1}^d}{1+\pi_t^c}\right) d_{t-1}^h \frac{A_{t-1} N_{t-1}}{A_t N_t} \\ + \left(c_t + \frac{p_t^x}{p_t^c} x_t\right) \vartheta(v_t) \qquad & \qquad + z_t^h - \left(\frac{1+i_{t-1}^z}{1+\pi_t^c}\right) z_{t-1}^h \frac{A_{t-1} N_{t-1}}{A_t N_t} \end{aligned}$$

Household's first order conditions

$$
\begin{aligned}
\lambda_t \left(1 + (1+\vartheta_1)\vartheta_0 (v_t)^{\vartheta_1}\right) &= \chi_t^u \left[c_t - \phi c_{t-1}\right]^{-\sigma} \\
&\quad -\beta\phi E_t A_{t+1}^u \frac{N_{t+1}}{N_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma}\left[c_{t+1} - \phi c_t\right]^{-\sigma} \\
r_t^k &= \frac{\gamma_t}{\lambda_t}\left(b\,(u_t)^{\Upsilon}\right) \\
\gamma_t &= \lambda_t \frac{p_t^x}{p_t^c}\left(1 + (1+\vartheta_1)\vartheta_0 (v_t)^{\vartheta_1}\right) \\
\gamma_t &= \beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1} r_{t+1}^k u_{t+1} \\
&\quad +\beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\psi^X (k_{t+1} - k_t)\frac{A_{t+1}}{A_t}\frac{N_{t+1}}{N_t} \\
&\quad +\beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\gamma_{t+1}\left(1 - \delta(u_{t+1})\right) \\
&\quad -\lambda_t \left(\psi^X (k_t - k_{t-1})\right) \\
\lambda_t &= \beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\frac{1+i_t^d}{1+\pi_{t+1}^c}\right) \\
&\quad +\beta E_t \left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\vartheta_0\vartheta_1 (v_{t+1})^{1+\vartheta_1}
\end{aligned}
$$

where $\lambda_t(h)$ and $\gamma_t(h)$ are the Lagrange multipliers associated to the budget constraint and to the capital accumulation constraint of household h.

$$
z_t^h = 0
$$

Wage setting

$$
\begin{aligned}
f_t^1 &= w_t^{opt}\lambda_t(\theta^w - 1)\left((1 - TD_t)\,TBP_t n_t^d \left(\frac{w_t^{opt}}{w_t}\right)^{-\theta^w}\right) + E_t(\beta\epsilon^w)\frac{N_{t+1}}{N_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma}\left(\frac{w_t^{opt}}{w_{t+1}^{opt}}\frac{1+\pi_t^c}{1+\pi_{t+1}^c}\right)^{1-\theta^w} f_{t+1}^1 \\
f_t^2 &= z_t^h \left((1 - TD_t)\,TBP_t n_t^d \left(\frac{w_t^{opt}}{w_t}\right)^{-\theta^w}\right)^{1+\varsigma}\theta^w + E_t(\beta\epsilon^w)\frac{N_{t+1}}{N_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma}\left(\frac{w_t^{opt}}{w_{t+1}^{opt}}\frac{1+\pi_t^c}{1+\pi_{t+1}^c}\right)^{-\theta^w(1+\varsigma)} f_{t+1}^2
\end{aligned}
$$

$$
f_1 = f_2
$$

$$
w_t = \left[\epsilon^w \left(w_{t-1}\left(\frac{1+\pi_{t-1}^c}{1+\pi_t^c}\right)\right)^{1-\theta_t^w} + (1-\epsilon^w)\left(w_t^{opt}\right)^{1-\theta_t^w}\right]^{\frac{1}{1-\theta_t^w}}
$$

Firms

$$q_t^s = \chi_t^{qs}\left[\alpha^{\frac{1}{\rho}}\left(v_t^q\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}}\chi_t^z\left(z_{t-1}^f\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

$$v_t^q = \chi_t^v\left[\alpha_v^{\frac{1}{\rho_v}}\left(k_t^s\right)^{\frac{\rho_v-1}{\rho_v}} + (1-\alpha_v)^{\frac{1}{\rho_v}}\left(A_t\left(1-TD_t\right)TBP_tn_t\right)^{\frac{\rho_v-1}{\rho_v}}\right]^{\frac{\rho_v}{\rho_v-1}}$$

$$\Pi_t^{qs} = \frac{p_t^{qs}}{p_t^c}q_t^s + z_t^f + \left(\frac{1+i_{t-1}^d}{1+\pi_t^c}\right)d_{t-1}^f\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}$$

$$-w_t\left(1-TD_t\right)TBP_tn_t - r_t^kk_t^s - \left(\frac{1+i_{t-1}^z}{1+\pi_t^c}\right)z_{t-1}^f\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} - d_t^f$$

$$r_t^k = \frac{p_t^{qs}}{p_t^c}\chi_t^{qs}\chi_t^v\left(\frac{\alpha q_t^s}{\chi_t^qv_t^q}\right)^{\frac{1}{\rho}}\left(\frac{\alpha_vv_t^q}{\chi_t^vk_t^s}\right)^{\frac{1}{\rho_v}}$$

$$w_t\left(1+\varphi\Gamma_t\right) = \frac{p_t^{qs}}{p_t^c}\chi_t^{qs}\chi_t^v\left(\frac{\alpha q_t^s}{\chi_t^qv_t^q}\right)^{\frac{1}{\rho}}\left(\frac{(1-\alpha_v)\,v_t^q}{\chi_t^v\left(1-TD_t\right)TBP_tn_t}\right)^{\frac{1}{\rho_v}}$$

$$\lambda_t = \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\frac{1+i_t^z}{1+\pi_{t+1}^c}\right)$$

$$-\beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\frac{p_{t+1}^{qs}}{p_{t+1}^c}\chi_{t+1}^q\chi_{t+1}^z\left(\frac{(1-\alpha)\,q_{t+1}^s}{\chi_{t+1}^qz_t^f}\frac{A_{t+1}}{A_t}\frac{N_{t+1}}{N_t}\right)^{\frac{1}{\rho}}$$

$$\lambda_t = \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\left(\frac{1+i_t^d}{1+\pi_{t+1}^c}\right)+\Gamma_{t+1}\right)$$

where Γ_t is the Lagrange multiplier associated to the working capital constraint. It is assumed that $\Gamma_t > 0$, so that the constraint is always binding

$$d_{t-1}^f\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} = \varphi w_t\left(1-TD_t\right)TBP_tn_t$$

$$k_t^s = k_{t-1}u_t$$

$$q_t^d = c_t^d + x_t^d + g_t + e_t + \xi_t\eta\left(z_t, d_t\right)$$

$$\frac{p_t^{qd}}{p_t^c} = \frac{1+\pi_t^{qd}}{1+\pi_t^c}\frac{p_{t-1}^{qd}}{p_{t-1}^c}$$

Price setting

$$E_t\Theta_t^q = \theta_t^q\frac{p_t^{qs}}{p_t^c}q_t^d\left(\chi_t^{qd}\right)^{\theta_t^q} + E_t\Lambda_{t+1,t}\epsilon^q\left(1+\pi_{t+1}^{qd}\right)^{\theta_{t+1}^{qd}}\left(\left\{\prod_{k=1}^n\left(1+\pi_{t-k+1}^{qd}\right)^{\gamma_{qk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{m=1}^n\gamma_{qm}\right)}\right)^{-\theta_{t+1}^q}\Theta_{t+1}^q$$

$$E_t\Psi_t^q = (\theta_t^q - 1)\frac{p_t^{qd}}{p_t^c} q_t^d \left(\chi_t^{qd}\right)^{\theta_t^q} + E_t\Lambda_{t+1,t}\epsilon^q \left(1+\pi_{t+1}^q\right)^{\theta_{t+1}^q - 1}\left(\left\{\prod_{k=1}^n \left(1+\pi_{t-k+1}^{qd}\right)^{\gamma_{qk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{j=1}^n \gamma_{qm}\right)}\right)^{1-\theta_{t+1}^q} \Psi_{t+1}^q$$

$$\frac{p_t^{qopt}}{p_t^{qd}} = \frac{E_t\Theta_t^q}{E_t\Psi_t^q}$$

$$\left(1+\pi_t^{qd}\right)^{1-\theta_t^q} = (1-\epsilon^q)\left(\frac{p_t^{qopt}}{p_t^{qd}}\right)^{1-\theta_t^q}\left(1+\pi_t^{qd}\right)^{1-\theta_t^q} + \epsilon^q\left[\prod_{k=1}^n \left(1+\pi_{t-k}^{qd}\right)^{\gamma_{qk}}(1+\overline{\pi})^{1-\sum_{m=1}^n \gamma_{qm}}\right]^{1-\theta_t^q}$$

$$q_t^s = \int_0^1 q_t(j)\,dj = q_t^d\left(\chi_t^{qd}\right)^{-\theta_t^q} v_t^{qd}$$

$$v_t^{qd} = \epsilon^q\left(\left(1+\pi_t^{qd}\right)^{-1}\prod_{k=1}^n \left(1+\pi_{t-k}^{qd}\right)^{\gamma_{qk}}(1+\overline{\pi})^{1-\sum_{m=1}^n \gamma_{qm}}\right)^{-\theta_t^q} v_{t-1}^{qd} + (1-\epsilon^q)\left(\frac{p_t^{qopt}}{p_t^{qd}}\right)^{-\theta_t^q}$$

$$\Pi_t^{qd} = \int_0^1 \Pi_t^{qd}(j)\,dj = \frac{p_t^{qd}}{p_t^c} q_t^d - \frac{p_t^{qs}}{p_t^c} q_t^s$$

Financial system

$$rb_t = \tau_t^d d_t$$

$$\begin{aligned}
\left(\frac{1+i_{t-1}^z}{1+\pi_t^c}\right) z_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} \quad & \quad \left(\frac{1+i_{t-1}^d}{1+\pi_t^c}\right) d_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} + z_t \\
+\frac{s_t p_t^\star}{p_t^c} f_t + d_t + b_t \quad = & \quad +\left(\frac{1+i_{t-1}^f}{1+\pi_t^\star}\right)\frac{s_t p_t^\star}{p_t^c} f_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} + \Pi_t^b \\
+rb_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} - \frac{p_t^{qd}}{p_t^c}\xi_t\eta(z_t, d_t) \quad & \quad +\left(\frac{1+i_{t-1}^{cb}}{1+\pi_t^c}\right) b_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t} + rb_t
\end{aligned}$$

$$\eta\left(z_t\left(j^b\right), d_t\left(j^b\right)\right) = \left[\nu_z\left(z_t\left(j^b\right)\right)^\nu + \nu_d\left(d_t\left(j^b\right)\right)^\nu\right]^{\frac{1}{\nu}}$$

$$\begin{aligned}
\lambda_t &= \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\frac{1+i_t^{cb}}{1+\pi_{t+1}^c}\right) \\
\lambda_t\frac{s_t p_t^\star}{p_t^c} &= \beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\frac{s_{t+1}p_{t+1}^\star}{p_{t+1}^c}\left(\frac{1+i_t^f}{1+\pi_{t+1}^\star}\right)
\end{aligned}$$

$$\beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\left(\frac{1+\theta^d}{\theta^d}\right)\left(\frac{1+i_t^d}{1+\pi_{t+1}^c}\right) - \tau_t^d\right) = \lambda_t\left(\left(1-\tau_t^d\right) - \frac{p_t^{qd}}{p_t^c}\xi_t\nu_d\left[\nu_z(z_t)^\nu + \nu_d(d_t)^\nu\right]^{\frac{1}{\nu}-1}(d_t)^{\nu-1}\right)$$

$$\beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\left(\frac{1+i_t^z}{1+\pi_{t+1}^c}\right) = \lambda_t\frac{\theta^z}{\theta^z-1}\left(1+\frac{p_t^{qd}}{p_t^c}\xi_t\nu_z\left[\nu_z(z_t)^\nu + \nu_d(d_t)^\nu\right]^{\frac{1}{\nu}-1}(z_t)^{\nu-1}\right)$$

$$d_t = d_t^h + d_t^f$$

Exports and imports

$$\frac{p_t^{qd}}{p_t^c} = \frac{s_t p_t^\star}{p_t^c}\frac{(p_t^{exp})^\star}{p_t^\star}$$

$$e_t = \left(\frac{(p_t^{exp})^\star}{p_t^\star}\right)^{-\mu} c_t^\star$$

$$\frac{p_t^{mdock}}{p_t^c} = \frac{s_t p_t^\star}{p_t^c}\frac{(p_t^m)^\star}{p_t^\star}$$

$$m_t = c_t^m + x_t^m$$

Import's price setting

$$E_t\Theta_t^m \quad = \quad \theta^m \frac{p_t^{mdock}}{p_t^c} m_t + E_t\Lambda_{t+1,t}\epsilon^m \left(1+\pi_{t+1}^m\right)^{\theta^m} \left(\left\{\prod_{k=1}^n \left(1+\pi_{t-k+1}^m\right)^{\gamma_{mk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{j=1}^n \gamma_{mj}\right)}\right)^{-\theta^m} \Theta_{t+1}^m$$

$$E_t\Psi_t^m \quad = \quad (\theta^m - 1)\frac{p_t^m}{p_t^c} m_t + E_t\Lambda_{t+1,t}\epsilon^m \left(1+\pi_{t+1}^m\right)^{\theta^m-1} \left(\left\{\prod_{k=1}^n \left(1+\pi_{t-k+1}^m\right)^{\gamma_{mk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{j=1}^n \gamma_{mj}\right)}\right)^{1-\theta^m} \Psi_{t+1}^m$$

$$\frac{p_t^{mopt}}{p_t^m} = \frac{E_t\Theta_t^m}{E_t\Psi_t^m}$$

$$(1+\pi_t^m)^{1-\theta_t^m} = (1-\epsilon^m)\left(\frac{p_t^{mopt}}{p_t^m}\right)^{1-\theta_t^m}(1+\pi_t^m)^{1-\theta_t^m} + \epsilon^m\left[\prod_{k=1}^n \left(1+\pi_{t-k}^m\right)^{\gamma_{mk}}(1+\overline{\pi})^{1-\sum_{j=1}^n \gamma_{mj}}\right]^{1-\theta_t^m}$$

$$m_t^{dock} = \int_0^1 m_t\left(j^m\right)dj^m = m_t v_t^m$$

$$v_t^m = \epsilon^m\left((1+\pi_t^m)^{-1}\prod_{k=1}^n \left(1+\pi_{t-k}^m\right)^{\gamma_{mk}}(1+\overline{\pi})^{1-\sum_{j=1}^n \gamma_{mj}}\right)^{-\theta_t^m} v_{t-1}^m + (1-\epsilon^m)\left(\frac{p_t^{mopt}}{p_t^m}\right)^{-\theta_t^m}$$

$$\Pi_t^m = \int_0^1 \Pi_t^m\left(j^m\right)dj^m = \frac{p_t^m}{p_t^c}m_t - \frac{p_t^{mdock}}{p_t^c}m_t^{dock}$$

Investment

$$x_t = \chi_t^x\left[(\gamma^x)^{\frac{1}{\omega^x}}\left(x_t^d\right)^{\frac{\omega^x-1}{\omega^x}} + (1-\gamma^x)^{\frac{1}{\omega^x}}\left(x_t^m\right)^{\frac{\omega^x-1}{\omega^x}}\right]^{\frac{\omega^x}{\omega^x-1}}$$

$$x_t^d = (\gamma^x)\left(\frac{p_t^{qd}/p_t^c}{\chi_t^x p_t^x/p_t^c}\right)^{-\omega^x}\frac{x_t}{\chi_t^x}$$

$$x_t^m = (1-\gamma^x)\left(\frac{p_t^m/p_t^c}{\chi_t^x p_t^x/p_t^c}\right)^{-\omega^x}\frac{x_t}{\chi_t^x}$$

$$\left(1+\pi_t^x\right)=\left(\chi_t^x\right)^{-1}\left[\left(\gamma^x\right)\left(1+\pi_t^q\right)^{1-\omega^x}\left(\frac{p_{t-1}^q}{p_{t-1}^x}\right)^{1-\omega^x}+\left(1-\gamma^x\right)\left(1+\pi_t^m\right)^{1-\omega^x}\left(\frac{p_{t-1}^m}{p_{t-1}^x}\right)^{1-\omega^x}\right]^{\frac{1}{1-\omega^x}}$$

Fiscal and monetary authorities

$$\left(1+i_t\right)=\left(1+i_{t-1}\right)^{\rho_i}\left(\left(1+\bar{i}\right)\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\rho_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{\rho_y}\right)^{1-\rho_i}\epsilon_t^i$$

$$\left(\frac{1+i_{t-1}^f}{1+\pi_t^\star}\right)\frac{s_t p_t^\star}{p_t^c}a_{t-1}^{cb}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+\left(\frac{1+i_{t-1}^{cb}}{1+\pi_t^c}\right)b_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+rb_t=\frac{s_t p_t^\star}{p_t^c}a_t^{cb}+rb_{t-1}\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+b_t+\Pi_t^{cb}$$

$$\Pi_t^{cb}=\left(\frac{1+i_{t-1}^f}{1+\pi_t^\star}\frac{s_t p_t^\star}{p_t^c}a_{t-1}^{cb}+\left(\frac{1+i_{t-1}^{cb}}{1+\pi_t^c}-1\right)b_{t-1}\right)\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}-\frac{s_t p_t^\star}{p_t^c}a_t^{cb}$$

$$\frac{p_t^{qd}}{p_t^c}g_t=\left(c_t+\frac{p_t^x}{p_t^c}x_t\right)\vartheta\left(v_t\right)+\tau_t+\frac{s_t p_t^\star}{p_t^c}f_t^g-\left(\frac{1+i_{t-1}^g}{1+\pi_t^\star}\right)\frac{s_t p_t^\star}{p_t^c}f_{t-1}^g\frac{A_{t-1}}{A_t}\frac{N_{t-1}}{N_t}+\Pi_t^{cb}$$

$$\lambda_t\frac{s_t p_t^\star}{p_t}=\beta E_t\left(\frac{A_{t+1}}{A_t}\right)^{-\sigma}\lambda_{t+1}\frac{s_{t+1}p_{t+1}^\star}{p_{t+1}}\left(\frac{1+i_t^g}{1+\pi_{t+1}^\star}\right)$$

Foreign sector and national accounts

$$i_t^f=i_t^s+\psi^b\left[\exp\left(\frac{s_t p_t^\star}{p_t^c}\frac{fs_t}{gdp_t}-\overline{fs}\right)-1\right]$$

$$fs_t=f_t-acb_t$$

$$i_t^g=i_t^s+\psi^g\left[\exp\left(\frac{s_t p_t^\star}{p_t^c}\frac{f_t^g}{gdp_t}-\overline{fg}\right)-1\right]$$

$$gdp_t=c_t+\frac{p_t^x}{p_t^c}x_t+\frac{p_t^{qd}}{p_t^c}g_t+\frac{p_t^{qd}}{p_t^c}e_t-\frac{p_t^{mdock}}{p_t^c}m_t^{dock}$$

$$\Pi_t=\Pi_t^{qs}+\Pi_t^{qd}+\Pi_t^m+\Pi_t^b$$

Exogenous variables

$$\log\left(N_t\right)=n+\log\left(N_{t-1}\right)+\epsilon_t^N$$

$$\log\left(A_t\right)=a+\log\left(A_{t-1}\right)+\epsilon_t^A$$

$$\log\left(TD_t\right)=\rho_{TD}\log\left(TD_{t-1}\right)+\left(1-\rho_{TD}\right)\log\left(TD\right)+\epsilon_t^{TD}$$

$$\log\left(TBP_t\right)=\rho_{TBP}\log\left(TBP_{t-1}\right)+\left(1-\rho_{TBP}\right)\log\left(TBP\right)+\epsilon_t^{TBP}$$

$$\ln\left(\xi_t\right)=\left(1-\rho_\xi\right)\ln\left(\overline{\xi}\right)+\rho_\xi\ln\left(\xi_{t-1}\right)+\epsilon_t^\xi$$

$$\ln\left(g_t\right)=\left(1-\rho_g\right)\ln\left(\overline{g}\right)+\rho_g\ln\left(g_{t-1}\right)+\epsilon_t^g$$

$$\ln\left(\tau_t^d\right)=\left(1-\rho_\tau\right)\ln\left(\overline{\tau^d}\right)+\rho_\tau\ln\left(\tau_{t-1}^d\right)+\epsilon_t^\tau$$

$$\ln\left(acb_t\right) = (1-\rho_{acb})\ln\left(\overline{acb}\right) + \rho_{acb}\ln\left(acb_{t-1}\right) + \epsilon_t^{acb}$$

$$\ln\left(tr_t\right) = (1-\rho_{tr})\ln\left(\overline{tr}\right) + \rho_{tr}\ln\left(tr_{t-1}\right) + \epsilon_t^{tr}$$

$$\ln\left(c_t^{\star}\right) = (1-\rho_{c\star})\ln\left(\overline{c^{\star}}\right) + \rho_{c\star}\ln\left(c_{t-1}^{\star}\right) + \epsilon_t^{c\star}$$

$$\ln\left(1+\pi_t^{\star}\right) = (1-\rho_{\pi\star})\ln\left(1+\overline{\pi^{\star}}\right) + \rho_{\pi\star}\ln\left(1+\pi_{t-1}^{\star}\right) + \epsilon_t^{\pi\star}$$

$$\ln\left(\frac{\left(p_t^m\right)^{\star}}{p_t^{\star}}\right) = (1-\rho_{pmstar})\ln\left(\frac{\overline{p^{m\star}}}{p^{\star}}\right) + \rho_{pmstar}\ln\left(\frac{\left(p_{t-1}^m\right)^{\star}}{p_{t-1}^{\star}}\right) + \epsilon_t^{pmstar}$$

$$i_{t+1}^s = (1-\rho_s)i^s + \rho_s i_t^s + \epsilon_{t+1}^s$$

$$\ln\left(\chi_t^j\right) = (1-\rho_j)\ln\left(\overline{\chi}^j\right) + \rho_j\ln\left(\chi_{t-1}^j\right) + \epsilon_t^j$$

for $j \in \{u, h, qs, z, v, qd, x\}$.

B. Aggregation and equilibrium conditions

Wage stickiness

The optimal wage of household h satisfies:

$$w_t^{opt}(h) = \frac{E_t\sum_{i=0}^{\infty}\left(\beta\epsilon^w\right)^i\frac{N_{t+i}}{N_t}\left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma}\chi_{t+i}^h\left(\left(1-TD_{t+i}\right)TBP_{t+i}n_{t+i}^d\left(\frac{w_t^{opt}(h)}{w_{t+i}}\pi_{t,i}^c\right)^{-\theta_{t+i}^w}\right)^{1+\varsigma}\theta_{t+i}^w}{E_t\sum_{i=0}^{\infty}\left(\beta\epsilon^w\right)^i\frac{N_{t+i}}{N_t}\left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma}\lambda_{t+i}\left(\theta_{t+i}^w-1\right)\pi_{t,i}^c\left(\left(1-TD_{t+i}\right)TBP_{t+i}n_{t+i}^d\left(\frac{w_t^{opt}(h)}{w_{t+i}}\pi_{t,i}^c\right)^{-\theta_{t+i}^w}\right)}$$

where $\pi_{t,i}^c = \prod_{k=1}^{i}\left(\frac{1+\pi_{t+k-1}^c}{1+\pi_{t+k}^c}\right)$.

Let

$$f_t^1 = w_t^{opt}(h)\,E_t\sum_{i=0}^{\infty}\left(\beta\epsilon^w\right)^i\frac{N_{t+i}}{N_t}\left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma}\lambda_{t+i}\left(\theta_{t+i}^w-1\right)\pi_{t,i}^c\left(\left(1-TD_{t+i}\right)TBP_{t+i}n_{t+i}^d\left(\frac{w_t^{opt}(h)}{w_{t+i}}\pi_{t,i}^c\right)^{-\theta_{t+i}^w}\right)$$

and

$$f_t^2 = E_t\sum_{i=0}^{\infty}\left(\beta\epsilon^w\right)^i\frac{N_{t+i}}{N_t}\left(\frac{A_{t+i}}{A_t}\right)^{1-\sigma}\chi_{t+i}^h\left(\left(1-TD_{t+i}\right)TBP_{t+i}n_{t+i}^d\left(\frac{w_t^{opt}(h)}{w_{t+i}}\pi_{t,i}^c\right)^{-\theta_{t+i}^w}\right)^{1+\varsigma}\theta_{t+i}^w$$

and the first order condition is summarized as

$$f_t^1 = f_t^2$$

Under the assumption that $\theta_{t+i}^w = \theta^w$ (constant elasticity of substitution between labour varieties) it can found a recursive expression for f_t^1 and f_t^2:

$$f_t^1 = w_t^{opt}\lambda_t(h)(\theta^w-1)\left((1-TD_t)TBP_t n_t^d\left(\frac{w_t^{opt}}{w_t}\right)^{-\theta^w}\right)+E_t(\beta\epsilon^w)\frac{N_{t+1}}{N_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma}\left(\frac{w_t^{opt}}{w_{t+1}^{opt}}\frac{1+\pi_t^c}{1+\pi_{t+1}^c}\right)^{1-\theta^w} f_{t+1}^1$$

$$f_t^2 = \chi_t^h\left((1-TD_t)TBP_t n_t^d\left(\frac{w_t^{opt}}{w_t}\right)^{-\theta^w}\right)^{1+\varsigma}\theta^w+E_t(\beta\epsilon^w)\frac{N_{t+1}}{N_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma}\left(\frac{w_t^{opt}}{w_{t+1}^{opt}}\frac{1+\pi_t^c}{1+\pi_{t+1}^c}\right)^{-\theta^w(1+\varsigma)} f_{t+1}^2$$

Retailers' price stickiness

The optimal price is characterized by

$$\frac{p_t^{qopt}(j)}{p_t^{qd}} = \frac{E_t\sum_{i=0}^{\infty}(\epsilon^q)^i\Lambda_{t+i,t}\left[\theta_{t+i}^q\frac{p_{t+i}^{qs}}{p_{t+i}^c}\left(\pi_{t,i,\{\gamma_{qk}\}_{k=1}^n}\right)^{-\theta_{t+i}^q}\left(\frac{p_{t+i}^{qd}}{p_t^{qd}}\right)^{\theta_{t+i}^{qd}}q_{t+i}^d\left(\chi_{t+i}^{qd}\right)^{\theta_{t+i}^q}\right]}{E_t\sum_{i=0}^{\infty}(\epsilon^q)^i\Lambda_{t+i,t}\left[\left(\theta_{t+i}^q-1\right)\frac{p_{t+i}^{qd}}{p_{t+i}^c}\left(\pi_{t,i,\{\gamma_{qk}\}_{k=1}^n}\right)^{1-\theta_{t+i}^j}\left(\frac{p_{t+i}^{jm}}{p_t^{jm}}\right)^{\theta_{t+i}^j-1}q_{t+i}^d\left(\chi_{t+i}^{qd}\right)^{\theta_{t+i}^q}\right]}$$

Under the assumption that $\theta_t^q=\theta_{t+i}^q$ for all $i>0$ it can be obtained a recursive expression. From the numerator, let:

$$E_t\Theta_t^q=\theta_t^q\frac{p_t^{qs}}{p_t^c}q_t^d\left(\chi_t^{qd}\right)^{\theta_t^q}+E_t\Lambda_{t+1,t}\epsilon^q\left(1+\pi_{t+1}^{qd}\right)^{\theta_{t+1}^{qd}}\left(\left\{\prod_{k=1}^n\left(1+\pi_{t-k+1}^{qd}\right)^{\gamma_{qk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{m=1}^n\gamma_{qm}\right)}\right)^{-\theta_{t+1}^q}\Theta_{t+1}^q$$

From the denominator, let:

$$E_t\Psi_t^q=(\theta_t^q-1)\frac{p_t^{qd}}{p_t^c}q_t^d\left(\chi_t^{qd}\right)^{\theta_t^q}+E_t\Lambda_{t+1,t}\epsilon^q\left(1+\pi_{t+1}^q\right)^{\theta_{t+1}^q-1}\left(\left\{\prod_{k=1}^n\left(1+\pi_{t-k+1}^{qd}\right)^{\gamma_{qk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{j=1}^n\gamma_{qm}\right)}\right)^{1-\theta_{t+1}^q}\Psi_{t+1}^q$$

and

$$\frac{p_t^{qopt}}{p_t^{qd}}=\frac{E_t\Theta_t^q}{E_t\Psi_t^q}$$

Importers' price stickiness

The optimal price is characterized by

$$\frac{p_t^{mopt}}{p_t^m} = \frac{E_t\sum_{i=0}^{\infty}(\epsilon^m)^i\Lambda_{t+i,t}\left[\theta_{t+i}^m\frac{p_{t+i}^{mdock}}{p_{t+i}^c}\left(\pi_{t,i,\{\gamma_{mk}\}_{k=1}^n}\right)^{-\theta_{t+i}^m}\left(\frac{p_{t+i}^m}{p_t^m}\right)^{\theta_{t+i}^m}m_{t+i}\right]}{E_t\sum_{i=0}^{\infty}(\epsilon^m)^i\Lambda_{t+i,t}\left[\left(\theta_{t+i}^m-1\right)\frac{p_{t+i}^m}{p_{t+i}^c}\left(\pi_{t,i,\{\gamma_{mk}\}_{k=1}^n}\right)^{1-\theta_{t+i}^m}\left(\frac{p_{t+i}^m}{p_t^m}\right)^{\theta_{t+i}^m-1}m_{t+i}\right]}$$

where $\pi_{t,i,\{\gamma_{mk}\}_{k=1}^n}=\prod_{l=1}^i\left\{\prod_{k=1}^n\left(1+\pi_{t-k+l}^m\right)^{\gamma_{mk}}\right\}(1+\overline{\pi})^{i\left(1-\sum_{j=1}^n\gamma_{mj}\right)}$.

From the numerator (assuming $\theta_t^m=\theta_{t+i}^m$ for all $i>0$) let:

$$E_t\Theta_t^m = \theta^m \frac{p_t^{mdock}}{p_t^c} m_t + E_t\Lambda_{t+1,t}\epsilon^m \left(1+\pi_{t+1}^m\right)^{\theta^m} \left(\left\{\prod_{k=1}^{n}\left(1+\pi_{t-k+1}^m\right)^{\gamma_{mk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{j=1}^{n}\gamma_{mj}\right)}\right)^{-\theta^m} \Theta_{t+1}^m$$

And from the denominator:

$$E_t\Psi_t^m = \left(\theta^m-1\right) \frac{p_t^m}{p_t^c} m_t + E_t\Lambda_{t+1,t}\epsilon^m \left(1+\pi_{t+1}^m\right)^{\theta^m-1} \left(\left\{\prod_{k=1}^{n}\left(1+\pi_{t-k+1}^m\right)^{\gamma_{mk}}\right\}(1+\overline{\pi})^{\left(1-\sum_{j=1}^{n}\gamma_{mj}\right)}\right)^{1-\theta^m} \Psi_{t+1}^m$$

Then the optimal price is

$$\frac{p_t^{mopt}}{p_t^m} = \frac{E_t\Theta_t^m}{E_t\Psi_t^m}$$

Retailers' aggregation

Each retailer firm buys a fraction $q_t(j)$ of the good q_t^s. In order to define the aggregate equilibrium of the output market it is needed to "add up" all these demands of the retailers. The equilibrium condition for the product market is

$$q_t^s = \int_0^1 q_t(j)\,dj = q_t^d\left(\chi_t^{qd}\right)^{-\theta_t^q} v_t^{qd}$$

with

$$v_t^{qd} = \epsilon^q\left(\left(1+\pi_t^{qd}\right)^{-1}\prod_{k=1}^{n}\left(1+\pi_{t-k}^{qd}\right)^{\gamma_{qk}}(1+\overline{\pi})^{1-\sum_{m=1}^{n}\gamma_{qm}}\right)^{-\theta_t^q} v_{t-1}^{qd} + (1-\epsilon^q)\left(\frac{p_t^{qopt}}{p_t^{qd}}\right)^{-\theta_t^q}$$

an adjustment factor that must be introduced due to the price stickiness.

The real profits of the retailer j are $\Pi_t^{qd}(j) = \left(\frac{p_t^q(j)}{p_t^c}\right) q_t(j) - \frac{p_t^{qs}}{p_t^c} q_t(j)$ and integrating over firms it is obtained the aggregate real profits of this sector:

$$\Pi_t^{qd} = \int_0^1 \Pi_t^{qd}(j)\,dj = \frac{p_t^{qd}}{p_t^c} q_t^d - \frac{p_t^{qs}}{p_t^c} q_t^s$$

Importers' aggregation

Each importer firm buys a fraction $m_t(j^m)$. In order to define the aggregate equilibrium of the output market it is needed to "add up" all these demands of the retailers. The equilibrium condition in the imports is

$$m_t^{dock} = \int_0^1 m_t(j^m)\,dj^m = m_t v_t^m$$

with

$$v_t^m = \epsilon^m \left((1+\pi_t^m)^{-1} \prod_{k=1}^{n} \left(1+\pi_{t-k}^m\right)^{\gamma_{mk}} (1+\overline{\pi})^{1-\sum_{j=1}^{n}\gamma_{mj}} \right)^{-\theta_t^m} v_{t-1}^m + (1-\epsilon^m) \left(\frac{p_t^{mopt}}{p_t^m} \right)^{-\theta_t^m}$$

an adjustment factor that must be introduced due to the price stickiness.

The real profits of the importer j^m are $\Pi_t^m(j^m) = \left(\frac{p_t^m(j^m)}{p_t^c}\right) m_t(j) - \frac{p_t^{mdock}}{p_t^c} m_t(j^m)$ and integrating over firms it is obtained the aggregate real profits of this sector:

$$\Pi_t^m = \int_0^1 \Pi_t^m(j^m)\, dj^m = \frac{p_t^m}{p_t^c} m_t - \frac{p_t^{mdock}}{p_t^c} m_t^{dock}$$