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# Debt taxes during crises, a blessing in disguise?

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Debt taxes during crises, a blessing in disguise?

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**Abstract**

Models with an occasionally binding credit constraint have been used to analyze financial crises and previous literature has highlighted that the specific form of this constraint is decisive for policymaking conclusions. What are the welfare effects of implementing a policy that is appropriate for a specific type of constraint when the economy is actually facing a different one? We provide an answer by analyzing the implementation either of ex ante (or macroprudential) vs. ex post debt taxes in four possible collateral constraint cases (depending on whether creditors assess current or future and total or disposable income of debtors). Our main conclusion is that a debt tax applied only during potentially constrained periods (i.e., ex post) is a more favorable intervention if the policymaker does not know which credit constraint is facing or if it is more likely to be facing a disposable-income constraint (either for current or future income).

Keywords: macroprudential policies; ex post policies; debt tax; financial constraint; financial crisis; sudden stops.

JEL Classification: E32, E44, F34, F38, F41, G01

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# Impuestos a la deuda durante las crisis, ¿bendición encubierta?

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**Resumen**

Para analizar las crisis financieras, se han utilizado modelos con una restricción crediticia ocasionalmente vinculante y la literatura previa ha destacado que la forma específica de esta restricción es decisiva para la formulación de políticas. ¿Cuáles son los efectos en el bienestar de la implementación de una política que es apropiada para un tipo específico de restricción cuando la economía se enfrenta en realidad a otra diferente? Damos una respuesta analizando la implementación de impuestos ex-ante (o macroprudenciales) sobre la deuda frente a impuestos ex post sobre la deuda en cuatro posibles casos de restricción de colateral (dependiendo de si los acreedores evalúan la renta actual o futura y la renta total o disponible de los deudores). Nuestra principal conclusión es que un impuesto sobre la deuda aplicado sólo durante períodos de crisis (es decir, ex post) es una intervención más favorable si el formulador de políticas no saber a qué restricción crediticia se enfrenta o si es más probable que se enfrente a una que depende del ingreso disponible (ya sea para ingresos corrientes o futuros).

Palabras clave: política macroprudencial; políticas ex post ; impuesto a la deuda; restricción financiera; crisis financieras; parada súbita.

Clasificación JEL: E32, E44, F34, F38, F41, G01

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# 1 Introduction

A strand of economic literature has analyzed financial crises in the context of open economies that are occasionally credit constrained. Some recent papers (Vargas & Parra-Polania, 2021; Ottonello et al., 2022) have called attention to the significance for this analysis of the information used by creditors in assessing the borrowing capacity of potential debtors, i.e., the specific characteristics of the credit constraint. Do creditors evaluate borrowing capacity of debtors based on their current or on their future income? Besides, do creditors mainly consider pre- or after-tax income (i.e., total or disposable income) of debtors?

From the relevance of the specific form of the financial constraint, several questions arise. In particular, one question pertains to the welfare consequences of implementing a policy that is appropriate for a particular type of credit constraint in an economy that is actually facing a different one (e.g., implementing a macroprudential tax, suggested by the related literature as the convenient one for a current-income borrowing constraint, in an economy actually facing a future-income constraint). This paper provides an answer to this question by analyzing the welfare effects of implementing either of two policies, ex ante (or macroprudential) debt tax vs. ex post debt tax, in the four possible collateral constraint cases (derived from combining current/future and total/disposable income). Since there are many possible intervention policies that could be considered, we limit the analysis to those two that are shown to equalize the decentralized equilibrium to the one obtained by a benevolent social planner under discretion.[1]

We find that imposing a debt tax during periods when the economy is potentially constrained[2] (an ex post debt tax, for short) is a more favorable intervention policy -in terms of welfare- than a macroprudential one (i.e., a debt tax that applies during normal periods only) if the policymaker does not know which of the four possible credit constraints is the economy facing or if it is more likely to be facing a disposable-income constraint (either for current or future income). The macroprudential debt tax is welfare improving when the economy faces a current-income credit constraint but welfare reducing with a future-income one (either for total or disposable income). With a current-income collateral constraint, a macroprudential debt tax reduces the variability of debt levels which helps to mitigate the negative effects when the credit constraint binds. However, since under a future-income collateral constraint borrowing decisions are already constrained efficient, the macroprudential policy ends up distorting those decisions and thus reduces welfare. Instead, an ex post debt tax (returned to households via a lump-sum subsidy) increases disposable income but does not affect total income, and hence has a welfare-improving impact when the economy faces a disposable-income credit constraint and no effect when it is a total-income one.

To give more context to our findings and contribution let us briefly recapitulate some

[1] This excludes, for instance, optimal policies under commitment.

[2] We refer to "potentially" constrained periods since, as a result of imposing the policy, the borrowing constraint may become non-binding for some of those (otherwise constrained) periods.

results obtained in the previous literature. From previous papers we know that a macroprudential debt tax makes private agents internalize the social cost of their borrowing decisions when they face a current-total-income credit constraint (e.g., Bianchi, 2011; Korinek, 2011). This result is preserved for the current-disposable-income case (Vargas and Parra-Polania, 2021). We also know that when facing a future-total-income constraint private agents make constrained-efficient decisions (Ottonello et al., 2022) and hence no policy intervention is required to internalize their social cost (despite the fact their shadow value of borrowing is different from that of the social planner). In the present paper we show that in that case, the one where the economy faces a future-total-income constraint, there is an ex post debt tax that entirely equalizes the private agents' equilibrium to that of the social planner: not only their borrowing decisions but also their shadow values of borrowing become equal. In this case, this is a mere theoretical curiosity with no practical effects (since in the absence of such a policy borrowing decisions were already the same). In contrast, when facing a future-disposable-income constraint this ex post debt tax increases debt capacity as it affects future disposable income positively: it is expected to be collected in every potentially constrained period and to be returned to households via a lump-sum subsidy.

To obtain these results we use a very standard and simple small open economy model (proposed by Mendoza, 2002) for which we only change the collateral constraint form to consider the four possible abovementioned cases. We find these results from imposing state-contingent debt taxes on debt acquired in the current period, either as a macroprudential policy, or as an ex post debt tax. Then we use standard parameter values to illustrate our theoretical results, simulate the different scenarios, and calculate the welfare effect of each policy in each case.

The remainder of this paper is organized as follows. Section 2 presents the theoretical framework and derives this paper's core results. In section 3, numerical examples are presented and discussed. Section 4 concludes.

## 2 Theoretical Framework

We adopt a canonical small open economy model with tradable and nontradable goods and borrowing subject to collateral constraint.

The economy is populated by a continuum of households of size one. A representative household seeks to maximize its lifetime expected utility function expressed by

$$U = E_1 \left[ \sum_{t=1}^{\infty} \beta^t u\left(C_t\right) \right], \tag{1}$$

where $E\left[\cdot\right]$ is the expectations operator, $u\left(\cdot\right)$ is the well-behaved period utility function, $\beta$ is the discount factor and $C_t$ is the consumption index which aggregates tradable ($T$) and nontradable ($N$) goods:

$$C_t = C\left(C_t^T, C_t^N\right). \tag{2}$$

Every period, this household receives a stochastic (and exogenous) bundle of tradable and nontradable goods, $Y_t^T$ and $Y_t^N$, and has access to international credit markets through one-period loans $B_{t+1}$ at an interest rate $r$ ($R \equiv 1 + r$). The budget constraint, expressed in units of tradable goods, is

$$C_t^T + P_t^N C_t^N + RB_t = Y_t^T + P_t^N Y_t^N + B_{t+1}, \tag{3}$$

where $P_t^N$ is the price of nontradables; the price of tradable goods operates as the numeraire.

The household faces a collateral constraint; that is, it can borrow $B_{t+1}$ up to a fraction $\kappa$ of its income, such that $B_{t+1} \leq \kappa \times$ (INCOME). We consider four types of collateral constraints depending on the income used for assessing borrowing capacity. These four types arise from the combination of two different features: a) whether the relevant income is the current one of the next-period one; and b) whether the relevant income is the total or the disposable one. This way, the four possible collateral constraints are:

1. Current-total-income collateral constraint:

$$B_{t+1} \leq \kappa\left(Y_t^T + P_t^N Y_t^N\right). \tag{4}$$

2. Current-disposable-income collateral constraint:

$$B_{t+1} \leq \kappa\left(Y_t^T + P_t^N Y_t^N - T_t\right), \tag{5}$$

   i.e., borrowers' debt capacity is evaluated considering income after deducting taxes ($T_t > 0$) or adding subsidies ($T_t < 0$). $T$ is taken as exogenous by the household.

3. Next-period-total-income collateral constraint:[3]

$$B_{t+1} \leq \kappa E_t\left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N\right]. \tag{6}$$

4. Next-period-disposable-income collateral constraint:

$$B_{t+1} \leq \kappa E_t\left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N - T_{t+1}\right]. \tag{7}$$

[3] For this and the next case we consider the expected value operator ($E[\cdot]$) as in Devereux et al. (2019). In contrast, Ottonello et al. (2022) use the minimum operator ($\min[\cdot]$) but, as they remark, their results also hold for the expected-value case. In terms of the environment to support the microfoundation of these constraint forms, the minimum operator ensures debt repayment in every possible state; however, as shown by Ottonello et al. (2022), the economy does not feature sudden stops and hence they need to add other type of shocks (either stochastic volatility or shocks to the share of income pledge as collateral). No additional shocks are needed in the expected-value case for the economy to feature sudden stops.

As mentioned in the introduction, the specific form of the collateral constraint is decisive for the analysis of financial crises. An economy with a collateral constraint determined by current income exhibits a pecuniary externality and overborrowing. Instead, in an economy with a collateral constraint determined by future income, the decentralized (DC) equilibrium is constrained efficient, and therefore there is no need for policy intervention to equalize it to the allocations obtained by a benevolent social planner (SP).

### 2.1 Solution to the household's maximization problem

We denote by $\mu_t$ and $\lambda_t$ the Lagrange multipliers associated to the budget and credit constraints, respectively. Non-tradable consumption is determined by the market-clearing condition of that market, i.e., $C_t^N = Y_t^N$. The other first-order conditions for maximization in this decentralized (DC) economy make up the following equation system:

$$\mu_t = R\beta E_t \mu_{t+1} + \lambda_t, \tag{8}$$

$$u_{T,t}\left(C_t^T\right) = \mu_t, \tag{9}$$

$$P_t^N = \frac{u_{N,t}\left(C_t^T\right)}{u_{T,t}\left(C_t^T\right)}, \tag{10}$$

$$C_t^T + RB_t = Y_t^T + B_{t+1}, \tag{11}$$

$$\lambda_t \left[\kappa \times (\text{INCOME}) - B_{t+1}\right] = 0, \tag{12}$$

which includes the market-clearing condition for tradables, i.e., Equation (11), and where $u_{T,t}\left(C_t^T\right) \equiv u'\left(C_t\right)\left(\partial C_t / \partial C_t^T\right)$ and $u_{N,t}\left(C_t^T\right) \equiv u'\left(C_t\right)\left(\partial C_t / \partial C_t^N\right)$. This five-equation system provides a solution for $C_t^T$, $\mu_t$, $\lambda_t$, $B_{t+1}$ and $P_t^N$ for given values of $\left\{B_t, Y_t^T, Y_t^N\right\}$ and the (consistent) expected values of future variables.

If the economy is financially unconstrained in period $t$, then $B_{t+1} \leq \kappa \times (\text{INCOME})$ and hence $\lambda_t = 0$, from Equation (12). Given this (and the expected value of $\mu_{t+1}$), $\mu_t$ is determined by Equation (8), in turn $C_t^T$ is determined by Equation (9), and then $P_t^N$ and $B_{t+1}$ are determined by Equations (10) and (11), respectively.

If, instead, the economy is constrained, then $\lambda_t \geq 0$ and Equation (12) turns into $B_{t+1} = \kappa \times (\text{INCOME})$. The solution of the system depends then on whether the relevant income in the collateral constraint is the current or the future one. If it is the current one, then $C_t^T$, $B_{t+1}$ and $P_t^N$ are determined by the sub-system of Equations (10)-(12). Next, Equation (9) determines $\mu_t$, and finally Equation (8) determines $\lambda_t$ (given the expected value of $\mu_{t+1}$). If instead the relevant income in the collateral constraint is the future one, $B_{t+1}$ is determined by Equation (12) (given the expected value of either total or disposable next-period income), then $C_t^T$ is determined by Equation (11), $P_t^N$ in turn is determined by Equation (10), subsequently Equation (9) determines $\mu_t$, and finally Equation (8) determines $\lambda_t$ (given the expected value of $\mu_{t+1}$).

## 2.2 Policy interventions and welfare effects

This class of models, with occasionally binding collateral constraints that are themselves a function of aggregate endogenous variables of the economy, are common in the study of sudden stops. With occasionally binding constraints, individual private agents may not internalize the effect of their decisions on the aggregate borrowing limit, leading to pecuniary externalities and inefficient borrowing. In this environment, a social planner (SP) who faces the same borrowing constraint as the private agents (i.e., a constrained SP) but takes into account the consequences of her choices on the market value of collateral would face a lower probability of being financially constrained relative to a DC economy.

These models have been extensively used for the analysis of optimal policy interventions that would reduce the probability and severity of sudden stops, as well as their welfare implications. The interventions are focused in two general sets of state-contingent policies: (a) macroprudential or ex ante policies (i.e., policies implemented in good times to mitigate the frequency and severity of financial crises in the future)[4], and (b) ex post policies aimed at dealing with the financial crises once it is in motion (i.e., policies implemented in potentially[5] constrained periods)[6]. Within these two general sets (ex ante vs. ex post), there are also many possible policies depending on whether they are subsidies or taxes, whether they are levied on nontradable or tradable consumption, or on their corresponding prices, or on debt. Regarding optimal interventions, they can be derived as optimal under discretion or under commitment.

Since there are many possible intervention policies that could be considered, in the present paper we limit the analysis to those two that equalize the DC equilibrium to the one obtained by a benevolent SP under discretion: an ex ante debt tax, that equalizes DC and SP equilibria with current-income credit constraints (e.g., Bianchi, 2011; Korinek, 2011; Vargas and Parra-Polania, 2021), and an ex post debt tax, that equalizes DC and SP equilibria with future-income credit constraints (as shown in Proposition 3 below).

More specifically, the interventions we evaluate are state-contingent debt taxes ($\tau_t$) which are issued on debt acquired in period $t$ (i.e., $B_{t+1}$). As a macroprudential policy, the debt tax would be positive during unconstrained times and nil during credit-constrained periods. As an ex post debt tax, it would be positive only during potentially constrained periods and nil during the unconstrained ones. In both cases, the corresponding tax is

[4] e.g., Bianchi, 2011 ; Bianchi and Mendoza, 2018; Jeanne and Korinek, 2019.

[5] By using the expression "potentially constrained periods", we want to emphazise that the ex post debt tax applies only in those periods when, were the tax not issued, the economy would be financially constrained. As a result of the tax, during those periods the economy might still be financially constrained but with lower severity, or it might not be financially constrained altogether.

[6] e.g., Benigno et al. 2016; Bianchi 2016; Jeanne and Korinek 2020

returned to the household in the same period through a lump-sum subsidy

$$T_t = -\tau_t B_{t+1} < 0.$$

Either as an ex post or an ex ante policy, the debt tax $\tau_t$ changes Equation (8) in the system of first-order conditions to

$$\left(1 - \tau_t\right) \mu_t = R\beta E_t \mu_{t+1} + \lambda_t. \tag{8.1}$$

Equations (9)-(11) remain unchanged.[7]

In what follows, we describe the theoretical implications of implementing these debt-tax policies in each one of the models determined by the collateral-constraint types described in Equations (4) to (7).

*Current-total-income collateral and macroprudential debt tax*

In an economy described by Equations (1) - (3) and a collateral constraint determined by current-total income, as in Equation (4), a macroprudential debt tax implements the SP solution in the DC economy (e.g., Bianchi, 2011; Korinek, 2011). That is, a macroprudential debt tax reduces the probability of being financially constrained, implements constrained-efficient allocations, and thus increases social welfare.

*Current-total-income collateral and ex post debt tax*

**Proposition 1** *In the economy described by Equations (1) - (3) and the collateral constraint that depends on current -total income, as in Equation (4), the ex post debt tax does not change the equilibrium allocation. Therefore, social welfare is not affected by levying this tax.*

**Proof.** During constrained periods, the debt tax $\tau_t$ is nonzero but it only affects the determination of $\lambda_t$ (as explained above, during constrained periods $C_t^T$, $B_{t+1}$ and $P_t^N$ are determined independently of Equation (8.1)). During unconstrained periods, the debt tax $\tau_t$ is nil. Therefore, the final and unique effect of $\tau_t$ on the equilibrium is rescaling $\lambda_t$ during constrained periods. ■

The current-total-income collateral constraint is the most commonly used financial constraint in the related literature. As a conclusion from the above results, when the collateral constraint is determined by current-total income, the policy intervention that implements the SP solution and increases social welfare is a macroprudential debt tax. Instead, as shown in Proposition 1, the ex post debt tax causes no effect on social welfare and in that sense it is equivalent to implementing no policy.

[7] Notice that as the debt tax is returned to the household as a lump-sum transfer in the same period, they cancel each out in the budget constraint.

*Current-disposable-income collateral and macroprudential debt tax*

Vargas and Parra-Polania (2021, Proposition 4) show that the SP solution can be implemented in a DC economy described by Equations (1) - (3) and facing a current-disposable-income constraint (Equation (5)) by means of the same macroprudential debt tax that implements such a solution in that DC economy but facing a current-total-income constraint. Consequently, imposing the macroprudential tax is welfare improving in this case.

*Current-disposable-income collateral and ex post debt tax*

**Proposition 2** *In an economy described by Equations (1) - (3) and with collateral constraint defined as in Equation (5), the ex post debt tax increases borrowing capacity during potentially-constrained periods. As a result, welfare is improved. The higher the debt tax, the higher the impact on borrowing capacity.*

**Proof.** The initial part of the proposition is straightforward and follows from the corresponding collateral constraint, Equation (5): during potentially-constrained periods (i.e. periods in which the economy would be constrained in the absence of any policy intervention), the debt tax is collected ($\tau_t > 0$) and it is returned to households via a lump-sum transfer $-T_t = \tau_t B_{t+1}$, increasing borrowing capacity and reducing (or in some periods even eliminating) the negative impact of the binding constraint. The last part can be seen from Equation (5) as well. This collateral constraint with an ex-post debt tax can be rewritten as follows $B_{t+1} \leq [\kappa / (1 - \kappa \tau_t)] \left(Y_t^T + P_t^N Y_t^N\right)$. The right hand side of this equation (i.e. the borrowing capacity) is increasing in $\tau_t$. ■

Although borrowing capacity cannot be infinitely increased ( if $\tau_t \longrightarrow 1$ borrowing capacity tends to $[\kappa / (1 - \kappa)] \left(Y_t^T + P_t^N Y_t^N\right)$), for some parameters the impact might be sufficiently high such that the collateral constraint is not binding in any period, as we show in our numerical example below (see Section 3). However, as remarked by Vargas and Parra-Polania (2021) regarding this type of results, implementation issues should be considered as it seems unfeasible to use debt of a specific period to increase the borrowing capacity on which that same debt depends, particularly considering that such a capacity is assessed before the loan is disbursed.

In conclusion, when the collateral constraint is determined by current-disposable income, both policies (a macroprudential and an ex post debt tax) increase social welfare. The macroprudential debt tax implements the SP allocation and the ex post debt tax could, under certain circumstances, make the economy financially unconstrained. Although both interventions increase social welfare, the macroprudential debt tax gives a specific welfare gain, i.e., the one derived from implementing the SP allocation in the decentralized economy. Instead, the welfare gain from the ex post debt tax arises from increasing repayment capacity (mitigating the negative impact of the financial constraint). This positive welfare effect is, in general, increasing in the level of the debt tax. Consequently, in many cases the policymaker could raise the tax level to obtain a welfare gain greater than the one that can be obtained with the macroprudential policy. In fact, as we illustrate in Section 3 for a specific but standard set of parameter values, it is possible for the policymaker to make the economy financially unconstrained, through a sufficiently high ex-post debt tax.

*Future-total-income collateral and ex post debt tax*

Ottonello et al. (2022) show that the DC equilibrium in an economy with a future-income collateral constraint (as in Equation (6)) is already constrained efficient; that is, there is no difference between the SP equilibrium allocation and that of the DC economy (these equilibria only differ in their shadow values of borrowing $\lambda_t^{SP}$ vs $\lambda_t$). Therefore, implementing the ex-post policy in this case does not affect the DC allocation. It only rescales $\lambda$ in the same way that such a policy does in the economy with a collateral constraint that depends on current-total income.

For the subsequent discussion it is useful to show that there is a policy intervention that equalizes all values of both equilibria (DC and SP), i.e., including the Lagrange multipliers $\lambda_t^{SP}$ and $\lambda_t$. To this purpose let us start by recalling that since each household has an insignificant impact on the market, it takes prices as given. Instead a SP, subject to the same financial constraint, internalizes the effect of borrowing and consumption decisions on prices. Following the constrained-efficiency criterion (i.e., we assume the SP is constrained by the same pricing rule of the DC equilibrium), it can be verified that the first-order conditions for the SP equilibrium are equal to those for the DC equilibrium, Equations (6) and (10)-(11), except for the case of Equation (8) that turns into

$$\mu_t^{SP} = R\beta E_t \mu_{t+1}^{SP} + \lambda_t^{SP} \left(1 + E_t \psi_{t+1}^{SP}\right), \tag{8.2}$$

where $E_t \psi_{t+1}^{SP} \equiv -\kappa E_t \left[ Y_{t+1}^{N} \left( \partial P_{t+1}^{N,SP} / \partial C_{t+1}^{T,SP} \right) \left( \partial C_{t+1}^{T,SP} / \partial B_{t+1}^{SP} \right) \right]$. Remember that during unconstrained periods $\lambda = 0$, and hence these equations, (8) and (8.2), only differ during constrained periods.

**Proposition 3** *For the DC economy described by Equations (1) - (3) and with the future-total-income financial constraint (6), there exist an ex post debt tax rate on debt that implements the SP equilibrium.*

**Proof.** To implement the SP equilibria it will be enough to equalize Equations (8.1) and (8.2) during constrained periods. It can be easily verified that the following debt tax fulfills that purpose:

$$\tau_t = \frac{\lambda_t E_t \psi_{t+1}}{\mu_t},$$

where we have taken into account that, as a result of this tax, both equilibria are exactly the same, including $\psi_{t+1} = \psi_{t+1}^{SP}$. Notice that during normal times (i.e., $\lambda_t = 0$) the tax is nil. ■

In this case, the implementation of this debt tax that equalizes the SP and the DC equilibria is merely a theoretical curiosity with no practical relevance since the SP and DC allocations are already equal in the absence of any policy, and the only difference between both equilibria is the fact that the shadow value of borrowing for the SP is a rescaled version of that for the DC economy.

*Future-total-income collateral and macroprudential debt tax*

As mentioned in the previous case, decisions in a DC economy facing a future-income constraint are already constrained efficient, and therefore no intervention is required to equalize SP and DC allocations.

**Proposition 4** *Implementing the macroprudential debt tax in a DC economy described by Equations (1) - (3) and with future-total-income collateral constraint (as in Equation (6)) causes welfare reduction*

**Proof.** The macroprudential debt tax only distorts decisions that are, in the absence of the tax, constrained efficient (during unconstrained periods due to its impact on Equation (8.1) which affects $C_t^T$, $B_{t+1}$ and $P_t^N$ equilibrium values), without causing any other effect on borrowing capacity. ■

In conclusion, when the collateral constraint is determined by future-total income, the DC equilibrium is constrained efficient and hence equilibrium allocations are equal to those of the SP equilibrium without the need of any intervention policy. On the one hand, implementing a macroprudential debt tax reduces social welfare since private agents' decisions deviate from the constrained efficient ones. On the other, the ex post debt tax causes no effect on consumption and future debt decisions and in that sense it is equivalent to implementing no policy (as in the current total income case). Such a tax is a mere theoretical curiosity that equalizes the shadow value of borrowing of the DC equilibrium to that of the SP, with no effect on allocations.

*Future-disposable-income collateral and ex post debt tax*

**Proposition 5** *If the economy is described by Equations (1) - (3) and its collateral constraint is determined by future-disposable income as defined in Equation (7), the ex-post debt tax increases the borrowing capacity of the economy during potentially-constrained periods (as long as in those periods there is a positive probability that the economy will remain constrained in the next period). As a result welfare is improved.*

**Proof.** It follows from Equation (7) and the fact that the relevant income for determining current borrowing capacity is the one expected for the next period. To the extent that there is a positive probability that the credit constraint binds in the next period, there is also a positive expected value of tax collection ($E_t \tau_{t+1} > 0$) that will be returned to households via a lump-sum transfer $-E_t T_{t+1} = E_t \tau_{t+1} B_{t+2} > 0$, increasing current borrowing capacity and reducing (or in some periods even eliminating) the negative impact of the binding constraint.[8] ■

In general, a small positive effect on welfare is expected in this case (as illustrated below in our numerical example) since the impact on borrowing capacity depends not on a transfer that occurs for sure, like in the current-disposable-income case, but on a expected transfer (i.e., it will occur with some probability). Furthermore, since decisions in the future-income models are constrained efficient in the absence of interventions, it is in general expected that the space for welfare improvement be smaller than the one in the current-income models.

*Future-disposable-income collateral and macroprudential debt tax*

Like in the scenario with future-total-income collateral constraint, the DC equilibrium is constrained efficient. Therefore, implementing the macroprudential debt tax has a negative effect on social welfare since it distorts decisions that are, in the absence of the intervention, constrained efficient. However, because in this scenario the relevant income for the collateral constraint is the future-disposable one, the macroprudential debt tax has a positive effect since the borrowing capacity of the economy is increased due to the probable next period transfer (which will certainly occur if the economy is unconstrained in the next period). The final effect on welfare is in principle ambiguous and will depend on the parameters. But, due to the arguments given in the previous case (i.e., in general the positive effect is small), it is generally expected that the final effect be negative. In our numerical example below, for very standard parameters, the final effect on welfare of implementing a macroprudential tax in this context is negative (see next section).

In conclusion, when the collateral constraint is determined by future-disposable income, there are positive welfare effects expected from an ex post debt tax; and, in general or at least for standard parameter values, negative welfare effects expected from a macroprudential debt tax.

Table 1 summarizes the above social welfare results of applying either a macroprudential debt tax or an ex post debt tax in each of the four types of collateral constraints considered. The macroprudential debt tax increases welfare in economies with a collateral constraint that depends on current income, either total or disposable one. Instead, in economies with future income, the macroprudential debt tax distorts decisions that are already constrained efficient, reducing welfare. With regard to the ex post debt tax, this policy has no effect on welfare when the collateral constraint depends on total income; however, it increases welfare, by increasing borrowing capacity, when the collateral constraint depends on disposable income.

[8] This requires the credible promise that if the credit constraint binds again in $t+1$ the debt tax will apply. Notice that the policymaker does not have incentives to break this promise since implementing the tax during $t+1$ causes no harm (althought it does not bring benefits either) to the economy in that period (i.e., it is not a time-inconsistent policy).

Another relevant consideration for policy makers would be the size of the welfare effects obtained from implementing one or the other policy in each context. In the next section, we present numerical results based on simulations for the scenarios considered under standard parameter values.

**Table 1. Welfare effects of debt tax policies for each collateral constraint type**

| | Macroprudential debt tax | Ex-post debt tax |
|---|---|---|
| **Current total income** | Implements SP allocation. Increases welfare. | No effect on allocations. No effect on welfare. |
| **Current disposable income** | Implements SP allocation. Increases welfare. Same effect as with current-total-income. | Positive effect on borrowing capacity. Effect increasing on debt tax level. Increases welfare. |
| **Future total income** | It distorts agents' decisions during unconstrained periods, reducing welfare. | No effect on allocations. Therefore, no effect on welfare. |
| **Future disposable income** | Effect is ambiguous and depends on parameters. For standard parameter values, there is reduction in welfare. | Positive effect on borrowing capacity. Increases welfare. |

# 3 Numerical Examples

For the simulations, we solve the model using a global nonlinear method similar to the one described by Bianchi (2011). It is based on a basic iteration algorithm that takes into account the existence of the credit constraint and its occasional activation. Initial values are assumed for the endogenous variables and for the relevant expectations of future variables (according to each case). An initial solution of the equation system is obtained for each state - i.e., for given values of $\{B_t, Y_t^T, Y_t^N\}$- (as described in subsection 2.1). The consistency of this solution (expectations and the binding/nonbinding condition of the constraint in each state) is verified and (according to a tolerance level) it is determined if a new iteration is required until there is convergence.

We suppose that the household's utility function is of a CRRA form, the total consumption aggregator is a CES function between tradable and nontradable consumption. For the models with the financial constraint expressed in terms of current income, we set the same parameter values assigned by Bianchi (2011), with the non-tradable endowment $Y_t^N$ normalized to one and the tradable endowment $Y_t^T$ following a log AR(1) process (See Table 2). Under the current-total-income financial constraint these parameter values imply a frequency of crisis equal to 6.2%.[9] For the models with the constraint expressed in

[9] As standard in the literature, a crisis period is defined by the presence of two events: a) the collateral constraint is binding, and b) the current account value is at least one standard deviation above its steady state average.

terms of future income, we use the same parameter set except for two that we recalibrate to match the same frequency of crisis: the discount factor $\beta$, and the coefficient in the credit constraint $\kappa$. The resulting values are equal to those used by Ottonello et al. (2022) in their model with future income and financial shocks, i.e., $\beta = 0.93$ and $\kappa = 0.29$.

The following analysis is based on the results for the stochastic steady states of the corresponding models, i.e., using the ergodic distribution of $\{B, Y^T\}$, obtained from 100 thousand-draw simulations. Specifically, we compute the welfare gain (loss) of implementing a specific policy following the consumption-compensating variation, that is to say, as the percentage reduction (increase) in consumption -across all periods and states- that would make the consumer indifferent between the equilibrium with the policy implemented and the one with no intervention.

**Table 2. Parameter values for each type of collateral constraint**

| | Parameter | Value | |
|---|---|---|---|
| Interest rate | R | 1.04 | |
| Coefficient of relative risk aversion | σ | 2.00 | |
| Intertemporal elasticity of substitution | ξ | 0.83 | |
| Average value of tradable endowment process | yT | 1.00 | |
| Autocorrelation of tradable endowment | ρT | 0.50 | |
| Standard deviation shocks to tradable endowment | $\sigma_T$ | 0.04 | |
| Weight of tradables in CES | ω | 0.31 | |
| | | | |
| | | Current Income | Future Income |
| Subjective discount factor | β | 0.91 | 0.93 |
| Credit constraint parameter | κ | 0.32 | 0.29 |

*Results for collateral constraints with current income*

For the current-income collateral economy, as predicted by the abovementioned theoretical results, welfare gains obtained from implementing the macroprudential debt tax are equal for both the total and the disposable-income financial constraints. The mean of the welfare gain is 0.12% of consumption and the standard deviation 0.01%.[10] The left panel in Figure 1 shows the (ergodic) distribution of the welfare gain, which takes values between 0.09% and 0.15%, and the right panel illustrates how the welfare gain varies according to the initial level of debt and for three different initial levels of income: low (dashed line), medium (solid line), and high (dash-dotted line) which are, respectively, the lowest, the average, and the highest levels of tradable endowment $Y_t^T$ in the distribution considered. The right panel shows that the welfare gain from implementing a macroprudential debt tax is, in general, increasing in the initial level of debt; however it becomes decreasing

[10] Figure A1, in the appendix, displays the tax value varying (between 0% and 14.8%) across levels of income and debt.

for combinations of low initial levels of income and high initial levels of debt. For those combinations, the crisis probability is high; therefore, the macroprudential debt tax has a low probability of mitigating or preventing the upcoming crisis and hence the welfare gain is lower (of course, it still has a mitigation effect on more distant potential crises).

For the case of an ex-post debt tax implemented in an economy with current-total-income financial constraint, as we know from the abovementioned theoretical results, there is no effect on the equilibrium allocation, and thus there is no welfare change.

**Figure 1. Welfare Gain: macroprudential tax in the current-income case**

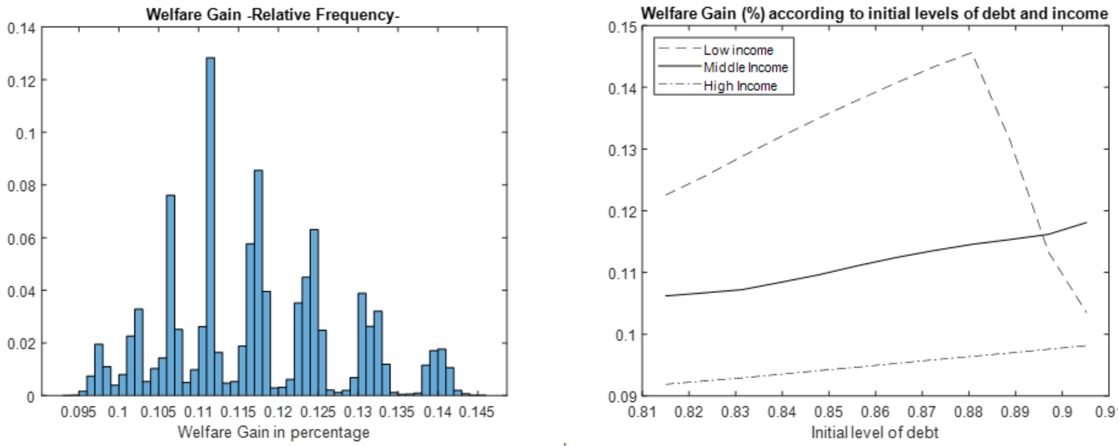
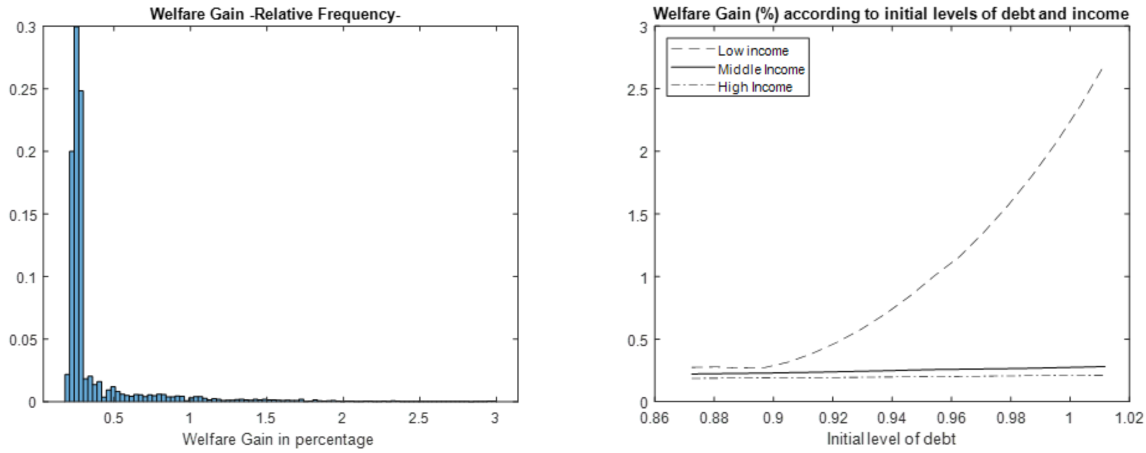



**Figure 2. Welfare Gain: ex post tax in the current-disposable-income case**




Instead, issuing an ex post debt tax when current-disposable income is the relevant one in the collateral constraint, results in a welfare gain which, in addition, increases with the level of the debt tax. It is even possible to avoid being financially constrained. For the parameters considered, we find that with an ex post debt tax between 1% and 44.5% (increasing in the level of debt and decreasing in the level of income) that applies only during those periods in which the financial constraint would bind (in the absence of the tax), it is possible to reach a nonbinding collateral constraint equilibrium.[11] The mean of the welfare gain is 0.38% and the standard deviation 0.34%. The left panel of Figure 2 shows the (ergodic) distribution of the welfare gain, which takes values between 0.19% and 3.0%, and right panel illustrates how the welfare gain varies according to the level of initial debt and for low (dashed line), medium (solid line) and high (dash-dotted line) initial levels of income. The welfare gain from an ex post debt tax in this economy is increasing in the initial level of debt. Since in this case the debt tax prevents being financially constrained, the greatest impact is reached for combinations of low initial levels of income and high initial levels of debt.

**Figure 3. Welfare Gain: macroprudential tax in the future-total-income case**

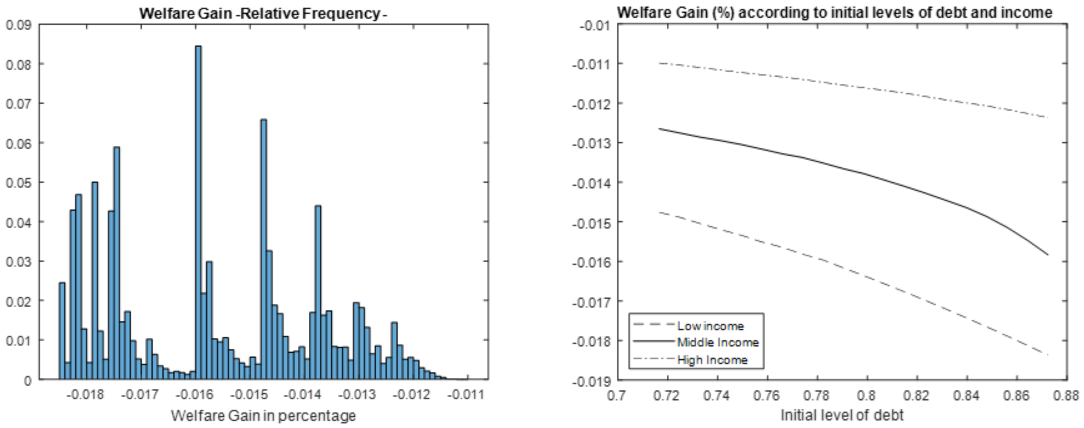
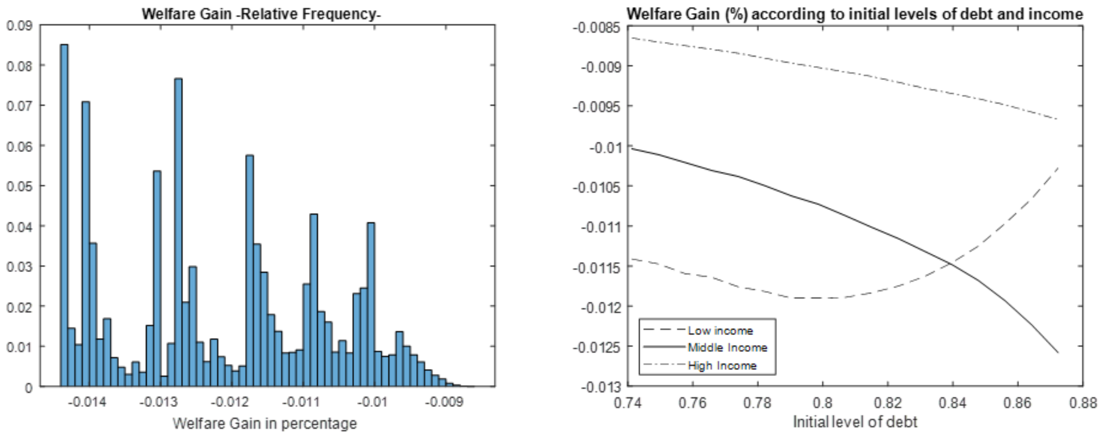



**Figure 4. Welfare Gain: macroprudential tax in the future-disposable-income case**



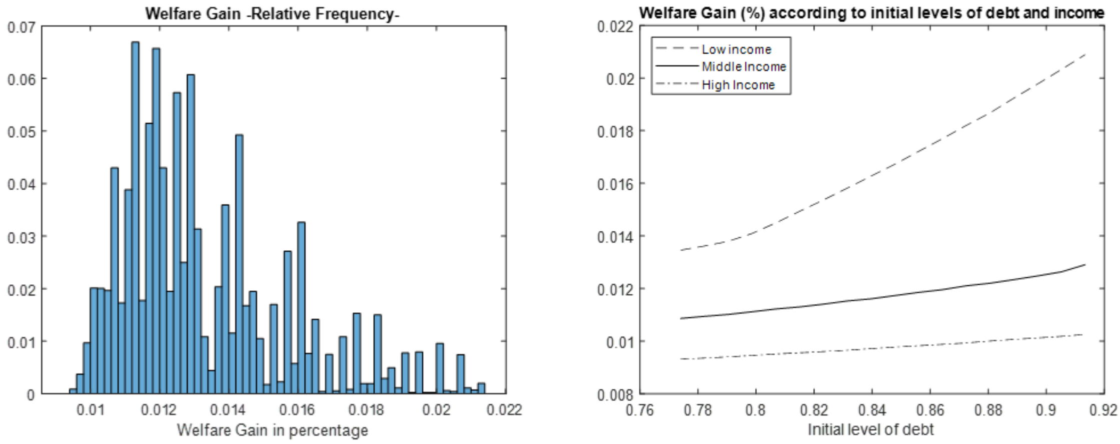

*Results for collateral constraint that depends on future income*

For the future-income collateral constraint economy, we know from the result found by Ottonello et al. (2022) that there is no difference, in terms of welfare, between the DC equilibrium and the one obtained by the SP (under the same constraint). In the case of

[11] Figure A2, in the appendix, displays the tax value varying across levels of income and debt.

the future-total-income financial constraint, the ex-post debt tax has no effect on welfare and the macroprudential debt tax only distorts constrained-efficient decisions, thereby deteriorating welfare. By implementing the same values of the macroprudential tax from the current-income case, we obtain a reduction in social well-being.[12] The mean of the welfare loss is 0.016% and the standard deviation 0.002%. The left panel of Figure 3 shows the (ergodic) distribution of the welfare loss, which takes values between 0.01% and 0.02%, and the right panel illustrates how the welfare loss from a macroprudential debt tax varies according to the initial level of debt and for low (dashed line), medium (solid line) and high (dash-dotted line) initial levels of income. The welfare loss is increasing in the initial level of debt and decreasing in the initial level of income.

**Figure 5. Welfare Gain: ex post tax in the future-disposable-income case**




Results are somewhat similar for the case of the future-disposable-income financial constraint. The mean of the welfare loss is 0.012% and the standard deviation 0.002%. However, as mentioned above, the main difference in this case is that debt capacity is increased due to the expected lump-sum transfers (which reimburse future debt tax payments) and the welfare loss is therefore lower than in the total-income case. The benefit from increasing debt capacity is greater in cases with low initial levels of income and high initial levels of debt such that for those states the welfare loss becomes decreasing in the initial level of debt -at least for the particular calibration considered here- (see Figure 4).

Finally, let us consider the case of the ex-post debt tax implemented in an economy with a future-disposable-income financial constraint. Such a policy causes no distortion to constrained-efficient decisions (unlike the macroprudential policy) and increases debt capacity due to the expected transfers. As a result, there will be welfare gains although small, at least for the the particular calibration considered here[13]: the mean of the welfare

[12] We assume that the policymaker applies the optimal macroprudential debt taxes from an economy with current-income collateral constraint in this economy that actually faces a future-total-income collateral constraint.

[13] We consider an ex post tax level between 0.06% and 30.7% -increasing in the level of debt and decreasing in the level of income-. With such a tax the frequency of crisis reduces to 2:2%. Figure A3, in the appendix, displays the tax value varying across levels of income and debt.

gain is 0.013% and the standard deviation 0.002% (see Figure 5).

**Table 3. Welfare gain (consumption-compensating variation in %) by collateral constraint and debt tax policy**

| | Welfare gain | Macroprudential debt tax | Ex post debt tax |
|---|---|---|---|
| Current total income | Average | 0.12 | 0 |
| | Standard deviation | 0.01 | |
| Current disposable income | Average | 0.12 | 0.38 |
| | Standard deviation | 0.01 | 0.34 |
| Future total income | Average | -0.016 | 0 |
| | Standard deviation | 0.002 | |
| Future disposable income | Average | -0.012 | 0.013 |
| | Standard deviation | 0.002 | 0.002 |

# 4 Conclusion

The specific form of the collateral constraint is not innocuous for the analysis of financial crises. As previous literature has shown, an economy with a collateral constraint determined by current income exhibits a pecuniary externality and overborrowing. Instead, in an economy with a collateral constraint determined by future income, the DC equilibrium is constrained efficient and, therefore, there is no need for policy intervention to equalize the DC and SP optimal allocations.

In this paper, we study the welfare consequences of implementing a policy that is appropriate for a particular type of credit constraint in an economy that is actually facing a different one (e.g., implementing a macroprudential tax - suggested by the related literature as the convenient one for a current-income borrowing constraint - in an economy actually facing a future-income constraint). Particularly, we analyze the welfare effects of implementing either of two policies, ex ante (or macroprudential) debt tax vs. ex post debt tax, in the four possible collateral constraint scenarios (derived from combining current/future and total/disposable income).

We find that imposing an ex post debt tax is a more favorable intervention policy -with regard to welfare- than a macroprudential debt tax if the policymaker does not know which of the four possible credit constraints is the economy facing or if it is more likely to be facing a disposable-income constraint (either for current or future income). The macroprudential debt tax is welfare improving when the economy faces a current-income credit constraint but welfare reducing with a future-income one (either for total or disposable income). With a current-income collateral constraint, a macroprudential debt tax reduces the variability of debt levels which helps to mitigate the negative effects when the credit constraint binds. However, since borrowing decisions are already constrained efficient under a future-income collateral constraint, the macroprudential policy ends up distorting those decisions with this type of constraint, reducing welfare. Instead, an ex post debt tax (returned to households via a lump-sum subsidy) increases disposable income but does not affect total income, and hence has a welfare-improving impact when the economy faces a disposable-income credit constraint and no effect when it is a total-income one.

For the scenarios considered, the policy that would reduce welfare is the macroprudential debt tax if issued in an economy with collateral constraint determined by future income. In that sense, in order to avoid unexpectedly reducing welfare, the most favorable policy is an ex post debt tax; at a minimum, it does not affect welfare, but it could increase it if the collateral constraint depends on disposable income.

## REFERENCES


1. Benigno, Gianluca & Chen, Huigang & Otrok, Christopher & Rebucci, Alessandro & Young, Eric R., 2016. "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective," Journal of Monetary Economics, Elsevier, vol. 84(C), pages 147-165.

2. Bianchi, Javier, 2011. "Overborrowing and Systemic Externalities in the Business Cycle," American Economic Review, American Economic Association, vol. 101(7), pages 3400-3426, December.

3. Bianchi, Javier, 2016. "Efficient Bailouts?," American Economic Review, American Economic Association, vol. 106(12), pages 3607-3659, December.

4. Bianchi, Javier & Mendoza, Enrique, 2018. "Optimal Time-Consistent Macroprudential Policy," Journal of Political Economy, University of Chicago Press, vol. 126(2), pages 588-634.

5. Devereux, Michael B. & Young, Eric R. & Yu, Changhua, 2019. "Capital controls and monetary policy in sudden-stop economies," Journal of Monetary Economics, Elsevier, vol. 103(C), pages 52-74.

6. Jeanne, Olivier & Korinek, Anton, 2019. "Managing credit booms and busts: A Pigouvian taxation approach," Journal of Monetary Economics, Elsevier, vol. 107(C), pages 2-17.

7. Jeanne, Olivier & Korinek, Anton, 2020. "Macroprudential Regulation versus mopping up after the crash," Review of Economic Studies, Oxford University Press, vol. 87(3), pages 1470-1497.

8. Korinek, Anton. (2011). The new economics of Prudential capital controls: A research agenda. IMF Economic Review, 59(3), 523–561.

9. Mendoza, Enrique (2002). Credit, prices, and crashes: Business cycles with a sudden stop. In S. by Edwards & J. A. Frankel (Eds.), Preventing currency crises in emerging

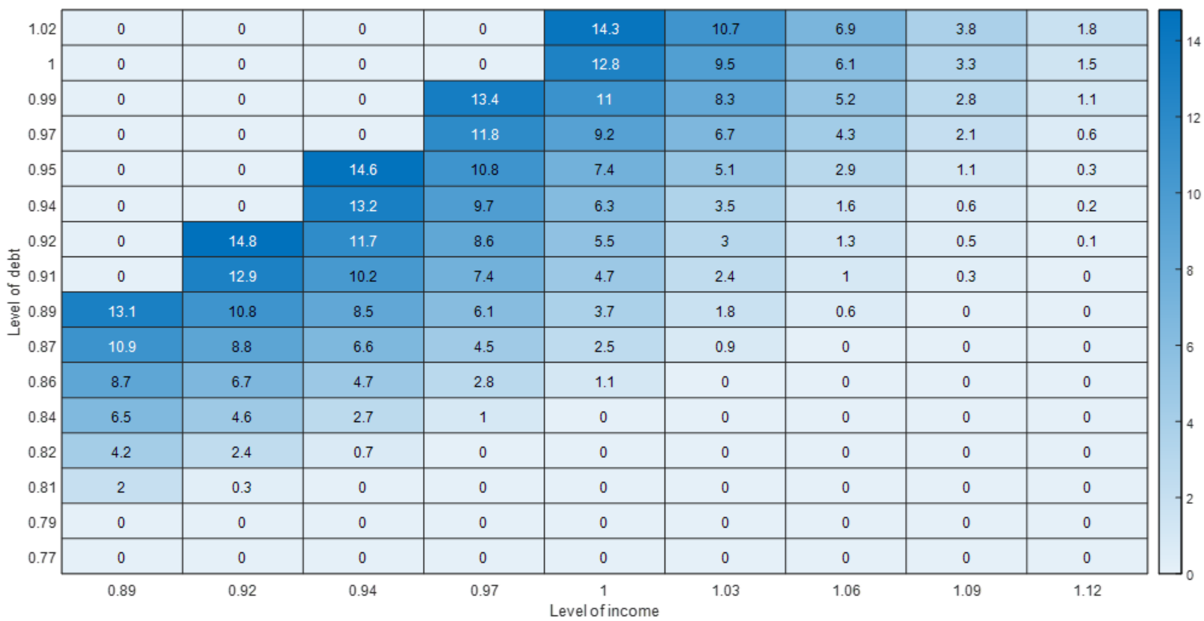
markets. Chicago, IL: University of Chicago Press and National Bureau of Economic Research.

10. Ottonello, Pablo & Perez, Diego J. & Varraso, Paolo, 2022. "Are collateral-constraint models ready for macroprudential policy design?," Journal of International Economics, Elsevier, vol. 139(C).

11. Vargas, Carmiña O. & Julian A. Parra-Polania, 2021. "Analysis of macro-prudential and ex post financial crisis interventions: Relevance of the fiscal-policy setup," International Journal of Finance & Economics, John Wiley & Sons, Ltd., vol. 26(3), pages 3759-3769, July.

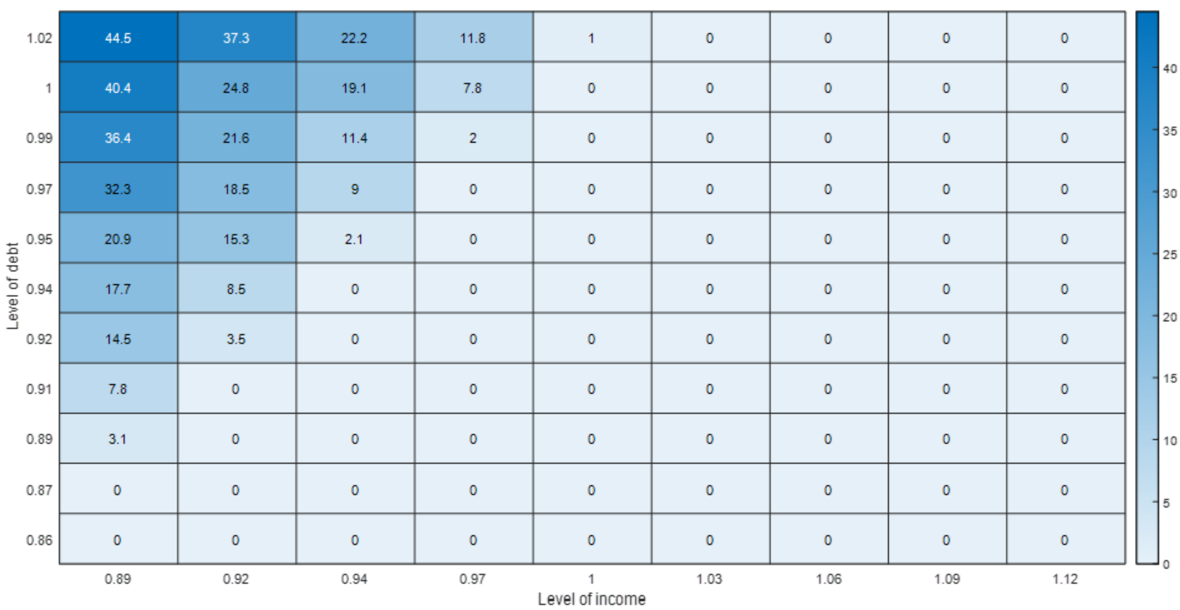
## APPENDIX

### Figure A1. Macroprudential tax values



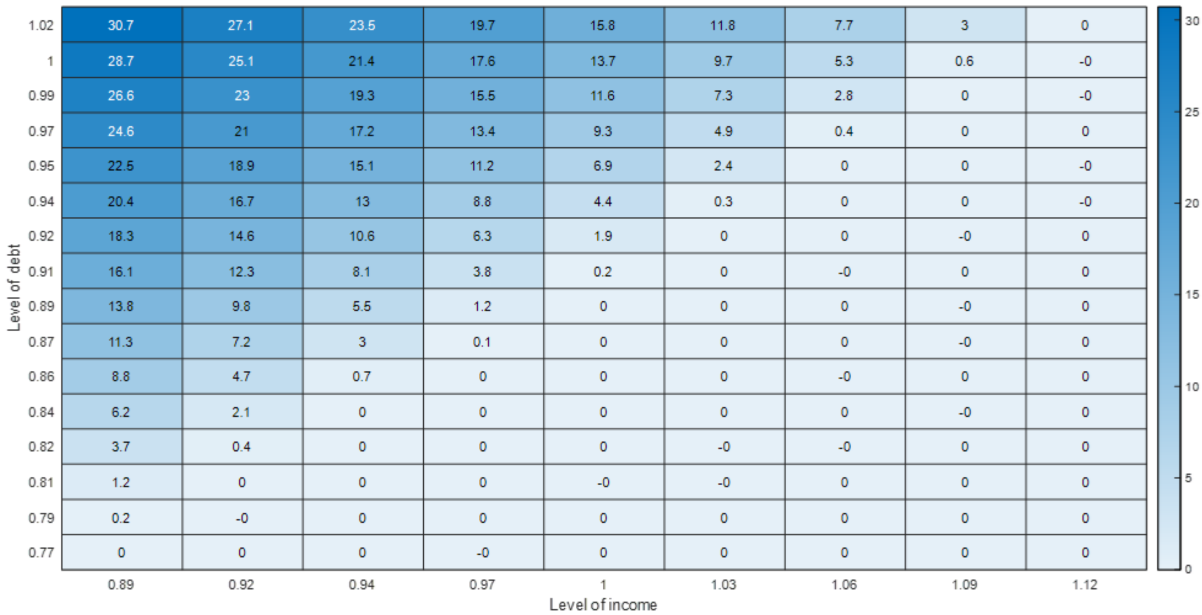

Darker blue indicates a higher level of tax (the value in percentage is inside the cell).
The macroprudential debt tax is by definition nil for constrained periods (i.e., for combinations of high debt an low income)

### Figure A2. Ex post tax values for the current-disposable-income case




Darker blue indicates a higher level of tax (the value in percentage is inside the cell)
The debt tax is nonzero for potentially constrained periods only.

**Figure A3. Ex post tax values for the future-disposable-income case**




Darker blue indicates a higher level of tax (the value in percentage is inside the cell)

The debt tax is nonzero for potentially constrained periods only.