BORRADORES DE ECONOMÍA

Colombian Framework for Fiscal Policy Economic Evaluation

By:
Andrés Nicolás Herrera-Rojas
David Camilo López-Valenzuela
Juan J. Ospina-Tejeiro
Jesús Bejarano

No. 1324
2025

BANCO DE LA REPUBLICA
COLOMBIA

Colombian Framework for Fiscal Policy Economic Evaluation

Andrés Nicolás Herrera-Rojas

David Camilo López-Valenzuela

Juan J. Ospina-Tejeiro

Jesús Bejarano *

Abstract

Fiscal policy and monetary policy are intricately connected, as government policies influence various macroeconomic variables, affecting prices and, consequently, Central Bank decisions. To elucidate this relationship, this document outlines the model employed by Banco de la República's technical staff to analyze and quantify the impact of fiscal policy on key macroeconomic variables, featuring a detailed fiscal structure that distinguishes it from other models. Furthermore, as an illustration of the analyses conducted with this model, the document presents a simulation that assumes an increase in the risk premium, examining two potential responses the government might consider under tighter financial conditions: adjusting public investment to comply with the fiscal rule, or not adjusting public expenses and incurring in additional debt, leading to non-compliance with the fiscal rule. When the adjustment involves reducing public investment, domestic demand, inflation, and the monetary policy rate decrease. Conversely, financing the revenue shortfall with public debt, thereby failing to comply with the fiscal rule, exacerbates negative effects on private demand.

Key Words: Risk premium, fiscal adjustments, dynamic stochastic general equilibrium model, fiscal rule.
JEL Classification: E44, E62, E63, H30

*Economists of the Banco de la República. Emails: aherrero@banrep.gov.co, dlopezva@banrep.gov.co, jospinte@banrep.gov.co jbejarro@banrep.gov.co.
We thank Andrés González, Diego Rodríguez and Juan Pablo Ángel who participated in an initial version of the development of the model presented in this paper through an IMF mission.

Modelo para el estudio de los efectos macroeconómicos de la política fiscal en Colombia

Herrera-Rojas Andrés Nicolás.

López-Valenzuela David Camilo.

Ospina-Tejeiro Juan J.

Bejarano Jesús.

Resumen

La política fiscal y la política monetaria están estrechamente relacionadas, debido a que las diferentes medidas fiscales adoptadas tienen un efecto sobre el resto de las variables macroeconómicas, afectando los precios y con ello las decisiones del Banco Central. Para entender mejor esta relación, en este documento se muestra el modelo de equilibrio general usado por el Banco de la República para analizar y cuantificar el impacto de la política fiscal sobre las principales variables económicas y que a diferencia de otros modelos incluye un detallado bloque fiscal. Adicionalmente, como parte de los análisis realizados con el modelo, se presenta un ejercicio de simulación que supone un incremento de la prima de riesgo, explorando dos caminos alternativos que el gobierno podría llevar a cabo: ajustar el gasto de inversión para cumplir con la regla fiscal o no ajustar el gasto y recurrir a mayor deuda, lo cual conllevaría a un incumplimiento de la regla fiscal. Cuando el ajuste se hace por medio de una reducción en la inversión pública, la demanda doméstica, la inflación y la tasa de interés son menores. Sin embargo, financiar el menor ingreso por medio de un mayor endeudamiento, incumpliendo la regla fiscal, profundiza los efectos negativos sobre la demanda privada.

Palabras Clave: Prima de riesgo, ajuste fiscal, modelo de equilibrio general dinámico estocástico, regla fiscal.
Clasificación JEL: E44, E62, E63, H30

1 Introduction

COFFEE (Colombian Framework for Fiscal Policy Economic Evaluation) is a newly developed dynamic macroeconomic model designed to enhance policy analysis in Colombia's small open economy. The model is the result of a collaborative effort between Banco de la República (Colombia's central bank), the Ministry of Finance and Public Credit, and the International Monetary Fund (IMF), with technical support from the IMF's Institute for Capacity Development. COFFEE provides a unified and internally consistent framework for evaluating fiscal policy, debt dynamics, and external shocks in conjunction with monetary policy. In response to recent macroeconomic challenges—including commodity price volatility, the COVID-19 pandemic, and heightened global financial uncertainty—Colombian policymakers identified the need for an integrated tool capable of simulating complex policy interactions. COFFEE meets this need by combining detailed fiscal architecture with robust macroeconomic foundations, making it a valuable platform for scenario analysis and medium-term policy assessment.

A distinguishing feature of COFFEE is its granular fiscal structure, which enables simulation of a broad range of tax and expenditure reforms. Unlike conventional DSGE models, COFFEE explicitly incorporates multiple revenue instruments and spending categories, thereby allowing a more nuanced analysis of fiscal trade-offs. The model's key fiscal channels include:

- Consumption taxes, including VAT and excise duties, applied to both domestic production and imports.
- Labor income taxes, which affect household take-home pay and labor supply decisions.
- Corporate and capital income taxes, including taxes on firm profits, dividends, and household wealth.
- Public expenditure composition, with a distinction between productive investment (which augments public capital) and non-productive spending (e.g., government consumption and transfers).

By modeling these components separately, COFFEE can evaluate the macroeconomic implications of diverse policy packages. For instance, the model has been applied internally at Banco de la República to assess the macroeconomic effects of recent Colombian tax reforms—such as the Ley de Inversión Social and the Reforma Tributaria para la Igualdad y Justicia—highlighting how different tax instruments propagate through consumption, investment, and the labor market. This fiscal detail provides policymakers with a versatile "policy laboratory" to test options such as adjusting the mix of tax burdens or reallocating spending across sectors, and to quantify their impacts on output, inflation, and public debt over time.

COFFEE is built on a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework, tailored to reflect Colombia's economic structure. It includes standard nominal and real frictions—such as Rotemberg-style price and wage rigidity, habit formation, and investment adjustment costs—ensuring that monetary policy has realistic short-run effects. The model features heterogeneous households: Ricardian households, who optimize intertemporally, and non-Ricardian households, who are liquidity-constrained and respond more directly to disposable income. On the production side, the model includes various firm types—domestic goods producers, capital goods producers, and importers—enabling it to simulate a broad set of transmission mechanisms.

This structure allows COFFEE to capture both the direct and indirect effects of fiscal changes. For example, a labor tax hike reduces labor supply incentives and disposable income, particularly affecting non-Ricardian households, while a change in VAT directly influences consumption prices and demand. The model's micro foundations offer a high degree of realism for short-run dynamics, without sacrificing theoretical coherence or a well-defined long-run equilibrium.

A core innovation of COFFEE is its detailed treatment of fiscal rules and public debt dynamics—critical features for a fiscally constrained emerging economy. The model embeds Colombia's institutional fiscal rule, which targets a structural fiscal balance adjusted for the economic cycle and commodity price fluctuations. The fiscal authority's budget constraint explicitly links spending and revenues to borrowing, allowing simulation of debt accumulation under various scenarios. Public debt can be financed through either domestic or external issuance, and the model projects the debt-to-GDP trajectory over time, taking into account policy choices and macroeconomic conditions.

Importantly, COFFEE endogenizes the interaction between public debt and sovereign risk. As public debt rises, the model assumes that sovereign risk premia increase, making external borrowing more expensive—an effect grounded in empirical findings. This feedback mechanism allows simulation of external financial shocks, such as a global risk-off event or sudden stop in capital flows, and their transmission to the domestic economy through rising borrowing costs, reduced investment, exchange rate depreciation, and constrained private credit. This channel is crucial for understanding how fiscal vulnerabilities can limit monetary policy space.

As a small open economy model, COFFEE also incorporates trade and commodity price channels. Colombia's oil-exporting status is modeled explicitly, with an oil sector generating revenue for both the public and private sectors. Oil price fluctuations affect government revenues and export income, feeding into the broader macroeconomic outlook. The model also includes imported goods for consumption and investment, enabling exchange rate dynamics and foreign inflation to pass through to domestic prices. Traditional open-economy features—such as uncovered interest parity, external current account balances, and terms-of-trade shocks—are integrated to provide a full representation of Colombia's external linkages. This makes COFFEE capable of analyzing scenarios such as tightening global financial conditions, commodity price crashes, or demand shocks from trading partners.

COFFEE represents an important evolution in Colombia's macroeconomic modeling toolkit. Earlier models like PATACON (Policy Analysis Tool Applied to Colombian Needs) provided a solid New Keynesian DSGE foundation for monetary policy analysis and medium-term forecasting. More recently, the 4GM model introduced semi-structural features tailored to an oil-exporting economy, incorporating inflation decomposition and stylized shocks. COFFEE complements these frameworks by adding greater fiscal granularity and debt realism, and by better capturing the joint dynamics of fiscal, external, and monetary sectors.

In the international context, COFFEE is aligned with advanced macroeconomic models developed by peer institutions. It is comparable in structure and scope to the MEDEA model of the Bank of Spain, which incorporates detailed fiscal instruments, nominal rigidities, and an open economy framework (Bank of Spain (2019)). Similarly, it shares core features with the Bank of Canada's Global Economy Model (GEM), particularly the integration of monetary and fiscal interactions in a New Keynesian DSGE setting (Bank of Canada (2006)). Moreover, COFFEE builds on conceptual foundations present in the IMF's Global Projection Model (GPM) and the Global Integrated Monetary and Fiscal Model (GIMF), both widely used for cross-country scenario analysis and fiscal-monetary coordination (Freedman, C., Kumhof, M., Laxton, D., Muir, D., and Mursula, S. (2010); Muir, D., and Laxton, D. (2017)). COFFEE benefits directly from collaboration with the IMF and draws on global modeling expertise to reflect international best practices.

In summary, COFFEE is a policy-relevant, technically rigorous, and institutionally grounded tool designed to support macroeconomic decision-making in Colombia. It offers a flexible platform to assess the macroeconomic and fiscal implications of tax reforms, spending adjustments, external shocks, and monetary-fiscal interactions. By enabling structured scenario analysis and consistent projection of key variables—such as output, inflation, debt, and interest rates—COFFEE enhances Colombia's capacity for evidence-based policy design. The following sections describe the model's formal structure and demonstrate its application to critical policy questions facing the Colombian economy.

The rest of the document is composed by the description of the model in section 2, model calibration in section 3, simulation of the increase in risk premium and results in section 4 and final remarks in section 5.

2 Model Structure

This section outlines the theoretical and mathematical features of the Dynamic Stochastic General Equilibrium (DSGE) model. The model represents a New Keynesian small open economy, incorporating a fiscal framework that encompasses government revenues, expenditures, and public debt. These elements facilitate an interaction between fiscal policy and monetary policy.

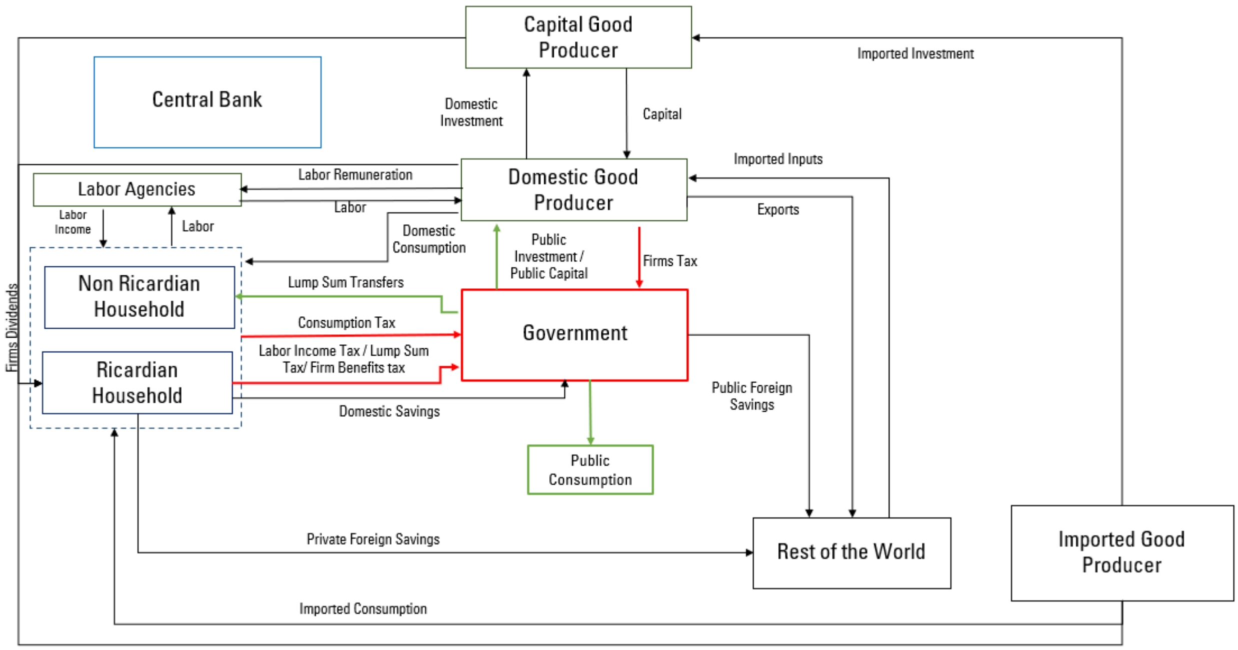

Figure 1: Economic Flows in the Model

This model, specifically calibrated for the Colombian context in quarterly frequency, encompasses nominal rigidities in prices and wages, adjustment costs related to investment and production factors and a dynamic interaction among different economic agents. These agents are composed by two types of households (Ricardian and non-Ricardian), three categories of firms (domestic, capital, and imported goods producers), a central bank making policy decisions based on a Taylor rule and a fiscal authority managing revenues from taxes, public debt, and oil income through a fiscal rule. As a small open economy, it also considers international relations via exports, imports, and external credit. Notably, the oil sector plays a crucial role, with exogenous oil production generating income distributed between households with access to the credit market and the government. Additionally, a portion of the oil income is exported in the form of remittances of profits.

Figure (1) illustrates the linkages among the different agents in our model. Ricardian households have the option to save using public domestic bonds at a domestic interest rate or external bonds at a foreign interest rate. Additionally, they hold ownership stakes in domestic goods firms and partially in oil endowments. In contrast, Non-Ricardian households do not own any firms and consequently do not receive firm utilities or oil income. Instead, they rely on lump-sum government transfers and remittances. These households face consumption constraints due to their lack of access to credit markets, resulting in the consumption of all received income within each period.

Together, Ricardian and Non-Ricardian households collectively demand a consumption bundle comprising both domestic and imported goods. They supply labor to labor agencies, which aggregate differentiated labor supplies and negotiate wages with domestic goods producers. The presence of intermediate labor firms introduces wage disparities between domestic goods producers and households. These wage differences arise from the margin charged by labor agencies for their intermediation services, with the remaining amount allocated as wages to households. Furthermore, households pay taxes on labor income, which, combined with consumption taxes, influences consumption-labor decisions and impacts the overall budget constraint.

Domestic producers of goods use physical capital, imported inputs from the global market, and public investments in infrastructure in their production processes. Private capital, sourced from capital producers, incurs rental fees. Imported inputs are acquired at external prices. Public capital, such as government investments in infrastructure, acts as a positive externality, enhancing production. These combined inputs enable firms to produce domestic goods for consumption, investment, and exports.

Regarding domestic good production, it represents the output of aggregating a continuum of differentiated domestic goods produced by many different firms operating under monopolistic competition. These latter firms face adjustment costs in pricing and are subject to dividend taxes

Domestic investment serves as a crucial resource for capital producers, enabling them to make up private capital that is subsequently leased to domestic goods firms. Additionally, capital producers engage in imported investment, sourced from imported goods firms. Therefore, domestic and imported investment jointly compose the total investment bundle.

Within this framework, capital firms play an important role. They make investment decisions, providing capital to domestic good producers in exchange for rent. However, this arrangement comes with certain obligations: capital firms must pay taxes on the rent they receive, and they also face adjustment costs related to their investment decisions.

Imported goods firms have a role in the sale of imported investment and consumption goods. Consequently, the consumption bundle comprises both domestically produced consumption (by domestic firms) and imported consumption. Similar to domestic firms, the imported goods firms aggregate a continuum of differentiated imported goods. These goods are individually and slightly transformed by monopolistic firms and face adjustment costs on prices like domestic good firms do. In contrast, exports consist of a portion of the domestic goods specifically allocated for this purpose, along with oil production. The export of domestic goods depends on exchange rates and foreign demand. The difference between exports and imports constitutes the trade balance in the model, which, in conjunction with remittances, interest payments, and debt flows, shapes the current account and the balance of payments.

The monetary authority makes decisions regarding the policy interest rate based on a Taylor rule, which relies on the expected inflation gap and the output growth gap. In contrast, the fiscal authority receives revenue from tax payments and allocates it to finance public expenses. These public expenses can be allocated as either productive or nonproductive. Productive expenses contribute to the accumulation of public capital, which, in turn, supports domestic goods production. On the other hand, nonproductive expenses lack these positive externalities.

Additional sources of resources for financing these public expenses include oil revenues, domestic debt (associated with Ricardian households), and external debt resulting from interactions with the rest of the world. Meanwhile, total expenses encompass transfers to non-Ricardian households and debt services, specifically interest payments related to public debt.

Since our model represents a small open economy, external prices are considered exogenous. They are directly multiplied by the exchange rate to convert them into domestic currency. The foreign interest rate, which impacts both Ricardian households and government debt payments, comprises a risk-free rate and a risk premium. The latter is determined by an exogenous shock and deviations in the foreign net assets-to-GDP ratio from a long-run value. We assume technological progress, ensuring productivity growth at a constant rate, and standardize the model for stationary variables. Notably, the final goods price implicitly reflects the value added tax, resulting in a discrepancy between the prices set by firms and those paid by consumers for the goods they consume.

2.1 Households

The model incorporates two different types of households. The first difference between these households lies in their access to the credit market. Specifically, we have Ricardian households, who can use their assets to smooth consumption, and non-Ricardian households, who lack this capability. Ricardian agents, who also have a role as firm owners, receive benefits and fulfill their tax obligations to the government. In contrast, non-Ricardian agents do not possess ownership stakes in firms but instead receive lump-sum transfers from the government and remittances from the global economy. Consequently, Ricardian households constitute a fraction of the entire economy $(1-f)$, while non-Ricardian households account for the remaining f fraction.

2.1.1 Ricardian Households

The problem of the household consists in maximize the discounted sum of his utility subject to a budget constraint. It is assumed that household has GHH preferences similar to González, A., López, M., Rodríguez, N. and Téllez, S. (2013) on consumption C_t^o and labor N_t^o. Besides, shocks on marginal utility z_t^c and labor supply z_t^N are included:

$$U(C_t^o, N_t^o) = z_t^c \frac{1}{1-\sigma} \left\{ C_t^o - z_t^N \frac{1}{1+\eta_o} A_t \left(N_t^o\right)^{1+\eta_o} \right\}^{1-\sigma}$$

where σ is the intertemporal elasticity of substitution and η_o is the inverse of Frish elasticity.

The household's budget constraint is a composite of expenses and incomes. We assume that the household allocates its spending toward a consumption bundle priced at P_t (implicitly influenced by the value added tax, as detailed in section 2.1.3). Additionally, given the access to credit markets, the household can save either in domestic bonds with the government (B_t^o) denominated in local currency or in private external bonds with the rest of the world $(B_t^{o,f})$ valued in foreign currency and multiplied by the nominal exchange rate S_t. Consequently, the total household expenses consist of the sum of these savings, tax payments T_t^o, and the consumption bundle.

Household incomes are derived from labor wages V_t, the interest earned on domestic and foreign savings (i_{t-1}, i_{t-1}^f), firms benefits D_t^o[1] and a fraction of the revenues coming from oil production $(1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{o,oil}$. Oil revenues are composed by the nominal exchange rate, external oil price and oil production: $\Pi_t^{o,oil} = S_t P_t^{oil} Y_t^{oil}$ [2]. Finally, household pay taxes for consumption τ_t^c, labor incomes $\tau_t^{N,o}$ and firms benefits τ_t^D.

The Ricardian household selects consumption, labor supply and domestic and foreign savings to maximize their utility, discounted by a discount factor β:

$$\max_{C_t^o, N_t^o, B_t^o, B_t^{o,f}} E_0 \sum_{t=0}^{\infty} \beta^t z_t^c \frac{1}{1-\sigma} \left\{ C_t^o - z_t^N \frac{1}{1+\eta_o} A_t \left(N_t^o\right)^{1+\eta_o} \right\}^{1-\sigma}$$

subject a budget constraint:

$$\begin{aligned}(1+\tau_t^c)P_t C_t^o + B_t^o + S_t B_t^{o,f} + T_t^o = \\ (1-\tau_t^{N,o})V_t N_t^o + (1+i_{t-1})z_{t-1}^{SW} B_{t-1}^o + (1+i_{t-1}^f)z_{t-1}^{SW} S_t B_{t-1}^{o,f} + (1-\tau_t^D)D_t^o + (1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{o,oil}\end{aligned}$$

z_t^{SW} is a exogenous shock that affects the private demand since it increases the interest rate agents face. The first order conditions of the problem are:

$$[C_t^o] : z_t^c \left\{ C_t^o - z_t^N \frac{1}{1+\eta_o} A_t \left(N_t^o\right)^{1+\eta_o} \right\}^{-\sigma} = \Lambda_t^o (1+\tau_t^c) P_t \tag{1}$$

[1] Includes benefits of domestic good producers after taxes $D_t^{o,d,a}$, capital producers $D_t^{o,k}$, labor agencies $D_t^{o,w}$ and imported good firms $D_t^{o,f}$: $D_t^o = D_t^{o,d,a} + D_t^{o,k} + D_t^{o,w} + D_t^{o,f}$

[2] Assume one part of the of the oil revenues is exported to the rest of the world in form of remittances. The other part stays inside the country and is distributed between the government and the ricardian household. Details in section 2.5. Besides, the oil income is distributed annually.

$$[N_t^o] : z_t^c \left\{ C_t^o - z_t^N \frac{1}{1+\eta_o} A_t \left(N_t^o\right)^{1+\eta_o} \right\}^{-\sigma} A_t z_t^N (N_t^o)^{\eta_o} = \Lambda_t^o (1-\tau_t^{N,o}) V_t \tag{2}$$

$$[B_t] : \Lambda_t^o = \beta \Lambda_{t+1}^o (1+i_t) z_t^{SW} \tag{3}$$

$$[B_t^f] : \Lambda_t^o S_t = \beta \Lambda_{t+1}^o (1+i_t^f) z_t^{SW} S_{t+1} \tag{4}$$

Λ_t^o is the lagrange multiplier of the problem. Combining equations (1) and (2) labor supply is found:

$$(N_t^o)^{\eta_o} = \frac{(1-\tau_t^{N,o})}{(1+\tau_t^c)} \frac{V_t}{P_t} \frac{1}{A_t z_t^N} \tag{5}$$

Using equation (3) and (4) the parity condition between domestic and external interest rate is obtained:

$$(1+i_t) = (1+i_t^f) \frac{S_{t+1}}{S_t} \tag{6}$$

2.1.2 Non Ricardian Households

In contrast to Ricardian households, these non-Ricardian households lack access to credit markets, preventing them from saving or incurring debt through domestic or foreign bonds to smooth their consumption. Additionally, they do not hold ownership stakes in firms, which means they do not receive benefits or oil revenues. Consequently, their incomes consist of labor wages (V_t), lump-sum transfers from the government (T_t^l) and foreign remittances from the rest of the world (REM_t^l). Their expenses are primarily driven by consumption and the consumption tax.

Non-Ricardian households' preferences are similar to Ricardian agents preferences concerning consumption and labor. Here, η_l represents the inverse of the Frisch elasticity. Their problem lies in selecting the consumption and labor choices that maximize the expected value of utility:

$$\max_{C_t^l, N_t^l} E_0 \sum_{t=0}^{\infty} \beta^t z_t^c \frac{1}{1-\sigma} \left\{ C_t^l - z_t^N \frac{1}{1+\eta_l} A_t \left(N_t^l\right)^{1+\eta_l} \right\}^{1-\sigma}$$

subject to their budget constraint[3]:

$$(1+\tau_t^c) P_t C_t^l = (1-\tau_t^{N,l}) V_t N_t^l + T_t^l + REM_t^l$$

First order conditions are:

$$[C_t^l] : z_t^c \left\{ C_t^l - z_t^N \frac{1}{1+\eta_l} A_t \left(N_t^l\right)^{1+\eta_l} \right\}^{-\sigma} = \Lambda_t^l (1+\tau_t^c) P_t \tag{7}$$

$$[N_t^l] : z_t^c \left\{ C_t^l - z_t^N \frac{1}{1+\eta_l} A_t \left(N_t^l\right)^{1+\eta_l} \right\}^{-\sigma} A_t z_t^N (N_t^l)^{\eta_l} = \Lambda_t^l (1-\tau_t^{N,l}) V_t \tag{8}$$

Λ_t^l is the lagrange multiplier of the problem. Combining equations (7) and (8), the labor supply of non ricardian household is found:

$$(N_t^l)^{\eta_l} = \frac{(1-\tau_t^{N,l})}{(1+\tau_t^c)} \frac{V_t}{P_t} \frac{1}{A_t z_t^N} \tag{9}$$

[3]Taxes on income labor to non ricardian households are included in the algebra of the model, however in the calibration of tax collection, non ricardian households do not pay this kind of taxes (that say, $\tau_t^{N,l} = 0$) so all tax revenue of income labor comes from ricardian households.

2.1.3 Consumption Bundle

The total consumption bundle consists of domestic consumption C_t^d and imported consumption C_t^f. Consequently, households solve an expense minimization problem to determine the optimal paths for each type of consumption, considering domestic prices P_t^d and imported prices P_t^f. This optimization is subject to an aggregator bundle represented by a CES (Constant Elasticity of Substitution) function:

$$\min_{C_t^d, C_t^f} (1+\tau_t^{vat})P_t^d C_t^d + (1+\tau_t^{vat})P_t^f C_t^f$$
$$\text{s.a } C_t = \left[\omega_c^{\frac{1}{\theta_c}} {C_t^d}^{\frac{\theta_c - 1}{\theta_c}} + (1-\omega_c)^{\frac{1}{\theta_c}} {C_t^f}^{\frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}}$$

The aggregated value tax (VAT), τ_t^{vat}, holds an important role in our model. Furthermore, we consider two parameters: ω_c, representing the fraction of domestic consumption within the overall consumption, and θ_c, which characterizes the elasticity of substitution between domestic and imported consumption. Solving the optimization problem provides insights into the demand for each consumption type.

$$C_t^d = \omega_c \left[\frac{(1+\tau_t^{vat})P_t^d}{P_t} \right]^{-\theta_c} C_t \tag{10}$$

$$C_t^f = (1-\omega_c) \left[\frac{(1+\tau_t^{vat})P_t^f}{P_t} \right]^{-\theta_c} C_t \tag{11}$$

Replacing the demands (10) and (11) in the aggregate bundle, the price index is obtained:

$$P_t = \left[\omega_c \left[(1+\tau_t^{vat})P_t^d \right]^{1-\theta_c} + (1-\omega_c) \left[(1+\tau_t^{vat})P_t^f \right]^{1-\theta_c} \right]^{\frac{1}{1-\theta_c}}$$

Defining $P_t^{dF} = (1+\tau_t^{vat})P_t^d$ and $P_t^{fF} = (1+\tau_t^{vat})P_t^f$, it is possible to rewrite the previous expression as:

$$P_t = \left[\omega_c (P_t^{dF})^{1-\theta_c} + (1-\omega_c)(P_t^{fF})^{1-\theta_c} \right]^{\frac{1}{1-\theta_c}} \tag{12}$$

where P_t^d, P_t^f are the prices before taxes and P_t^{dF}, P_t^{fF} are the prices after taxes [4].

2.2 Labor Agencies and Wages Rigidities

Labor relationships between households and domestic goods firms operate through labor agencies that act as intermediaries bridging both agents. Initially, a labor agency combines Ricardian and non-Ricardian labor, subsequently selling this composite labor to another aggregator agency that supplies it to domestic goods producers. The model assumes no differences in the capacities of the two household types. Consequently, labor agencies are indifferent to hiring either Ricardian or non-Ricardian labor. However, there exist $N_{i,t}$ labor varieties, uniformly distributed across agents. These labor varieties (indexed by i) are aggregated using a the function:

$$N_t = \left[\int_0^1 (N_{i,t})^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}}$$

Here, θw represents the elasticity of substitution between labor varieties. Aggregated labor is sold to domestic goods firms, and the labor aggregator agency receives a remuneration denoted as W_t. Consequently, the labor aggregator agency faces the problem of maximizing its benefits by selecting the labor varieties i purchases at a price $W_{i,t}$, all subject to the constraints imposed by the aggregation function.

$$\max_{N_{i,t}} W_t N_t - \int_0^1 W_{i,t} N_{i,t} di$$

[4] This helps to reflects the additional increase of final prices given the aggregate value tax. Note that the numerary price of the model, P_t, will be affected by this tax, that means, the final price is composed by the prices (domestic and imported) .

By solving the problem, we determine the demand for each labor variety i. This demand depends negatively on its cost and positively on the total labor available

$$N_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} N_t$$

Labor agencies operate in monopolistic competition, combining Ricardian and non-Ricardian labor to sell it to the aggregator labor agency at a price $W_{i,t}$. These labor agencies compensate households with remuneration denoted as V_t, and the difference between $W_{i,t}$ and V_t constitutes the labor agencies' benefits. Given the demand for each labor variety, the labor agency selects the price at which to sell labor, aiming to maximize its benefits while considering adjustment costs related to $W_{i,t}$.

$$\max_{W_{i,t}} E_0 \sum_{j=0}^{\infty} \beta^j \left(\frac{\Lambda^o_{t+j}}{\Lambda^o_t}\right) \left\{ (W_{i,t+j} - V_{t+j})N_{i,t+j} - \frac{\zeta_w}{2}\left(\frac{W_{i,t+j}}{(1+\pi^w_{t+j-1})^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w} W_{i,t+j-1}} - 1\right)^2 W_{t+j}N_{t+j} \right\}$$

s.a

$$N_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} N_t$$

where ι_w is the wage inflation index and ζ_w is the adjustment cost parameter (the larger ζ_w is, the cost to adjust $W_{i,t}$ is bigger). Solving the problem:

$$\begin{aligned}
&(1-\theta_w)\left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} N_t + \theta_w V_t \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} \frac{N_t}{W_{i,t}} - \zeta_w W_t N_t \left(\frac{W_{i,t}}{(1+\pi^w_{t-1})^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w} W_{i,t-1}} - 1\right) \\
&\left(\frac{1}{(1+\pi^w_{t-1})^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w} W_{i,t-1}}\right) + \beta\left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t}\right)\zeta_w \left(\frac{W_{t+1}}{W_{i,t}}\right) N_{t+1} \left(\frac{W_{i,t+1}}{(1+\pi^w_t)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w} W_{i,t}} - 1\right) \\
&\qquad \left(\frac{W_{i,t+1}}{(1+\pi^w_t)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w} W_{i,t}}\right) = 0
\end{aligned}$$

$\frac{W_t}{W_{t-1}} = (1+\pi^w_t)$ and at equilibrium $W_t = W_{i,t}$, so Phillips curve of wages is:

$$\begin{aligned}
&\zeta_w \left(\frac{(1+\pi^w_t)}{(1+\pi^w_{t-1})^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)\left(\frac{(1+\pi^w_t)}{(1+\pi^w_{t-1})^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}\right) W_t N_t = (1-\theta_w) W_{i,t} N_t + \theta_w V_t N_t \\
&\quad + \beta\left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t}\right)\zeta_w \left(\frac{(1+\pi^w_{t+1})}{(1+\pi^w_t)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)\left(\frac{(1+\pi^w_{t+1})}{(1+\pi^w_t)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}\right) W_{t+1}N_{t+1} \qquad (13)
\end{aligned}$$

The mark-up θ_w can be expressed as $\theta_w = \overline{\theta_w} * exp(z^w_t)$ where z^w_t is a exogenous shock over the wage inflation. Finally, aggregated benefits are:

$$D^w_t = (W_t - V_t)N_t - \frac{\zeta_w}{2}\left(\frac{(1+\pi^w_t)}{(1+\pi^w_{t-1})^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)^2 W_t N_t \qquad (14)$$

2.3 Firms

In the model there are 3 different types of firms: those producing domestic goods, imported goods and capital goods. The household with access to the financial market is owner of the firms and therefore receive the benefits of each one. The problem of the firms is described below.

2.3.1 Domestic Goods Producing Firms

In the context of domestic goods production, there are two different types of firms: those involved in producing final goods and those specializing in intermediate goods. Firms engaged in final goods production operate within a perfectly competitive market. Their central problem lies in determining the optimal quantities of intermediate goods $Y^d_{i,t}$, priced at $P^d_{i,t}$. These intermediate goods are subsequently aggregated into a final good Y^d_t, which serves multiple purposes: domestic consumption (both private and public), domestic investment (both private and public), and exports. The goal for these firms is profit maximization where revenues stem from selling the final good at price P^d_t, while costs are directly associated with the acquisition of intermediate goods.[5]:

$$\max_{Y^d_{i,t}} P^d_t Y^d_t - \int_0^1 P^d_{i,t} Y^d_{i,t} di$$

Subject to the aggregation function of intermediate goods:

$$Y^d_t = \left[\int_0^1 (Y^d_{i,t})^{\frac{\theta_d - 1}{\theta_d}} di\right]^{\frac{\theta_d}{\theta_d - 1}}$$

Where θ_d represents the elasticity of substitution of intermediate goods, solving the problem yields the demand for intermediate goods produced by the firm manufacturing the final product.

$$Y^d_{i,t} = \left(\frac{P^d_{i,t}}{P^d_t}\right)^{-\theta_d} Y^d_t \tag{15}$$

Intermediate goods firms produce this good using labor, capital, imported raw materials and public capital. Labor is demanded from labor unions and pays the wage W_t; capital is demanded from the firm producing the capital good and pays the rent R^k_t; raw materials are imported from abroad at the price P^M_t[6]; and public capital acts as a positive externality on production. The production function is assumed to be of Cobb-Douglas type represented by the following expression:

$$Y^d_{i,t} = z_t \left(A_t N_{i,t}\right)^{1-\alpha_k-\alpha_{M^d}} \left(K_{i,t-1}\right)^{\alpha_k} \left(M^d_{i,t}\right)^{\alpha_{M^d}} \left(\frac{K^g_{t-1}}{A_t}\right)^{\psi} \tag{16}$$

α_k represents the share of capital in production, while α_{M^d} represents the share of imported raw materials in production. The remaining $1 - \alpha_k - \alpha_{M^d}$ corresponds to the share of labor in production and ψ represents the elasticity of the public capital externality in production. z_t denotes a productivity shock and A_t refers to technical progress that is neutral in labor. Additionally, intermediate goods firms operate in monopolistic competition, which gives them market power so they can choose the price at which they sell their products to the final goods firm. However, there exists a quadratic adjustment cost similar to the one analyzed by Rincón, H., Rodríguez, D., Toro, J. and Téllez, S. (2014):

$$\varphi^d_{i,t} = \frac{\zeta_d}{2}\left(\frac{P^d_{i,t+j}}{(1+\pi^d_{t+j-1})^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d} P^d_{i,t+j-1}} - 1\right)^2 P^d_{t+j} Y^d_{t+j}$$

Where ι_d is the indexation to domestic inflation and ζ_d is the adjustment cost parameter (similar to what happens in labor aggregation agencies, the higher ζ_d the higher the adjustment costs of domestic prices). Intermediate goods firms face adjustment costs on the factors of production: labor and raw materials. Profits are calculated as the difference between income and remuneration to each of the factors and the adjustment costs on prices, labor and raw materials. Thus, the problem of the intermediate firm i consists of maximizing

[5] It is assumed that they have no costs for the aggregation of the intermediate good.

[6] The price of raw materials is assumed to be in foreign currency and is multiplied by the nominal exchange rate to transform it in local currency: $P^M_t = S_t P^{M*}_t$

its profits, net of the corporate income tax τ_t^k:

$$\max_{N_{i,t},K_{i,t-1},M^d_{i,t},P^d_{i,t}} E_0 \sum_{j=0}^{\infty} \beta^j \left(\frac{\Lambda^o_{t+j}}{\Lambda^o_t}\right) (1-\tau^k_{t+j}) \Big\{ P^d_{i,t+j} Y^d_{i,t+j} - W_{t+j} N_{i,t+j} - R^k_{t+j} K_{i,t+j-1} - P^M_{t+j} M^d_{i,t+j}$$
$$- \frac{\zeta_N}{2}\left(\frac{N_{i,t+j}}{N_{i,t+j-1}} - 1\right)^2 W_{t+j} N_{i,t+j} - \frac{\zeta_{M^d}}{2}\left(\frac{M^d_{i,t+j}}{(1+g_t) M_{i,t+j-1}} - 1\right)^2 P^M_{t+j} M^d_{i,t+j}$$
$$- \frac{\zeta_d}{2}\left(\frac{P^d_{i,t+j}}{(1+\pi^d_{t+j-1})^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d} P^d_{i,t+j-1}} - 1\right)^2 P^d_{t+j} Y^d_{t+j} \Big\}$$

Subject to the demand made by firms for final domestic goods (15) and to the production function (16):

$$Y^d_{i,t} = \left(\frac{P^d_{i,t}}{P^d_t}\right)^{-\theta_d} Y^d_t$$

$$Y^d_{i,t} = z_t \left(A_t N_{i,t}\right)^{1-\alpha_k-\alpha_{M^d}} \left(K_{i,t-1}\right)^{\alpha_k} \left(M^d_{i,t}\right)^{\alpha_{M^d}} \left(\frac{K^g_{t-1}}{A_t}\right)^{\psi}$$

ζ_N and ζ_{M^d} correspond to the parameter of the adjustment costs of labor and imported inputs respectively. From the solution of the problem, and denoting $MC_{i,t}$ as the marginal cost of the firm, we obtain the demands for the factors of production:

$$[K_{i,t-1}] : R^k_t = \alpha_k MC_{i,t} \frac{Y^d_{i,t}}{K_{i,t-1}} \tag{17}$$

$$[N_{i,t}] : W_t = (1-\alpha_k-\alpha_{M^d}) MC_{i,t} \frac{Y^d_{i,t}}{N_{i,t}} - \frac{\zeta_N}{2} W_t \left(\frac{N_{i,t}}{N_{i,t-1}} - 1\right)^2 - \zeta_N W_t \frac{N_{i,t}}{N_{i,t-1}} \left(\frac{N_{i,t}}{N_{i,t-1}} - 1\right)$$
$$+ \beta \left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t}\right) \zeta_N W_{t+1} \left(\frac{1-\tau^k_{t+1}}{1-\tau^k_t}\right) \left(\frac{N_{i,t+1}}{N_{i,t}}\right)^2 \left(\frac{N_{i,t+1}}{N_{i,t}} - 1\right) \tag{18}$$

$$[M^d_{i,t}] : P^M_t = \alpha_{M^d} MC_{i,t} \frac{Y^d_{i,t}}{M^d_{i,t}} - \frac{\zeta_{M^d}}{2} P^M_t \left(\frac{M^d_{i,t}}{(1+g_t) M^d_{i,t-1}} - 1\right)^2 - \zeta_{M^d} P^M_t \frac{M^d_{i,t}}{(1+g_t) M^d_{i,t-1}} \left(\frac{M^d_{i,t}}{(1+g_t) M^d_{i,t-1}} - 1\right)$$
$$+ \beta \left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t}\right) \zeta_{M^d} P^M_{t+1} \left(\frac{1-\tau^k_{t+1}}{1-\tau^k_t}\right) \left(\frac{(M^d_{i,t+1})^2}{(1+g_{t+1})(M^d_{i,t})^2}\right) \left(\frac{M^d_{i,t+1}}{(1+g_{t+1}) M^d_{i,t}} - 1\right) \tag{19}$$

Defining the following expressions:

$$\widetilde{W_t} = W_t + \frac{\zeta_N}{2} W_t \left(\frac{N_{i,t}}{N_{i,t-1}} - 1\right)^2 + \zeta_N W_t \frac{N_{i,t}}{N_{i,t-1}} \left(\frac{N_{i,t}}{N_{i,t-1}} - 1\right)$$
$$- \beta \left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t}\right) \left(\frac{1-\tau^k_{t+1}}{1-\tau^k_t}\right) \zeta_N W_{t+1} \left(\frac{N_{i,t+1}}{N_{i,t}}\right)^2 \left(\frac{N_{i,t+1}}{N_{i,t}} - 1\right) \tag{20}$$

$$\widetilde{P^M_t} = P^M_t + \frac{\zeta_{M^d}}{2} P^M_t \left(\frac{M^d_{i,t}}{(1+g_t) M^d_{i,t-1}} - 1\right)^2 + \zeta_{M^d} P^M_t \frac{M^d_{i,t}}{(1+g_t) M^d_{i,t-1}} \left(\frac{M^d_{i,t}}{(1+g_t) M^d_{i,t-1}} - 1\right)$$
$$- \beta \left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t}\right) \left(\frac{1-\tau^k_{t+1}}{1-\tau^k_t}\right) \zeta_{M^d} P^M_{t+1} \left(\frac{(M^d_{i,t+1})^2}{(1+g_{t+1})(M^d_{i,t})^2}\right) \left(\frac{M^d_{i,t+1}}{(1+g_{t+1}) M^d_{i,t}} - 1\right) \tag{21}$$

it is possible to rewrite the demands for factors of production (17),(18),(19) as follows:

$$K_{i,t-1} = \alpha_k MC_{i,t} \frac{Y_{i,t}^d}{R_t^k}$$

$$N_{i,t} = (1-\alpha_k-\alpha_{M^d}) MC_{i,t} \frac{Y_{i,t}^d}{\widetilde{W}_t}$$

$$M_{i,t}^d = \alpha_{M^d} MC_{i,t} \frac{Y_{i,t}^d}{\widetilde{P_t^M}}$$

replacing in the production function:

$$Y_{i,t}^d = z_t \left(A_t (1-\alpha_k-\alpha_{M^d}) MC_{i,t} \frac{Y_{i,t}^d}{\widetilde{W}_t} \right)^{1-\alpha_k-\alpha_{M^d}} \left(\alpha_k MC_{i,t} \frac{Y_{i,t}^d}{R_t^k} \right)^{\alpha_k} \left(\alpha_{M^d} MC_{i,t} \frac{Y_{i,t}^d}{\widetilde{P_t^M}} \right)^{\alpha_{M^d}} \left(\frac{K_{t-1}^g}{A_t} \right)^{\psi}$$

Clearing $MC_{i,t}$ gives the marginal cost of the firm:

$$MC_{i,t} = \left[\frac{1}{z_t} \right] \left[\frac{\widetilde{W}_t}{(1-\alpha_k-\alpha_{M^d}) A_t} \right]^{1-\alpha_k-\alpha_{M^d}} \left[\frac{R_t^k}{\alpha_k} \right]^{\alpha_k} \left[\frac{\widetilde{P_t^M}}{\alpha_{M^d}} \right]^{\alpha_{M^d}} \left[\frac{A_t}{K_{t-1}^g} \right]^{\psi} \tag{22}$$

On the other hand, of the first-order condition with respect to prices we obtain:

$$(1-\theta_d) \left(\frac{P_{i,t}^d}{P_t^d} \right)^{-\theta_d} Y_t^d - \zeta_d \left(\frac{P_{i,t}^d}{(1+\pi_{t-1}^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d} P_{i,t-1}^d} - 1 \right) \frac{P_t^d Y_t^d}{(1+\pi_{t-1}^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d} P_{i,t-1}^d}$$
$$+ \theta_d MC_{i,t} \left(\frac{P_{i,t}^d}{P_t^d} \right)^{-\theta_d} \frac{Y_t^d}{P_{i,t}^d} + \beta \left(\frac{\Lambda_{t+1}^o}{\Lambda_t^o} \right) \left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k} \right) \zeta_d \left(\frac{P_{t+1}^d}{P_{i,t}^d} \right) Y_{t+1}^d \left(\frac{P_{i,t+1}^d}{(1+\pi_t^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d} P_{i,t}^d} - 1 \right)$$
$$\left(\frac{P_{i,t+1}^d}{(1+\pi_t^d)^{\iota_d} (1-\overline{\pi^d})^{1-\iota_d} P_{i,t}^d} \right) = 0$$

The Phillips curve of domestic prices is obtained, where $(1+\pi_t^d) = \frac{P_t^d}{P_{t-1}^d}$:

$$\zeta_d P_t^d \left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d}} - 1 \right) \left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d}} \right) = \beta \left(\frac{\Lambda_{t+1}^o}{\Lambda_t^o} \right) \left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k} \right) \zeta_d P_{t+1}^d \left(\frac{Y_{t+1}^d}{Y_t^d} \right)$$
$$\left(\frac{(1+\pi_{t+1}^d)}{(1+\pi_t^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d}} - 1 \right) \left(\frac{(1+\pi_{t+1}^d)}{(1+\pi_t^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d}} \right) + (1-\theta_d) P_{i,t}^d + \theta_d MC_{i,t} \tag{23}$$

The mark-up θ_d can be expressed as $\theta_d = \overline{\theta_d} * exp(z_t^{pd})$ where z_t^{pd} is an exogenous *domestic cost push* shock. Finally, the pre-tax and aggregate dividends of the domestic goods-producing companies are as follows:

$$D_t^{d,b} = \Big\{ P_t^d Y_t^d - W_t N_t - R_t^k K_{t-1} - P_t^M M_t^d$$
$$- \frac{\zeta_N}{2} \left(\frac{N_t}{N_{t-1}} - 1 \right)^2 W_t N_t - \frac{\zeta_{M^d}}{2} \left(\frac{M_t^d}{(1+g_t) M_{t-1}} - 1 \right)^2 P_t^M M_t^d$$
$$- \frac{\zeta_p}{2} \left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d} (1+\overline{\pi^d})^{1-\iota_d}} - 1 \right)^2 P_t^d Y_t^d \Big\} \tag{24}$$

After-tax dividends:

$$D_t^{d,a} = (1-\tau_t^k) D_t^{d,b} \tag{25}$$

2.3.2 Capital Goods Producing Firms

These firms are responsible for producing the capital that intermediate goods firms utilize in their production process. To achieve this, they determine the optimal amounts of domestic and imported investment by solving a profit maximization problem. Their income is composed of the rent they receive for the rental of the capital (minus the legal income tax) and an additional depreciation allowance. Meanwhile, their expenses consist of the domestic and imported investment, valued at prices P_t^{dF} and P_t^{fF}, respectively. Additionally, they take into account an investment subsidy denoted as τ_t^I.

$$\max_{K_t, I_t, I_t^d, I_t^f} E_0 \sum_{j=0}^{\infty} \beta^t \left(\frac{\Lambda_{t+j}^o}{\Lambda_t^o}\right) \left\{(1-\tau_{t+j}^k) R_{t+j}^k K_{t+j-1} + \tau_{t+j}^k \delta Q_{t+j} K_{t+j-1} - (1-\tau_{t+j}^I) P_{t+j}^{dF} I_{t+j}^d - (1-\tau_{t+j}^I) P_{t+j}^{fF} I_{t+j}^f\right\}$$

Subject to the law of capital accumulation with investment adjustment costs:

$$K_t = (1-\delta)K_{t-1} + z_t^I I_t \left(1 - \frac{\zeta_I}{2}\left(\frac{I_t}{(1+g_t)I_{t-1}} - 1\right)^2\right) \tag{26}$$

and an investment aggregator basket (which follows a CES-type function):

$$I_t = \left[\omega_I^{\frac{1}{\theta_I}} {I_t^d}^{\frac{\theta_I - 1}{\theta_I}} + (1-\omega_I)^{\frac{1}{\theta_I}} {I_t^f}^{\frac{\theta_I - 1}{\theta_I}}\right]^{\frac{\theta_I}{\theta_I - 1}}$$

where δ is the capital depreciation, z_t^I a shock to investment, Q_t is the Tobin Q, ζ_I is the investment adjustment cost parameter, ω_I is the share of domestic investment in the investment basket and θ_I is the elasticity of substitution between domestic and imported investment. Defining P_t^I as the price of investment, the first order conditions with respect to capital and total investment are:

$$[K_t] : Q_t = \beta \frac{\Lambda_{t+1}^o}{\Lambda_t^o} \left\{(1-\tau_{t+1}^k) R_{t+1}^k + \tau_{t+1}^k \delta Q_{t+1} + Q_{t+1}(1-\delta)\right\} \tag{27}$$

$$\begin{aligned}[I_t] : P_t^I = Q_t z_t^I &\left[1 - \frac{\zeta_I}{2}\left(\frac{I_t}{(1+g_t)I_{t-1}} - 1\right)^2 - \zeta_I \frac{I_t}{(1+g_t)I_{t-1}}\left(\frac{I_t}{(1+g_t)I_{t-1}} - 1\right)\right] \\ &+ \beta\left(\frac{\Lambda_{t+1}^o}{\Lambda_t^o}\right) Q_{t+1} z_{t+1}^I \zeta_I \left(\frac{(I_{t+1})^2}{(1+g_{t+1})(I_t)^2}\right)\left(\frac{I_{t+1}}{(1+g_{t+1})I_t} - 1\right)\end{aligned} \tag{28}$$

And the demand for domestic and imported investment corresponds to:

$$[I_t^d] : I_t^d = \omega_I \left(\frac{(1-\tau_t^I)P_t^{dF}}{P_t^I}\right)^{-\theta_I} I_t \tag{29}$$

$$[I_t^f] : I_t^f = (1-\omega_I) \left(\frac{(1-\tau_t^I)P_t^{fF}}{P_t^I}\right)^{-\theta_I} I_t \tag{30}$$

Where the price of investment goods is equal to:

$$P_t^I = \left[\omega_I((1-\tau_t^I)P_t^{dF})^{1-\theta_I} + (1-\omega_I)((1-\tau_t^I)P_t^{fF})^{1-\theta_I}\right]^{\frac{1}{1-\theta_I}} \tag{31}$$

Finally, the aggregate dividends of capital goods firms are defined as:

$$D_t^k = (1-\tau_t^k) R_t^k K_{t-1} + \tau_t^k \delta Q_t K_{t-1} - (1-\tau_t^I) P_t^{dF} I_t^d - (1-\tau_t^I) P_t^{fF} I_t^f \tag{32}$$

2.3.3 Imported Goods Producing Firms

Similar to the production of domestic goods, there is a firm producing imported final goods that purchases imported intermediate goods $Y^f_{i,t}$ at a price $P^f_{i,t}$. These intermediate goods are then aggregated to produce a final imported good Y^f_t valued at a price P^f_t that is used for both consumption and imported investment. The main objective is to maximize profit:

$$\max_{Y^f_{i,t}} P^f_t Y^f_t - \int_0^1 P^f_{i,t} Y^f_{i,t} di$$

subject to the aggregation function of imported intermediate goods:

$$Y^f_t = \left[\int_0^1 \left(Y^f_{i,t} \right)^{\frac{\theta_f - 1}{\theta_f}} di \right]^{\frac{\theta_f}{\theta_f - 1}}$$

Where θ_f is the elasticity of substitution of imported intermediate goods. Solving the problem, the demand for intermediate goods is found:

$$Y^f_{i,t} = \left(\frac{P^f_{i,t}}{P^f_t} \right)^{-\theta_f} Y^f_t \tag{33}$$

Firms importing intermediate goods assume a cost $MC^f_{i,t}$ equal to the external price P^{f*}_t multiplied by the exchange rate and sell the intermediate goods to the firm producing the final imported goods. Additionally, they operate under monopolistic competition, which allows them to choose the price $P^f_{i,t}$ at which they sell the intermediate good. However, they also face an adjustment cost in price setting. In addition, they have a tariff (τ^a_t) for the goods they import from abroad. The problem of firm i consists of maximizing its benefit:

$$\max_{P^f_{i,t}} E_0 \sum_{j=0}^{\infty} \beta^j \left(\frac{\Lambda^o_{t+j}}{\Lambda^o_t} \right) \left\{ P^f_{i,t+j} Y^f_{i,t+j} - (1+\tau^a_t) MC^f_{i,t+j} Y^f_{i,t+j} - \frac{\zeta_f}{2} \left(\frac{P^f_{i,t+j}}{(1+\pi^f_{t+j-1})^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f} P^f_{i,t+j-1}} - 1 \right)^2 P^f_{t+j} Y^f_{t+j} \right\}$$

subject to the demand for imported intermediate goods (33) and their cost $MC^f_{i,t}$:

$$Y^f_{i,t} = \left(\frac{P^f_{i,t}}{P^f_t} \right)^{-\theta_f} Y^f_t$$

$$MC^f_{i,t} = S_t P^{f*}_t$$

Solving the problem:

$$(1-\theta_f) \left(\frac{P^f_{i,t}}{P^f_t} \right)^{-\theta_f} Y^f_t + \theta_f (1+\tau^a_t) MC^f_{i,t} \left(\frac{P^f_{i,t}}{P^f_t} \right)^{-\theta_f} \frac{Y^f_t}{P^f_{i,t}} - \zeta_f P^f_t Y^f_t \left(\frac{P^f_{i,t}}{(1+\pi^f_{t-1})^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f} P^f_{i,t-1}} - 1 \right)$$
$$\left(\frac{1}{(1+\pi^f_{t-1})^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f} P^f_{i,t-1}} \right) + \beta \left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t} \right) \zeta_f \frac{P^f_{t+1}}{P^f_{i,t}} Y^f_{t+1} \left(\frac{P^f_{i,t+1}}{(1+\pi^f_t)^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f} P^f_{i,t}} - 1 \right)$$
$$\left(\frac{P^f_{i,t+1}}{(1+\pi^f_t)^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f} P^f_{i,t}} \right) = 0$$

The Phillips curve of import prices is obtained:

$$\zeta_f P^f_t \left(\frac{(1+\pi^f_t)}{(1+\pi^f_{t-1})^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f}} - 1 \right) \left(\frac{(1+\pi^f_t)}{(1+\pi^f_{t-1})^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f}} \right) = \beta \left(\frac{\Lambda^o_{t+1}}{\Lambda^o_t} \right) \zeta_f P^f_{t+1} \left(\frac{Y^f_{t+1}}{Y^f_t} \right)$$
$$\left(\frac{(1+\pi^f_{t+1})}{(1+\pi^f_t)^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f}} - 1 \right) \left(\frac{(1+\pi^f_{t+1})}{(1+\pi^f_t)^{\iota_f} (1+\overline{\pi^f})^{1-\iota_f}} \right) + (1-\theta_f) P^f_{i,t} + \theta_f (1+\tau^a_t) MC^f_{i,t} \tag{34}$$

the mark-up θ_f can be expressed as $\theta_f = \overline{\theta_f} * exp(z_t^{pf})$ where z_t^{pf} is an exogenous *imported cost push* shock on imported prices. Finally, the aggregate dividends of the companies are as follows:

$$D_t^f = P_t^f Y_t^f - (1+\tau_t^a) MC_t^f Y_t^f - \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)^2 P_t^f Y_t^f \tag{35}$$

2.4 Monetary Policy

Monetary policy follows a Taylor rule on the expectative annual inflation gap and annual output growth to set the annual policy interest rate:

$$1 + i_t^{an} = (1 + i_{t-1}^{an})^{\phi_i}\left[(1+\bar{i})\left(\frac{1+\pi_{t+4}^{an}}{1+\overline{\pi}}\right)^{\phi_\pi}\left(\frac{1+\triangle PIB_t^{an}}{1+\overline{g}}\right)^{\phi_y}\right]^{1-\phi_i} exp(z_t^i) \tag{36}$$

Where ϕ_i is the weight of the interest rate in previous periods, ϕ_π is the weight of the inflation gap and ϕ_y the weight of the GDP growth gap ($\triangle PIB_t$). z_t^i is a shock that deviates monetary policy from its rule.

2.5 Fiscal Policy

2.5.1 Expenditures

The government has an unproductive expense C_t^g and a productive expense I_t^g. These expenses are assumed to follow exogenous processes. Consequently, the primary expenditure, denoted as G_t^p, is defined as the sum of these two types of expenditures

$$G_t^p = P_t^{dF} C_t^g + P_t^{dF} I_t^g \tag{37}$$

Unproductive spending is associated with public consumption, while productive spending is associated with public investment. The latter produces public capital that has positive effects on the private production of the domestic good. The accumulation of public capital is given by:

$$K_t^g = (1-\delta^g)K_{t-1}^g + \varphi I_t^g \tag{38}$$

δ^g is the depreciation of public capital and φ is the efficiency of public investment.

In addition, since lump-sum transfers are made to households without access to the financial market T_t^l, total non-interest government spending is:

$$G_t = P_t^{dF} C_t^g + P_t^{dF} I_t^g + T_t^{l,agr} \tag{39}$$

Finally, the government has a domestic debt with households with access to the financial market, denominated in local currency, and an external debt with the rest of the world, denominated in foreign currency. Therefore, it must pay interest on that debt, which constitutes its debt service expense:

$$G_t^b = i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^{g,f} \tag{40}$$

Thus, total government spending would be the sum of its non-interest spending plus interest payments:

$$G_t^T = G_t + G_t^b = \left[P_t^{dF} C_t^g + P_t^{dF} I_t^g + T_t^{l,agr}\right] + \left[i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^{g,f}\right] \tag{41}$$

2.5.2 Revenues

A portion of government revenue comes from rents generated by the oil sector: it is assumed that oil is produced and exported in the economy, where a proportion $(1-\omega_{oil})$ of oil revenues is sent abroad in the form of profit remittances. Of the proportion ω_{oil} that remains in the country, a percentage ψ_{oil} goes to the government and the remaining $(1-\psi_{oil})$ to households with access to the financial market. Thus, government income from oil revenues is defined as:

$$Rec_t^{oil} = \omega_{oil}\psi_{oil} S_{t-4} P_{t-4}^{oil} Y_{t-4}^{oil} \tag{42}$$

where P_t^{oil}, Y_t^{oil} is the external price and oil production respectively. The variables are lagged one year to reflect the fact that the oil sector's profits are paid the year after they are earned. This means that the profits distributed in the period t were generated in period $t-4$.

In addition to this oil income, the government obtains revenues through each of the taxes it levies on private agents (households and firms), which include the value added tax VAT (τ_t^{vat}), consumption taxes (τ_t^c), tax to labor income ($\tau_t^{N,o}, \tau_t^{N,l}$)[7], to the corporate income (τ_t^k), to dividends (τ_t^D) and to import duties (τ_t^a). This is reduced by investment subsidies and depreciation discounts, thus obtaining the non-oil tax revenue::

$$Rec_t^{\tau} = \tau_t^c P_t C_t + (1-f)\tau_t^{N,o} V_t N_t^o + f\tau_t^{N,l} V_t N_t^l + \tau_t^k \left(D_t^{d,b} + R_t^k K_{t-1} - \delta Q_t K_{t-1} \right) + \tau_t^a MC_t^f Y_t^f + \tau_t^{vat}(P_t^d Y_t^d + P_t^f Y_t^f) + \tau_t^D D_t - \tau_t^I (P_t^{dF} I_t^d + P_t^{fF} I_t^f) \quad (43)$$

Finally, the government receives lump-sum taxes $T_t^{o,agr}$ paid by households with access to the financial market. These resources will reflect the revenues associated with other capital resources in the fiscal accounts.

With all of the above, total government revenues are as follows [8]:

$$Rec_t^{total} = Rec_t^{\tau} + Rec_t^{oil} + T_t^{o,agr} \quad (44)$$

2.6 Deficit and Fiscal Rule

The government's budget constraint is:

$$P_t^{dF} C_t^g + P_t^{dF} I_t^g + T_t^{l,agr} + (1+i_{t-1}) z_{t-1}^{SW} B_{t-1} + (1+i_{t-1}^f) S_t B_{t-1}^{g,f} = Rec_t^{\tau} + Rec_t^{oil} + T_t^{o,agr} + B_t + S_t B_t^{g,f} \quad (45)$$

Where the primary deficit is defined as:

$$DP_t = P_t^{dF} C_t^g + P_t^{dF} I_t^g + T_t^{l,agr} - Rec_t^{\tau} - Rec_t^{oil} - T_t^{o,agr} \quad (46)$$

and total deficit as the sum of primary deficit and debt service:

$$DT_t = DP_t + G_t^b \quad (47)$$

In the model it is assumed that the government has a fiscal rule that can be set on the primary balance (as a proportion of GDP) or the total balance (as a proportion of GDP) and depends on an auto-regressive component, the output gap, the oil price, and the debt.

$$\frac{DD_t}{PIB_t} = (1-\rho_D)\overline{\left(\frac{DD}{PIB}\right)} + \rho_D \frac{DD_{t-1}}{PIB_{t-1}} - \rho_{PIB}\left(\frac{PIB_t}{\overline{PIB}} - 1\right) - \rho_B \left(\frac{B_{t-1}^T / PIB_{t-1}}{\overline{B^T/PIB}} - 1\right) - \rho_{oil}\left(\frac{S_{t-1} P_{t-1}^{oil}}{\overline{SP^{oil}}} - 1\right) \quad (48)$$

where $DD_t = \{DP_t, DT_t\}$ and B_t^T is the total debt:

$$B_t^T = B_t + S_t B_t^{g,f} \quad (49)$$

The government can use different instruments to consolidate its fiscal accounts and comply with the fiscal target. For example, it can make adjustments through transfers to households, public consumption, public

[7] Recall that the proportion $(1-f)$ corresponds to households with access to the financial market and the remaining f to households without access to the financial market. Since it is assumed that labor income taxes are different for each household, total personal income tax collection should take into account the proportion of each type of household in the total population, as well as the rate paid by the household. In the case that taxes are the same for both types of households, $\tau_t^N = \tau_t^{N,o} = \tau_t^{N,l}$, the total labor income collection can be written as $\tau_t^N V_t N_t$ (where $N_t = (1-f)N_t^o + fN_t^l$, as shown below in the model aggregation).

[8] The government also obtains income from the debt it incurs at home and abroad. It is not taken into account in this part but in the government's budget constraint

investment or taxes. Since the effects of each instrument are different in the general equilibrium of the model, the economic results of any simulation or shock will vary depending on the fiscal consolidation instrument.

Since a proportion of total government debt is in local currency and another in foreign currency, based on the government's restriction, it is possible to define the accumulation of domestic debt as follows:

$$B_t = \phi^B \left[DP_t + i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^f \right] + z_{t-1}^{SW} B_{t-1} \tag{50}$$

where ϕ^B is the proportion of domestic debt in total debt. Similarly, the accumulation of the external debt will be:

$$S_t B_t^{g,f} = (1 - \phi^B) \left[DP_t + i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^{g,f} \right] + S_t B_{t-1}^{g,f} \tag{51}$$

2.7 External Definitions and Domestic Product

The foreign interest rate that ricardian households and government pay for the external bonds depends on a risk free rate i_t^* and a risk premium $prem_t$:

$$(1 + i_t^f) = (1 + i_t^*)(1 + prem_t) \tag{52}$$

The risk-free rate is exogenous, while the risk premium depends on the deviation of total public debt (as a proportion of GDP) and private external debt (as a proportion of GDP) from their long-run values. As this deviation increases, the value of the risk premium also rises. This deviation is then multiplied by an elasticity Φ that relates the risk premium to the debt gap:

$$(1 + prem_t) = exp \left(\varPhi \left[\frac{S_t (B_t^{g,f} - B_t^f)}{PIB_t} - \overline{B^f} \right] + \varPhi \left[\frac{B_t}{PIB_t} - \overline{B} \right] \right) (1 + z_t^{prem}) \tag{53}$$

Keep in mind that in the previous expression, the risk premium depends on domestic and foreign public debt $(B_t^{g,f} + B_t)$ and the private net foreign assets (B_t^f). Alternatively, the risk premium may solely depend on foreign debt (public and private):

$$(1 + prem_t) = exp \left(\varPhi \left[\frac{S_t (B_t^{g,f} - B_t^f)}{PIB_t} - \overline{B^f} \right] \right) (1 + z_t^{prem})$$

The risk premium equation also includes a shock z_t^{prem} that represents a displacement of the premium curve.

On the other hand, exports demand X_t is given by:

$$X_t = \left(\frac{P_t^{dF}}{S_t P_t^*} \right)^{-\theta_x} Y_t^* \tag{54}$$

where θ_x is the elasticity to the exchange rate, P_t^* is the external price and Y_t^* is the foreign demand for exports. Both external price and foreign demand are exogenous process. Because there is an oil sector that exports oil, total exports are:

$$EXPORTS_t = P_t^{dF} X_t + S_t P_t^{oil} Y_t^{oil} \tag{55}$$

Besides, imports in the model are defines by:

$$IMPORTS_t = S_t P_t^{f*} Y_t^f + P_t^M M_t \tag{56}$$

So the trade balance is:

$$TB_t = EXPORTS_t - IMPORTS_t \tag{57}$$

Otherwise, domestic product is given by:

$$Y_t^d = C_t^d + I_t^d + C_t^g + I_t^g + X_t + \frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}} - 1\right)^2 \frac{W_t}{P_t^{dF}} N_t + \frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}} - 1\right)^2 \frac{P_t^M}{P_t^{dF}} M_t$$
$$+ \frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}} - 1\right)^2 \frac{P_t^d}{P_t^{dF}} Y_t^d + \frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)^2 \frac{W_t}{P_t^{dF}} N_t \quad (58)$$

and the external product is:

$$Y_t^f = C_t^f + I_t^f + \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)^2 \frac{P_t^f}{P_t^{fF}} Y_t^f \quad (59)$$

Aggregating all the restrictions of the different agents in the model, and replacing previous expressions, the balance of payments is found:

$$\underbrace{S_t(B_t^f - z_{t-1}^{SW} B_{t-1}^f) - S_t(B_t^{g,f} - B_{t-1}^{g,f})}_{\text{Foreign Debt Flows}} = \underbrace{EXPORTS_t - IMPORTS_t}_{\text{Trade Balance}} + \underbrace{REM_t}_{\text{Remittances}} - \underbrace{(S_t P_t^{oil} Y_t^{oil} - \omega_{oil} S_{t-4} P_{t-4}^{oil} Y_{t-4}^{oil})}_{\text{Remittances of Profits}}$$
$$- \underbrace{i_{t-1}^f S_t B_{t-1}^{g,f} + i_{t-1}^f z_{t-1}^{SW} S_t B_{t-1}^f}_{\text{Interest Payments by Foreign Debt}} \quad (60)$$

where the right side of the expression corresponds to the current account CC_t:

$$CC_t = EXPORTS_t - IMPORTS_t + REM_t - (S_t P_t^{oil} Y_t^{oil} - \omega_{oil} S_{t-4} P_{t-4}^{oil} Y_{t-4}^{oil}) - i_{t-1}^f S_t B_{t-1}^{g,f} + i_{t-1}^f z_{t-1}^{SW} S_t B_{t-1}^f \quad (61)$$

The aggregation of the model is detailed in appendix.

3 Model Calibration

The calibration of the macroeconomic and fiscal variables, as a percentage of GDP, of the model is presented below. For the macroeconomic variables, quarterly series for consumption, investment, and GDP between the first quarter of 2005 and the fourth quarter of 2019, as reported by the National Administrative Department of Statistics (DANE), were used[9]. Table 1 shows the calibration of the macroeconomic variables[10]

Table 1. Macroeconomic Variables (%GDP)

Variable	**Observed**	**Calibrated**
Private Consumption	66,2	66,2
Private Investment	19	18,9
Public Consumption	14	14
Public Investment	2,8	2,8
Exports	17	18,7
Imports	19	20,6
Trade Balance	2,1	1,9

Source: DANE and Banco de la República, Authors'calculations.

[9]Public investment is taken from DANE's Economic Accounts by Institutional Sectors and Private investment is calculated as residual between total and public investment. Regarding external accounts (exports and imports), data is taken from the Balance of Payments (BoP) series published by Central Bank of Colombia.

[10]In the DANE data, private consumption to GDP is 67%. However, if the share of exports and imports from the Balance of Payments are used, the sum of the GDP components does not add up to 1 because imports in BoP are measured differently than in DANE's National Accounts. To make sure that the sum of GDP components gives 1, the share of consumption and private investment are adjusted.

Our analysis spans the years from 2005 to 2019, drawing data from the Ministry of Finance and Public Credit (MHCP). The key components are Value Added Tax (VAT), which encompasses both internal and external VAT, as well as taxes on gasoline, diesel, and carbon; Income Tax, for which, despite the MHCP's lack of differentiation between individual and corporate income, we rely on OECD data for calibration[11]; Tariff Collections, which comprise tariff levies and import surcharges; and Tax on Dividends, calculated as approximately 3% of declared income, using OECD income collection data. The model calibration for each tax type is meticulously presented below

Table 2. Tax Revenue (%GDP)

Tax Revenue	**Observed**	**Calibrated**
Value Added Tax - VAT	5,7	5,7
Personal Income Tax	1,07	1,07
Corporate Income Tax	4,8	4,8
Consumption Tax	0,1	0,1
Dividends	0,03	0,03
Tariff	0,6	0,6
Tax Income	12,3	12,3

Source: MHCP, Authors'calculations.

It should be noted that according to the observed data, total tax revenues are 13.6% of GDP. In addition to the taxes mentioned above, data tax base includes collections from the tax on financial movements, stamp tax and withholding tax on real estate. For the purposes of simplification in our analytical model, we incorporate those latter collections and non-tax revenues, such as earmarked funds and other capital inflows, which together account for an additional 2.2% of GDP, into a lump-sum tax variable that is calibrated in the steady state to match that value.

Government expenditures encompass public consumption, public investment, and interest payments. These outlays, informed by the series from the Department of National Accounts and Economic Environment (DANE) and the Ministry of Finance and Public Credit (MHCP), as delineated in Table 1, culminate in a primary expenditure equivalent to 16% of GDP. The aggregate of these public revenues and expenditures generates a fiscal deficit of 3.5% of GDP.[12]

Note that from the government constraint, as a percentage of GDP, standardized and in steady state, the following expression for the definition of debt is achieved:

$$B^{total} = dp\frac{(1+g)}{r-g}$$

Given a primary balance of -0.7% of GDP, a growth rate of 3.3% and a real interest rate of 2%, a total debt to GDP of 57.5% is found, consistent with the fiscal rule's target. Assuming that 70% of total debt is domestic and the remaining is external, consistent with the policy objective of Colombian government, domestic public debt to GDP would be 40.25% and external public debt to GDP 17.25%.

4 Tightening of Financial Conditions and Results

This section describes the scenario with tighter external financial conditions, considering that over the past two years, global financial markets have grappled with heightened uncertainty and volatility. Factors such

[11]For the calibration of tax income of individuals, we assume that non ricardians agents do not pay income tax ($\tau_t^{N,l} = 0$), so the tax income of individuals come only by ricardian agents

[12]Considering the revenues and expenditures calibrated in the model, a primary deficit of 0.7% of GDP and interest expenditure of 2.75% of GDP are calculated. This results in a total deficit of 3.5% of GDP. When compared to the MHCP data, these figures are not significantly different. Specifically, the MHCP data indicates interest expenditure of 2.9% of GDP, primary expenditure of 15.7% of GDP, and a total fiscal deficit of 3.2% of GDP.

as more restrictive monetary policies in developed economies, increased risk perception in emerging markets, global geopolitical conflicts, and fears of a global economic slowdown have collectively shaped this landscape. Notably, concerns about the banking systems in the United States and Europe during the first half of 2023 triggered a fresh spike in market volatility. Consequently, we have witnessed a generalized appreciation of the dollar, higher emerging market risk premiums, and an overall tightening of financial conditions compared to 2021. (see figure 2).

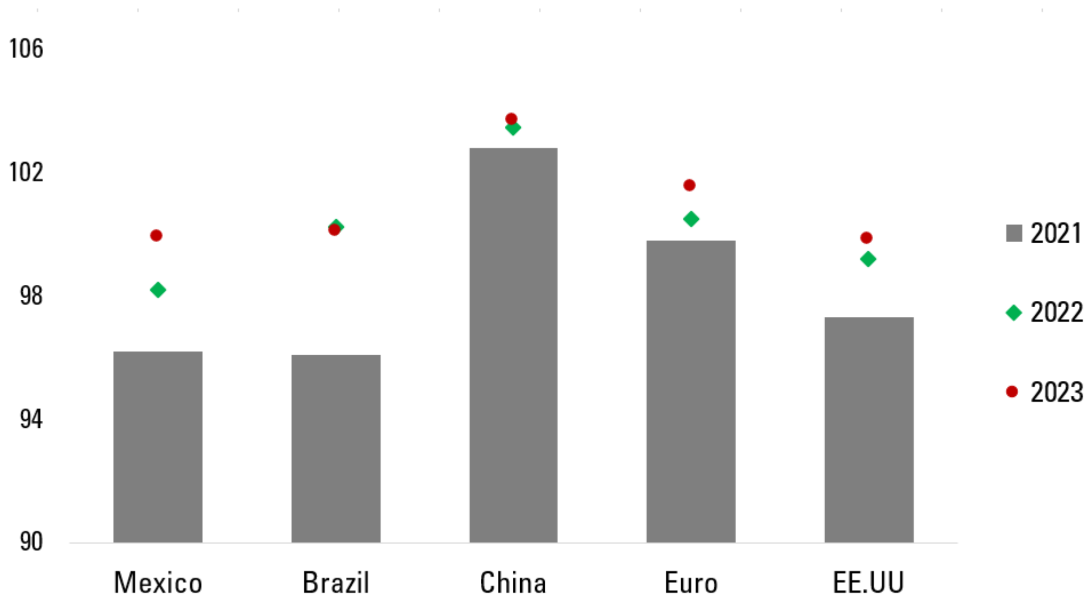

Figure 2: Financial Conditions (Index)
Source: Authors'calculations based on Goldman Sachs and Bloomberg. The index is a weighted average of risk-free interest rates, exchange rates, equity valuations and credit spreads.

The Colombian economy has not remained immune to the impact of international financial markets. Specifically, compared to 2021 and to prepandemic period, there have been notable increases in risk premiums, debt placement interest rates, and a depreciation of the peso against the dollar. These combined factors have led to a tightening of financial conditions both internally and externally. Additionally, local uncertainty has further contributed to this challenging environment. Between January 2021 and September 2022, Colombia's risk premium, as measured by the five-year Credit Default Swap (CDS), exhibited an upward trend, surging by nearly 180 basis points (bps) (see figure 3). It is crucial to recognize that similar magnitudes of increase occurred in previous periods, excluding the COVID-19 period. Notably, between September 2014 and February 2016, the risk premium rose by 205 bps, and during the 2009 financial crisis, it reached 310 bps.

Although the risk premium has moderated in 2023 compared to the previous year as it shows figure 3, it remains significantly above its historical average observed before the pandemic, standing at around 249 bps (versus 134 bps reached on average between 2010 and 2019)—a level comparable to that recorded in 2009 during the global financial crisis (245 bps). This persistently high level reflects heightened country risk perception and underscores the impact of both external and internal factors.

The potential impacts of a prolonged high risk premium on the Colombian economy would include downside risks to growth and upside risks to inflation, alongside its effects on foreign capital flows and the current account. From a public finance perspective, a persistent high risk premium affects fiscal accounts through various channels. In all scenarios, the government's fiscal balance worsens. On one hand, tightening financial conditions lead to reduced economic growth and, consequently, lower tax revenue. On the other hand, this results in increased financing costs for both the public and private sectors, leading to higher interest payments on debt. Figure 4 illustrates this correlation by plotting the Credit Default Swap (CDS) and fixed interest

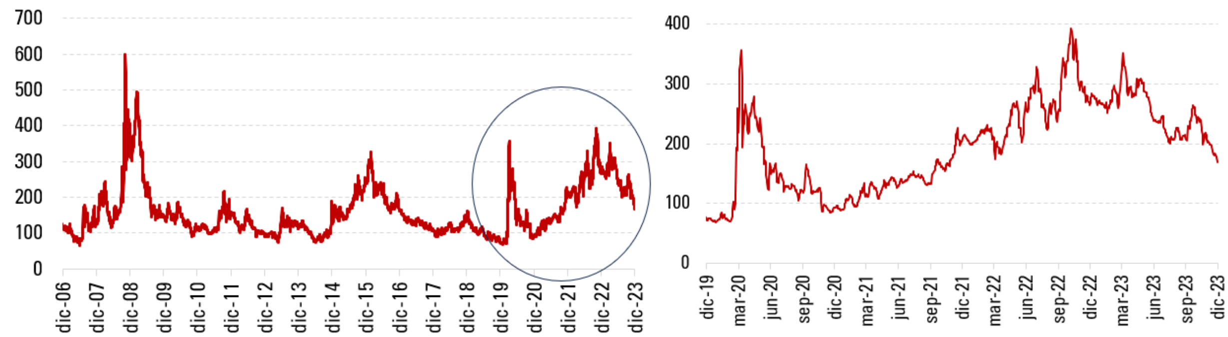

Figure 3: Colombia risk premium (CDS)
Source: Authors'calculations based on Bloomberg. The risk premium corresponds to the five-year Credit Default Swap.

rates of new credit loans obtained by the public sector [13].

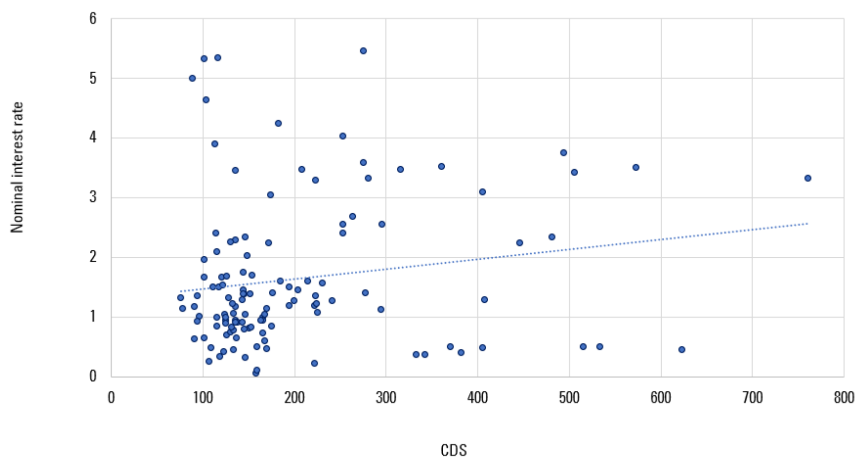

Figure 4: CDS and fixed interest rate of new credit loans
Source: Authors'calculations based on Bloomberg and Banco de la República.

Despite maintaining good fiscal discipline, countries remain susceptible to external shocks affecting risk premiums and exchange rates. Consequently, the imperative arises to create more flexible fiscal spaces while safeguarding access to credit markets. However, when fiscal space tightens—due, for instance, to compliance with fiscal rules—governments face a dilemma. They must choose between cutting spending or raising taxes. Alternatively, they may opt for increased borrowing to finance higher deficits, albeit at the risk of missing fiscal targets. These scenarios have distinct implications for macroeconomic variables, which constitutes a significant contribution to the literature. As fiscal policy significantly influences countries' risk premiums, prudent fiscal action becomes essential. Developing countries like Colombia regularly grapple with adverse financing scenarios characterized by reduced credit market access and elevated risk premiums. These conditions translate into higher financing costs, impacting short-term macroeconomic stability and, consequently, public finances(Lozano, I. (coordinator) (2019)).

A fiscal policy consistent with public debt sustainability has significant macroeconomic benefits. Macro

[13] These interest rates, weighted by the value (in dollars) of disbursed credit, are calculated using monthly data published by Banco de la República on external debt between January 2003 and June 2023. Each dot on the graph represents a month. It's worth noting that new loans are not acquired every month, so only months with recorded disbursements are included.

stabilization supported by a fiscal policy that promotes sustainability and solvency is essential for economic policy management and ends up favoring economic growth. In practice, the benefits of a good macro environment in terms of credibility and confidence translate into lower risk premiums and greater access to local and international financial markets. In addition, when fiscal policy cooperates with macroeconomic stability, the burden of countercyclical policies does not fall exclusively on monetary policy, thus avoiding possible overreactions through interest rates. Besides, a low cost of using capital reduces volatility in financial markets and creates a better environment for investment. Over time, these factors create a virtuous circle that strengthens long-term economic growth (Aizenman, J.; Jinjarak, Y., Estrada, G. and Tian, S. (2018); Stiglitz, J (2016)).

In a scenario without a liquidity trap (Zero Lower Bound), several authors have studied the interactions between the risk premium, public debt, and fiscal policy, concluding that fiscal austerity measures aimed at balancing the fiscal balance in the short term fail to contain the risk premium. However, a long-term fiscal reform plan can mitigate the effect of a rising risk premium (see Bianchi, J., Ottonello, P., and Presno, I (2023); Novelli, A. C., and Barcia, G. (2021); Bi, H. (2012)). In addition, introducing fiscal rules has also played an essential role in reducing sovereign default risk (Gomez-Gonzalez, J. E., Valencia, O. M., and Sánchez, G. A. (2022)). International investors perceive that countries that implement fiscal rules are less risky, and their governments pay lower yield spreads than similar countries that have not implemented fiscal rules.

Tighter financing conditions make it more difficult for an economy's agents to access credit and negatively affect households' savings and investment decisions. It also makes the government's interest payments more expensive and reduces its tax collection due to the economy's lower revenues, which deteriorates its fiscal balance. With a cost on the economy, that differs from the type of policy, reducing spending or increasing taxes would help to keep the deficit unchanged and reduce the risk premium (Lalik, M. (2017)). Transitory cuts in government spending in highly indebted countries expand the fiscal space, reducing the risk premium and facilitating a faster recovery in output(Andrés, J., Burriel, P. and Shen, W. (2020)).

When financial conditions become more restrictive, access to credit markets becomes challenging for economic agents. This has adverse effects on household saving and consumption decisions, as well as firms' investment choices. Additionally, it leads to increased interest payments on both domestic and foreign debt by governments and households. To assess the resulting impact on macroeconomic and fiscal variables, we employ a model that simulates an increase in the risk premium. We base this simulation on higher Credit Default Swap (CDS) rates observed in Colombia during the periods of heightened market uncertainty and volatility in 2016 and 2023.

Initially, economic agents perceive the elevated risk premium as an enduring shock. However, as time elapses, the shock gradually dissipates, prompting agents to adjust their expectations regarding its persistence. Based on this observation, they anticipate a less persistent shock, which swiftly reverts to its initial value. The consequences are significant: consumption, investment, and GDP all suffer adverse effects. Government revenue decreases, while expenditures rise due to increased debt service. Consequently, the fiscal deficit expands. The government faces a critical decision: adhere to fiscal rules by adjusting expenses (or taxes), or deviate by incurring additional debt. Our simulation explores both scenarios.

In the first scenario, the government chooses to adjust public investment to maintain a constant total fiscal deficit relative to GDP. In the second scenario, both taxes and expenditures remain fixed over time, while the government deliberately increases its debt to finance a higher fiscal deficit.

The results, shown in Figure (5), reveal that a higher risk premium leads to increased foreign interest rates and depreciation of the exchange rate, ultimately resulting in higher inflation. In response, the central bank raises the policy interest rate. Higher interest rates, both domestic and foreign, negatively affect private consumption and contribute to a decline in GDP. For Ricardian households, elevated interest rates make financing more expensive, leading them to reduce their consumption. Additionally, reduced economic activity results in lower firm dividends, which contributes in the negative impact household incomes.

Furthermore, decreased production leads to a contraction in the demand for productive factors (labor and capital), resulting in reduced wages for households and exacerbating the decline in Ricardian household incomes. Lower wages also impact non-Ricardian households, as they lack access to credit markets and must adjust consumption due to wage reductions.

The reduced demand for capital in domestic production, coupled with elevated interest rates, adversely impacts private investment. Besides, depreciation of the exchange rate makes more expensive the costs associated with imported inputs used by capital-producing firms, further exacerbating the decline in investment and capital. These factors amplify the negative effects on production, wages, and household consumption.

However, the magnitude of impact on economic variables can be different based on fiscal policy. In the scenario where the government adjusts public investment to maintain a constant total fiscal deficit relative to GDP, there is a more significant contraction in domestic demand in the short term. This helps to mitigate the increase in inflation and the policy interest rate set by the central bank. Lower interest rates not only stimulate the private consumption of Ricardian households (who can smooth consumption using credit markets) but also incentives private investment, thereby mitigating the negative effects of depreciation on the demand for import inputs used in capital goods production.

The contraction in public investment is partially compensated by the increase in private investment, resulting in a moderate decline in total investment. Simultaneously, larger private consumption and private investment contribute to a faster recovery of economic activity and production. Consequently, in the mid-term, there is an increase in demand for productive factors, wages, and household consumption.

In contrast, when the government doesn't adjust and chooses to take on debt, it increases the risk premium. This leads to exchange rate depreciation, inflation, and higher interest rates. As a result, the reduction in Ricardian consumption and investment is more significant compared to when the government adheres to fiscal rules. In this situation, private investment becomes the primary driver, as public investment remains unchanged. Consequently, the effects are more persistent, so in the mid-term, this negatively impacts production more than when the government follows fiscal rules.

Finally, it is essential to note that the effects of fiscal policy on the risk premium will depend on the type of household preferences (Shi, L. (2013)). So far, we have assumed preferences without an income effect on labor supply. Alternatively, we can consider preferences with an income effect and simulate the same increase in the risk premium with the two possible fiscal roles. While the conclusions remain consistent, the magnitudes might vary. For detailed information and results, please refer to Appendix 6.3

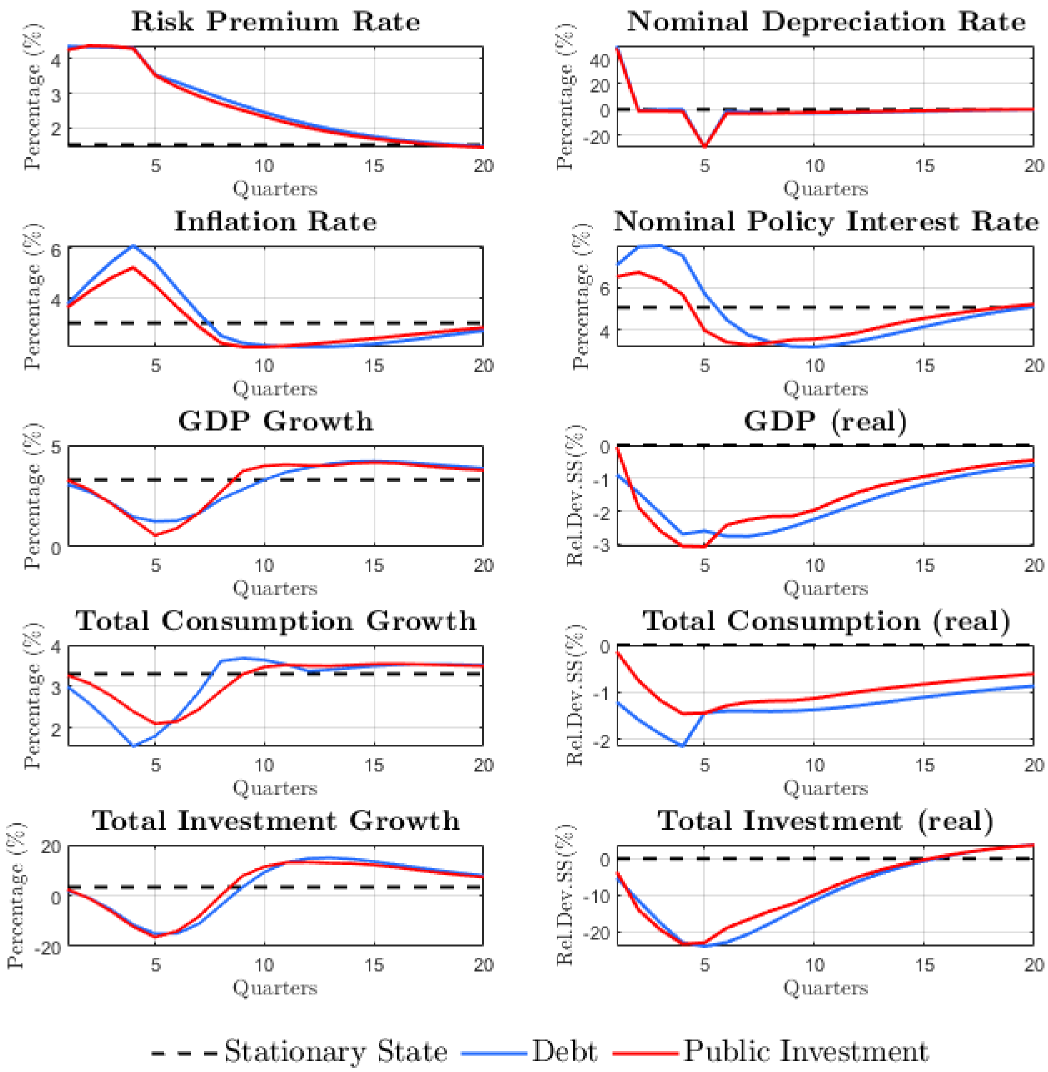

Figure 5: Macroeconomic Effects of Tightening of Financial Conditions

5 Conclusions

Over the past two years, global financial markets have grappled with heightened uncertainty and volatility, leading to a tightening of external financial conditions. Factors such as more restrictive monetary policies in developed economies, increased risk perception in emerging markets, global geopolitical conflicts and fears of a global economic slowdown have explained this economic outlook. The Colombian economy has not remained immune to the impact of international financial markets. Specifically, there have been notable increases in risk premiums, debt placement interest rates and a depreciation of the peso against the dollar, compared to 2021 and the pre-Covid19 period. Besides, local uncertainty has further contributed to this challenging environment. Between January 2021 and September 2022, Colombia's risk premium, measured by the five-year Credit Default Swap (CDS), exhibited an upward trend, surging by nearly 180 basis points (bps). During 2022 and 2023, the CDS for Colombia reached on average 259 bps and 249 bps, respectively, similar to the level observed during the 2009 financial crisis (245 bps).

These higher risk premiums and constrained financial markets present formidable challenges for an economy, as they drive up debt interest rates and make more difficult access to financing. Consequently, economic agents are compelled to adjust their consumption and investment strategies, precipitating a contraction in economic activity. Furthermore, the government is not immune to these dynamics, with the surge in debt interest potentially exacerbating the fiscal landscape. With diminished economic activity dampening tax collection, fiscal revenues become insufficient to cover the burgeoning interest payments. Thus, the government is compelled to undertake adjustments to fulfill fiscal objectives, such as adhering to a fiscal rule.

The manner in which the government takes these adjustments bears substantial effects on the economy, since the important role of fiscal policy. Hence, this paper quantifies the effects of various fiscal adjustments, given an upsurge in the risk premium, on macroeconomic variables. To this end, scenarios are simulated utilizing a dynamic stochastic general equilibrium model tailored for a small open economy and calibrated to reflect the main characteristics of Colombia's economic landscape. The model encompasses two categories of households—those with access to credit markets and those without—alongside domestic firms engaged in monopolistic competition, a monetary authority, and a fiscal policy governed by a fiscal rule.

The simulation initiates with a rise in the risk premium, triggering higher interest rates and the depreciation of the exchange rate. Households with access to credit markets reduce their consumption in response to the exacerbated financial conditions, while private investment drops due to escalated costs of imported materials. Consequently, there is a contraction in economic activity and tax revenue. The government is presented with two adjustment options. Firstly, it can pare down public investment to counterbalance the diminished revenue and keep constant the public deficit, thereby adhering to the fiscal rule. This approach precipitates further short-term downturns in economic activity and domestic demand, notwithstanding the positive externality of public investment on domestic production. However, lower demand leads to price reductions and prompts a decline in the policy interest rate, thereby stimulating private consumption and investment.

Alternatively, the government may keep constant its expenditure levels and finance the diminished revenue through debt. Nonetheless, heightened public debt increases the risk premium and make higher the depreciation of the exchange rate, amplifying adjustments made by households with access to financial markets and firms. Moreover, inflation surpasses levels observed in previous fiscal adjustment, leading a larger policy interest rate that further constrains private consumption and investment, thereby prolonging the economic downturn.

References

Aizenman, J.; Jinjarak, Y., Estrada, G. and Tian, S. (2018). Flexibility of Adjustment to Shocks: Economic Growth and Volatility of Middle-Income Countries before and After the Global Financial Crisis of 2008. *Emerging Markets Finance and Trade*, 54(5):1112–1131.

Andrés, J., Burriel, P. and Shen, W. (2020). Debt sustainability and fiscal space in a heterogeneous Monetary Union: standard times vs the zero lower bound. *CEMLA*.

Bank of Canada (2006). The Bank of Canada´s New Global Economy Model (GEM). *Technical Report*.

Bank of Spain (2019). MEDEA: A DSGE Model for the Spanish Economy. *Occasional Papers, Banco de España*.

Bi, H. (2012). Sovereign default risk premia, fiscal limits, and fiscal policy. *European Economic Review*, (56):389–410.

Bianchi, J., Ottonello, P., and Presno, I (2023). Fiscal stimulus under sovereign risk. *Journal of Political Economy*, 131(9).

Freedman, C., Kumhof, M., Laxton, D., Muir, D., and Mursula, S. (2010). Global Integrated Monetary and Fiscal Model (GIMF)—Theoretical Structure. *IMF Working Paper No. 10/34*.

Gomez-Gonzalez, J. E., Valencia, O. M., and Sánchez, G. A. (2022). How can fiscal rules reduce sovereign debt default risk? . *Emerging Markets Review*, 50.

González, A., López, M., Rodríguez, N. and Téllez, S. (2013). Fiscal Policy in a Small Open Economy with Oil Sector and Non Ricardian Agents . *Borradores de Economía. Banco de la República de Colombia*, (759).

González, A., Mahadeva, L., Prada, J.D. and Rodríguez, D. (2011). Policy Analysis Tool Applied to Colombian Needs: PATACON model description. *Borradores de Economía. Banco de la República de Colombia*, (656).

Lalik, M. (2017). Interactions between fiscal multipliers and sovereign risk premium during fiscal consolidation: model based assessment for the euro area. *ECB Working Paper*.

Lozano, I. (coordinator) (2019). Fiscal policy and macroeconomic stabilization in Colombia. *Ensayos Sobre Política Económica. Banco de la República de Colombia*, (90).

Muir, D., and Laxton, D. (2017). Modeling with GIMF for Policy Analysis. *IMF, Institute for Capacity Development*.

Novelli, A. C., and Barcia, G. (2021). Sovereign risk, public investment, and the fiscal policy stance. *Journal of Macroeconomics*, 67.

Rincón, H., Rodríguez, D., Toro, J. and Téllez, S. (2014). FISCO: Modelo Fiscal para Colombia. *Borradores de Economía. Banco de la República de Colombia*, (855).

Shi, L. (2013). Avoiding debt traps: Fiscal consolidation, financial backstops, and structural reforms.

Stiglitz, J (2016). he Theory of Credit and Macro-economic Stability. *Working Paper. National Bureau of Economic Research*, (22837).

6 Appendix

6.1 Model Aggregation

Total consumption is defined as the aggregation of the consumption of all households. However, since there is a fraction $(1-f)$ of ricardian households with access to credit markets and a fraction f of non ricardian households, total consumption is defined by:

$$C_t = \int_0^1 C_t(j)dj$$

$$C_t = \int_0^f C_t^l(j)dj + \int_f^1 C_t^o(j)dj$$

$$C_t = fC_t^l + (1-f)C_t^o$$

Similarly, aggregate labor is:

$$N_t = fN_t^l + (1-f)N_t^o$$

Domestic bonds:

$$B_t = (1-f)B_t^o$$

Private foreign bonds:

$$B_t^f = (1-f)B_t^{o,f}$$

Dividends:

$$D_t = (1-f)D_t^o$$

Oil revenues:

$$\Pi_t^{oil} = (1-f)\Pi_t^{o,oil}$$

Lump sum taxes:

$$T_t^{o,agr} = (1-f)T_t^o$$

Lump sum transfers:

$$T_t^{l,agr} = fT_t^l$$

Remittances:

$$REM_t = fREM_t^l$$

Aggregating ricardian household budget restriction:

$$\int_f^1 (1+\tau_t^c)P_tC_t^o dj + \int_f^1 B_t^o dj + \int_f^1 S_tB_t^{o,f}dj + \int_f^1 T_t^o dj = \int_f^1 (1-\tau_t^{N,o})V_tN_t^o dj$$
$$+ \int_f^1 (1+i_{t-1})z_{t-1}^{SW}B_{t-1}^o dj + \int_f^1 (1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^{o,f}dj + \int_f^1 (1-\tau_t^D)D_t^o dj + \int_f^1 (1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{o,oil}dj$$

$$(1-f)(1+\tau_t^c)P_tC_t^o + (1-f)B_t^o + (1-f)S_tB_t^{o,f} + (1-f)T_t^o = (1-f)(1-\tau_t^{N,o})V_tN_t^o$$
$$+(1-f)(1+i_{t-1})z_{t-1}^{SW}B_{t-1}^o + (1-f)(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^{o,f} + (1-f)(1-\tau_t^D)D_t^o + (1-f)(1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{o,oil}$$

$$(1-f)(1+\tau_t^c)P_tC_t^o + B_t + S_tB_t^f + T_t^{o,agr} = (1-f)(1-\tau_t^{N,o})V_tN_t^o$$
$$+ (1+i_{t-1})z_{t-1}^{SW}B_{t-1} + (1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f + (1-\tau_t^D)D_t + (1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{oil}$$

Aggregating non ricardian budget restriction:

$$\int_0^f (1+\tau_t^c)P_tC_t^l dj = \int_0^f (1-\tau_t^{N,l})V_tN_t^l dj + \int_0^f T_t^l dj + \int_0^f REM_t^l dj$$

$$f(1+\tau_t^c)P_tC_t^l = f(1-\tau_t^{N,l})V_tN_t^l + fT_t^l + fREM_t^l$$
$$f(1+\tau_t^c)P_tC_t^l = f(1-\tau_t^{N,l})V_tN_t^l + T_t^{l,agr} + REM_t$$

Combing both budget restrictions:

$$\begin{aligned}
P_tC_t + \tau_t^cP_tC_t + B_t + S_tB_t^f + T_t^{o.agr} &= V_tN_t - (1-f)\tau_t^{N,o}V_tN_t^o - f\tau_t^{N,l}V_tN_t^l + (1+i_{t-1})z_{t-1}^{SW}B_{t-1} \\
&+ (1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f + (1-\tau_t^k)\left[P_t^dY_t^d - R_t^kK_{t-1} - W_tN_t - P_t^MM_t - \frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t\right. \\
&\left. -\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t - \frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d\right] + \\
&\left[W_tN_t - V_tN_t - \frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t\right] + \left[(1-\tau_t^k)R_t^kK_{t-1} + \tau_t^k\delta Q_tK_{t-1} - (1-\tau_t^I)P_t^{dF}I_t^d\right. \\
&\left. -(1-\tau_t^I)P_t^{fF}I_t^f\right] + \left[P_t^fY_t^f - (1+\tau_t^a)MC_t^fY_t^f - \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota^f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)P_t^fY_t^f\right] - \tau_t^DD_t \\
&+ (1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{oil} + T_t^{l,agr} + REM_t
\end{aligned}$$

Simplifying:

$$\begin{aligned}
P_tC_t + \tau_t^cP_tC_t + B_t + S_tB_t^f + T_t^{o,agr} &= -(1-f)\tau_t^{N,o}V_tN_t^o - f\tau_t^{N,l}V_tN_t^l + (1+i_{t-1})z_{t-1}^{SW}B_{t-1} \\
&+ (1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f + P_t^dY_t^d - P_t^MM_t - \tau_t^kD_t^{d,b} - \tau_t^kR_t^kK_{t-1} + \tau_t^k\delta Q_tK_{t-1} - P_t^{dF}I_t^d - P_t^{fF}I_t^f \\
&+ \tau_t^IP_t^{dF}I_t^d + \tau_t^IP_t^{fF}I_t^f + P_t^fY_t^f - MC_t^fY_t^f - \tau_t^aMC_t^fY_t^f - \tau_t^DD_t + (1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{oil} + REM_t + T_t^{l,agr} \\
&- \frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t - \frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t - \frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d \\
&- \frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t - \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_t^f
\end{aligned}$$

Clearing the taxes:

$$\begin{aligned}
&\tau_t^cP_tC_t + (1-f)\tau_t^{N,o}V_tN_t^o + f\tau_t^{N,l}V_tN_t^l + \tau_t^kD_t^{d,b} + \tau_t^kR_tK_{t-1} - \tau_t^k\delta Q_tK_{t-1} - \tau_t^IP_t^{dF}I_t^d - \tau_t^IP_t^{fF}I_t^f + \tau_t^DD_t + \tau_t^aMC_t^fY_t^f = \\
&- P_tC_t - B_t - S_tB_t^f - T_t^{o,agr} + (1+i_{t-1})z_{t-1}^{SW}B_{t-1} + (1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f + P_t^dY_t^d - P_t^MM_t - P_t^{dF}I_t^d - P_t^{fF}I_t^f \\
&+ P_t^fY_t^f - MC_t^fY_t^f + (1-\psi_{oil})\omega_{oil}\Pi_{t-4}^{oil} + REM_t + T_t^{l,agr} \\
&- \frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t - \frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t - \frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d \\
&- \frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t - \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_t^f
\end{aligned}$$

From the government restriction:

$$\begin{aligned}
&\tau_t^cP_tC_t + (1-f)\tau_t^{N,o}V_tN_t^o + f\tau_t^{N,l}V_tN_t^l + \tau_t^kD_t^{d,b} + \tau_t^kR_t^kK_{t-1} - \tau_t^k\delta Q_tK_{t-1} + \tau_t^DD_t - \tau_t^IP_t^{dF}I_t^d - \tau_t^IP_t^{fF}I_t^f \\
&+ \tau_t^aMC_t^fY_t^f = -\tau_t^{vat}(P_t^dY_t^d + P_t^fY_t^f) - \psi_{oil}\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil} - T_t^{o,agr} - B_t - S_tB_t^{g,f} + P_t^{dF}C_t^g + P_t^{dF}I_t^g \\
&+ T_t^{l.agr} + (1+i_{t-1})z_{t-1}^{SW}B_{t-1} + (1+i_{t-1}^f)S_tB_{t-1}^{g,f}
\end{aligned}$$

Replacing and simplifying:

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f = -P_tC_t+P_t^dY_t^d+\tau_t^{vat}P_t^dY_t^d-P_t^MM_t-P_t^{dF}I_t^d-P_t^{dF}C_t^g$$
$$-P_t^{dF}I_t^g-P_t^{fF}I_t^f+P_t^fY_t^f+\tau_t^{vat}P_t^fY_t^f-MC_t^fY_t^f+REM_t+(1-\psi_{oil})\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}+\psi_{oil}\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$
$$-\frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t-\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t$$
$$-\frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d-\frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t$$
$$-\frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_t^f$$

Given that $P_t^{dF}=(1+\tau_t^{vat})P_t^d$ y $P_t^{fF}=(1+\tau_t^{vat})P_t^f$, previous expression can be rewrite as:

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f = -P_tC_t+P_t^{dF}Y_t^d-P_t^MM_t-P_t^{dF}I_t^d-P_t^{dF}C_t^g$$
$$-P_t^{dF}I_t^g-P_t^{fF}I_t^f+P_t^{fF}Y_t^f-MC_t^fY_t^f+REM_t+(1-\psi_{oil})\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}+\psi_{oil}\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$
$$-\frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t-\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t$$
$$-\frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d-\frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t$$
$$-\frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_t^f$$

Given $P_tC_t=P_t^{dF}C_t^d+P_t^{fF}C_t^f$:

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f = -P_t^{dF}C_t^d-P_t^{fF}C_t^f+P_t^{dF}Y_t^d-P_t^MM_t-P_t^{dF}I_t^d$$
$$-P_t^{dF}C_t^g-P_t^{dF}I_t^g-P_t^{fF}I_t^f+P_t^{fF}Y_t^f-MC_t^fY_t^f+REM_t+(1-\psi_{oil})\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}+\psi_{oil}\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$
$$-\frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t-\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t$$
$$-\frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d-\frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t$$
$$-\frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_t^f$$

Definition: *Domestic Product.*:

$$Y_t^d=C_t^d+I_t^d+C_t^g+I_t^g+X_t+\frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2\frac{W_t}{P_t^{dF}}N_t+\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2\frac{P_t^M}{P_t^{dF}}M_t$$
$$+\frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2\frac{P_t^d}{P_t^{dF}}Y_t^d+\frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2\frac{W_t}{P_t^{dF}}N_t$$

Definition: *Foreign Product*:

$$Y_t^f=C_t^f+I_t^f+\frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2\frac{P_t^f}{P_t^{fF}}Y_t^f$$

Replacing

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f=-P_t^{dF}C_t^d-P_t^{fF}C_t^f-P_t^MM_t-P_t^{dF}I_t^d-P_t^{fF}I_t^f-MC_t^fY_t^f-P_t^{dF}C_t^g$$
$$-P_t^{dF}I_t^g+\Bigg\{P_t^{dF}C_t^d+P_t^{dF}I_t^d+P_t^{dF}C_t^g+P_t^{dF}I_t^g+P_t^{dF}X_t+\frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t+\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t$$
$$+\frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d+\frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t\Bigg\}$$
$$+\left\{P_t^{fF}C_t^f+P_t^{fF}I_t^f+\frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_{tf}\right\}$$
$$+REM_t+(1-\psi_{oil})\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}+\psi_{oil}\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$
$$-\frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}}-1\right)^2W_tN_t-\frac{\zeta_{M^d}}{2}\left(\frac{M_t}{(1+g_t)M_{t-1}}-1\right)^2P_t^MM_t$$
$$-\frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)^2P_t^dY_t^d-\frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}-1\right)^2W_tN_t$$
$$-\frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}-1\right)^2P_t^fY_t^f$$

Simplifying:

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f=-P_t^MM_t-MC_t^fY_t^f+REM_t+P_t^{dF}X_t$$
$$+(1-\psi_{oil})\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}+\psi_{oil}\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f=-P_t^MM_t-MC_t^fY_t^f+REM_t+P_t^{dF}X_t$$
$$+\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$

Adding and subtracting $S_tP_t^{oil}Y_t^{oil}$

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f=-P_t^MM_t-MC_t^fY_t^f+REM_t+P_t^{dF}X_t$$
$$+S_tP_t^{oil}Y_t^{oil}+\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}-S_tP_t^{oil}Y_t^{oil}$$

Definition: *Total Exports*:

$$EXPORTS_t=P_t^{dF}X_t+S_tP_t^{oil}Y_t^{oil}$$

Definition: *Total Imports*:

$$IMPORTS_t=P_t^MM_t+MC_t^fY_t^f$$

Replacing

$$-S_tB_t^{g,f}+(1+i_{t-1}^f)S_tB_{t-1}^{g,f}+S_tB_t^f-(1+i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f=EXPORTS_t-IMPORTS_t+REM_t$$
$$+\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}-S_tP_t^{oil}Y_t^{oil}$$

Definition: *Trade Balance*:

$$TB_t=EXPORTS_t-IMPORTS_t$$

Definition: *Remittance of Profits*:

$$REM_t^{util}=S_tP_t^{oil}Y_t^{oil}-\omega_{oil}S_{t-4}P_{t-4}^{oil}Y_{t-4}^{oil}$$

Replacing

$$-\ S_tB_t^{g,f}\ +\ (1\ +\ i_{t-1}^f)S_tB_{t-1}^{g,f}\ +\ S_tB_t^f\ -\ (1\ +\ i_{t-1}^f)z_{t-1}^{SW}S_tB_{t-1}^f\ =\ TB_t\ +\ REM_t\ -\ REM_t^{util}$$

The expression can be rewrite as:

$$S_t(B_t^f - z_{t-1}^{SW}B_{t-1}^f) - S_t(B_t^{g,f} - B_{t-1}^{g,f}) = TB_t + REM_t - REM_t^{util} - i_{t-1}^f S_tB_{t-1}^{g,f} + i_{t-1}^f z_{t-1}^{SW}S_tB_{t-1}^f$$

6.2 Appendix. Standardized Model

To standardize the model, it's important to take account the numerarie P_t to kepp variables in real terms and the technological growth $\frac{A_t}{A_{t-1}} = (1 + g_t)$.

6.2.1 Ricardian Households

Taking the first order condition of the consumption (1):

$$z_t^c\left[C_t^o\frac{A_t}{A_t} - z_t^N\frac{1}{1+\eta_o}A_t\left(N_t^o\right)^{1+\eta_o}\right]^{-\sigma} = \Lambda_t^o(1+\tau_t^c)P_t$$

Given the standardized consumption $c_t = C_t/A_t$, previous expression can be write as:

$$z_t^c\left[c_t^o - z_t^N\frac{1}{1+\eta_o}\left(N_t^o\right)^{1+\eta_o}\right]^{-\sigma} = \lambda_t^o(1+\tau_t^c)$$

Where $\lambda_t = \Lambda_t P_t A_t^\sigma$. From the first order condition with respect to labor(2):

$$z_t^c\left\{C_t^o\frac{A_t}{A_t} - z_t^N\frac{1}{1+\eta_o}A_t\left(N_t^o\right)^{1+\eta_o}\right\}^{-\sigma}A_tz_t^N(N_t^o)^{\eta_o} = (1-\tau_t^{N,o})\Lambda_t^oV_t\frac{P_t}{P_t}$$

$$z_t^c\left\{c_t^o - z_t^N\frac{1}{1+\eta_o}\left(N_t^o\right)^{1+\eta_o}\right\}^{-\sigma}z_t^N(N_t^o)^{\eta_o} = (1-\tau_t^{N,o})\lambda_t^ov_t$$

Where $v_t = \frac{V_t}{P_tA_t}$. From equation (3):

$$\Lambda_t^o = \beta(1+i_t)z_t^{SW}\Lambda_{t+1}^o$$

$$\Lambda_t^oP_tA_t^\sigma = \beta(1+i_t)z_t^{SW}\Lambda_{t+1}^oP_t\frac{P_{t+1}}{P_{t+1}}A_t^\sigma\frac{A_{t+1}^\sigma}{A_{t+1}^\sigma}$$

$$\lambda_t^o = \beta\frac{(1+i_t)z_t^{SW}}{(1+\pi_{t+1})}(1+g_{t+1})^{-\sigma}\lambda_{t+1}^o$$

Where $\frac{P_{t+1}}{P_t} = (1+\pi_{t+1})$ and the domestic real interest rate is defined by $(1+r_t) = \frac{1+i_t}{1+\pi_{t+1}}$. Keep in mind that in the stationary state, $\lambda_t^o = \lambda_{t+1}^o$, so $\beta = \frac{1}{1+r}(1+g_t)^\sigma$. For the calibration, a value of the real interest rate is defined and the discount factor is found

From equation (4):

$$\Lambda_t^oS_t = \beta(1+i_t^f)z_t^{SW}S_{t+1}\Lambda_{t+1}^o$$

$$\Lambda_t^oS_tP_tA_t^\sigma\frac{P_t^*}{P_t} = \beta(1+i_t^f)z_t^{SW}S_{t+1}\Lambda_{t+1}^oP_t^*A_t^\sigma\frac{A_{t+1}^\sigma}{A_{t+1}^\sigma}\frac{P_{t+1}^*}{P_{t+1}^*}\frac{P_{t+1}}{P_{t+1}}$$

$$\lambda_t^o = \beta\frac{(1+i_t^f)z_t^{SW}}{1+\pi_{t+1}^*}dz_{t+1}(1+g_{t+1})^{-\sigma}\lambda_{t+1}^o$$

Where $dz_{t+1} = \frac{Z_{t+1}}{Z_t}$ is the real depreciation, $Z_t = S_t \frac{P_t^*}{P_t}$ is the real exchange rate and $\frac{P_{t+1}^*}{P_t^*} = (1 + \pi_{t+1}^*)$. The foreign real interest rate is defined as $(1 + r_t^f) = \frac{1+i_t^*}{1+\pi_{t+1}^*}$.

From the labor supply(5):

$$(N_t^o)^{\eta_o} = \frac{(1 - \tau_t^{N,o})}{(1 + \tau_t^c)} \frac{v_t}{z_t^N}$$

Finally, from the household budget constraint::

$$(1 + \tau_t^c) \frac{P_t C_t^o}{A_t P_t} + \frac{B_t^o}{A_t P_t} + \frac{S_t B_t^{o,f}}{A_t P_t} \frac{P_t^*}{P_t^*} + \frac{T_t^o}{A_t P_t} = (1 - \tau_t^{N,o}) \frac{V_t N_t^o}{A_t P_t} + (1 + i_{t-1}) z_{t-1}^{SW} \frac{B_{t-1}^o}{A_t P_t} \frac{A_{t-1} P_{t-1}}{A_{t-1} P_{t-1}}$$
$$+ (1 + i_{t-1}^f) z_{t-1}^{SW} \frac{S_t B_{t-1}^{o,f}}{A_t P_t} \frac{P_t^*}{P_t^*} \frac{P_{t-1}^*}{P_{t-1}^*} \frac{A_{t-1}}{A_{t-1}} + (1 - \tau_t^D) \frac{D_t^o}{A_t P_t} + (1 - \psi_{oil}) \omega_{oil} \frac{\Pi_{t-4}^{o,oil}}{A_t P_t}$$

$$(1 + \tau_t^c) c_t^o + b_t^o + Z_t b_t^{o,f} + t_t^o =$$
$$(1 - \tau_t^{N,o}) v_t N_t^o + \frac{(1 + i_{t-1}) z_{t-1}^{SW}}{(1 + \pi_t)} \frac{b_{t-1}^o}{(1 + g_t)} + \frac{(1 + i_{t-1}^f) z_{t-1}^{SW}}{(1 + \pi_t^*)} \frac{Z_t b_{t-1}^{o,f}}{(1 + g_t)} + (1 - \tau_t^D) d_t^o$$
$$+ (1 - \psi_{oil}) \omega_{oil} \frac{\pi_{t-4}^{o,oil}}{(1 + \pi_t)(1 + \pi_{t-1})(1 + \pi_{t-2})(1 + \pi_{t-3})(1 + g_t)(1 + g_{t-1})(1 + g_{t-2})(1 + g_{t-3})}$$

Where $c_t^o = \frac{C_t^o}{A_t}$, $t_t^o = \frac{T_t^o}{P_t A_t}$, $b_t^o = \frac{B_t^o}{P_t A_t}$,$b_t^{o,f} = \frac{B_t^{o,f}}{P_t^* A_t}$, $v_t = \frac{V_t}{P_t A_t}$, $d_t^o = \frac{D_t^o}{P_t A_t}$. Dividends are defined as:

$$D_t^o = D_t^{o,d,a} + D_t^{o,w} + D_t^{o,k} + D_t^{o,f}$$

standardized:

$$d_t^o = d_t^{o,d,a} + d_t^{o,w} + d_t^{o,k} + d_t^{o,f}$$

While the oil incomes are:

$$\frac{\Pi_{t-4}^{o,oil}}{P_t A_t} = \frac{S_{t-4} P_{t-4}^{oil} Y_{t-4}^{oil}}{P_t A_t} \frac{P_{t-4}^*}{P_{t-4}^*} \frac{P_{t-1}}{P_{t-1}} \frac{P_{t-2}}{P_{t-2}} \frac{P_{t-3}}{P_{t-3}} \frac{P_{t-4}}{P_{t-4}} \frac{A_{t-1}}{A_{t-1}} \frac{A_{t-2}}{A_{t-2}} \frac{A_{t-3}}{A_{t-3}} \frac{A_{t-4}}{A_{t-4}}$$
$$= \frac{Z_{t-4} p_{t-4}^{oil} y_{t-4}^{oil}}{(1 + \pi_t)(1 + \pi_{t-1})(1 + \pi_{t-2})(1 + \pi_{t-3})(1 + g_t)(1 + g_{t-1})(1 + g_{t-2})(1 + g_{t-3})}$$

6.2.2 Non Ricardian Households

From the first order condition with respect to consumption (7):

$$z_t^c \left\{ c_t^l - z_t^N \frac{1}{1 + \eta_l} \left(N_t^l \right)^{1+\eta_l} \right\}^{-\sigma} = (1 + \tau_t^c) \lambda_t^l$$

From the first order condition with respect to labor(8):

$$z_t^c \left\{ c_t^l - z_t^N \frac{1}{1 + \eta_l} \left(N_t^l \right)^{1+\eta_l} \right\}^{-\sigma} z_t^N (N_t^l)^{\eta_l} = (1 - \tau_t^{N,l}) \lambda_t^l v_t$$

and from the labor supply (9):

$$(N_t^l)^{\eta_l} = \frac{(1 - \tau_t^{N,l})}{(1 + \tau_t^c)} \frac{v_t}{z_t^N}$$

Finally, from the household budget constraint:

$$(1 + \tau_t^c) c_t^l = (1 - \tau_t^{N,l}) v_t N_t^l + t_t^l + rem_t^l$$

Where $c_t^l = \frac{C_t^l}{A_t}$, $t_t^l = \frac{T_t^l}{P_t A_t}$, $rem_t^l = \frac{REM_t^l}{P_t A_t}$

6.2.3 Consumption Bundle

Taking equation (10):

$$c_t^d = \omega_c \left[p_t^{dF}\right]^{-\theta_c} c_t$$

with $c_t^d = \frac{C_t^d}{A_t}$, $p_t^{dF} = \frac{P_t^{dF}}{P_t}$, $c_t = \frac{C_t}{A_t}$. Similarly, taking the demand for imported consumption (11):

$$c_t^f = (1-\omega_c)\left[p_t^{fF}\right]^{-\theta_c} c_t$$

and from the price index(12)[14]:

$$P_t = \left[\omega_c (P_t^{dF})^{1-\theta_c} + (1-\omega_c)(P_t^{fF})^{1-\theta_c}\right]^{\frac{1}{1-\theta_c}}$$

$$1 = \left[\omega_c (p_t^{dF})^{1-\theta_c} + (1-\omega_c)(p_t^{fF})^{1-\theta_c}\right]^{\frac{1}{1-\theta_c}}$$

6.2.4 Labor Agencies and Wages Rigidities

Taking the Phillips curve of wages (13):

$$(1-\theta_w)\frac{W_t N_t}{P_t A_t} + \theta_w \frac{V_t N_t}{P_t A_t} - \zeta_w \left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}\right)\frac{W_t N_t}{P_t A_t}$$
$$+\beta\left(\frac{\Lambda_{t+1}^o}{\Lambda_t^o}\right)\left(\frac{A_{t+1}}{A_{t+1}}\right)^{\sigma}\left(\frac{A_t}{A_t}\right)^{\sigma}\left(\frac{P_{t+1}}{P_{t+1}}\right)\left(\frac{P_t}{P_t}\right)\zeta_w\left(\frac{(1+\pi_{t+1}^w)}{(1+\pi_t^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)\left(\frac{(1+\pi_{t+1}^w)}{(1+\pi_t^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}\right)$$
$$\frac{W_{t+1}N_{t+1}}{P_t A_t}\frac{A_{t+1}}{A_{t+1}} = 0$$

Given $w_t = \frac{W_t}{P_t A_t}$ and simplifying:

$$\zeta_w \left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}\right) = \theta_w\left[\frac{v_t}{w_t} - \frac{\theta_w - 1}{\theta_w}\right] +$$
$$\beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\zeta_w \frac{w_{t+1}}{w_t}\frac{N_{t+1}}{N_t}\left(\frac{(1+\pi_{t+1}^w)}{(1+\pi_t^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)\left(\frac{(1+\pi_{t+1}^w)}{(1+\pi_t^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}}\right)$$

The aggregate dividends of labor agencies (14):

$$d_t^w = (w_t - v_t)N_t - \frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)^2 w_t N_t$$

6.2.5 Domestic Good Firms

Taking the production function(16):

$$y_t^d = z_t (N_t)^{1-\alpha_k-\alpha_{M^d}}\left(\frac{k_{t-1}}{1+g_t}\right)^{\alpha_k}(m_t^d)^{\alpha_{M^d}}\left(\frac{k_{t-1}^g}{1+g_t}\right)^{\psi}$$

Where $y_t^d = \frac{Y_t^d}{A_t}$, $k_t = \frac{K_t}{A_t}$ (same way to public capital).

From the capital demand (17)

$$r_t^k = \alpha_k mc_t (1+g_t)\frac{y_t^d}{k_{t-1}}$$

[14]Keep in mind that equation (12) can be rewrite as $P_t = (1+\tau_t^{vat})\left[\omega_c(P_t^d)^{1-\theta_c} + (1-\omega_c)(P_t^f)^{1-\theta_c}\right]^{\frac{1}{1-\theta_c}}$. Defining $P_t^c = \left[\omega_c(P_t^d)^{1-\theta_c} + (1-\omega_c)(P_t^f)^{1-\theta_c}\right]^{\frac{1}{1-\theta_c}}$ as the aggregate price before taxes, $P_t = (1+\tau_t^{vat})P_t^c$ and normalizing using P_t, $p_t^c = \frac{1}{1+\tau_t^{vat}}$. Thus, $(1+\pi_t^c) = \frac{p_t^c}{p_{t-1}^c}$ is the inflation of prices before the value added tax.

with $r_t^k = \frac{R_t^k}{P_t}$, $mc_t = \frac{MC_t}{P_t}$. From labor demand (18)

$$w_t = (1-\alpha_k-\alpha_M)mc_t\frac{y_t^d}{N_t} - \frac{\zeta_N}{2}w_t\left(\frac{N_t}{N_{t-1}}-1\right)^2 - \zeta_N w_t\frac{N_t}{N_{t-1}}\left(\frac{N_t}{N_{t-1}}-1\right) \\ + \beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\zeta_N w_{t+1}\left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k}\right)\left(\frac{N_{t+1}}{N_t}\right)^2\left(\frac{N_{t+1}}{N_t}-1\right)$$

Taking the imported inputs demand (19)

$$p_t^M = \alpha_{M^d}mc_t\frac{y_t^d}{m_t^d} - \frac{\zeta_{M^d}}{2}p_t^M\left(\frac{m_t^d}{m_{t-1}^d}-1\right)^2 - \zeta_{M^d}p_t^M\frac{m_t^d}{m_{t-1}^d}\left(\frac{m_t^d}{m_{t-1}^d}-1\right) \\ + \beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\zeta_{M^d}p_{t+1}^M\left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k}\right)\left(\frac{m_{t+1}^d}{m_t^d}\right)^2\left(\frac{m_{t+1}^d}{m_t^d}-1\right)$$

with $p_t^M = \frac{P_t^M}{P_t}$ y $m_t^d = \frac{M_t^d}{A_t}$. Besides, given $P_t^M = S_t P_t^{M*}$, $p_t^m = Z_t p_t^{M*}$ where $p_t^{M*} = \frac{P_t^{M*}}{P_t^*}$.

Taking the Phillips curve of domestic prices (23):

$$\zeta_d\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}\right) = \theta_d\left(\frac{mc_t}{p_t^d}-\frac{\theta_d-1}{\theta_d}\right)$$

$$\beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k}\right)\zeta_d\frac{p_{t+1}^d}{p_t^d}\left(\frac{y_{t+1}^d}{y_t^d}\right)\left(\frac{(1+\pi_{t+1}^d)}{(1+\pi_t^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}-1\right)\left(\frac{(1+\pi_{t+1}^d)}{(1+\pi_t^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}}\right)$$

To find marginal cost, first take equation (20):

$$\widetilde{w_t} = w_t + \frac{\zeta_N}{2}w_t\left(\frac{N_t}{N_{t-1}}-1\right)^2 + \zeta_N w_t\frac{N_t}{N_{t-1}}\left(\frac{N_t}{N_{t-1}}-1\right) \\ - \beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k}\right)\zeta_N w_{t+1}\left(\frac{N_{t+1}}{N_t}\right)^2\left(\frac{N_{t+1}}{N_t}-1\right)$$

and equation (21):

$$\widetilde{p_t^M} = p_t^M + \frac{\zeta_{M^d}}{2}p_t^M\left(\frac{m_t^d}{m_{t-1}^d}-1\right)^2 + \zeta_{M^d}p_t^M\frac{m_t^d}{m_{t-1}^d}\left(\frac{m_t^d}{m_{t-1}^d}-1\right) \\ - \beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\left(\frac{1-\tau_{t+1}^k}{1-\tau_t^k}\right)\zeta_{M^d}p_{t+1}^M\left(\frac{m_{t+1}^d}{m_t^d}\right)^2\left(\frac{m_{t+1}^d}{m_t^d}-1\right)$$

Where $\widetilde{p_t^M} = \frac{\widetilde{P_t^M}}{P_t}$ y $\widetilde{w_t} = \frac{\widetilde{W_t}}{P_t A_t}$. Thus, marginal cost (22):

$$MC_t = \left[\frac{1}{z_t}\right]\left[\frac{\widetilde{W_t}}{(1-\alpha_k-\alpha_{M^d})A_t}\right]^{1-\alpha_k-\alpha_{M^d}}\left[\frac{R_t^k}{\alpha_k}\right]^{\alpha_k}\left[\frac{\widetilde{P_t^M}}{\alpha_{M^d}}\right]^{\alpha_{M^d}}\left[\frac{A_t}{K_{t-1}^g}\right]^{\psi}$$

$$\frac{MC_t}{P_t} = \left[\frac{1}{z_t}\right]\left[\frac{\widetilde{W_t}}{(1-\alpha_k-\alpha_{M^d})P_tA_t}\right]^{1-\alpha_k-\alpha_{M^d}}\left[\frac{R_t^k}{P_t\alpha_k}\right]^{\alpha_k}\left[\frac{\widetilde{P_t^M}}{P_t\alpha_{M^d}}\right]^{\alpha_{M^d}}\left[\frac{A_t}{K_{t-1}^g}\left(\frac{A_t}{A_t}\right)\left(\frac{A_{t-1}}{A_{t-1}}\right)\right]^{\psi}$$

$$mc_t = \left[\frac{1}{z_t}\right]\left[\frac{\widetilde{w_t}}{(1-\alpha_k-\alpha_{M^d})}\right]^{1-\alpha_k-\alpha_{M^d}}\left[\frac{r_t^k}{\alpha_k}\right]^{\alpha_k}\left[\frac{\widetilde{p_t^M}}{\alpha_{M^d}}\right]^{\alpha_{M^d}}\left[\frac{(1+g_t)}{k_{t-1}^g}\right]^{\psi}$$

Where $k^g_{t-1} = \frac{K^g_{t-1}}{A_{t-1}}$. Once $\widetilde{w_t}$ and $\widetilde{p^M_t}$ are found, labor and imported inputs demands can be expressed as:

$$\widetilde{w_t} = (1 - \alpha_k - \alpha_{M^d}) mc_t \frac{y^d_t}{N_t}$$

$$\widetilde{p^M_t} = \alpha_{M^d} mc_t \frac{y^d_t}{m^d_t}$$

Finally, firm benefits before taxes (24):

$$d^{d,b}_t = \Bigg\{ p^d_t y^d_t - w_t N_t - r^k_t \frac{k_{t-1}}{(1+g_t)} - p^M_t m^d_t - \frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}} - 1\right)^2 w_t N_t - \frac{\zeta_{M^d}}{2}\left(\frac{m^d_t}{m_{t-1}} - 1\right)^2 p^M_t m^d_t - \frac{\zeta_d}{2}\left(\frac{(1+\pi^d_t)}{(1+\pi^d_{t-1})^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}} - 1\right)^2 p^d_t y^d_t \Bigg\}$$

and firm benefits after taxes (25):

$$D^{d,a}_t = (1 - \tau^k_t) D^{d,b}_t$$
$$d^{d,a}_t = (1 - \tau^k_t) d^{d,b}_t$$

where $d^{d,i}_t = \frac{D^{d,i}_t}{P_t A_t}$, $i = \{b, a\}$.

6.2.6 Capital Good Producer Firms

Taking the private capital accumulation (26):

$$k_t = (1-\delta)\frac{k_{t-1}}{(1+g_t)} + z^I_t \hat{I}_t \left(1 - \frac{\zeta_I}{2}\left(\frac{\hat{I}_t}{\hat{I}_{t-1}} - 1\right)^2\right)$$

where $\hat{I}_t = \frac{I_t}{A_t}$. From the first order condition with respect to the capital(27):

$$q_t = \frac{(1+\pi_{t+1})}{(1+i_t) z^{SW}_t} \left\{(1 - \tau^k_{t+1}) r^k_{t+1} + \tau^k_{t+1} \delta q_{t+1} + q_{t+1}(1-\delta)\right\}$$

where $q_t = \frac{Q_t}{P_t}$. Taking the investment demand (28):

$$p^I_t = q_t z^I_t \left[1 - \frac{\zeta_I}{2}\left(\frac{\hat{I}_t}{\hat{I}_{t-1}} - 1\right)^2 - \zeta_I \frac{\hat{I}_t}{\hat{I}_{t-1}}\left(\frac{\hat{I}_t}{\hat{I}_{t-1}} - 1\right)\right] + \frac{(1+\pi_{t+1})}{(1+i_t) z^{SW}_t}(1+g_{t+1}) q_{t+1} z^I_{t+1} \zeta_I \left(\frac{\hat{I}_{t+1}}{\hat{I}_t}\right)^2 \left(\frac{\hat{I}_{t+1}}{\hat{I}_t} - 1\right)$$

where $p^I_t = \frac{P^I_t}{P_t}$. Defining $\hat{I}^i_t = \frac{I^i_t}{A_t}, i = \{d, f\}$, domestic and imported investment demand (29) (30) are:

$$\hat{I}^d_t = \omega_I \left(\frac{(1 - \tau^I_t) p^{dF}_t}{p^I_t}\right)^{-\theta_I} \hat{I}_t$$

$$\hat{I}^f_t = (1 - \omega_I) \left(\frac{(1 - \tau^I_t) p^{fF}_t}{p^I_t}\right)^{-\theta_I} \hat{I}_t$$

and the investment price (31):

$$p^I_t = \left[\omega_I((1 - \tau^I_t p^{dF}_t)^{1-\theta_I} + (1 - \omega_I)((1 - \tau^I_t) p^{fF}_t)^{1-\theta_I}\right]^{\frac{1}{1-\theta_I}}$$

Finally, the dividends of the capital good producer firm (32) are:

$$D_t^k = (1-\tau_t^k)R_t^k K_{t-1} + \tau_t^k \delta Q_t K_{t-1} - (1-\tau_t^I)P_t^{dF} I_t^d - (1-\tau_t^I)P_t^{fF} I_t^f$$

$$\frac{D_t^k}{P_t A_t} = (1-\tau_t^k)\frac{R_t^k K_{t-1}}{P_t A_t}\frac{A_{t-1}}{A_{t-1}} + \tau_t^k \delta \frac{Q_t K_{t-1}}{P_t A_t}\frac{A_{t-1}}{A_{t-1}} - (1-\tau_t^I)\frac{P_t^{dF} I_t^d}{P_t A_t} - (1-\tau_t^I)\frac{P_t^{fF} I_t^f}{P_t A_t}$$

$$d_t^k = (1-\tau_t^k)r_t^k \frac{k_{t-1}}{(1+g_t)} + \tau_t^k \delta q_t \frac{k_{t-1}}{(1+g_t)} - (1-\tau_t^I)p_t^{dF}\hat{I}_t^d - (1-\tau_t^I)p_t^{fF}\hat{I}_t^f$$

6.2.7 Imported Good Producer Firms

Taking the Phillips curve of imported prices (34):

$$\zeta_f P_t^f \left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}\right) = \beta\left(\frac{\Lambda_{t+1}^o}{\Lambda_t^o}\right)\zeta_f P_{t+1}^f \left(\frac{Y_{t+1}^f}{Y_t^f}\right)$$
$$\left(\frac{(1+\pi_{t+1}^f)}{(1+\pi_t^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)\left(\frac{(1+\pi_{t+1}^f)}{(1+\pi_t^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}\right) + (1-\theta_f)P_t^f + \theta_f(1+\tau_t^a)MC_t^f$$

$$\zeta_f \left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}\right) = \theta_f\left[\frac{(1+\tau_t^a)mc_t^f}{p_t^f} - \frac{\theta_f - 1}{\theta_f}\right]$$
$$+\beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right)(1+g_{t+1})^{1-\sigma}\zeta_f \frac{p_{t+1}^f}{p_t^f}\left(\frac{y_{t+1}^f}{y_t^f}\right)\left(\frac{(1+\pi_{t+1}^f)}{(1+\pi_t^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)\left(\frac{(1+\pi_{t+1}^f)}{(1+\pi_t^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}}\right)$$

In this case, $mc_t^f = \frac{MC_t^f}{P_t} = \frac{S_t P_t^{f*}}{P_t}\frac{P_t^*}{P_t^*} = Z_t p_t^{f*}$. Besides, benefits (35) are:

$$D_t^f = P_t^f Y_t^f - (1+\tau_t^a)MC_t^f Y_t^f - \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)^2 P_t^f Y_t^f$$

$$d_t^f = p_t^f y_t^f - (1+\tau_t^a)mc_t^f y_t^f - \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)^2 p_t^f y_t^f$$

where $d_t^f = \frac{D_t^f}{P_t A_t}$.

6.2.8 Government Expenditures

Defining standardized public consumption and investment as $c_t^g = \frac{C_t^g}{A_t}$ and $\hat{I}_t^g = \frac{I_t^g}{A_t}$ respectively, primary expenditure (37) is:

$$k_t^g = (1-\delta^g)\frac{k_{t-1}^g}{(1+g_t)} + \varphi \hat{I}_t^g$$

Taking account that transfers to non ricardian households $T_t^{l,agr}$, standardized total expenditure without interest payments (39) is:

$$g_t = p_t^{dF} c_t^g + p_t^{dF}\hat{I}_t^g + t_t^{l,agr}$$

with $g_t = \frac{G_t}{P_t A_t}$. On the other hand, taking the public expenditure in interest payment (40):

$$G_t^b = i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^{g,f}$$

$$\frac{G_t^b}{P_t A_t} = i_{t-1} z_{t-1}^{SW}\frac{B_{t-1}}{P_t A_t}\frac{P_{t-1}A_{t-1}}{P_{t-1}A_{t-1}} + i_{t-1}^f \frac{S_t B_{t-1}^{g,f}}{P_t A_t}\frac{P_{t-1}^* P_t^*}{P_{t-1}^* P_t^*}\frac{A_{t-1}}{A_{t-1}}$$

$$g_t^b = i_{t-1} z_{t-1}^{SW} \frac{b_{t-1}}{(1+\pi_t)(1+g_t)} + i_{t-1}^f \frac{Z_t b_{t-1}^{g,f}}{(1+\pi_t^*)(1+g_t)}$$

where $g_t^b = \frac{G_t^b}{P_t A_t}$. Finally, total public expenditure (41):

$$g_t^T = g_t + g_t^b = \left[p_t^{dF} c_t^g + p_t^{dF} \hat{I}_t^g + t_t^{l,agr} \right] + \left[i_{t-1} z_{t-1}^{SW} \frac{b_{t-1}}{(1+\pi_t)(1+g_t)} + i_{t-1}^f \frac{Z_t b_{t-1}^{g,f}}{(1+\pi_t^*)(1+g_t)} \right]$$

with $g_t^T = \frac{G_t^T}{P_t A_t}$.

6.2.9 Government Revenues

First taking the revenues from oil sector (42):

$$rec_t^{oil} = \omega_{oil} \psi_{oil} \frac{Z_{t-4} p_{t-4}^{oil} y_{t-4}^{oil}}{(1+\pi_t)(1+\pi_{t-1})(1+\pi_{t-2})(1+\pi_{t-3})(1+g_t)(1+g_{t-1})(1+g_{t-2})(1+g_{t-3})}$$

with $rec_t^{oil} = \frac{Rec_t^{oil}}{P_t A_t}$. From the tax revenues (43):

$$rec_t^{\tau} = \tau_t^c c_t + (1-f) \tau_t^{N,o} v_t N_t^o + f \tau_t^{N,l} v_t N_t^l + \tau_t^k \left(d_t^{d,b} + r_t^k \frac{k_{t-1}}{(1+g_t)} - \delta q_t \frac{k_{t-1}}{(1+g_t)} \right) + \tau_t^a mc_t^f y_t^f + \tau_t^{vat} (p_t^d y_t^d + p_t^f y_t^f) + \tau_t^D d_t - \tau_t^I (p_t^{dF} \hat{I}_t^d + p_t^{fF} \hat{I}_t^f)$$

Thus, total revenues (44) are:

$$rec_t^{total} = rec_t^{\tau} + rec_t^{oil} + t_t^{o,agr}$$

where $rec_t^{total} = \frac{Rec_t^{total}}{P_t A_t}$ y $rec_t^{\tau} = \frac{Rec_t^{\tau}}{P_t A_t}$

6.2.10 Government Budget Restriction and Fiscal Deficit

Following the previous expenditures and revenues, the standardized fiscal budget restriction (45) is:

$$p_t^{dF} c_t^g + p_t^{dF} \hat{I}_t^g + t_t^{l,agr} + (1+i_{t-1}) z_{t-1}^{SW} \frac{b_{t-1}}{(1+\pi_t)(1+g_t)} + (1+i_{t-1}^f) \frac{Z_t b_{t-1}^{g,f}}{(1+\pi_t^*)(1+g_t)} = rec_t^{\tau} + rec_t^{oil} + t_t^{o,agr} + b_t + Z_t b_t^{g,f}$$

The primary deficit (46) is:

$$dp_t = p_t^{dF} c_t^g + p_t^{dF} \hat{I}_t^g + t_t^{l,agr} - rec_t^{\tau} - rec_t^{oil} - t_t^{o,agr}$$

and the total deficit (47):

$$dt_t = dp_t + g_t^b$$

where $dp_t = \frac{DP_t}{P_t A_t}$ y $dt_t = \frac{DT_t}{P_t A_t}$. Remember that there is fraction ϕ^B of the total public debt that corresponds to the domestic public debt, while the fraction $1 - \phi^B$ is the foreign public debt, so the domestic public debt accumulation is

$$B_t = \phi^B \left[DP_t + i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^f \right] + z_{t-1}^{SW} B_{t-1}$$

$$\frac{B_t}{P_t A_t} = \phi^B \left[\frac{DP_t}{P_t A_t} + i_{t-1} z_{t-1}^{SW} \frac{B_{t-1}}{P_t A_t} \frac{P_{t-1} A_{t-1}}{P_{t-1} A_{t-1}} + i_{t-1}^f \frac{S_t B_{t-1}^f}{P_t A_t} \frac{P_{t-1}^* A_{t-1}}{P_{t-1}^* A_{t-1}} \frac{P_t^*}{P_t^*} \right] + z_{t-1}^{SW} \frac{B_{t-1}}{P_t A_t} \frac{P_{t-1} A_{t-1}}{P_{t-1} A_{t-1}}$$

$$b_t = \phi^B \left[dp_t + i_{t-1} z_{t-1}^{SW} \frac{b_{t-1}}{(1+\pi_t)(1+g_t)} + i_{t-1}^f \frac{Z_t b_{t-1}^f}{(1+\pi_t^*)(1+g_t)} \right] + z_{t-1}^{SW} \frac{b_{t-1}}{(1+\pi_t)(1+g_t)}$$

and the foreign public debt accumulation (51):

$$S_t B_t^{g,f} = (1-\phi^B) \left[DP_t + i_{t-1} z_{t-1}^{SW} B_{t-1} + i_{t-1}^f S_t B_{t-1}^{g,f} \right] + S_t B_{t-1}^{g,f}$$

$$\frac{S_t B_t^{g,f}}{P_t A_t}\frac{P_t^*}{P_t^*} = (1-\phi^B)\left[\frac{DP_t}{P_t A_t} + i_{t-1} z_{t-1}^{SW}\frac{B_{t-1}}{P_t A_t}\frac{P_{t-1}A_{t-1}}{P_{t-1}A_{t-1}} + i_{t-1}^f \frac{S_t B_{t-1}^{g,f}}{P_t A_t}\frac{P_{t-1}^* A_{t-1}}{P_{t-1}^* A_{t-1}}\frac{P_t^*}{P_t^*}\right] + \frac{S_t B_{t-1}^{g,f}}{P_t A_t}\frac{P_{t-1}^* A_{t-1}}{P_{t-1}^* A_{t-1}}\frac{P_t^*}{P_t^*}$$

$$Z_t b_t^{g,f} = (1-\phi^B)\left[dp_t + i_{t-1} z_{t-1}^{SW}\frac{b_{t-1}}{(1+\pi_t)(1+g_t)} + i_{t-1}^f \frac{Z_t b_{t-1}^{g,f}}{(1+\pi_t^*)(1+g_t)}\right] + \frac{Z_t b_{t-1}^{g,f}}{(1+\pi_t^*)(1+g_t)}$$

Finally, defining $b_t^T = \frac{B_t^T}{P_t A_t}$ total debt (49) is:

$$b_t^T = b_t + Z_t b_t^{g,f}$$

6.2.11 Aggregation and External Definitions

Taking the aggregation of consumption, domestic debt, foreign debt, dividends and transfers:

$$c_t = f c_t^l + (1-f) c_t^o$$

$$b_t = (1-f) b_t^o$$

$$b_t^f = (1-f) b_t^{o,f}$$

$$d_t = (1-f) d_t^o$$

$$\pi_t^{oil} = (1-f)\pi_t^{o,oil}$$

$$t_t^{o,agr} = (1-f) t_t^o$$

$$t_t^{l,agr} = f t_t^l$$

$$rem_t = f rem_t^l$$

Given that $x_t = \frac{X_t}{A_t}$ y $y_t^* = \frac{Y_t^*}{A_t}$, exports demand (54) is:

$$x_t = \left(\frac{p_t^{dF}}{Z_t p_t^*}\right)^{-\theta_x} y_t^*$$

So total exports are (55):

$$EXPORTS_t = P_t^{dF} X_t + S_t P_t^{oil} Y_t^{oil}$$

$$exports_t = p_t^{dF} x_t + Z_t p_t^{oil} y_t^{oil}$$

and total imports (56) are:

$$IMPORTS_t = S_t P_t^{f*} Y_t^f + P_t^M M_t$$

$$imports_t = Z_t p_t^{f*} y_t^f + p_t^M m_t$$

where $exports_t = \frac{EXPORTS_t}{P_t A_t}$ y $imports_t = \frac{IMPORTS_t}{P_t A_t}$. Thus, $tb_t = \frac{TB_t}{P_t A_t}$ and the trade balance (57) is:

$$tb_t = exports_t - imports_t$$

On the other hand, domestic product (58) is:

$$y_t^d = c_t^d + \hat{I}_t^d + c_t^g + \hat{I}_t^g + x_t + \frac{\zeta_N}{2}\left(\frac{N_t}{N_{t-1}} - 1\right)^2 \frac{w_t}{p_t^{dF}} N_t + \frac{\zeta_{M^d}}{2}\left(\frac{m_t}{m_{t-1}} - 1\right)^2 \frac{p_t^M}{p_t^{dF}} m_t$$
$$+ \frac{\zeta_d}{2}\left(\frac{(1+\pi_t^d)}{(1+\pi_{t-1}^d)^{\iota_d}(1+\overline{\pi^d})^{1-\iota_d}} - 1\right)^2 \frac{p_t^d}{p_t^{dF}} y_t^d + \frac{\zeta_w}{2}\left(\frac{(1+\pi_t^w)}{(1+\pi_{t-1}^w)^{\iota_w}(1+\overline{\pi^w})^{1-\iota_w}} - 1\right)^2 \frac{w_t}{p_t^{dF}} N_t$$

and external product (59):

$$y_t^f = c_t^f + \hat{I}_t^f + \frac{\zeta_f}{2}\left(\frac{(1+\pi_t^f)}{(1+\pi_{t-1}^f)^{\iota_f}(1+\overline{\pi^f})^{1-\iota_f}} - 1\right)^2 \frac{p_t^f}{p_t^{fF}} y_t^f$$

Defining $cc_t = \frac{CC_t}{P_t A_t}$ the current account (61) is:

$$cc_t = exports_t - imports_t + REM_t - i^f_{t-1}\frac{Z_t b^{g,f}_{t-1}}{(1+\pi^*_t)(1+g_t)} + i^f_{t-1} z^{SW}_{t-1}\frac{Z_t b^f_{t-1}}{(1+\pi^*_t)(1+g_t)}$$
$$-\left(Z_t p^{oil}_t y^{oil}_t - \omega_{oil}\frac{Z_{t-4} p^{oil}_{t-4} y^{oil}_{t-4}}{(1+\pi_t)(1+\pi_{t-1})(1+\pi_{t-2})(1+\pi_{t-3})(1+g_t)(1+g_{t-1})(1+g_{t-2})(1+g_{t-3})}\right)$$

and the balance of payments (60):

$$-S_t B^{g,f}_t + S_t B^f_t = EXPORTS_t - IMPORTS_t + REM_t - (S_t P^{oil}_t Y^{oil}_t - \omega_{oil} S_{t-4} P^{oil}_{t-4} Y^{oil}_{t-4})$$
$$-(1+i^f_{t-1}) S_t B^{g,f}_{t-1} + (1+i^f_{t-1}) z^{SW}_{t-1} S_t B^f_{t-1}$$

$$-\frac{S_t B^{g,f}_t}{P_t A_t}\frac{P^*_t}{P^*_t} + \frac{S_t B^f_t}{P_t A_t}\frac{P^*_t}{P^*_t} = \frac{EXPORTS_t}{P_t A_t} - \frac{IMPORTS_t}{P_t A_t} + \frac{REM_t}{P_t A_t} - \left(\frac{S_t P^{oil}_t Y^{oil}_t}{P_t A_t}\frac{P^*_t}{P^*_t} - \omega_{oil}\frac{S_{t-4} P^{oil}_{t-4} Y^{oil}_{t-4}}{P_t A_t}\right)$$
$$-(1+i^f_{t-1})\frac{S_t B^{g,f}_{t-1}}{P_t A_t}\frac{P^*_t}{P^*_t}\frac{P^*_{t-1}}{P^*_{t-1}}\frac{A_{t-1}}{A_{t-1}} + (1+i^f_{t-1}) z^{SW}_{t-1}\frac{S_t B^f_{t-1}}{P_t A_t}\frac{P^*_t}{P^*_t}\frac{P^*_{t-1}}{P^*_{t-1}}\frac{A_{t-1}}{A_{t-1}}$$

$$-Z_t b^{g,f}_t + Z_t b^f_t = exports_t - imports_t + rem_t - (1+i^f_{t-1})\frac{Z_t b^{g,f}_{t-1}}{(1+\pi^*_t)(1+g_t)} + (1+i^f_{t-1}) z^{SW}_{t-1}\frac{Z_t b^f_{t-1}}{(1+\pi^*_t)(1+g_t)}$$
$$-\left(Z_t p^{oil}_t y^{oil}_t - \omega_{oil}\frac{Z_{t-4} p^{oil}_{t-4} y^{oil}_{t-4}}{(1+\pi_t)(1+\pi_{t-1})(1+\pi_{t-2})(1+\pi_{t-3})(1+g_t)(1+g_{t-1})(1+g_{t-2})(1+g_{t-3})}\right)$$

6.3 Appendix. Income Effect on Preferences

In the structure of the model we used preferences without income effect to avoid the relation between consumption and labor in the labor supply. However, we can assume other kind of preferences that include income effect and simulate the increase in risk premium to estimate the effects on macroeconomic and fiscal variables with the two possible fiscal adjustments that government can take: adjust public investment to keep constant total fiscal deficit to GDP or get into debt to finance bigger fiscal deficit.

Assume that ricardian and non ricardian households have a non separable preferences with exogenous habit consumption similar to González, A., Mahadeva, L., Prada, J.D. and Rodríguez, D. (2011):

$$U(C_t, N_t) = z^c_t\left\{\frac{1}{1-\sigma}\left[C_t - \chi(1+g_t)\overline{C_{t-1}}\right]^{1-\sigma} - z^N_t\frac{1}{1+\eta}A^{1-\sigma}_t (N_t)^{1+\eta}\right\}$$

In this case, labor supply (standardized) of ricardian households is defined by:

$$\frac{(N^o_t)^{\eta_o}}{\left[c^o_t - \chi\overline{c^o_{t-1}}\right]^{-\sigma}} = \frac{(1-\tau^{N,o}_t)}{(1+\tau^c_t)}\upsilon_t\frac{1}{z^N_t}$$

and labor supply (standardized) of non ricardian households would be:

$$\frac{(N^l_t)^{\eta_l}}{\left[c^l_t - \chi\overline{c^l_{t-1}}\right]^{-\sigma}} = \frac{(1-\tau^{N,l}_t)}{(1+\tau^c_t)}\upsilon_t\frac{1}{z^N_t}$$

Now the consumption decisions affect labor supply, so when there is an increase in risk premium and ricardian households reduce their consumption due to hardest financial condition, they increase their labor supply to look for labor income and avoid biggest falls in consumption. In this case, the highest risk premium also has a negative effect on credit market access and increase the interest payments that agents have to pay for their debt with the rest of the world. There is also a depreciation in exchange rate which increase inflation and central bank responds with a higher interest rate that discourage private consumption and investment.

When government decides to adjust investment to keep constant fiscal deficit to GDP, there is also a contraction in the domestic demand that turn down the inflation and policy interest rate, encouraging private consumption and investment. On the other hand, when government uses debt to finance the higher deficit, there is not a contraction in domestic demand, so inflation and policy interest rate is higher while private consumption and investment is lower. However, due to the income effect in labor supply, falls in consumption encourage labor supply and help in the midterm to return to the initial conditions, so the adjustment is not as persistent as in the case where we use preferences without income effect.

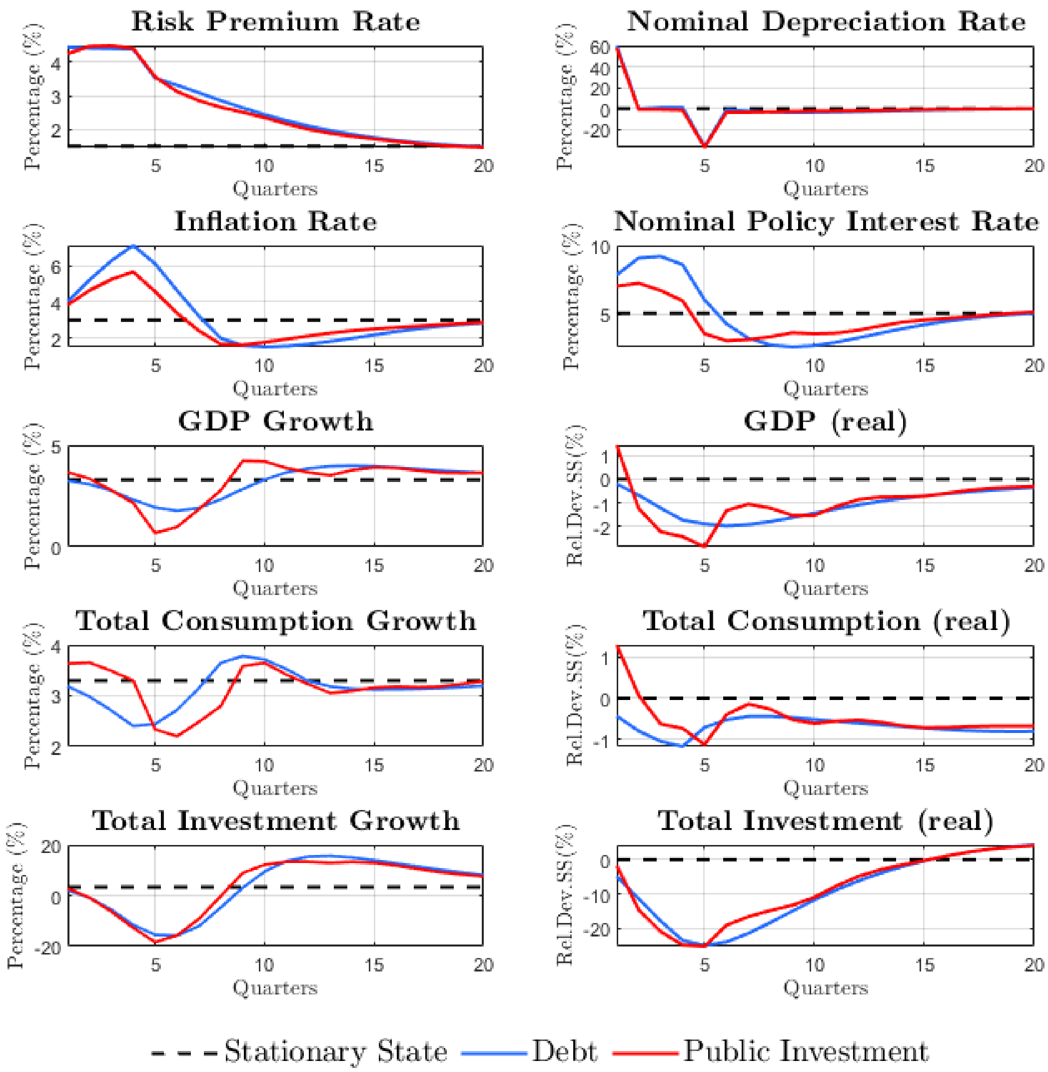

Figure 6: Macroeconomic Effects of Tightening of Financial Conditions with Income Effect Preferences

6.4 Appendix. Model Parameters

The following table shows the values of the other parameters used in the model[15]:

Table 3. Model Parameters

Parameter	Name	Value
σ	Intertemporal consumption elasticity	1,1
η_o	Inverse of the Frisch elasticity of the household with access to the financial market.	0.5
η_l	Inverse of the Frisch elasticity of the household without access to the financial market	0.5
ω_c	Proportion of domestic consumption in total consumption	0,9
ω_I	Proportion of domestic investment in the investment basket	0,5
θ_c	Elasticity of substitution between domestic and imported consumption	1,5
θ_I	Elasticity of substitution between domestic and imported investment	1,5
α_k	Share of capital in production	0.24
α_{M^d}	Share of imported inputs in production	0.07
ψ	Elasticity of the externality of public capital in production	0,028
δ	Depreciation of capital	0.011
ζ_w	Wage adjustment costs	50
ζ_d	Domestic price adjustment costs	20
ζ_f	Adjustment cost of imported prices	40
ζ_{M^d}	Adjustment costs of imported inputs	2.5
ζ_N	Labor adjustment costs	2,5
ζ_I	Investment adjustment costs	1,05
$\overline{\theta_w}$	Elasticity of substitution between work varieties	5
$\overline{\theta_d}$	Elasticity of substitution between intermediate domestic goods	5
$\overline{\theta_f}$	Elasticity of substitution for imported intermediate goods	5
ι_w	Wage indexation to inflation	1
ι_d	Indexation to domestic inflation	0.5
ι_f	Indexation to imported inflation	0,5
ϕ_i	Weight of the interest rate for the previous period	0,7
ϕ_π	Weight of the inflation gap	2,5
ϕ_y	Weight of the GDP growth gap	0,8
ω_{oil}	Proportion of oil revenues that stay in the country	0,8
ψ_{oil}	Proportion of oil revenue received by the government	0,42
δ^g	Depreciation of public capital	0,025
φ	Public investment efficiency	0,5
ϕ^B	Domestic debt as a proportion of total debt	0,7
Φ	Elasticity of the risk premium to the debt gap	0,01
θ_x	Elasticity of exports to the exchange rate	0,01
f	Proportion of non-optimizing households	0,45

[15]In this case the exports elasticity to exchange rate is low to avoid Mundell Fleming effects since the depreciation of exchange rate