

Interbank relationship lending in
Colombia

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Interbank relationship lending in Colombia^α

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Abstract

Stable and strong lending relations among financial institutions have been credited as the mainstay of interbank markets and financial stability. They are an indicator of trust among financial institutions. From a network perspective, we measure how stable relations are in the Colombian unsecured interbank lending market. We calculate the *survival ratio* of interbank networks, which corresponds to the fraction of linkages found in consecutive interbank lending networks. From January 2014 to September 2019, on average, about 58 per cent of linkages survive from one day to the next. About 36, 28, and 22 per cent of linkages survive during a 5-, 10-, and 20-day period, respectively. Regarding the strength of lending relations, results are robust to the exclusion of low-value linkages and non-banking institutions. A non-parametric test discards randomness as a plausible source of observed survival ratios. Preliminary examination of survival ratios during the first weeks of the financial turmoil caused by Covid-19 pandemic and falling oil prices suggests that trust in the interbank market was not seriously affected. Therefore, it is fair to conclude that stable and strong interbank lending relations exist in the Colombian market. From a financial stability perspective, the survival ratio may aid financial authorities in their quest for monitoring financial institutions' willingness to exchange funds among them.

JEL Classification: G10, G21, G23, L14

Keywords: interbank, lending, survival, linkage, network.

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Relaciones de préstamo interbancario en Colombia^α

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Resumen

Las relaciones estables y fuertes entre las instituciones financieras son consideradas como fundamentales para los mercados interbancarios y la estabilidad financiera. Estas relaciones son un indicador de confianza entre instituciones financieras. En este artículo medimos cuán estables son las relaciones en el mercado interbancario colombiano no colateralizado desde una perspectiva de análisis de redes. Para tal fin calculamos el coeficiente de sobrevivencia de las redes interbancarias, el cual se obtiene como la fracción de conexiones que se encuentran en redes de préstamo interbancario de manera consecutiva. Desde enero de 2014 hasta septiembre de 2019, encontramos que en promedio el 58 por ciento de las conexiones sobrevive de un día para otro. Para periodos de 5, 10 y 20 días este coeficiente se reduce a 36, 28 y 22 por ciento, respectivamente. Los resultados son robustos a la exclusión de conexiones de bajo monto y de instituciones no bancarias. Una prueba no paramétrica demuestra que los coeficientes de sobrevivencia obtenidos no son atribuibles a la aleatoriedad. Una revisión preliminar de los coeficientes de sobrevivencia durante las primeras semanas de estrés relacionado con la pandemia de Covid-19 y la caída de los precios del petróleo sugiere que la confianza en el mercado interbancario no resultó afectada seriamente. En consecuencia, puede concluirse que existen relaciones de préstamo interbancario en el mercado colombiano. Desde una perspectiva de estabilidad financiera, los coeficientes de sobrevivencia pueden ayudar a que las autoridades monitoreen la voluntad de intercambio de liquidez entre instituciones financieras.

Clasificación JEL: G10, G21, G23, L14

Palabras clave: interbancario, préstamo, sobrevivencia, conexiones, redes.

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1 Introduction

The presence of stable and strong interbank lending relationships is a well-documented fact that is commonly referred to in related literature as *interbank customer relationships* or *interbank relationship lending* (see Furfine, 1999, Furfine, 2001, Cocco, et al., 2009, Affinito, 2012, Afonso, et al., 2013, Hüser, 2016, Bräuning & Fecht, 2017, Kobayashi & Takaguchi, 2018).¹ From a theoretical perspective, the emergence of stable and strong interbank lending relations is an expected outcome as well (see Babus, 2012, Iori, et al., 2015, Hüser, 2016).

Several measures of interbank lending relations have been used. The number of transactions (see Furfine, 1999, Bräuning & Fecht, 2017) or the number of days with transactions (see Furfine, 2001) between banks during a certain period are common choices. The concentration of relations between banks, in the form of lending and borrowing preference indexes is also common (see Cocco, et al., 2009, Affinito, 2012, Temizsoy, et al., 2015, Capera-Romero, et al., 2015, Bräuning & Fecht, 2017). From a network modeling perspective, a measure of significant number of interbank trades –against a null network model– has been introduced by Kobayashi and Takaguchi (2018). Martínez (*forthcoming*) employs an econometric approach to identifying interbank lending relations.²

From a network perspective, the survival of linkages (i.e. relations) between nodes (i.e. elements) through time may be used to assess the existence of interbank relationship lending. This approach, introduced by Onnela, et al. (2003), yields a ratio that measures how stable a network is over time by computing the fraction of linkages found common in consecutive networks. This is what Onnela et al. refer to as the *survival ratio*.

The consecutive (i.e. repeated) nature of linkages between financial institutions is the sole determinant of the survival ratio. Therefore, from a network perspective, this ratio conveniently summarizes how persistent relations are in the interbank funding market. From a practical viewpoint, the survival ratio is a measure that is easy to calculate, interpret and monitor, that may be available as a high-frequency variable.

¹ The presence and benefits of stable funding relations between banks and their customers –commonly known as *relationship banking*– is also well-documented (see Petersen & Rajan, 1994, Ongena & Smith, 2000).

² Capera-Romero, et al. (2015) and Martínez (*forthcoming*) study the Colombian case. They both find evidence of interbank relationship lending.

Measuring the persistence or survival of interbank lending relations and its dynamics is a valuable tool for financial authorities. Interbank relationship lending plays a positive role in financial stability (Temizsoy, et al., 2015). Unexpected sharp decreases in the persistence of lending relations between financial institutions may signal changes in their willingness to borrow or lend funds from their peers. Eventually, such unexpected sharp decreases in interbank relationship lending may cause distress among borrowing financial institutions and increase systemic risk. Also, for research purposes, a measure of the persistence of interbank lending relations is convenient as an input for further understanding the interbank market in explanatory modeling.

Nevertheless, the survival of linkages in interbank networks has not been a recurrent research topic until recently. For instance, when studying core-periphery hierarchies in Italian and German interbank networks, Fricke and Lux (2014) and Craig and von Peter (2014), respectively, briefly remark that bilateral connections are persistent. Bargigli, et al. (2014) and Molina-Borboa, et al. (2015) study persistence in multi-layer interbank networks in the Italian and Mexican case, respectively; they both report high levels of persistence that depends on the type of interbank relation (e.g. secured, unsecured). The common choice for measuring persistence is the Jaccard index, also known as the *coefficient of community*. This may be an inconvenient choice because the Jaccard index is designed for cross-sectional data (see Jaccard, 1912), where the sequence of networks is inconsequential. Unlike the Jaccard index, the survival ratio ponders the sequence of networks.

We test the survival ratio on the Colombian unsecured interbank market. Unsecured interbank lending in Colombia is an over the counter market. The dataset consists of observed unsecured lending transactions between banking and non-banking financial institutions.³ Working on observed lending transactions –instead of exposures– is convenient as we are able to capture actual repeated funding. We extract data from the local large-value payments system data warehouse, and it is available daily, from January 2, 2014 to September 30, 2019. Intraday, overnight and term lending are available in the dataset. To the best of our knowledge, this is the first time the survival ratio is used to measure the persistence of linkages in financial networks; likewise, it is the first time the persistence of linkages is measured in interbank networks comprising banking and non-banking financial institutions, and including intraday interbank lending.

³ Non-banking financial institutions include securities brokers-dealers, pension funds and investment funds.

Linkages in Colombian interbank networks are persistent over time. On average, about 58 per cent of linkages survive from one day to the next. When measured over longer periods, linkages are persistent too. For instance, about 36 per cent of linkages survive during a 5-day period, whereas about 28 and 22 per cent survive during a 10- and 20-day period. A non-parametric test shows randomness is not a plausible source of observed survival ratios. Results are robust to the exclusion of low-value linkages and to the exclusion of intraday lending. Also, based on preliminary examination of survival ratios during the first weeks of turmoil caused by Covid-19 pandemic and falling oil prices, results suggest that trust among financial institutions in the interbank market was not seriously affected. Therefore, it is fair to conclude that stable and strong interbank lending relations exist in the Colombian financial system.

Results are consistent with interbank lending relationships literature. Results also concur with findings related to interbank lending relationship in the Colombian case. Additionally, results are consistent with previous findings related to the stability of linkages in interbank networks reported in Fricke and Lux (2014), Craig and von Peter (2014), Bargigli, et al. (2014) and Molina-Borboa, et al. (2015).

2 The interbank lending dataset

The interbank lending dataset is extracted from the Colombian large-value payment system (LVPS-CUD) data warehouse. LVPS-CUD is the country's single financial market infrastructure for settlement in local currency, owned and operated by the Central Bank of Colombia (Banco de la República). All types of financial institutions, banking and non-banking, local and foreign-owned, private and government-owned, may participate directly in the LVPS-CUD. Therefore, LVPS-CUD is a non-tiered payment system, with no settlement banks.⁴

It is mandatory for participating financial institutions and financial market infrastructures to report transactions based on a set of codes determined by Central Bank's LVPS-CUD. Therefore, unlike many other jurisdictions, interbank lending transactions are directly observable by extracting transactions reported with the corresponding code. In the case of interbank lending transactions reported by financial institutions directly, the standing set of codes was established in the second quarter of 2013.

⁴ There is no information about interbank transactions settled outside LVPS-CUD. However, as reported in León, et al. (2016), due to the non-tiered access scheme of LVPS-CUD and the absence of settlement banks, external settlement of interbank lending is expected to be unimportant –if any.

Interbank lending transactions corresponding to the local interbank reference rate formation program (Índice Bancario de Referencia - IBR) result from an auction among eight (annually selected) banks. They are reported to LVPS-CUD by a financial market infrastructure, with a distinct code. Therefore, as linkages from the interbank reference rate formation program do not result from interbank relationship lending –but an auction-, we exclude them.

The dataset consists of interbank lending transactions from January 2th, 2014 to September 30th, 2019, which total 1,400 observations (i.e. days). During this period, there are 66,509 interbank lending transactions. About 66 per cent of lending transactions (44,250) corresponds to intraday lending, whereas the remaining 34 per cent (22,259) corresponds to overnight or term lending. Regarding their value, intraday lending represents about 44 per cent, whereas the remaining 56 per cent corresponds to overnight or term lending. By value, about 81 per cent of non-intraday interbank lending corresponds to overnight lending, with about 18 and 1 per cent to 2-5 days and to more than 5 days, respectively.⁵

The contribution of unsecured lending to total lending between financial institutions in money markets is rather low. In the period under analysis, secured lending between financial institutions (i.e., sell/buy backs and repos) account for about 85 per cent of money market transactions by value, whereas unsecured interbank lending contributes with about 15 per cent. After we discard transactions corresponding to the local interbank reference rate formation program, unsecured interbank lending contributes with about 11 per cent of money market transactions by value.

The low contribution of unsecured interbank lending is consistent with findings by Martínez & León (2016). They point out that the Colombian money market appears to be a case of liquidity cross-underinsurance (see Castiglionesi & Wagner, 2013), in which strong negative potential externalities arising from counterparties' failure discourage lending without collateral. Nevertheless, as it is widely accepted in related literature, it is the absence of collaterals in unsecured interbank lending markets which creates powerful incentives for participants to monitor each other and to exert market discipline (Rochet & Tirole, 1996, Furfine, 2001). And, there is a well-established connection between market discipline, peer monitoring, and interbank lending relations (see Furfine, 2001, Affinito, 2012, Temizsoy, et al., 2015, Hüser, 2016, Bräuning & Fecht, 2017).

⁵ The breakdown of non-intraday interbank lending by maturity is calculated from data provided by the Financial Superintendence of Colombia. All other calculations are based on LVPS-CUD data.

3 The survival ratio

In financial networks literature, the survival ratio of linkages has been used to assess how stable a network is (see Onnela, et al., 2003, Coehlo, et al., 2007, Ji & Fan, 2017). To the best of our knowledge, the most common application of the survival ratio in financial networks is related to measuring the robustness of networks resulting from filtering correlation-based graphs into asset trees.⁶

For instance, Onnela, et al. (2003) measure the survival ratio of an asset tree consisting of 477 stocks traded at the NYSE from January 1980 to December 1999; Onnela, et al. is credited as the first research article to measure the survival ratio in financial networks. Coehlo, et al. (2003) measure the survival ratio of asset trees corresponding to stock markets from 53 countries from January 1997 to February 2006, whereas Jin & Fan (2017) use an asset tree of 24 oil-producing and oil-consuming countries from January 2000 to November 2011. In these three cases the aim is to measure how robust asset trees are; that is, to assess how stable cross-section interdependences (i.e. correlations) are through time. Unlike these three cases, Cepeda, et al. (2017) measure the survival ratio on a tree that retains the most salient features of world trade networks from 1995 to 2014. All four applications measure the survival ratio on trees built based on the minimal spanning tree method suggested by Mantegna (1999).

In the herein case, the survival ratio is used to measure how persistent interbank lending relations are over time. Persistency in lending relations is particularly important in financial literature because repeated relations have been documented as determinant of liquidity exchanges in money markets (see Furfine, 1999, Furfine, 2001, Cocco, et al., 2009, Affinito, 2012, Afonso, et al., 2013, Capera-Romero, et al., 2015, Hüser, 2016), and –thus- a vital factor in the evolution and stability of financial markets.

Onnela, et al. (2003) introduce two survival ratios, namely the *single-step survival ratio* and the *multi-step survival ratio*.⁷ The former ratio measures the fraction of linkages (i.e. lending transactions) found common in two consecutive networks; that is, it measures the persistence of connections from time

⁶ A graph is a representation of a network, with nodes corresponding to elements, and linkages to relations between elements. A tree is a special case of a graph, in which all nodes are connected to the network, with non-crossing linkages (i.e. a hierarchical graph) and without boucle of any size; in all trees, the number of nodes equals the number of edges plus one (Börner, et al., 2007). In a minimal spanning tree, the sum of all edges of the network is minimum.

⁷ Onnela et al. (2003) introduces the survival ratio for trees –a special case of networks (see previous footnote). Therefore, our calculation of survival ratios is slightly different from that introduced in Onnela et al. (2003). In our case, the denominator measures the number of connections in the first graph (i.e. the benchmark) instead of the number of connections in a tree, which is fixed by definition as number of participants minus one.

$t - 1$ to time t . Let E_t be the set of linkages of the network at time t , \cap the intersection operator, and $|\dots|$ the number of elements in the set, the single step survival ratio at time t is denoted s_t , and is calculated as⁸

$$s_t = \frac{1}{|E_{t-1}|} |E_t \cap E_{t-1}| \quad [1]$$

where

$$0 \leq s_t \leq 1$$

The multi-step survival ratio introduced by Onnela, et al. (2003) measures the long-term persistence of linkages in a network. Unlike the single step survival ratio (s_t), the multi-step survival ratio, \bar{s}_t , measures the fraction of linkages found common in a continuum of consecutive networks. By setting $k > 1$ in [2], it is possible to calculate the fraction of linkages that persisted for an entire period without any interruption; in the case of $k = 1$, the multi-step and the single-set survival ratio are identical.

$$\bar{s}_t = \frac{1}{|E_{t-k}|} |E_t \cap E_{t-1} \dots \cap E_{t-k}| \quad [2]$$

where

$$k > 1$$

$$0 \leq \bar{s}_t \leq 1$$

In the multi-step survival ratio, when the linkage between two nodes is missing even once in k steps it is considered as a non-survival. Therefore, it is a more rigorous measure of persistence of linkages in networks. Hence, $\bar{s}_t \leq s_t$.

As the interbank market is expected to show repeated interactions between participating financial institutions, we expect the survival ratio to be different from zero; otherwise, the generating process behind financial institutions' interactions could be described as -approximately- random. On the other hand, we

⁸ An alternative to the survival ratio is the Jaccard index (see Fricke & Lux, 2014, Craig & von Peter, 2014, Bargigli, et al., 2014, Molina-Borboa, et al., 2015). Nevertheless, in the Jaccard index the order of the networks is irrelevant. Let j_t be the Jaccard index at time t , and \cup the union operator, $j_t = |E_t \cap E_{t-1}| / |E_t \cup E_{t-1}|$. As the denominator is the union of two samples (instead of the first sample in $t - 1$, as in the survival ratio) the sequence of the networks is irrelevant; that is, persistence from t to $t - 1$ and from $t - 1$ to t are identical. This drawback also applies to the – less common- *topological overlap* (see Clauset & Eagle, 2007) reported in Molina-Borboa, et al. (2015). For comparison purposes, results attained with the Jaccard index are reported in the Appendix.

also expect the survival ratio to be different from one; otherwise, financial institutions' interactions could be described as fixed. All in all, we expect a survival ratio that deviates from both lower and upper limits.

4 Main results

We work on the 1,400 networks corresponding to the 1,400 daily observations in the sample. Working on daily networks is a natural choice because most interbank lending has a maturity lower or equal to one day.⁹ That is, as intraday and overnight lending are prevalent, we expect most interactions to be repeated daily.

The networks we work on are directed and unweighted. They are directed (i.e. the direction of the flows is relevant) because being a lender or a borrower is critical to the interbank market. Also, as we focus on the existence (or absence) of linkages between financial institutions, networks are unweighted.¹⁰ In order to avoid potential biases due to low-value linkages, we discard all linkages in the 10th percentile by value; thus, all linkages under about COP \$440 million, about USD \$130,000 as of October 2019 are discarded.¹¹ After filtering out low-value linkages, there are 101 participants along the sample, comprising banking and non-banking financial institutions. The dataset consists of an hypermatrix with 101 rows (lenders), 101 columns (borrowers), and 1400 layers (observations).

Before reporting the survival ratios, we report some of the main connective features of the networks in the dataset. Based on network analysis basics (see Börner, et al., 2007, Newman, 2009), Table 1 and Figure 1 display the following descriptive statistics of the networks: i) number of participants (i.e. nodes with at least one linkage); ii) mean geodesic distance, corresponding to the average minimal distance (i.e. shortest path) between all pairs of reachable nodes, measured in number of links; iii) density or connectedness, corresponding to the fraction of possible linkages observed in the network; iv) reciprocity, corresponding to the fraction of linkages that are reciprocated (i.e. dyads); and v) transitivity or clustering coefficient, corresponding to the fraction of possible triads (i.e. transitive relations) observed in the network.

⁹ Following Clauset and Eagle (2007), working on daily frequency is a principled choice based on how the interbank market works in Colombia –thus, it may be considered the *natural snapshot rate* of the interbank network. Our choice is the same as in Molina-Borboa, et al. (2015).

¹⁰ The unweighted nature of networks under analysis may seem inconvenient as the informational content of the weights is lost. However, it has been reported that unweighted properties often convey more information about real-world economic networks than that corresponding to weighted properties (see Squartini, et al., 2013).

¹¹ As shown in Table A4 and A5 (in Appendix), results are robust to other choices of this threshold (i.e. no limit, 15th percentile).

Statistic	Mean	St. dev.	Min	Max
Number of participants ^a	31.17	3.64	19.00	43.00
Mean geodesic distance ^b	1.83	0.36	1.00	3.63
Density ($\times 100$) ^c	0.34	0.07	0.16	0.55
Reciprocity ($\times 100$) ^d	3.00	4.41	0.00	24.24
Transitivity ($\times 100$) ^e	0.07	0.29	0.00	3.23

Table 1. Networks' descriptive statistics, from January 2nd, 2014 to September 30th, 2019. ^a Number of nodes with at least one linkage in the network; ^b average minimal distance (i.e. measured in number of links) between all pairs of reachable nodes; ^c fraction of possible linkages observed in the network, under the assumption of no self-connections; ^d fraction of linkages that are reciprocated; ^e fraction of transitive relations observed in the network. Source: authors' calculations, based on data from Banco de la República.

As reported in León, et al. (2016) and Sarmiento, et al. (2017), the number of participants in the Colombian interbank market is rather low. On average, about 31 financial institutions borrow or lend each day, with a minimal of 19 financial institutions. Furthermore, as discussed below, if intraday lending is excluded, the average drops to about 14 financial institutions.

Colombian interbank networks share most of the connective features reported in related literature (see Bech & Atalay 2010, Fricke & Lux, 2014, Hüser, 2016).¹² Also, the most salient connective features correspond to those reported in previous studies related to the Colombian interbank market (León & Sarmiento, 2016, León, et al., 2018). It is a sparse (i.e. low density) network, in which only a small fraction (less than 1 per cent) of all potential relations is observed¹³. The mean geodesic distance is low, consistent with *small world networks*. Linkages do not tend to be reciprocal. This is an expected outcome as it should be uncommon for a participant to borrow and lend from another counterparty on the same day; yet, interestingly, it is not negligible in some days. And, linkages do not tend to be transitive.

¹² We do not report the distribution of linkages. Not only the distribution of linkages is unimportant in this case, but also the size of the networks is too small to properly fit a (e.g. power law, negative binomial) distribution. However, the visualization of the empirical distribution and the average fit of a power law distribution show that Colombian interbank networks are consistent with an inhomogeneous network (see Bollobás, et al., 2007); that is, the distribution of linkages is particularly skewed and fat-tailed, showing that only a few nodes are heavily connected, whereas most nodes are poorly connected. Therefore, as in León et al. (2018), it is fair to say that Colombian interbank networks also share the type of distribution documented for interbank networks (see Hüser, 2016).

¹³ The low density of Colombian interbank network (i.e. below 1 per cent) may seem extreme. However, density has been reported below 1 per cent in several interbank networks, such as the US (see Furfine, 1999, Bech & Atalay, 2010), Germany (see Craig & von Peter, 2014), and Mexico (Molina-Borboa, et al., 2015).

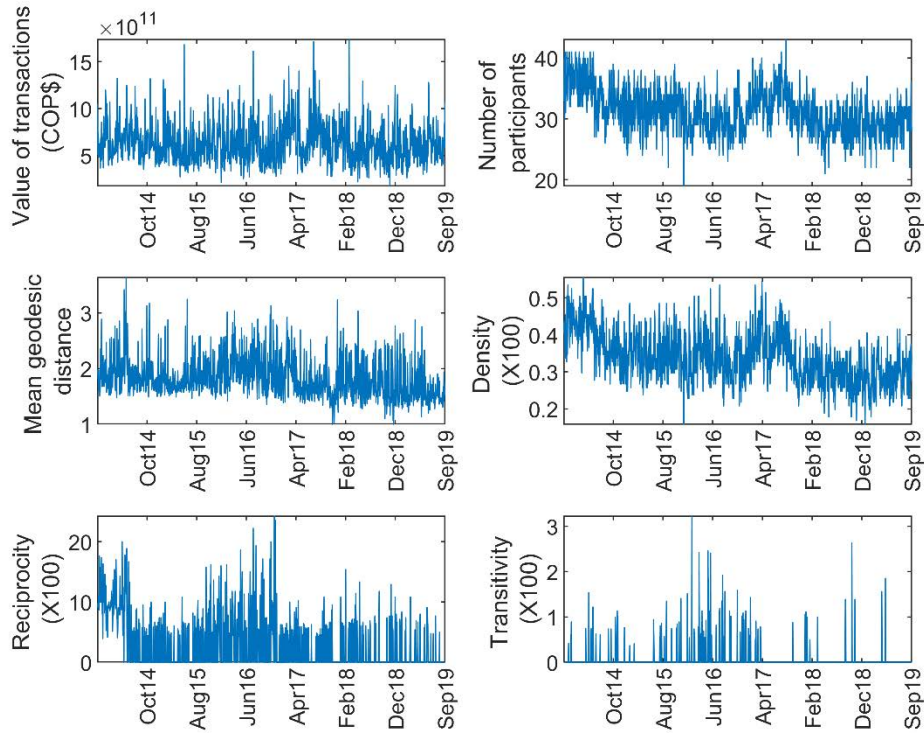


Figure 1. Networks' descriptive statistics, from January 2nd, 2014 to September 30th, 2019. Source: authors' calculations, based on data from Banco de la República.

Figure 2 displays the single-step survival ratio for the Colombian interbank networks. On average, about 58 per cent of linkages survive from one day to the next. This is consistent with interbank relationship lending empirical and theoretical literature. It suggests that the linkages between financial institutions in the Colombian interbank networks are not random or occasional, but fairly persistent in consecutive days.

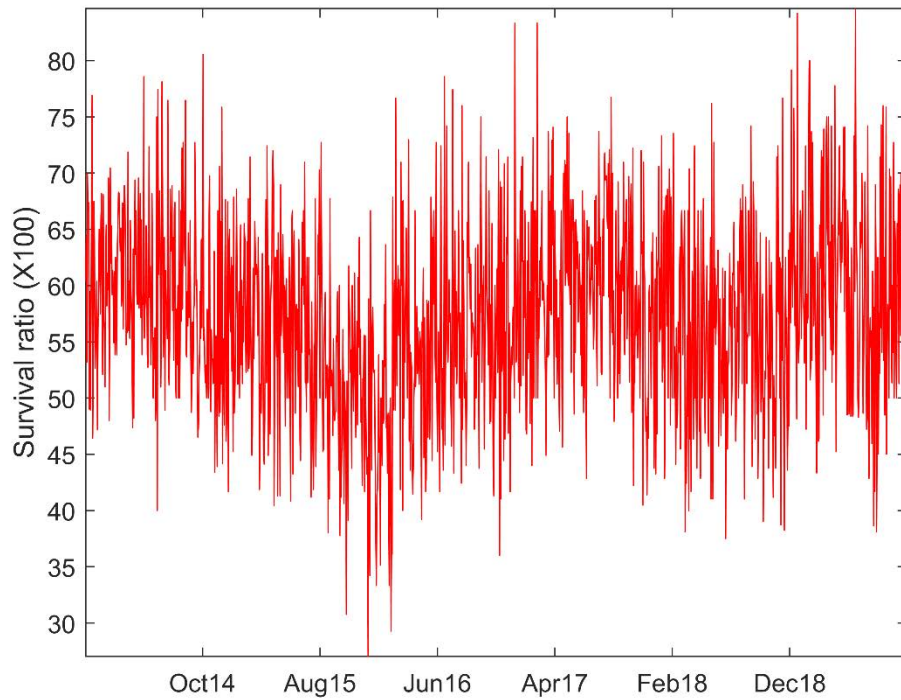


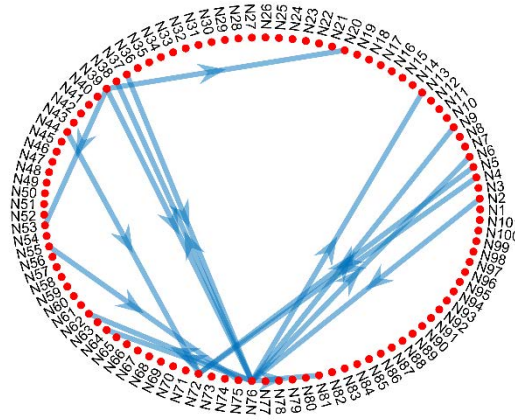
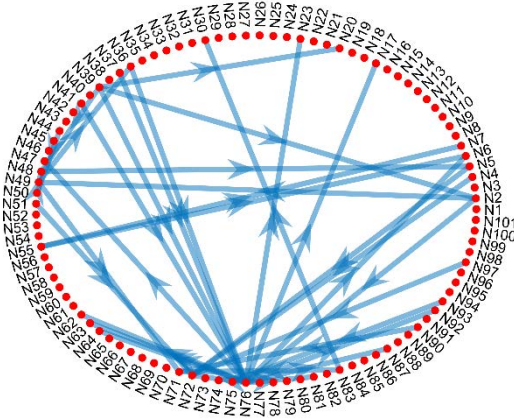
Figure 2. Single-step survival ratio, from January 2nd, 2014 to September 30th, 2019. Source: authors' calculations, based on data from Banco de la República.

The single step survival ratio reaches 27 per cent minimum and 85 per cent maximum. Figure 3 shows samples of both extreme cases. Each sample shows the network in $t - 1$ and t ; financial institutions are represented as nodes with a tag that conceals their identity (for confidentiality reasons), and they are portrayed in a circular fashion for visualization purposes; arrows represent funding relations in the interbank market, from the lender to the borrower.

a. $s_t = 0.27$ (i.e. 10 out of 37 linkages)

Wednesday, December 23, 2015

Thursday, December 24, 2015



b. $s_t = 0.85$ (i.e. 22 out of 26 linkages)

Wednesday, May 22, 2019

Thursday, May 23, 2019

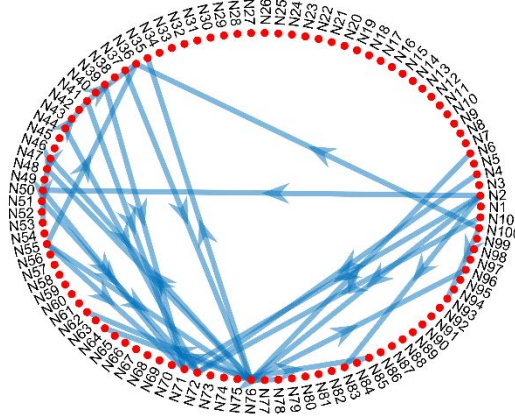
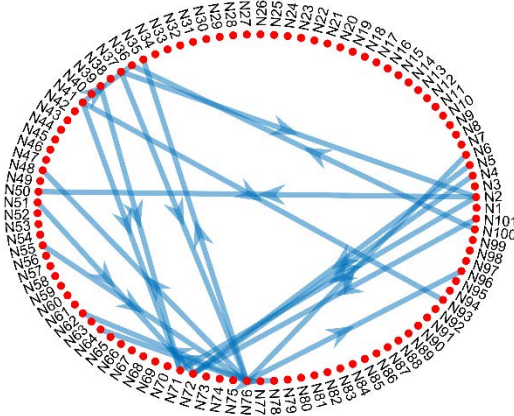


Figure 3. Networks corresponding to two extreme cases of survival. Financial institutions are represented as nodes with a tag that conceals their identity (for confidentiality reasons), and they are portrayed in a circular fashion for visualization purposes. Arrows represent funding relations in the interbank market, from the lender to the borrower. In the upper panel (a.), the survival ratio is 27 per cent; that is, 10 out of 37 linkages in the first date are present in the next one. In the lower panel (b.), the survival ratio is 85 per cent; that is, 22 out of 26 linkages in the first date are present in the next one. Source: authors' calculations, based on data from Banco de la República.

The single-step survival ratio is informative about how persistent relations are among financial institutions. However, the multi-step survival ratio provides a more rigorous measure of interbank lending relations. Figure 4 displays different choices of k for the multi-step survival ratio. The choices, $k = 5, 10, 20$, approximately correspond to a week, two weeks, and four weeks periods, respectively.

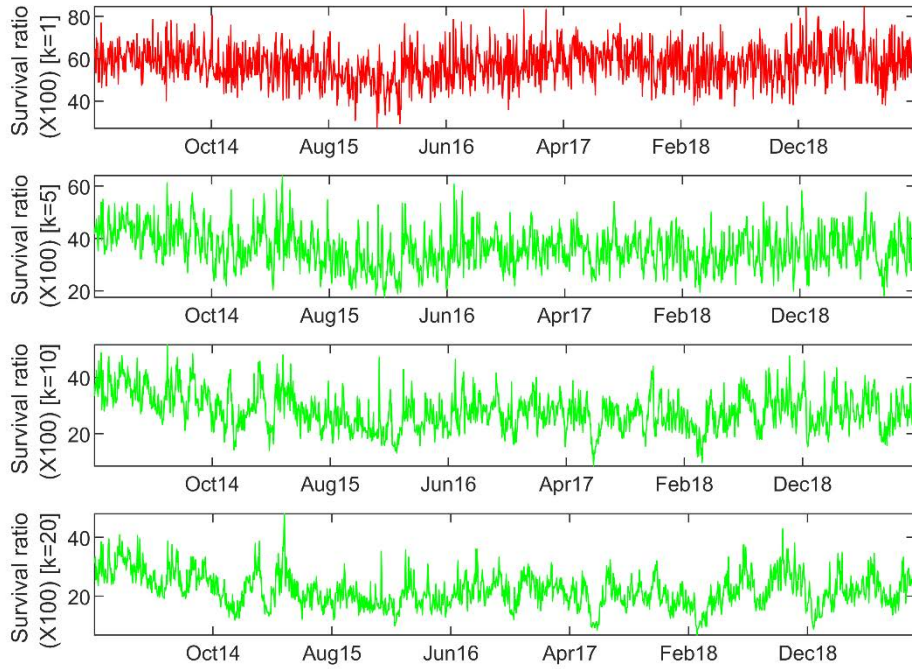


Figure 4. Single-step and multi-step survival ratio, from January 2nd, 2014 to September 30th, 2019. Source: authors' calculations, based on data from Banco de la República.

As expected, Figure 4 shows that the higher k , the lower the survival ratio. About 36 per cent of linkages survive during a 5-day period, whereas about 28 and 22 per cent survive during a 10- and 20-day period. That is, the longer the evaluation period in which linkages should persist without interruption, the lower the fraction of linkages between financial institutions that survive.

5 Tests

Three tests are implemented to further study the survival ratio in the Colombian case. First, we design a test for randomness of results; that is, to discard randomness as a plausible cause of our results. Second, we exclude intraday lending from the dataset to test whether results are dependent on the existence of very short-term lending to non-banking financial institutions. Third, we show how the survival ratio evolved amid the first weeks of turmoil caused by Covid-19 pandemic and falling oil prices.

5.1 Randomness of results

Although a single-step survival ratio of about 58 per cent appears to be high (i.e. distant from the lower limit, 0), we must test whether it may be the result of sheer chance. We build a null model for the networks in order to test whether the survival ratio of observed interbank networks differs from a random allocation of linkages in the network.

We perform a limited randomization of the networks by randomly reallocating linkages from lending to borrowing financial institutions in each observation.¹⁴ This would retain the main connective features of the networks while respecting the nature of financial institutions, which may be borrowers, lenders, both or none. This randomization is performed 100 times for each of the 1,400 networks in the dataset.¹⁵ The distribution of survival ratios attained from randomized networks are compared with that from observed networks using a two-sample Kolmogorov-Smirnov test, which is a standard non-parametric test of the equality of probability distributions.

Table 2 exhibits the statistics of the single-step and multi-step survival ratios calculated on the observed dataset. The last column (in italic) reports the mean survival ratio on randomized networks; a † marks when the null hypothesis of equal distribution with respect to observed networks is rejected at 0.99 confidence.

Survival ratio (X100)		Mean	St. dev.	Min	Max	<i>Null mean^a</i>
Single-step	$k = 1$	57.29	8.66	27.03	84.62	<i>17.99†</i>
	$k = 5$	36.12	7.70	0.00	64.00	<i>2.08†</i>
Multi-step	$k = 10$	27.93	7.07	0.00	51.61	<i>0.42†</i>
	$k = 20$	21.80	6.30	0.00	48.00	<i>0.00†</i>

Table 2. Survival ratio, from January 2nd, 2014 to September 30th, 2019. ^a Calculated on randomized networks. † Null hypothesis of equal distribution of observed and randomized data is rejected at 0.99 confidence (i.e. p-value < 0.01) with Kolmogorov-Smirnov non-parametric two-sample test. Source: authors' calculations, based on data from Banco de la República.

¹⁴ This entails randomly shuffling observed linkages among actual borrowing and lending institutions; thus, not only the number of linkages in the network tends to be preserved but also the number of linkages for each financial institution. Randomizing linkages within the entire network is not a sound choice because the existence of a small group of lenders and borrowers would be neglected. An alternative to the randomizing procedure herein implemented could be based on network modeling, such as the one introduced by Kobayashi & Takaguchi (2018). However, not only it is a more complex and demanding procedure, but it is uncertain to what extent it could replicate over time the small set of borrowing and lending institutions that is observed in the Colombian interbank network.

¹⁵ The Appendix includes randomized networks' descriptive statistics (Table A1) and survival ratios (Table A2 and A3).

As expected, the mean survival ratio for the randomized network is visibly lower than that for observed networks at all choices of k . For instance, the mean single-step survival ratio for the randomized network is about 18 per cent, whereas for the observed network it is about 58 per cent; that is, the observed single-step average survival is about 3.18 times that of the null network. In all cases the null hypothesis of equal distribution with respect to the randomized network is rejected. Therefore, it is fair to conclude that stable and strong interbank lending relations exist in the Colombian market.¹⁶

5.2 Intraday lending

Reported results correspond to the dataset that contains lending among banking and non-banking financial institutions to any term available, namely intraday, overnight and term lending. If intraday lending is excluded from the dataset, non-banking financial institutions are excluded too. Presumably, as documented in Martínez & León (2015), Colombian non-banking institutions and small banking institutions are unable to borrow without a collateral unless they refund in the same day due to counterparty risk aversion; this overlaps with empirical findings by Allen et al. (1989), who report that smaller banks in the US tend to borrow with collateral more than larger banks. On the other hand, large banking institutions may avoid opportunity costs by not pledging collateral among them in the unsecured interbank market, while they preserve unencumbered collateral to access central bank liquidity.

Table 3 shows main connective features of the networks after excluding intraday lending from the dataset. The main connective features remain (i.e. sparse, small-world, non-reciprocal, non-transitive). The average number of participants drops from about 31 to about 14.

¹⁶ Results are robust to other choices of filtering linkages by value (see Table A4 and A5). Also, for comparison purposes, results attained with the Jaccard index are reported in Table A6 (in the Appendix). As expected, conclusions concur with those extracted from Table 2.

Statistic	Mean	St. dev.	Min	Max
Number of participants ^a	14.66	3.45	4.00	28.00
Mean geodesic distance ^b	1.10	0.15	1.00	2.13
Density ($\times 100$) ^c	0.60	0.22	0.11	1.69
Reciprocity ($\times 100$) ^d	0.43	3.48	0.00	66.67
Transitivity ($\times 100$) ^e	0.04	1.41	0.00	50.00

Table 3. Networks' descriptive statistics, excluding intraday interbank lending, from January 2nd, 2014 to September 30th, 2019. ^a Number of nodes with at least one linkage in the network; ^b average minimal distance (i.e. measured in number of links) between all pairs of reachable nodes; ^c fraction of possible linkages observed in the network, under the assumption of no self-connections; ^d fraction of linkages that are reciprocated; ^e fraction of transitive relations observed in the network. Source: authors' calculations, based on data from Banco de la República.

Figure 5 shows results after excluding intraday lending. Persistence of linkages among financial institutions decreases. However, linkages remain persistent over time. On average, about 39 per cent of linkages surviving from one day to the next. Even for longer periods, the multi-step survival ratio is non-negligible, but lower than that corresponding to the all types of interbank lending.

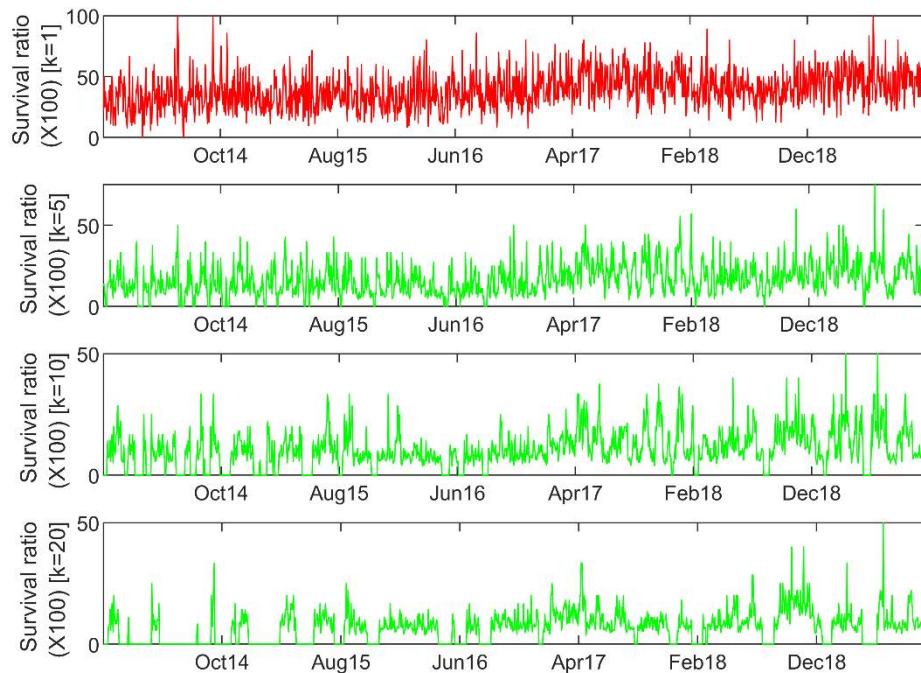


Figure 5. Single-step and multi-step survival ratio, excluding intraday interbank lending, from January 2nd, 2014 to September 30th, 2019. Source: authors' calculations, based on data from Banco de la República.

Table 4 shows that the null hypothesis of equal distribution with respect to the randomized network (without intraday lending) is rejected again. Once more, as expected, the mean survival ratio for the randomized network is visibly lower for all choices of k . For instance, the mean single-step survival ratio for the randomized network is about 16 per cent, whereas for the observed network it is about 39 per cent; that is, the observed single-step average survival is about 2.44 times that of the null network. Therefore, it is fair to conclude that interbank lending relations exist in the Colombian market –even after excluding intraday lending.

Survival ratio (X100) ^a		Mean	St. dev.	Min	Max	<i>Null mean</i> ^a
Single-step	$k = 1$	39.11	15.14	0.00	100.00	15.64†
	$k = 5$	16.62	10.13	0.00	75.00	0.76†
Multi-step	$k = 10$	10.57	7.83	0.00	50.00	0.02†
	$k = 20$	6.63	6.35	0.00	50.00	0.00†

Table 4. Survival ratio, excluding intraday interbank lending, from January 2nd, 2014 to September 30th, 2019. ^a Mean calculated on randomized networks. † Null hypothesis of equal distribution of observed and randomized data is rejected at 0.99 confidence (i.e. p-value < 0.01) with Kolmogorov-Smirnov non-parametric two-sample test. Source: authors' calculations, based on data from Banco de la República.

5.3 Covid-19 and falling oil prices

The first quarter of 2020 is a period of turmoil for world financial markets. First, on December 2019 a new type of coronavirus was discovered after an outbreak in Wuhan, People's Republic of China. On February 11th, 2020 it was named Covid-19, and on March 11th it was declared a pandemic by the World Health Organization. According to International Monetary Fund staff (see Bluedorn, et al., 2020), Covid-19 pandemic has pushed the world into a recession, and 2020 will be worse than the global financial crisis. Second, the failure of the OPEC+¹⁷ countries to reach an agreement on output cuts along with weakening global demand pushed oil prices downwards (IMF, 2020). Colombia is among several emerging markets at risk under this adverse scenario (see IMF, 2020).¹⁸

¹⁷ OPEC+ comprises the 13 members of the Organization of the Petroleum Exporting Countries (OPEC) plus ten other countries –including Russia.

¹⁸ Some events confirm the adversity of this scenario for Colombia. For instance, from March 5th to March 17th (to April 17th) the Colombian peso lost about 17% (13%) of its value against the US dollar, equivalent to a -25.63 (-19.78) standard deviation cumulative decrease in value; standard deviation was estimated on daily returns from January 2018 to December 2019. On March 26th, 2020, Standard & Poor's changed the perspective of Colombia credit rating (BBB-) to negative, whereas Fitch decreased the rating from BBB to BBB- on April 1st, 2020.

Figure 6 displays networks' descriptive statistics from March 3rd, 2020 to April 17th, 2020; the starting date corresponds to three days before the first case was confirmed in Colombia –about a week before Covid was declared a pandemic. The horizontal line represents the prevailing mean of descriptive statistics before March 3rd, 2020. Although the dataset for this period is small, it is apparent that some statistics are below their mean, such as the number of participants and the density. After mid-March, the number of participants and density levels are rather close to their corresponding minimal levels, 19 and 0.16, respectively (see Table 1).

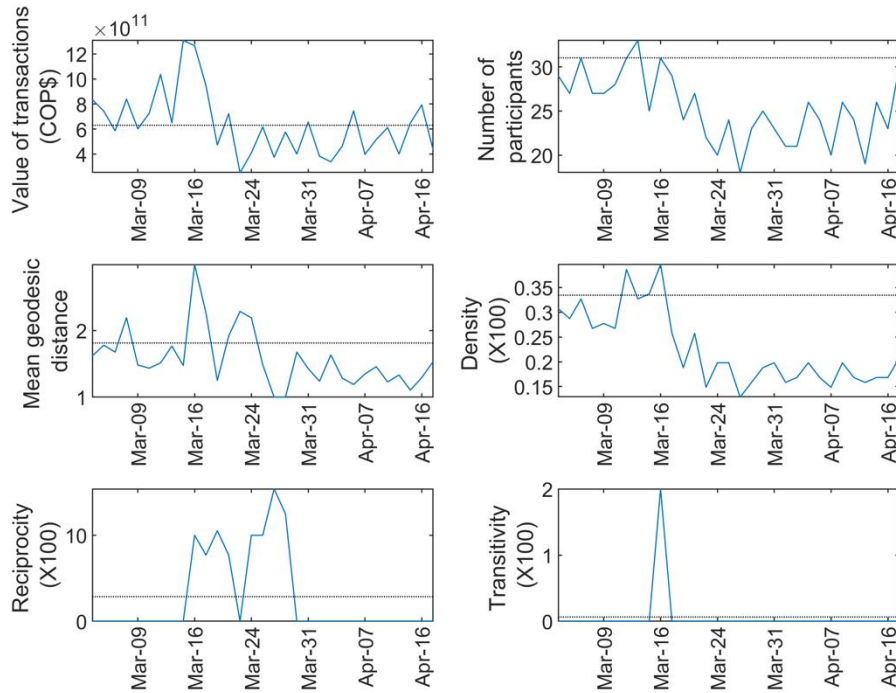


Figure 6. Networks' descriptive statistics, from March 3rd, 2020 to April 17th, 2020. Source: authors' calculations, based on data from Banco de la República.

Figure 7 displays the single-step and multi-step survival ratios from March 3rd, 2020 to April 17th, 2020. It is apparent that the single-step survival ratio fluctuates around the mean without a clear direction. Moreover, when compared to statistics reported in Table 2, the single-step survival levels do not approach observed minimum (27.03) or maximal (84.62) values. Multi-step survival ratio with $k = 5$ shows a behavior similar to that of the single-step survival ratio. However, with $k = 10, 20$, the multi-step survival ratio shows a distinctive behavior. When $k = 10$, the survival ratio drops for a brief period by the end of March and recovers a level close to its mean in April. When $k = 20$, this drop is longer, and the survival ratio reaches the mean at the end of the sample.

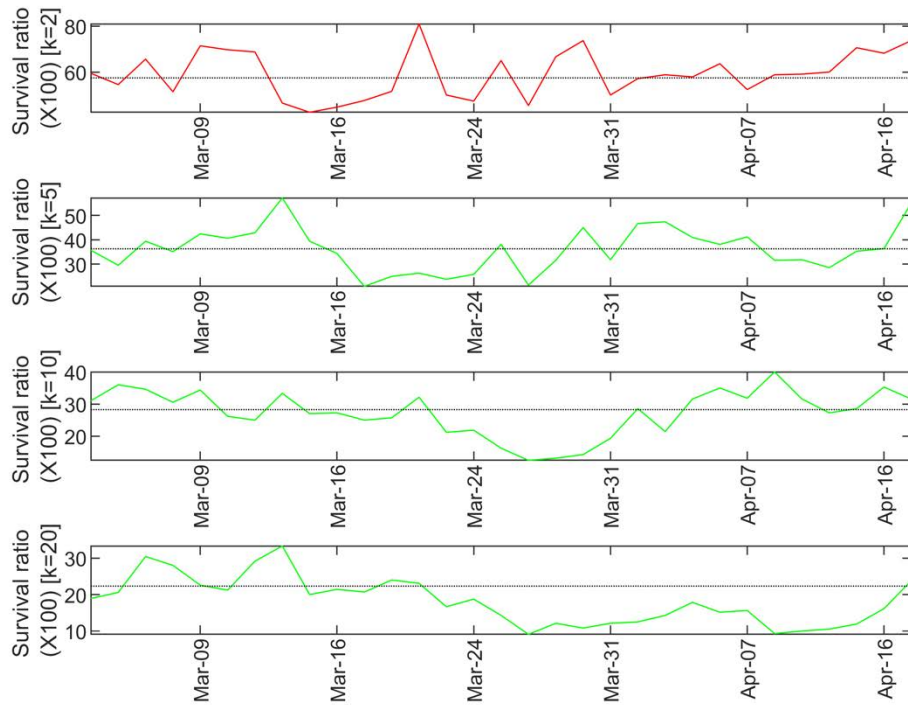


Figure 7. Single-step and multi-step survival ratio, from March 3rd, 2020 to April 17th, 2020. Source: authors' calculations, based on data from Banco de la República.

Although the number of days in the sample is rather small to make judicious inferences or statistical tests, a preliminary depiction of the evolution of the survival ratio during first weeks of the Covid-19 and oil prices turmoil period shows that interbank relationship lending was not severely affected. Nevertheless, it is apparent that the longer the evaluation period of survival ratios, the stronger the effect of the adverse scenario. An interesting trait during this scenario is the decrease in density and the stability of the single-step survival ratio; this may suggest that interbank linkages that vanished during the turmoil are those that are unstable –thus not relevant for the evolution of the survival ratio.

6 Final remarks

From a network perspective, about 58 per cent of unsecured interbank linkages survive from one day to the next in Colombia, whereas about 36, 28, and 22 per cent of those linkages survive during a 5-, 10-, and 20-day period, respectively. Results are robust to the exclusion of low-value linkages. And, based on a non-parametric test, randomness is not a plausible source of observed survival ratios. During the first weeks

of turmoil caused by Covid-19 pandemic and falling oil prices, the single-step survival ratio did not deviate from its norm. However, the multi-step survival ratio corresponding to a 20-day period showed a rather clear drop for about a month. It is apparent that trust among financial institutions in the interbank market was not severely affected during the first weeks of turmoil caused by Covid-19 pandemic and falling oil prices.

Our findings add to overall literature on interbank lending relationship. Also, concurrent with Capera-Romero, et al. (2015) and Martínez (*forthcoming*), we find that stable and strong interbank lending relations exist in Colombia. Furthermore, our findings overlap with literature related to the stability of linkages in interbank networks reported in Fricke and Lux (2014), Craig and von Peter (2014), Bargigli, et al. (2014), and Molina-Borboa, et al. (2015).

Relationship lending is an indicator of trust in the financial system and plays a positive role in financial stability (see Temizsoy, et al., 2015). Monitoring financial institutions' funding behavior is a key task for central banks to maintain the health of the financial markets, especially when markets are under stress (see Kobayashi & Takaguchi, 2018). The evolution of interbank networks' survival ratio may aid financial authorities in their quest for monitoring financial institutions' willingness to exchange funds among them. Because the survival ratio is easy to calculate and to interpret, and it is available as a high-frequency up-to-date variable¹⁹, its potential to continuously monitor financial institutions' willingness to fund each other may be used as a proxy of their behavior and –thus– as a key input in early warning models²⁰; Furthermore, from an academic viewpoint, as the repeated nature of linkages is the sole determinant of survival ratios, their potential as inputs in explanatory models of interbank markets is promising too.

Finally, although unweighted networks often convey more information about real-world economic systems (see Squartini, et al., 2013), incorporating weights to the interbank network is judicious. Two obvious weights are to be considered: the amount of lending and the cost of lending. By considering these two weights it is possible to incorporate the quantity and price of interbank funding as relevant factors of interbank relationship lending. This is particularly important when financial stability is in view.

¹⁹ In our case, an updated survival ratio is available at the end of the day.

²⁰ For instance, it is most likely that an unexpected drop in interbank network's survival ratio is related to a major change in financial institutions' trust in the interbank market. If this unexpected sharp drop is accompanied by a decrease in density, it is most likely that financial institutions are distrustful of the market as a whole; not only financial institutions are relocating their relations, but they are also reducing them. On the other hand, if density does not decrease, it is most likely that financial institutions are distrustful regarding one (or some) of their peers –and the drop in the survival ratio is a consequence of reallocation of relations in the market.

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8 Appendix

Statistic	Mean	St. dev.	Min	Max
Number of participants ^a	31.17	3.64	19.00	43.00
Mean geodesic distance ^b	1.84	0.40	1.00	4.20
Density ($\times 100$) ^c	0.34	0.06	0.14	0.63
Reciprocity ($\times 100$) ^d	6.08	4.41	0.00	34.15
Transitivity ($\times 100$) ^e	0.92	0.99	0.00	13.64

Table A1. Randomized networks' descriptive statistics, from January 2nd, 2014 to September 30th, 2019. ^a Number of nodes with at least one linkage in the network; ^b average minimal distance (i.e. measured in number of links) between all pairs of reachable nodes; ^c fraction of possible linkages observed in the network, under the assumption of no self-connections; ^d fraction of linkages that are reciprocated; ^e fraction of transitive relations observed in the network. Source: authors' calculations, based on data from Banco de la República.

Survival ratio (X100)		Mean	St. dev.	Min	Max
Single-step	$k = 1$	17.99	6.16	0.00	57.89
	$k = 5$	2.08	2.33	0.00	20.69
Multi-step	$k = 10$	0.42	1.10	0.00	12.00
	$k = 20$	0.00	0.41	0.00	8.00

Table A2. Survival ratio on randomized networks, from January 2nd, 2014 to September 30th, 2019. Source: authors' calculations, based on data from Banco de la República.

Survival ratio (X100)		Mean	St. dev.	Min	Max
Single-step	$k = 1$	15.64	11.39	0.00	100.00
	$k = 5$	0.76	2.83	0.00	66.67
Multi-step	$k = 10$	0.02	0.46	0.00	25.00
	$k = 20$	0.00	0.02	0.00	7.69

Table A3. Survival ratio on randomized networks, excluding intraday interbank lending, from January 2nd, 2014 to September 30th, 2019. Source: authors' calculations, based on data from Banco de la República.

Survival ratio (X100)		Mean	St. dev.	Min	Max	Null mean ^a
Single-step	$k = 1$	57.27	8.66	27.03	84.62	18.00†
	$k = 5$	36.10	7.70	0.00	64.00	2.11†
Multi-step	$k = 10$	27.92	7.07	0.00	51.61	0.43†
	$k = 20$	21.79	6.30	0.00	48.00	0.00†

Table A4. Survival ratio, without filtering linkages, from January 2nd, 2014 to September 30th, 2019. ^a Calculated on randomized networks. † Null hypothesis of equal distribution of observed and randomized data is rejected at 0.99 confidence (i.e. p-value < 0.01) with Kolmogorov-Smirnov non-parametric two-sample test. Source: authors' calculations, based on data from Banco de la República.

Survival ratio (X100)		Mean	St. dev.	Min	Max	Null mean ^a
Single-step	$k = 1$	57.31	8.67	27.03	84.62	18.02†
	$k = 5$	36.17	7.73	0.00	64.00	2.09†
Multi-step	$k = 10$	27.79	7.11	0.00	51.61	0.43†
	$k = 20$	21.83	6.34	0.00	48.00	0.00†

Table A5. Survival ratio, filtering linkages in the 15th percentile, from January 2nd, 2014 to September 30th, 2019. ^a Calculated on randomized networks. † Null hypothesis of equal distribution of observed and randomized data is rejected at 0.99 confidence (i.e. p-value < 0.01) with Kolmogorov-Smirnov non-parametric two-sample test. Source: authors' calculations, based on data from Banco de la República.

Jaccard index (X100)		Mean	St. dev.	Min	Max	Null mean ^a
Single-step	$k = 1$	28.42	3.55	14.81	39.34	8.82†
	$k = 5$	7.12	1.28	0.00	11.05	0.40†
Multi-step	$k = 10$	2.75	0.60	0.00	4.50	0.00†
	$k = 20$	1.07	0.27	0.00	1.90	0.00†

Table A6. Jaccard index, filtering linkages in the 10th percentile, from January 2nd, 2014 to September 30th, 2019. ^a Calculated on randomized networks. † Null hypothesis of equal distribution of observed and randomized data is rejected at 0.99 confidence (i.e. p-value < 0.01) with Kolmogorov-Smirnov non-parametric two-sample test. Source: authors' calculations, based on data from Banco de la República.

