



# Asymmetric Effects of Terms of Trade Shocks on Tradable and Non-tradable Investment Rates: The Colombian Case\*

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## Abstract

Terms of trade (ToT) shocks drive business cycles and have direct impact on the macroeconomic equilibrium conditions in commodity-exporter countries. ToT shocks also affect the dynamics of other variables such as national income and relative prices, and consequently, cause agents and firms to adjust their saving, spending and investment decisions accordingly. The latter is of special interest because of its link with potential GDP and the capital stock of the economy, relevant concepts when assessing sustainable and long-run growth. In this document we explore how tradable and nontradable investment rates respond asymmetrically to ToT shocks. We estimate a Threshold VAR (TVAR) in which the ToT are the transition variable. The empirical results suggest the existence of two regimes (low and high ToT levels) and that ToT shocks have different effects on tradable and nontradable investment rates depending not only the direction of the shock, but also on the levels from which the shock departs.

**Keywords:** Asymmetries, Terms of Trade, Shocks, Tradable and Nontradable Investment Rates, Threshold VAR, Nonlinearities, Colombia.

**JEL:** C32, F41, O11, P45, Q31, Q37.

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# Efectos Asimétricos de Choques de Términos de Intercambio sobre la Inversión Transable y No Transable: El Caso Colombiano\*

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## Resumen

Los choques de Términos de Intercambio (TdI) determinan en parte los ciclos económicos y tienen impacto directo en el equilibrio macroeconómico en los países exportadores de materias primas. Estos choques también afectan las dinámicas de variables como el ingreso nacional y los precios relativos, haciendo que los agentes y las firmas ajusten sus decisiones de ahorro, gasto e inversión. La inversión es de especial interés debido a su vínculo con el PIB potencial y el stock de capital de la economía, conceptos que resultan relevantes al evaluar el crecimiento sostenible y de largo plazo. En este documento se estudian cómo las inversiones transable y no transable reaccionan de forma asimétrica a choques de TdI. Estimamos un modelo VAR por umbrales (TVAR) en el cual la variable de transición son los TdI. Los resultados sugieren la existencia de dos regímenes (de altos y bajos TdI) y que los choques de TdI producen respuestas asimétricas en la inversión transable y no transable, dependiendo no sólo de la dirección del choque, sino del nivel de TdI desde el cual se parte cuando éste sucede.

**Palabras Clave:** Asimetrías, Términos de Intercambio, Choques, Inversión Transable y No Transable, VAR por umbrales, No lineal, Colombia.

**JEL:** C32, F41, O11, P45, Q31, Q37.

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# 1. Introduction

Terms of trade (ToT) shocks play a key role when assessing the external vulnerabilities that face commodity-exporter countries. There is a wide and extensive amount of literature that has documented how ToT dynamics are closely related to business cycles in these countries and how the volatility of macroeconomic aggregates increase due to ToT shocks (Mendoza, 1995; Kose, 2002; Chia and Alba, 2006). These shocks propagate across the economy through multiple channels, and this is precisely what allows for asymmetries and differentials in the responses of macroeconomic variables. For example, ToT shocks have different effects on both GDP levels and growth depending on the monetary policy stance, market and price rigidities, external variables, the financial system structure, and several other relevant characteristics of the economy. Different combinations of these factors can magnify or mitigate ToT shocks effects on the economy.

The Colombian case is of interest because, throughout the most part of the last decade, commodity prices of exported goods were historically high and international capital inflows were relatively abundant. This meant several years of favorable ToT levels, which, in turn, allowed for elevated production and spending growth rates and a higher potential GDP level. However, the ToT shock of mid-2014 had major impacts on the Colombian economy. Indeed, there has been an important economic slowdown and moderated national income dynamics, which have had significant effects on the current account deficit, inflation, public finances, and the private sector. In spite of its magnitude and persistence, the macroeconomic adjustment to the ToT shock has been smooth and, in fact, economic authorities expect in 2018 an improvement in GDP growth rates, a convergence of inflation rates to the long-run target, and a lower external deficit.

The analysis of ToT shock effects on investment rates is of particular interest as it provides hints about the medium-term dynamics of GDP and the future paths of potential growth. A recent study by Aslam et al. (2016) suggests that, in presence of ToT shocks, long run or potential GDP is ultimately affected by the short-run responses of investment and its impact on the working capital stock of the economy. Also, the empirical evidence therein presented shows that investment movements (due to ToT shocks) have more immediate consequences on demand, activity and current account levels.

In this document we explore the asymmetric effects of ToT shocks on investment rates, depending on whether this shock happens when the ToT levels are high or low. This distinction is relevant for the Colombian case as it is a small and open economy, highly specialized in commodities production and exports; and therefore, high ToT levels usually bring indirect effects on the economy that imply more capital inflows, risk premium indexes reductions, both real and nominal exchange rate appreciations, and credit expansions, all of which trigger positive dynamics of absorption and GDP growth rates. This situation tends to revert much quicker when a negative ToT shock occurs, and as a consequence, the macroeconomic adjustment is much more volatile and costly in terms of welfare (Hamann et al., 2016).

The contribution of this paper to the literature about asymmetric effects of ToT shocks on investment in commodity-exporter countries is twofold. First, by exploring the official figures of

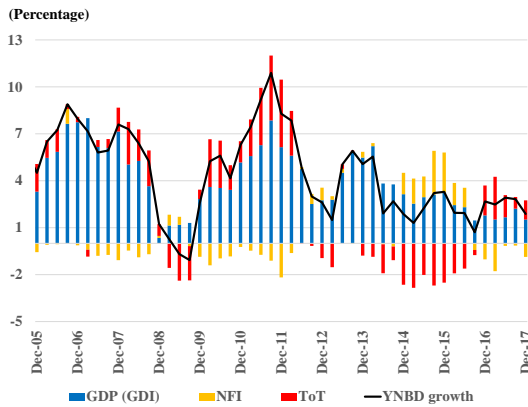
national accounts, we make an important distinction between tradable and nontradable capital goods investment. This refinement is relevant as the responses of these types of investment following a ToT shock are different in magnitude, persistence, and in many cases, direction. Moreover, these investments have different marginal productivities and implications on capital stock accumulation, total factor productivity, and both long-run and potential GDP growth. Second, we provide empirical evidence that the asymmetric effects of ToT shocks on the economy depend on the state (i.e. level) of the ToT index. Our results show that in presence of high ToT levels, the effects of negative ToT shock on macroeconomic variables are more persistent and greater in magnitude (i.e. more negative) than when in presence of low ToT levels. The latter suggests that positive spillovers derived from high ToT levels (i.e. higher real income, lower risk, more access to external funding, wealth effects, etc.) also imply positive aggregate consumption and investment dynamics, that, when negatively shocked, result in greater aggregate output losses. In other words, a reversal of the ToT is more costly in terms of welfare and GDP growth when the former are in the upper regime, as it was the case for Colombia an the mid-2014 oil prices shock, than when in the lower one. This approach can be easily extended to other commodity-producer countries case studies.

Following this introduction, in section 2 we present the propagation channels by which ToT shocks asymmetrically affect different macroeconomic variables, and ultimately the investment rates. This claim is jointly supported by a short literature review and by the empirical evidence for Colombia. The econometric approach we used in this paper to model the asymmetric regularities present in the data, a nonlinear vector autoregressive model, namely a Threshold VAR (TVAR) model, is shortly explained in section 3. Results and conclusions are presented in sections 4 and 5, respectively.

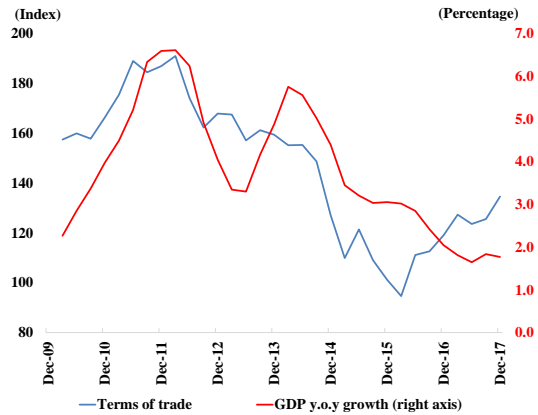
## 2. Literature Review and Empirical Evidence

There are several channels through which ToT shocks affect (directly and indirectly) investment rates. Firstly, there is a direct impact through the first-round effects on national income. Agénor and Aizenman (2004) show how changes in national disposable income (due to permanent ToT shocks) have asymmetric effects on private savings, and therefore, on subsequent investment decisions. Moreover, Adler et al. (2017) find that ToT shocks have a direct impact through the demand and current account channels. Indeed, for the Colombian case, negative ToT shocks were associated with lower national income growth rates over the past few years, as shown in Figure 1a. This, in turn, meant an overall decline in economic activity, as displayed in Figure 1b.

Secondly, negative (positive) ToT shocks usually imply lower (higher) profits and investment deterioration carried out by firms in the mining and oil sectors. Actually, in most cases, extractive activities become non-profitable when the international commodity prices are low. In this context, firms would reduce output even if the capital stock (already accumulated) remains unaffected in the short-run. This is particularly true for the oil industry case. Negative ToT shocks impact negatively both the firms' output and the sectoral marginal productivity of the capital, and as a result, oil-producing firms are forced to cut investments (Bornstein et al., 2017).



(a) Gross national income growth rate decomposition.



(b) ToT index (2000 = 100) and GDP growth.

Figure 1: Effects of negative ToT shocks on income and GDP growth.

Source: DANE, Banco de la República, authors' own estimations.

To the latter, one should add the fact that ToT shocks limit investment behavior of firms in the oil sector, this due to the irreversibility and the sunk costs of incurring in investment decisions (Pindyck, 1991).

In fact, official statistics for the oil industry in Colombia show a significant drop in extraction and exploration activities over the past few years. This situation worsened the already affected oil production. As displayed in Figure 2, oil sector firms reduced significantly their perforation activities shortly after the mid-2014 oil shock occurred. Similar evidence has been found for other commodity-exporting countries, such as Australia, Canada, Chile, Perú, and South Africa. For all of these, a strong and positive correlation between investment spending by firms in the mining sector and the international commodity price is documented by Fornero et al. (2016).

A third propagation channel is that related to the exchange rate (nominal and real) dynamics. A handful of research papers have found that a flexible exchange rate regime might help to better mitigate the effects of ToT fluctuations on GDP (see, e.g. Broda, 2004 or Edwards and Levy Yeyati, 2005). However, when it comes to investment rates, the response is twofold. On one hand, exchange rate depreciations (appreciations) generate changes in the relative prices of the economy that can foster (discourage) firms' investments in sectors producing (and exporting) tradable goods and services. On the other hand, a depreciated (appreciated) exchange rate augments (reduces) the costs (measured in local currency) of acquisition of capital goods, which are mainly imported in the case of emerging markets. In this sense, it is more expensive (cheaper) for firms to import capital goods and to pay off external debts, leading to drops (increases) in the investment spending. As shown in Figure 3a, the import price index for capital goods (in Colombian pesos, COP) rose steadily during 2014-2016, and currently remains at higher level than in the past. This was due to the exchange rate depreciation (both in real and nominal terms, Figure 3b) associated with the drop in the international oil prices.

A fourth mechanism by which ToT shocks affect investment is the financial propagation

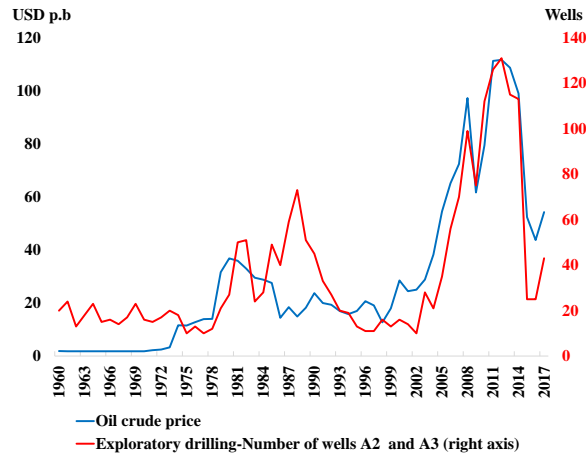


Figure 2: Nominal oil price (in USD) and oil-exploratory activities in Colombia.  
 Source: BP Statistical Review of World Energy (June 2017),  
 Ministry of Mines and Energy, Ecopetrol, ANH and ACP.

channel. In particular, negative ToT shocks are usually followed by increases in the cost and a tightening of the credit conditions, as access requirements imposed by the financial sector become stricter (Lown and Morgan, 2006). These changes in the credit markets amplify the shock’s adverse effects on the economy, a phenomenon known in macroeconomics as the financial accelerator (Bernanke et al., 1996; Bernanke et al., 1999). Higher costs and tighter lending conditions in the financial market, together with lower profits and incomes at the firm level, reduce the amount of resources that the firms can use in capital acquisition (investment). Moreover, negative ToT shocks might yield reductions of external fundings, as the risk premium of the commodity-exporting country rises accordingly (Hamann et al., 2016). The latter happens due to the fact that the country, at an aggregate level, is less capable of generating future output, and because of the lower valuation of the collateral (i.e. the country’s oil and mining reserves) backing the external debt (Barone and Descalzi, 2012).

For Colombia, the empirical evidence shows that after the mid-2014 oil shock, private banks tightened their lending requirements. Figure 4a suggests that there is a positive correlation between the banks’ lending conditions and the investment growth rate. Indeed, we point out that in 2017, the levels for the lending requirements index were similar to those observed in 2009, one year after the international financial crisis occurred. Furthermore, there was a raise in the interest rates for commercial credits, as shown by Vegh et al. (2017). Also, the risk premium for Colombia (measured by the 5-year CDS) displayed a significant increase between 2014 and late 2015 (Figure 4b), which meant more expensive external funding.

Another channel acknowledges the close relationship between public investment and ToT levels, as in Bems and Li (2015). For the vast majority of commodity-exporter countries, most of fiscal incomes depend heavily on the royalties and taxes paid by firms in the extractive sector (Davis et al., 2003; López-Enciso et al., 2013). This is also the case for the Colombian Government. Indeed, before the ToT shock of mid-2014, the oil sector had a share close to 20% on the Government’s revenues.

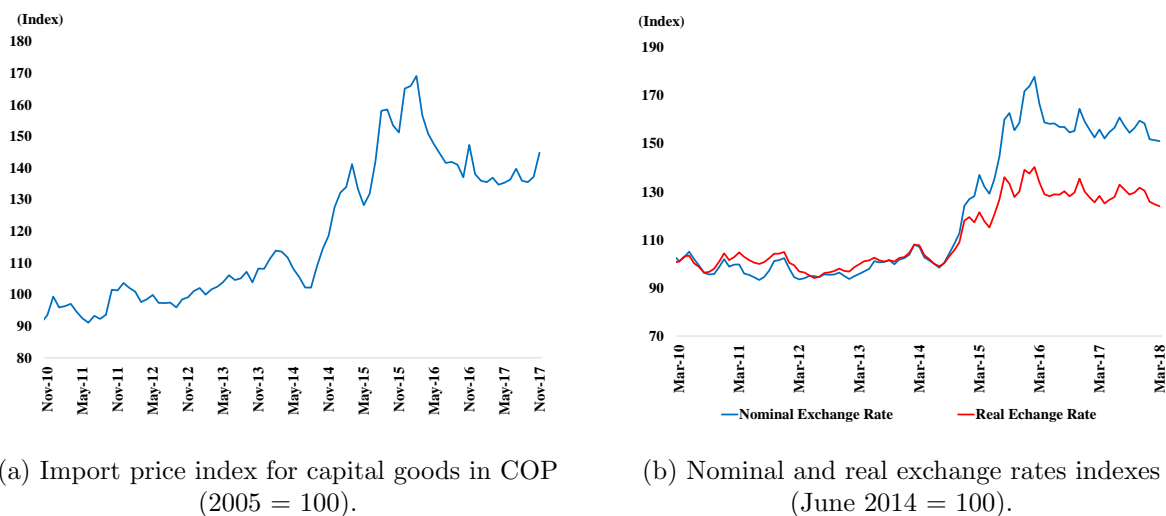


Figure 3: The exchange rate propagation channel.

Source: Banco de la República, authors' own estimations.

As a final remark, it has been shown that the duration of the ToT shock also plays a key role when assessing its effects on the economy (Cashin and Kent, 2003, Andrews and Rees, 2009). More persistent shocks can affect the agents' perspectives on the economic growth and future income, and therefore, their investment decisions and plans, as in Bernanke (1983). The empirical evidence suggests that agents interpreted the mid-2014 oil shock as permanent, and therefore they expect future GDP growth paths to be permanently lower (Figure 5). In fact, it has been described that for the Colombian case, this shock was non-anticipated, sudden, of considerable magnitude, and more importantly, persistent over time, as reported in Toro-Córdoba et al. (2015).

Said that, it is clear how ToT shocks can affect investment dynamics through a handful of propagation channels. In fact, economic literature has identified diverse sources of asymmetric responses of key macroeconomic variables (investment rates included) to ToT shocks. For example, An et al. (2014) presents how oil price shocks have different effects on a large number of economic series using a Factor Augmented VAR approach; while Donayre and Wilmot (2016) argue that these effects depend on the phase of the economic cycle. In this document we explore the asymmetric effects of ToT shocks on investment rates, depending on whether this shock happens when the ToT levels are high or low. It is important to bear in mind that not only the size, the direction of the shock, or the channel of interest through which the shock propagates, but also the interactions among these channels might cause nonlinearities in the response of the investment rates. A graphical representation of the propagation channels is presented in Figure 6, and explains how different interactions among these channels can lead to potential asymmetries that might yield heterogeneous effects of a ToT shock on the investment rates.

The empirical evidence for Colombia shows a deterioration of the investment rates in the quarters following the mid-2014 ToT shock, both in real and nominal terms (Figure 7a). In particular, real gross capital formation (GCF) registered an average annual growth rate of -0.7% between 2015 and 2017, after displaying an average of 10,3% between 2011 and 2014. The

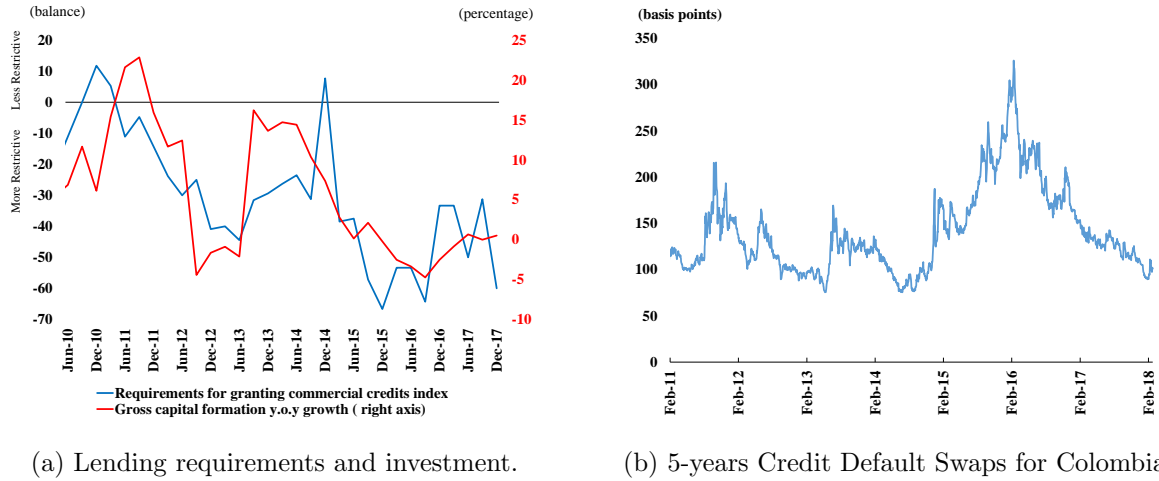


Figure 4: Empirical evidence of the financial propagation channel.

Source: Banco de la República, Bloomberg.

latter yielded a decrease of the real investment rate, measured as the ratio between GCF and real GDP levels. This rate reached historic highs in June 2014 (30,1%, see the Appendix section), a few months succeeding the ToT shock. After about three years of macroeconomic adjustment, the effects of the ToT drop on the real investment rate was of nearly three percentage points as of the fourth quarter of 2017. It is important to recall that the aforementioned reduction was initially concentrated in the tradable sector (e.g. mining, manufacturing, transportation, etc., as seen Figure 7b), and was only partially offset by the investment dynamics in non-tradable sectors (i.e. civil works, housing and construction).

In fact, expenditure on capital goods such as industrial machinery and transportation equipment (which are mostly imported for the Colombian case) dropped significantly in response to the mid-2014 ToT shock. This, in turn, had a negative impact on the tradable real investment rate ( $\text{TrInvR}^1$ ) of about 20% between the second quarter of 2014 and the fourth quarter of 2017 (Figure 8a). In contrast, expenditure on non-tradable capital goods (i.e. housing, buildings and civil works) remained somewhat stable along the period considered. This, due in part to the different range of counter-cyclical measures regarding housing and civil works investments implemented by the public sector between 2014 and 2017. As for the latter, the figures of the Civil Works Investment Index (CWII) published by the Colombian National Statistics Office (DANE) suggest investments in different infrastructure and road projects contributed to the civil works' expansion over the past few years. This compensated the cut in the mining civil works that, as already explained, was the result of the decrease in the exploration and exploitation activities due to the low international oil prices. Figure 8b shows how the mining CWII decrease in about 20% between 2014 and 2017, while the non-mining CWII increased around 40%.

A more detailed explanation of the investment rate adjustment that took place in Colombia after the mid-2014 ToT shock can be achieved by looking at the annual national accounts data

<sup>1</sup>Following DANE's official national account statistics, we define the  $\text{TrInvR}$  as the ratio of sum of the investment expenditure on industrial machinery, transportation equipment, inventories, capital goods for agriculture and services associated with the improvement of capital assets to GDP.

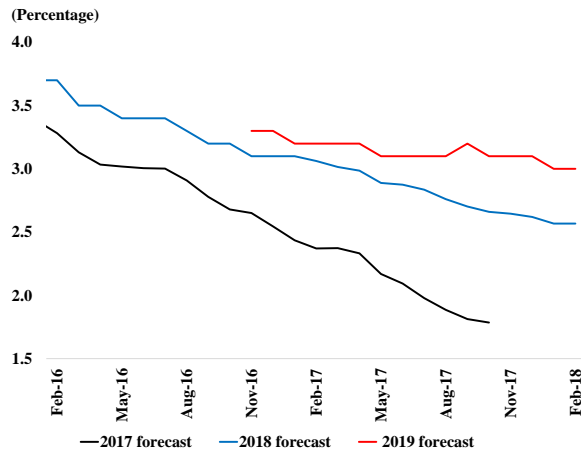


Figure 5: Analysts' GDP growth forecasts.  
Source: Focus Economics.

available up to 2016 published by the DANE. Firstly, non-official estimates by authors suggest that the investment rate reduction was the result of, to a large extent, the adjustment made by private sector (i.e. households and firms), more than that of the public sector (i.e. Government and NGOs). In the first case, the real investment rate went from 25,5% in 2014 to 23,6% in 2016, while in the second, went from 4,3% to 4,1% in the same period (Figure 9).

Secondly, after assessing the investment expenditure by economic sectors, the authors estimates' suggest that, as expected, the mining sector was the most affected due to the direct incidence of the ToT shock through the aforementioned propagation channels. Firms in other sectors producing tradable goods and services also decreased their investment expenditure since 2015, as it is the case of manufacturing and transportation (Figure 10a). However, capital goods expenditure by firms in the non-tradable sectors of the economy did not respond as quick as the former, and the investment rate dynamics for these sectors took longer to be affected by the ToT shock (Figure 10b). As already explained, the investment rates in these activities was stimulated by the allocation of resources by the public sector.

### 3. Empirical Approach

In the following subsections we describe the econometric framework, the nonlinearity testing and estimation procedures, and the computation of nonlinear impulse response functions (IRF). Since the main objective of this paper is to assess how asymmetric responses of investment rates to term o trade shocks emerge due to the presence of different propagation channels, we consider a nonlinear multivariate time series model that captures such features observed in the dataset. Said that, we estimate a threshold vector autoregressive model (TVAR). These type of models allow for observed variables (i.e. the threshold variable) to determine the dynamics of the system of equations, as opposed to other kind of nonlinear models (such as a Markov Switching VAR) in which the dynamics are determined by latent variables (e.g. probabilities). This distinction

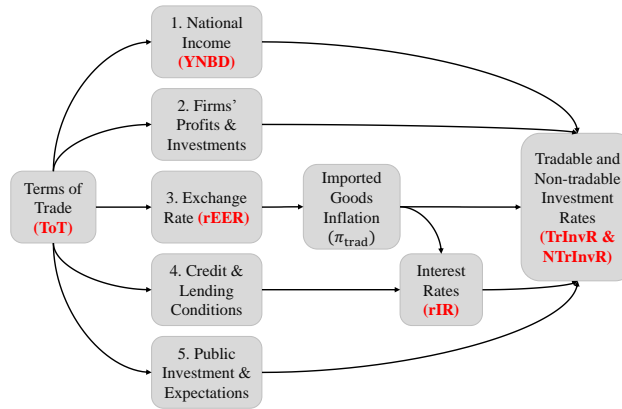


Figure 6: Channels through which a ToT shock impacts investment rates. Variables in bold are the ones included in the empirical exercise in Section 3.

Source: Authors' own design.

provides clear and measurable ways to assess possible changes in the system's coefficients.

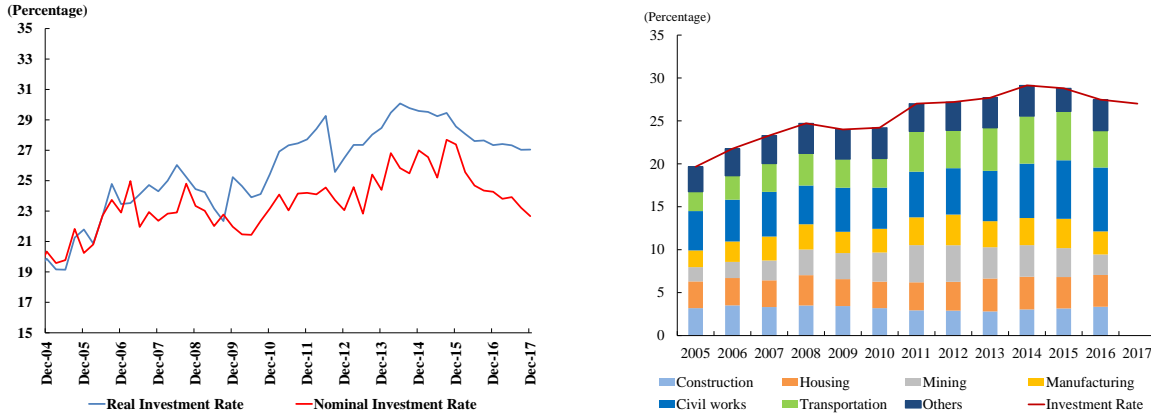
### 3.1 Data and Unit Root Tests

The empirical exercise considers the following quarterly series: terms of trade (ToT), real exchange rate (rEER), real national disposable income (YNBD), real interest rate (rIR), and real investment rates for both tradable and non-tradable capital goods (TrInvR and NTrInvR, respectively). All variables were detrended using quadratic trends, as in Schmitt-Grohé and Uribe (2016). The sample period spans from 1996-Q1 to 2017-Q4, a total of 88 observations. The detrended series in the dataset are estimations by the authors using information from the DANE and from Banco de la República. Moreover, we test for unit root presence. The null hypothesis of unit root is rejected for every series using an augmented Dickey-Fuller test (Dickey and Fuller, 1979, 1981). Descriptive statistics and unit-root test results are presented in section 4.1.

### 3.2 Nonlinearity tests

In this paper we follow the multivariate testing for (non)linearity approach in Lo and Zivot (2001), which is a multivariate extension of the tests proposed by Hansen (1997, 1999). Lo and Zivot showed that Hansen's method for testing linearity can be extended to the multivariate case as the distribution of the sup-Wald test statistic is asymptotically equivalent to that of the sup-LR test statistic (Casella and Berger, 2002, chapter 8 and Carrasco, 2002), the authors test the following statistic under the null hypothesis of linearity, against the alternative of a TVAR( $m$ ):

$$LR_{1,m} = T \times \left[ \ln(|\hat{\Sigma}|) - \ln(|\hat{\Sigma}_m(\hat{\gamma}, \hat{d})|) \right], \quad (1)$$



(a) Real and nominal investment rates in Colombia.

(b) Real investment rate by sector.

Figure 7: Investment Rates dynamics in Colombia.

Source: DANE, authors' own estimations.

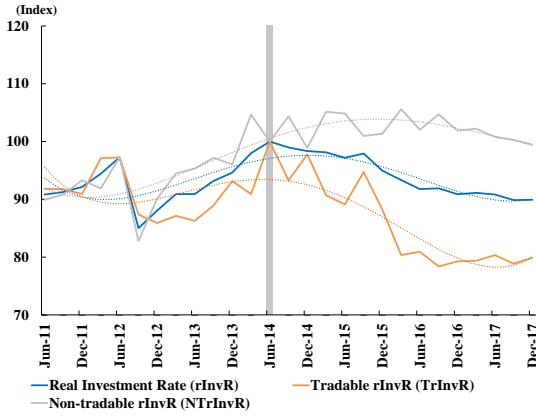
where  $\hat{\Sigma}$  and  $\hat{\Sigma}_m(\hat{\gamma}, \hat{d})$  are the estimated residual covariance matrices from the linear VAR( $m$ ) and the nonlinear TVAR( $m$ ), respectively. The asymptotic distribution for  $LR_{1,m}$  is approximated using bootstrap methods (Hansen, 1996). Results are reported in section 4.2.

### 3.3 The Threshold Vector Autoregression (TVAR) Model

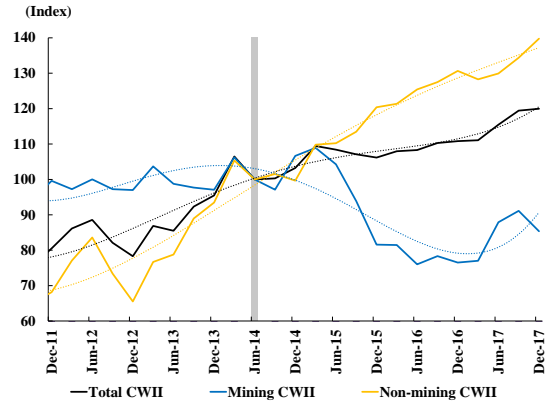
Conventional linear vector autoregression models (VAR) fail when the underlying data generating process follows nonlinear dynamics. This is often the case in macroeconomics. The responses of some macroeconomic variables to specific shocks are state dependent, asymmetric and usually vary across regimes. Following the work of Tsay (1998), in this paper we estimate a threshold vector autoregression (TVAR) model, a multivariate extension of the threshold autoregressive model developed by Tong and Lim (1980) and further discussed in detail in Tong (1983) and Tong (1990).

TVAR models are regime dependent linear VAR models with different autoregressive coefficient matrices for each regime. Switching between regimes depends on the value of a (lagged) stationary and continuous transition variable;  $z_{t-d}$ . This threshold variable is usually endogenous (modeled within the system of equations), but it can also be exogenous (not modeled within the system). In this paper we choose  $z_{t-d}$  to be the (detrended) terms of trade (ToT) index mainly because Colombia is a small, open, commodities-exporter country, which in turn means that ToT are exogenous and that business cycles are largely caused by ToT shocks. Also, this selection is supported by the results in Kilian and Vega (2011), which argue in favor of the price-taking nature of small open economies.

Let  $\mathbf{y}_t$  be a vector of  $K$  stationary endogenous variables, as in  $\mathbf{y}_t = (y_{1t}, \dots, y_{Kt})'$ . Let  $\mathbf{c}^{(i)}$  a vector of  $K \times 1$  constants and  $\mathbf{\Gamma}_p^{(i)}$  a matrix of  $K \times K$  coefficients, both corresponding to regime



(a) Total, tradable and non-tradable real investment rates (2014Q2 = 100).



(b) Civil Works Investment Index (2014Q2 = 100).

Figure 8: TrInvR and NTrInvR adjustment in Colombia after the mid-2014 ToT shock.  
Source: DANE, authors' own estimations.

$i \in I$ . The nonlinear TVAR( $P$ ) can be defined as

$$\mathbf{y}_t = \mathbf{c}^{(i)} + \sum_{p=1}^P \mathbf{\Gamma}_p^{(i)} \mathbf{y}_{t-p} + \mathbf{e}_t, \quad \text{whenever } \gamma_{i-1} < z_{t-d} \leq \gamma_i, \quad \text{for each } i = 1, \dots, I, \quad (2)$$

where  $\mathbf{e}_t$  is a  $K \times 1$  vector of innovations with zero mean, covariance matrix equal to  $\mathbf{\Sigma}_e$ . The thresholds,  $-\infty = \gamma_0 < \gamma_1, \dots, \gamma_{i-1} < \gamma_i = +\infty$ , take values over the same range the transition variable  $z_{t-d}$  does. An aggregate representation of the nonlinear TVAR can be written as

$$\mathbf{y}_t = \sum_{i=1}^I \left[ \left( \mathbf{c}^{(i)} + \sum_{p=1}^P \mathbf{\Gamma}_p^{(i)} \mathbf{y}_{t-p} + \mathbf{e}_t \right) \times \mathbb{I}_{(\gamma_{i-1}, \gamma_i]}(z_{t-d}) \right], \quad (3)$$

where  $\mathbb{I}_{(\gamma_{i-1}, \gamma_i]}(z_{t-d})$  is the indicator function that takes value of 1 whenever  $z_{t-d}$  falls in the interval  $(\gamma_{i-1}, \gamma_i]$  along the real line.

Equation (3) is a reduced form representation of a system of linear equations (one for each regime) in which the contemporaneous error terms are allowed to be correlated, that is,  $\mathbf{\Sigma}_e$  is not necessarily assumed to be diagonal. In this case, the structural shocks are not identified, since the structure of the covariance matrix increases the likelihood of other shocks to accompany that of interest and thus affect the inferences and conclusions derived from the model estimation. Motivated by economic analysis, for simplicity we identify the shocks using a Cholesky decomposition for structural identification in the error covariance matrix. This approach allows for the structural TVAR representation

$$\mathbf{A}^{(i)} \mathbf{y}_t = \sum_{i=1}^I \left[ \left( \mathbf{c}_0^{(i)} + \sum_{p=1}^P \mathbf{B}_p^{(i)} \mathbf{y}_{t-p} + \boldsymbol{\epsilon}_t \right) \times \mathbb{I}_{(\gamma_{i-1}, \gamma_i]}(z_{t-d}) \right], \quad (4)$$

where  $\boldsymbol{\epsilon}_t$  is a  $K \times 1$  random vector of structural errors assumed to be distributed as  $\mathbb{N}_K[\mathbf{0}, \mathbf{\Sigma}_\epsilon]$ , with covariance matrix equal to identity of rank  $K$ ;  $(\mathbf{A}^{(i)})^{-1}$  is an inferior triangular matrix

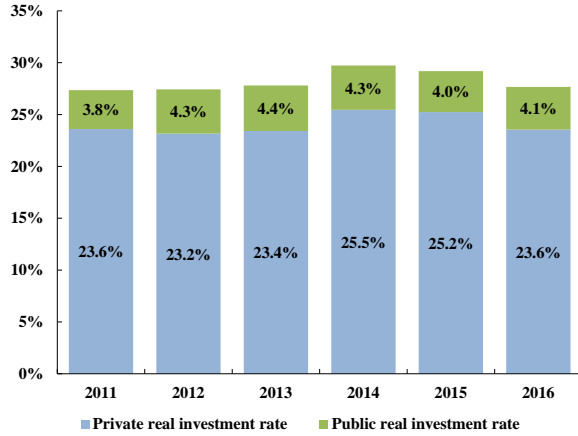


Figure 9: Private and public real investment rates.  
Source: DANE, authors' own estimations.

with ones along its diagonal;  $\mathbf{c}^{(i)} = (\mathbf{A}^{(i)})^{-1} \mathbf{c}_0^{(i)}$ ;  $\mathbf{\Gamma}_p^{(i)} = (\mathbf{A}^{(i)})^{-1} \mathbf{B}_p^{(i)}$ ; and  $\mathbf{e}_t = (\mathbf{A}^{(i)})^{-1} \epsilon_t$ . This Cholesky decomposition renders  $\mathbf{\Sigma}_e = (\mathbf{A}^{(i)})^{-1} \mathbf{\Sigma}_\epsilon ((\mathbf{A}^{(i)})^{-1})' = (\mathbf{A}^{(i)})^{-1} ((\mathbf{A}^{(i)})^{-1})'$ .

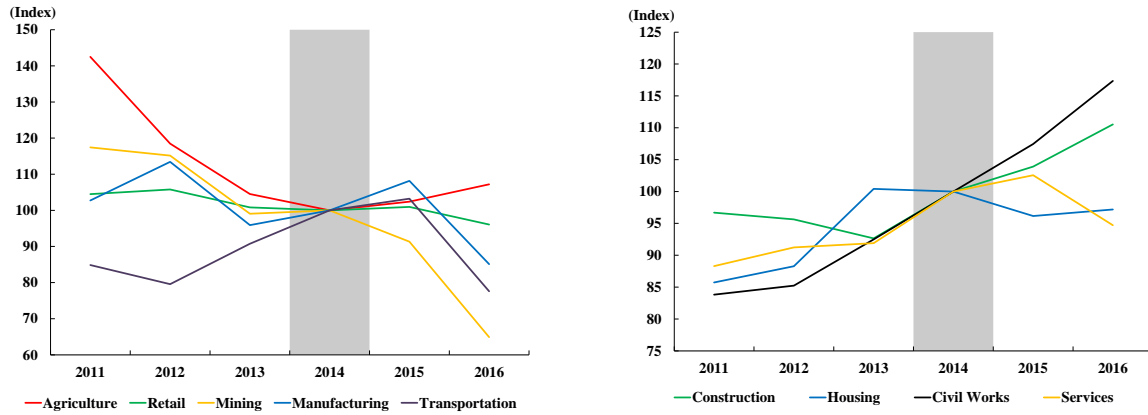
As in Tsay (1998), by assuming  $P$  and  $I$  known, and given the realizations of the transition variable  $\{z\}_{t=1}^T$ , a conditional least squares (CLS) estimation for the parameters in  $\mathbf{c}^{(i)}$ ,  $\mathbf{\Gamma}_p^{(i)}$ ,  $\mathbf{\Sigma}_e$ ,  $\gamma_i$ , and  $d$  for  $i = 1, \dots, I$  and  $p = 1, \dots, P$  is followed. The CLS algorithm follows as:

1. Conditional on  $d$  and the set of values for  $\gamma_i$ 's,  $\gamma$ , equation (3) reduces to a linear multivariate system of equations for which the least squares (LS) estimation of the parameters,  $\hat{\mathbf{c}}^{(i)}$  and  $\hat{\mathbf{\Gamma}}_p^{(i)}$  for  $p = 1, \dots, P$ , are straightforward. The covariance matrix estimator is  $\hat{\mathbf{\Sigma}}_e = (n_i - p)^{-1} \sum_t [\hat{\mathbf{e}}_t(\hat{\mathbf{c}}^{(i)}, \hat{\mathbf{\Gamma}}^{(i)}) \hat{\mathbf{e}}_t'(\hat{\mathbf{c}}^{(i)}, \hat{\mathbf{\Gamma}}^{(i)})]$ .
2. Denote the sum of squared residuals (SSR) as  $S(d, \gamma) = \sum_{i=1}^I S_i(d, \gamma_{i-1}, \gamma_i)$ , where  $S_i(d, \gamma_{i-1}, \gamma_i)$  is the trace of  $(n_i - p) \hat{\mathbf{\Sigma}}_e(d, \gamma_{i-1}, \gamma_i)$  for every regime. Recall that  $d \leq P$ .
3. Next, the CLS estimates of  $d$  and  $\gamma$  are obtained by

$$(\hat{d}, \hat{\gamma}) \in \arg \min_{\{d, \gamma\}} S(d, \gamma).$$

In practical terms, this minimization is done by implementing a grid search over trimmed domains of  $\{z\}_{t=1}^T$  and  $P$ .

Tsay (1998) derives the asymptotic properties of these estimators. He demonstrates that CLS estimators for the TVAR parameters are strongly consistent and asymptotically normally distributed. In order to achieve model stability and reliable estimates of the parameters for each regime, it is necessary to restrict threshold values,  $\gamma$ , so that each regime contains a minimum number of observations. This assumption, made explicit in both Tsay (1998) and Hansen (1999), states that the asymptotic theory suggests that  $\gamma_i$  should be set such that as  $n \rightarrow \infty$ ,  $n_i/n \geq \tau$ , for  $i = 1, \dots, I$  and for some  $\tau > 0$ . In our application we set  $\tau = 0.15$ . Results for the estimated TVARs are reported in section 4.3.



(a) Real investment rates in the tradable goods and services sectors (2014 = 100). (b) Real investment rates in the non-tradable goods and services sectors (2014 = 100).

Figure 10: Investment rates adjustments by sectors after the 2014 ToT shock.  
Source: DANE, authors' own estimations.

### 3.4 The Generalized Impulse Response Function (GIRF)

For nonlinear models, linear impulse response functions (IRF) are not valid and might provide results that lead to delusive inferences about the propagation mechanism of the model in the presence of (structural) shocks. Indeed, for the estimated TVARs the effects of these shocks on the future path of the random variables vector depend on numerous specific initial regime/shock characteristics, such as the complete history of the vector series, the period  $t$  when the shock occurs, and the sign, direction, and composition of the (nonstructural) shock (see Gallant 1993, Koop, 1996 and Koop et al., 1996). Moreover, in the TVAR framework and depending on the estimated values of the model parameters, it is likely that certain shocks allow for a regime change during the simulation period and therefore, its analysis needs special treatment.

More formally, IRF are linear functions that are i) history-independent, i.e. do not depend on a particular history of the data up to time  $t$ , when the shock occurs, ii) symmetric, i.e. a vector of shocks of size  $\epsilon_t$  has exactly the opposite effect of  $-\epsilon_t$ , and iii) proportional, i.e. if  $\epsilon_t$  is multiplied by a constant  $\alpha$ , then the expected effect is exactly  $\alpha$  times greater. These characteristics do not hold in nonlinear models, and therefore, IRF should not be applied in the TVAR case. For a clear, yet technical explanation on the latter issues see Galvão (2003) or Avdjiev and Zen (2014).

In order to avoid these shortcomings, we compute a generalized impulse response function (GIRF), as first introduced in Koop et al. (1996), which has the special feature that can be applied to both linear and nonlinear models (see, for example, Pesaran and Shin, 1998). Define  $\epsilon_t^{(i)}$  as a structural shock of a specific size  $\delta$  occurring under regime  $i$  at time  $t$ ,  $h$  as the simulation horizon, and  $\Omega_{t-1}$  the history (or information set) at time  $t-1$ ; the GIRF is defined as the difference between two expected paths of  $\mathbf{y}_{t+h}$ , conditional on the estimated model parameters

$\hat{\theta}$ ,  $\Omega_{t-1}$ , size and sign of  $\delta$ , and a sequence of shocks  $\epsilon_t^{(i_t)}, \dots, \epsilon_{t+h}^{(i_{t+h})}$  such that

$$\begin{aligned} \text{GIRF}(h, \epsilon_t^{(i_t)}, \dots, \epsilon_{t+h}^{(i_{t+h})}, \Omega_{t-1}, \hat{\theta}) &= \mathbb{E}(\mathbf{y}_{t+h}^\delta | \epsilon_t^{(i_t)} = \delta, \epsilon_{t+1}^{(i_{t+1})} = \mathbf{0}, \dots, \epsilon_{t+h}^{(i_{t+h})} = \mathbf{0}, \Omega_{t-1}, \hat{\theta}) \\ &- \mathbb{E}(\mathbf{y}_{t+h} | \epsilon_t^{(i_t)} = \mathbf{0}, \epsilon_{t+1}^{(i_{t+1})} = \mathbf{0}, \dots, \epsilon_{t+h}^{(i_{t+h})} = \mathbf{0}, \Omega_{t-1}, \hat{\theta}) \end{aligned} \quad (5)$$

where the initial regime at time  $t$ ,  $(i_t)$ , is determined by  $\Omega_{t-1}$ . It is important to note that  $i_t$  is not necessarily the same as  $i_{t+1}, \dots, i_{t+h}$ . Results of the GIRF for the estimated TVARs are presented in section 4.3. A description of the algorithm used to compute the GIRF is explained in the Appendix section.

## 4. Results

### 4.1 Unit Root Test Results

We conduct augmented Dickey-Fuller tests to check for non-stationary series in the TVAR dataset. Significance is assessed for the unit root parameter ( $\gamma$ ) and the autoregressive coefficients ( $\delta_p$ ) in the following model:

$$\Delta y_t = \gamma y_{t-1} + \sum_{p=1}^P \delta_p \Delta y_{t-p} + e_t,$$

Lag length  $P$  is set to 4, compatible with the Akaike Information Criterion (AIC). The results for the unit root estimated test statistic and critical values are displayed in Chart 1 for the TVAR selected variables.

	Descriptive Statistics			Augmented Dicky-Fuller Unit Root Test Results		
	Median	Min.	Max.	<b>t-stat.</b>	F-stat.	p-value
ToT	0,0006	-0,3437	0,2777	-1,9594	4,446	0,0012
rEER	0,0006	-0,2107	0,2601	-2,0774	2,088	0,0757
YNBD	-0,001	-0,0874	0,0707	-3,0638	2,530	0,0355
rIR	0,0014	-0,0513	0,1258	-3,1270	11,89	0,0000
TrInvR	0,0034	-0,0393	0,0492	-2,0310	1,209	0,3130
NTrInvR	0,0020	-0,0448	0,0396	-2,2273	2,561	0,0337

Chart 1: Descriptive statistics and unit-root test results for selected variables

The critical value for this one-tailed t-test is  $-1,95$  (with  $\alpha = 0,05$ ), which means that tests results allow for rejecting the null of unit root for every series in the TVAR.

## 4.2 Non-Linearity Test Results

As already mentioned, the transition variable  $z_{t-d}$  is the ToT series. We choose an optimal lag length for the linear VAR based on the Hannan-Quinn information criterion (HQ) and the Schwarz information criterion (SC). Both suggest that a lag order  $P = 1$  is optimal. Since  $d \leq P$ , we also set  $d = 1$ . Moreover, we run 10.000 simulations for the bootstrap exercise in order to approximate the aforementioned test distribution. Results are shown in table 2.

	sup-LR test	Bootstrapped Crit. Value			p-value
	statistic	90%	95%	99%	
VAR (1 reg.) vs TVAR (2 reg.)	<b>84,6260</b>	79,6080	84,5047	95,4927	<b>0,0491</b>
VAR (1 reg.) vs TVAR (3 reg.)	157,0283	154,3753	161,6012	175,8173	0,0776
TVAR (2 reg.) vs TVAR (3 reg.)	72,4022	89,5860	97,7566	118,7712	0,3823

Chart 2: sup-LR test results

Results for The test for the null of the data generating process following a linear VAR (i.e. 1 regime, no thresholds) against the alternative of a two-regime TVAR (i.e. 1 threshold) supports the estimation of a nonlinear model for the selected dataset. In spite of the null of a linear VAR against the alternative of a three-regime TVAR (i.e. 2 thresholds) not being rejected at a 0,10 significance level, the null of a two-regime TVAR versus a three-regime TVAR is rejected.

## 4.3 TVAR Estimation Results and GIRFs

Optimal lag length for the estimated TVAR follows the tests for the linear VAR. As already mentioned, the HQ and SC information criteria recommend a lag order  $P = 1$ . The grid search over the trimmed domain of  $z_{t-1}$  suggests that the threshold value that minimizes the SSR in the two-regime TVAR is  $\hat{\gamma} = -0,1214$  ( $SSR(\hat{\gamma}) = 0,4144$ ). The transition variable used (quadratic detrended ToT), the estimated threshold ( $\hat{\gamma}$ ), and the SSR for each possible threshold values are shown in Figures 11a and 11b. Given the estimated threshold, 20,7% of observations fall in the lower regime, i.e. *a period with low ToT*, and 79,3% fall in the upper regime, i.e. *a period with high ToT*.

We are interested in the asymmetric effects of ToT shocks on the economy, mostly on national income and investment rates, through the analysis of the GIRF results. The ordering of the variables in the system of equations is  $\mathbf{y}_t = (\text{ToT}, \text{rEER}, \text{YNBD}, \text{rIR}, \text{TrInvR}, \text{NTrInvR})'$ , which follows from the most exogenous to the most endogenous according to the different propagation channels described in section 2. This setting also follows the standard literature in which ToT are considered as exogenous for the case of small and open countries (see, e.g. Schmitt-Grohé and Uribe, 2016), as it is the case of Colombia.

Figure 12 displays the responses of both tradable and nontradable real investment rates (TrInvR and NTrInvR) to a one standard deviation shock in the (detrended) ToT level, along

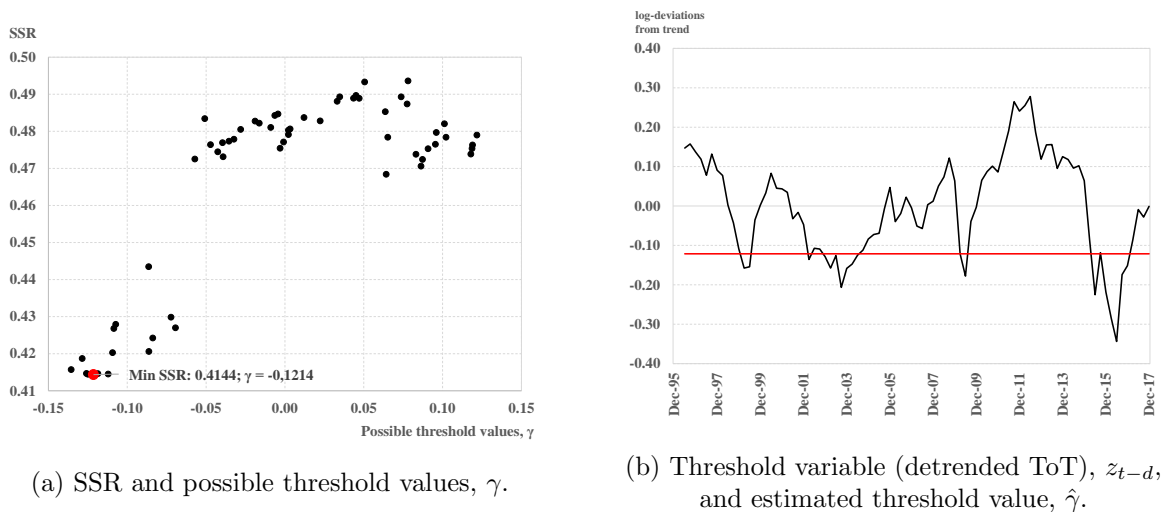
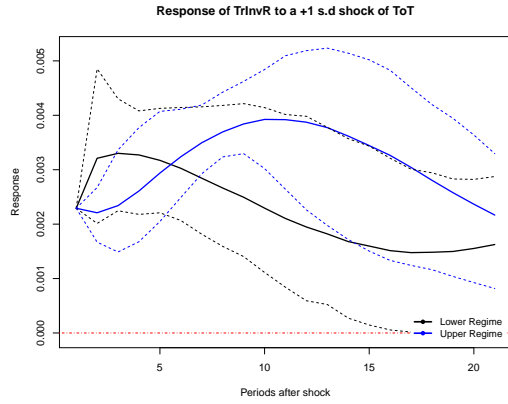


Figure 11: SSR, threshold variable,  $z_{t-d}$ , and estimated threshold,  $\hat{\gamma}$ .  
Source: Authors' own estimations.

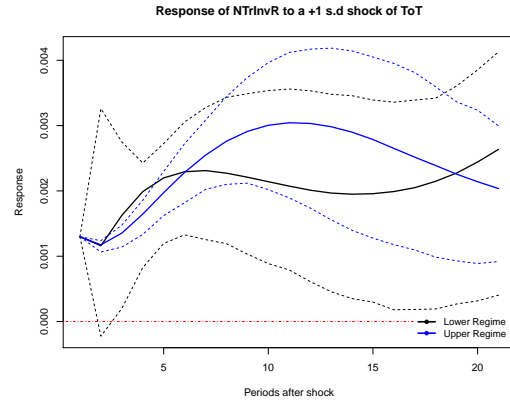
with a 68% bootstrap confidence interval. This interval is commonly used in the nonlinear VAR literature when the number of observations for a given regime is relatively small. See Sims (1987) and Sims and Zha (1999) for a detailed discussion. One important result is that responses are significantly different from zero (at least for the first 6 to 8 quarters), with expected sign, and asymmetric between both regimes and shock direction. It is important to bear in mind that the responses are to a one time (temporary) ToT shock. The analysis of the implications of a permanent ToT shock to the economy are out of the scope of this paper.

The instantaneous response of the TrInvR is greater than that of the NTrInvR (for both positive and negative -in absolute terms- shocks). This suggests that the former is more elastic to ToT fluctuations than the latter, as expected. However, the NTrInvR displays a lagged response, a result that suggests that, in addition to the direct effect, this type of investment is also affected by ToT shocks due to second round effects happening through the different propagation channels already discussed.

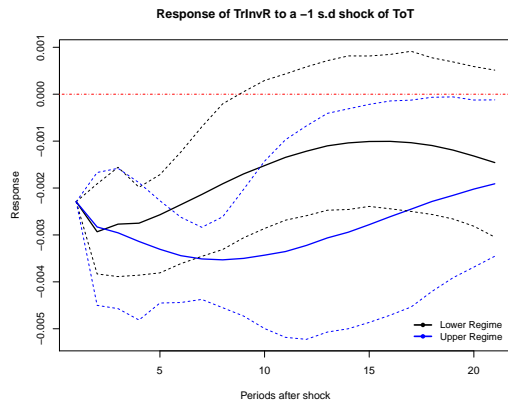
GIRF results in Figure 13 suggest that positive (denoted by  $\text{GIRF}_X^{(+)}$ , and displayed in Figures 12a and 12b) and negative (denoted by  $\text{GIRF}_X^{(-)}$ , and displayed in Figures 12c and 12d) shocks affect differently the TrInvR and NTrInvR, respectively. Firstly, Figure 13a shows how, in the presence of low ToT levels (lower regime, black line), the increases in TrInvR due to positive ToT shock outperform the decreases due to negative shocks. That is, when ToT are low, positive shocks have greater impact on this type of investment when compared to the adverse effects of negative shocks. When in the upper regime (i.e. high ToT, blue line), the response to a negative shock is greater than the effect of a positive shock for the first six quarters. From the seventh quarter onwards, the gains of a positive shock are greater than the losses of a negative shock. The latter suggest that TrInvR is more susceptible to ToT drops than to ToT increases when in presence of high commodity prices. The empirical evidence supports this claim, as presented in section 2.



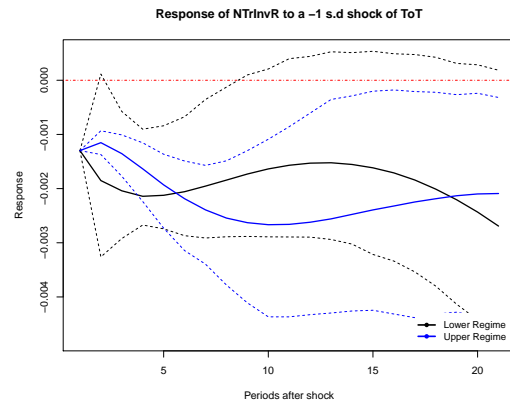
(a) TrInvR response to a positive ToT shock.



(b) NTrInvR response to a positive ToT shock.



(c) TrInvR response to a negative ToT shock.



(d) NTrInvR response to a negative ToT shock.

Figure 12: Responses of Tradable and Nontradable Real Investment Rates to a Terms of Trade Shock. Dashed lines represent the 68% bootstrapped confidence interval (C.I).

Source: Authors' own estimations.

Secondly, Figure 13b shows that when in the lower regime, during the first four periods after the shock, decreases in the NTrInvR due to a negative ToT shock are greater than the gains derived from an increase in commodity prices. From the fifth onwards, the positive effects outweigh the negative ones. Whereas in the upper regime, there is no significant difference between the responses of NTrInvR to positive or negative ToT shocks during the same time span. After the fifth period, the gains are greater than the losses.

The aforementioned dynamics of the tradable and nontradable real investment rates are strongly associated with the behavior of other (less endogenous) variables in the system: rEER, YNBD, and rIR. Figure 14 shows the responses to both transitory positive and negative ToT shocks for all the other variables in the TVAR model. Results suggest that in both regimes, a negative ToT shock causes, as expected, drops YNBD and also the rEER to depreciate. Note, however, the asymmetries that these responses display, since these are more pronounced in the upper regime than those for the lower one. Additionally, in the upper regime, the rIR increases due to the initial response of the monetary policy facing inflationary pressures derived from the

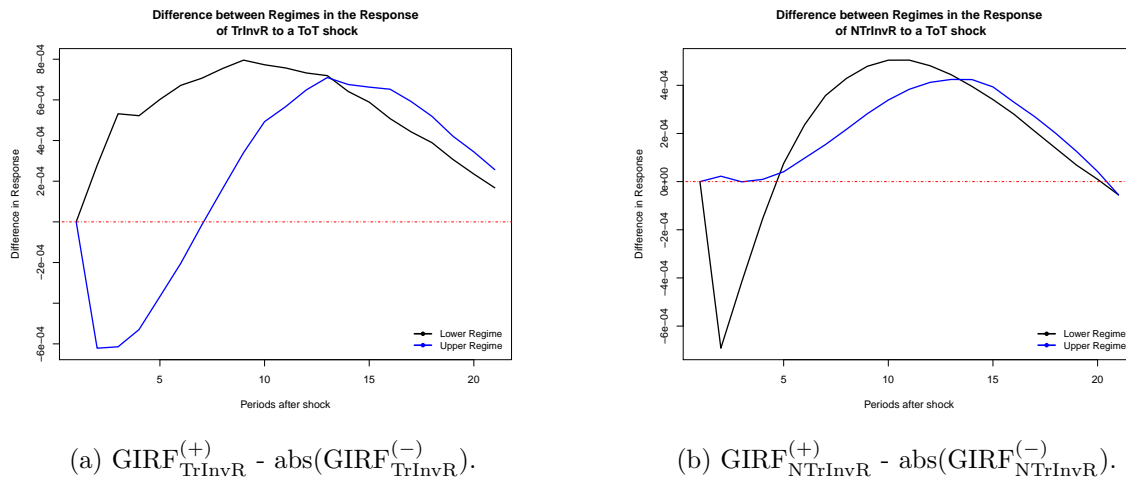


Figure 13: Difference in the responses of TrInvR and NTrInvR to positive and negative ToT shocks.  
Source: Authors' own estimations.

exchange rate depreciation. Said that, the drops in YNBD, summed to the rEER depreciation and the increases in the rIR, are second round determinants of the new levels of investment rates through the propagation channels of the ToT shock described in section 2. Responses of TrInvR and NTrInvR to negative shocks of YNBD, and positive shocks of rEER (depreciation) and rIR (monetary shock), together with the 68% bootstrapped confidence intervals are displayed in Figure 15. As expected, these shocks cause investment rates to drop.

## 5. Conclusions

Terms of trade (ToT) shocks have direct effects on commodity-exporter countries, and in particular, on investment rate dynamics. There are several propagation channels by which a negative ToT shock impacts investment rates, being the most relevant those related to: i) lower national income levels, ii) lower profits and drops in both productivity and investment by commodity-producing firms, iii) the exchange rate depreciation and the subsequent changes in relative prices of the economy, iv) the tightening in the lending conditions and the financial propagation channel, and v) the uncertainty of agents about the shock's persistence. Not only these channels, but the interactions among them and the second-round effects of the ToT shock yield asymmetric responses of investment rates. In order to capture the nonlinearities present in the data, we employ a Threshold VAR (TVAR) that allows for modeling the asymmetric effects of ToT shocks on the economy.

Our results suggest that for the Colombian case, an oil-exporter country, negative and temporary ToT shocks have immediate, direct, and asymmetric effects on investment. When a distinction between tradable and nontradable investment is made, the asymmetries in size, persistence and direction of the responses are much more evident. In fact, tradable investment rate responds quicker and more persistently to the ToT shock than the nontradable investment rate,

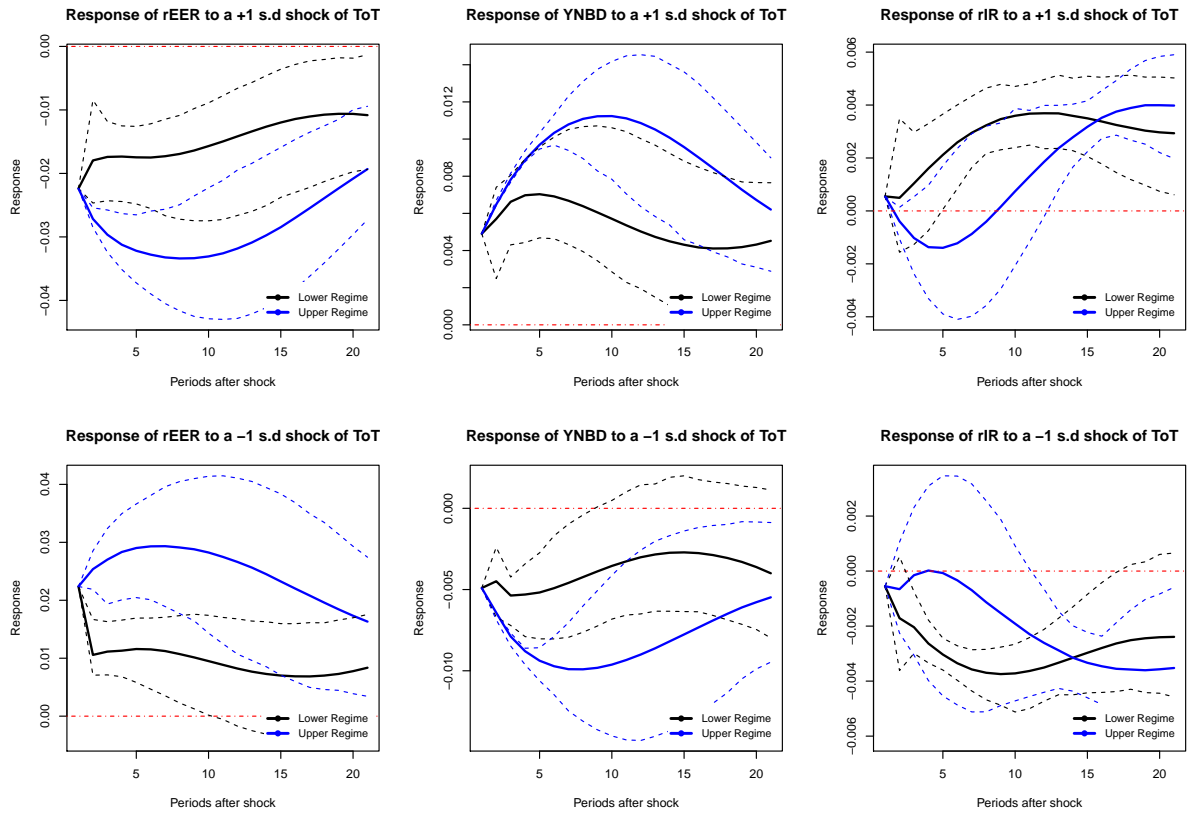


Figure 14: Responses of rEER, YNBD, and rIR to positive and negative ToT shocks ( $\pm 1$  s.d. Dashed lines represent the 68% bootstrapped C.I).  
Source: Authors' own estimations.

as expected and predicted by economic theory. The econometric results are consistent with the theoretical channels through which the shock propagates. In this sense, a negative ToT shock causes national income to fall, a depreciation of the real exchange rate, and a tightening of the monetary policy stance. Lastly, we emphasize that responses of the investment rates (and also those of the other variables included in the model) are more persistent and greater in magnitude when the ToT shock occurs in the upper regime (i.e. high ToT levels), as it was the case of the mid-2014 oil price shock, than when it occurs in the lower regime (i.e. low ToT levels).

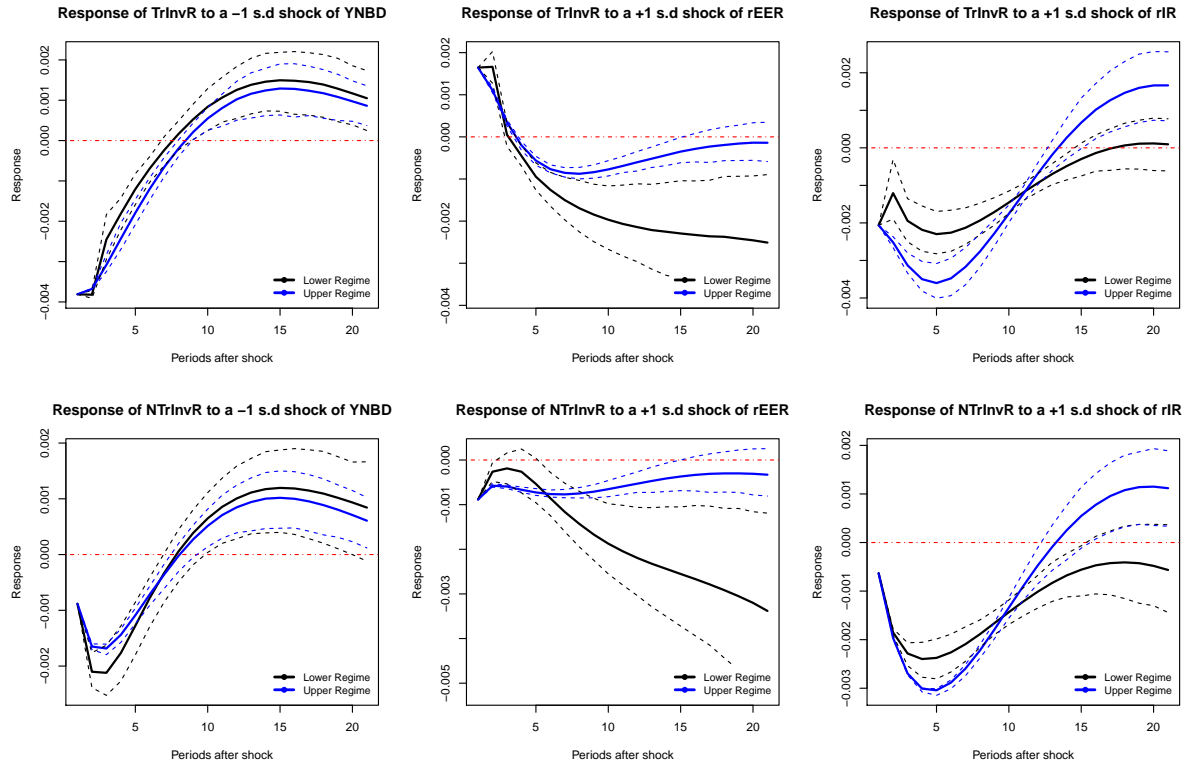


Figure 15: Responses of TrInvR and NTrInvR to shocks in YNBD (-1 s.d.), rEER (+1 s.d.) and rI (+1 s.d.). Dashed lines represent the 68% bootstrapped C.I. Source: Authors' own estimations.

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# Appendix

## Generalized Impulse Response Function Algorithm

We compute the generalized impulse response functions (GIRF) following the approach proposed by Koop et al. (1996). The algorithm, which departs from those presented in Galvão (2003) and Avdjiev and Zen (2014) consists of the following steps:

1. Consider the all the available observations, and construct the set of possible histories  $\mathbf{\Omega}$  of length  $p$ .  $\mathbf{\Omega}$  will contain  $T - p + 1$  histories  $\Omega_t$ . For each regime  $i = 1, \dots, I$ , randomly pick up a history  $\Omega_{t-1}^{(i)}$ .
2. Pick up a sequence of residuals  $\hat{\epsilon}_t^{(i)}, \dots, \hat{\epsilon}_{t+h}^{(i)}$ . These are taken from a bootstrapped sample from the estimated residuals  $\hat{\epsilon}_t^{(i)}$  of the TVAR. Take the estimated residual covariance matrix  $\hat{\Sigma}_{e^{(i)}}$ , compute the structural residuals  $\hat{\epsilon}_t^{(i)}, \dots, \hat{\epsilon}_{t+h}^{(i)}$  using the Cholesky decomposition of the covariance matrix, as  $\hat{\epsilon}_t^{(i)} = (\mathbf{A}^{(i)})^{-1} \hat{\epsilon}_t^{(i)}$ , where  $\hat{\Sigma}_{e^{(i)}} = (\hat{\mathbf{A}}^{(i)})^{-1} ((\hat{\mathbf{A}}^{(i)})^{-1})'$ .
3. Take the sequence of structural residuals chosen in step 3,  $\hat{\epsilon}_t^{(i)}, \dots, \hat{\epsilon}_{t+h}^{(i)}$ , and add a shock of size  $\delta$  to the  $k^{\text{th}}$  variable of the first element of the sequence. Keep the shocked structural residual sequence  $\hat{\epsilon}_t^{(i)} + \boldsymbol{\delta}, \dots, \hat{\epsilon}_{t+h}^{(i)}$  at hand, where  $\boldsymbol{\delta}$  is a vector with all its elements being 0 but the  $k^{\text{th}}$ , which is usually set to 1.
4. For the selected story  $\Omega_{t-1}^{(i)}$ , using the estimated coefficients of the TVAR and the sequences of structural residuals, both shocked and not shocked, simulate the paths of  $\mathbf{y}_{t+h}^\delta = \mathbf{y}_{t+h}(\Omega_{t-1}^{(i)}, \epsilon_t^{(i)} + \boldsymbol{\delta}, \dots, \epsilon_{t+h}^{(i)})$  and  $\mathbf{y}_{t+h} = \mathbf{y}_{t+h}(\Omega_{t-1}^{(i)}, \epsilon_t^{(i)}, \dots, \epsilon_{t+h}^{(i)})$ . Repeat steps 2 to 4 B times to get B estimates of the baseline ( $\mathbf{y}_{t+h}$ ) and shocked ( $\mathbf{y}_{t+h}^\delta$ ) paths.
5. Compute the GIRF in equation (5) by taking the average difference over the B estimates of the two paths, i.e.  $GIRF(\cdot) = B^{-1} \sum_{b=1}^B \mathbf{y}_{t+h}^\delta(b) - \mathbf{y}_{t+h}(b)$ . In this paper we set  $B = 1000$ .
6. For each regime  $i = 1, \dots, I$ , repeat steps 2 to 5 for R random draws of  $\Omega_{t-1}^{(i)}$ , and get R estimates of the GIRF as in step 5, i.e.  $GIRF(\cdot)_1, \dots, GIRF(\cdot)_R$ . Take the average over the R histories,  $GIRF(\cdot)^{(i)} = R^{-1} \sum_{r=1}^R GIRF(\cdot)_r$ , and compute the final GIRF path for a shock of size and sign  $\boldsymbol{\delta}$  for regime  $i$ . In this paper we set  $R = 500$ .

## Series: YNBD growth decomposition

		YNBD components						YNBD components			
		YNBD	GDP	NFI	ToT			YNBD	GDP	NFI	ToT
1997	Q1	0,29	0,81	-0,24	-0,28	2008	Q1	7,30	5,04	-0,46	2,72
	Q2	3,72	3,85	-0,14	0,01		Q2	6,38	5,27	-0,89	2,01
	Q3	4,69	4,29	-0,12	0,53		Q3	5,24	3,65	-0,69	2,28
	Q4	4,42	4,92	-0,06	-0,44		Q4	1,23	0,35	0,13	0,76
1998	Q1	5,64	5,73	0,17	-0,26	2009	Q1	0,26	1,14	0,69	-1,57
	Q2	1,58	2,27	0,28	-0,97		Q2	-0,69	1,19	0,51	-2,39
	Q3	-3,20	-0,95	0,60	-2,85		Q3	-1,07	1,30	-0,18	-2,19
	Q4	-6,81	-4,43	0,90	-3,27		Q4	2,57	2,83	-0,86	0,60
1999	Q1	-6,90	-6,02	1,06	-1,95	2010	Q1	5,25	3,60	-1,40	3,05
	Q2	-5,98	-6,98	1,65	-0,66		Q2	5,60	3,54	-0,96	3,02
	Q3	-2,00	-3,29	0,30	0,99		Q3	4,16	3,42	-0,84	1,58
	Q4	2,62	-0,73	0,53	2,83		Q4	6,29	5,17	-0,24	1,35
2000	Q1	4,59	2,55	-0,70	2,73	2011	Q1	7,45	5,57	-0,47	2,35
	Q2	3,23	3,11	-1,36	1,48		Q2	9,20	6,27	-0,74	3,66
	Q3	3,82	2,90	-0,17	1,10		Q3	10,89	7,86	-1,10	4,14
	Q4	3,00	3,32	-0,14	-0,18		Q4	8,29	6,14	-2,18	4,32
2001	Q1	-0,55	1,20	-0,11	-1,64	2012	Q1	7,84	5,60	-0,62	2,86
	Q2	1,15	1,43	0,93	-1,21		Q2	4,83	4,75	0,12	-0,03
	Q3	0,77	1,69	0,55	-1,47		Q3	2,99	2,51	0,65	-0,17
	Q4	0,67	2,45	0,28	-2,06		Q4	2,62	2,75	0,80	-0,94
2002	Q1	0,45	0,81	0,31	-0,67	2013	Q1	1,49	2,79	0,22	-1,52
	Q2	3,97	4,07	-0,34	0,24		Q2	5,04	4,51	0,25	0,28
	Q3	3,83	2,84	0,40	0,59		Q3	5,91	5,82	-0,02	0,11
	Q4	3,77	2,49	0,10	1,18		Q4	5,06	5,48	0,36	-0,78
2003	Q1	5,29	4,31	-0,17	1,15	2014	Q1	5,54	6,21	0,20	-0,86
	Q2	2,50	2,46	0,31	-0,26		Q2	1,91	3,82	-0,06	-1,85
	Q3	3,77	4,06	-0,18	-0,11		Q3	2,69	3,76	-0,20	-0,87
	Q4	5,57	5,12	0,15	0,30		Q4	1,87	3,14	1,36	-2,64
2004	Q1	5,77	6,29	-0,69	0,17	2015	Q1	1,30	2,54	1,60	-2,84
	Q2	5,02	4,46	-0,33	0,89		Q2	2,25	2,95	1,32	-2,02
	Q3	5,12	4,44	-0,52	1,20		Q3	3,22	3,12	2,79	-2,69
	Q4	7,69	6,42	-0,09	1,37		Q4	3,29	3,25	2,55	-2,51
2005	Q1	4,91	4,26	-0,06	0,70	2016	Q1	1,95	2,44	1,42	-1,92
	Q2	7,05	6,06	-0,55	1,53		Q2	1,94	2,31	1,24	-1,61
	Q3	5,86	5,40	-1,01	1,47		Q3	0,71	1,47	-0,43	-0,33
	Q4	4,51	3,31	-0,56	1,76		Q4	2,68	1,79	-1,02	1,91
2006	Q1	6,51	5,46	-0,09	1,13	2017	Q1	2,48	1,53	-1,77	2,73
	Q2	7,24	5,85	-0,06	1,44		Q2	2,92	1,66	-0,16	1,42
	Q3	8,88	7,64	0,94	0,30		Q3	2,82	2,22	-0,14	0,75
	Q4	7,96	7,74	-0,12	0,34		Q4	1,88	1,52	-0,87	1,23
2007	Q1	7,15	8,00	-0,41	-0,44						
	Q2	5,81	6,16	-0,80	0,45						
	Q3	5,93	6,08	-0,74	0,58						
	Q4	7,60	7,14	-1,08	1,53						

Chart A1: National Income (YNBD) y.o.y growth rate decomposition. Series expressed in percentage change (%).

Series: YNBD and real investment rates

		YNBD		Investment Rates				YNBD		Investment Rates	
				TrInvR	NTrInvR					TrInvR	NTrInvR
1996	Q1	11,13	11,42	14,90		2007	Q1	11,45	12,51	11,02	
	Q2	11,14	11,02	13,99			Q2	11,48	13,67	10,43	
	Q3	11,14	11,68	13,66			Q3	11,50	14,40	10,31	
	Q4	11,14	10,82	12,79			Q4	11,53	14,29	10,01	
1997	Q1	11,13	8,44	12,22		2008	Q1	11,52	11,91	13,09	
	Q2	11,17	10,47	11,79			Q2	11,54	12,47	13,56	
	Q3	11,18	11,43	12,22			Q3	11,55	10,91	14,33	
	Q4	11,18	13,11	12,13			Q4	11,55	11,74	12,71	
1998	Q1	11,19	12,38	11,49		2009	Q1	11,52	11,20	13,05	
	Q2	11,19	9,82	12,02			Q2	11,53	8,67	14,49	
	Q3	11,15	8,96	9,98			Q3	11,54	8,41	13,93	
	Q4	11,11	6,56	9,13			Q4	11,57	11,95	13,29	
1999	Q1	11,12	6,11	8,23		2010	Q1	11,57	11,45	13,20	
	Q2	11,13	4,47	7,44			Q2	11,59	11,10	12,80	
	Q3	11,13	5,34	7,08			Q3	11,58	11,35	12,78	
	Q4	11,14	6,48	5,73			Q4	11,63	11,50	13,93	
2000	Q1	11,16	5,98	8,62		2011	Q1	11,65	13,84	13,08	
	Q2	11,16	5,02	9,12			Q2	11,67	13,43	13,89	
	Q3	11,17	5,22	7,89			Q3	11,68	13,42	14,03	
	Q4	11,17	6,37	7,53			Q4	11,71	13,31	14,41	
2001	Q1	11,16	5,67	8,98		2012	Q1	11,72	14,21	14,20	
	Q2	11,17	6,49	8,39			Q2	11,72	14,23	15,03	
	Q3	11,17	5,96	8,06			Q3	11,71	12,78	12,79	
	Q4	11,17	6,26	7,45			Q4	11,74	12,57	13,92	
2002	Q1	11,16	5,64	8,06		2013	Q1	11,74	12,75	14,60	
	Q2	11,21	6,43	9,70			Q2	11,77	12,62	14,73	
	Q3	11,21	6,77	9,19			Q3	11,77	13,01	15,01	
	Q4	11,21	6,61	9,53			Q4	11,79	13,63	14,84	
2003	Q1	11,21	7,91	9,56		2014	Q1	11,79	13,30	16,17	
	Q2	11,23	6,79	9,65			Q2	11,79	14,63	15,45	
	Q3	11,25	7,30	9,63			Q3	11,80	13,65	16,12	
	Q4	11,27	6,83	10,75			Q4	11,81	14,31	15,28	
2004	Q1	11,27	8,15	10,66		2015	Q1	11,80	13,27	16,25	
	Q2	11,28	6,54	9,94			Q2	11,81	13,04	16,20	
	Q3	11,30	8,46	10,65			Q3	11,83	13,86	15,60	
	Q4	11,34	8,63	11,24			Q4	11,84	12,91	15,66	
2005	Q1	11,32	7,90	11,26		2016	Q1	11,82	11,76	16,32	
	Q2	11,35	9,17	9,98			Q2	11,83	11,84	15,76	
	Q3	11,36	10,46	10,80			Q3	11,84	11,47	16,18	
	Q4	11,38	10,85	10,94			Q4	11,86	11,60	15,74	
2006	Q1	11,38	10,65	10,22		2017	Q1	11,85	11,61	15,79	
	Q2	11,42	11,50	11,19			Q2	11,86	11,75	15,57	
	Q3	11,44	13,07	11,71			Q3	11,86	11,54	15,49	
	Q4	11,46	13,06	10,40			Q4	11,88	11,69	15,36	

Chart A2: National Income (YNBD, COP billions in logs), Tradable and Nontradable real Investment Rates (in percentages, %).

