# Estimating Credit Quality Transition Matrices for the Commercial Loan Portfolio in the Colombian Financial System

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## I. Introduction

Financial institutions rate loans as an expression of the risk they believe the client poses. With the data from those ratings, they can evaluate the current quality of their balance sheet and calculate the loan-loss provisions required for their loan portfolio. A loan rating also is an instrument for assessing and granting a loan, and for deciding how much to charge for it.

However, in a credit-risk management system, the forecast on client default and possible changes in client status also is extremely important.[1] For financial institutions, transition matrices are a fundamental tool in this respect, as they measure the likelihood of migration from one rating to another. This is done for each client and should be measured as precisely as possible.

In literature and conventional credit-risk models, transition matrices usually are measured in discreet time. Nonetheless, exploring more precise tools, such as those offered by duration models, is of interest. For that reason, a duration model is presented in this article and the transition matrices are estimated in continuous time.

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[1] The five ratings: A, B, C, D and E, are used in this article.

This article is divided into five sections, including the introduction. The data used to arrive at the estimate are described in Section II. The method used to estimate the transition matrices in discreet and continuous time, and the results of those estimates are contained in Section III. The duration model is presented in Section IV and the conclusions in Section V.

## II. Data Description

The transition matrix exercises outlined in Section III were developed with information on commercial loan-portfolio clients, taken from Financial Superintendent Form 341. Because the information in that database is divided by loan amounts and ratings, it allows for a detailed analysis of credit risk, without overlooking the dissimilarity among borrowers and loan agreements, which is a very important feature.

Given the wealth of information, the individuals in this analysis are the loans (not their amounts or the firm) and each is rated by the respective lender. There are five ratings (A, B, C, D and E); A is the best and E, the worst. The transition matrices are calculated on the basis of upward or downward migration between these ratings.

The database includes all commercial loans reported quarterly to the National Office of the Superintendent of Financial Institutions in Colombia during 1998-2006. Some entries were avoided for the sake of consistency in the data and to accomplish the objectives of the exercise. To begin with, leasing agreements were ruled out because their characteristics are different from those of ordinary loans. Secondly, the smallest 5% of the amounts on loan were ignored, as it was felt those loans might contain entry errors. Operations in foreign currency were eliminated, as were loans extended for no more than two consecutive quarters.

The figures for the duration model described in Section IV were provided by the National office of the Superintendent of Financial Institutions and the National Office of the Superintendent of Corporate Affairs. Data from the latter were used to construct the model's matrix of explanatory variables, which is why all borrowers in the commercial loan portfolio that reported balance and income sheet results to the Superintendent of Corporate Affairs are included.

As with the transition matrices, the individuals are the loans, although several additional features were added. Only migration to adjacent ratings is taken into account. This resulted in eight distinctive pairs: A to B; B to C; C to D; D to E; E to D; D to C; C to B, and B to A. Moreover, a loan that has migrated more than once during the period is distinguished in the model as if each migration were a different loan. For example, if a loan changed from A to B at a given moment, then from B to C at another point in time, it appears twice in the model.

## III. Estimating Transition Matrices[2]

Transition matrices are used to measure the probability of migration from one rating to another. The results of discrete and continuous time estimates of the quality of commercial loans in the Colombian financial system are presented in this section. The assumption assumed throughout is that a Markov chain[3] can be used to properly represent the stochastic process that produces actual momentum in migration.

In the system comprised of the commercial credit ratings, there is no absorbent state; consequently, it is unnecessary to clarify the periodicity whereby the system is assumed to be Markovian,[4] and the probabilities of transition from one state to another are (potentially) strictly positive.

### A. Discrete Time Estimates

Let us assume we have a sample of $N$ loans monitored during $T$ periods. At each point in time, the loans report a rating, and their number of ratings is finite. The change in these ratings over time can be characterized by migrations between states, which are assumed to be independent from one another. In this case, $n_i(t)$ is the number of loans in category $i$ at the start of period $t$, and $n_{ij}(t)$ is the number of loans that migrate from category i to category j between the times[5] $t$ and $t + 1$. The objective is to estimate the transition matrix. Its elements ($p_{ij}(t)$) represent the probability of a loan migrating from state $i$ to state $j$, for $i, j \in S$, where $S$ is the finite set of all possible states for a determined period of time. The verisimilitude function is provided by:

(1) $$L(p,\ n) = \prod_{(i,j)} (p_{ij}(t))^{n_{ij}(t)}$$

And the respective logarithmic function of verisimilitude is obtained by:

(2) $$\ell(p,n) = \sum_{(i,j)} n_{ij}(t)\log(p_{ij}(t))$$

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2 The methods outlined in this section are based on Gómez, González and Kiefer (2007b).

3 The validity of this assumption is assessed in Section IV, with a duration model.

4 In the event of an absorbent state, the Markov assumption implies that all individuals will migrate towards the absorbent state in the long run. For example, if the absorbent state is default, the Markov assumption implies that, in a stationary situation, all individuals would enter into default.

5 The unit of time in this article is a quarter.

The function presented in equation (2) is globally concave and the estimators of maximum verisimilitude of the elements in the transition matrix are obtained by maximizing this function with respect to each of the probabilities. These estimators are represented by:

$$(3) \qquad \hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)}, \quad i, j \in S$$

In other words, they are estimated by the proportion of loans shown in category *i* that migrate to category *j* between periods *t* and *t* + 1.

In some applications, the transition matrix is assumed to be homogeneous (or invariant) over time for a specific number of periods. For example, it is assumed that $p_{ij}(t) = p_{ij}$. $i. j \in S$, for $t = 1,..., T$, where $t < \infty$. This assumption is a convenient for developing forecasts, since the state of loans at some future point in time $\bar{t}$ can be predicted with only a limited amount of data (the state of loans in time 1 and the transition matrix). The only restriction is that $\bar{t} < T$. Nevertheless, the assumption of transition matrix homogeneity over time is not appropriate for longer periods in most empirical applications (see, for example, Gómez González and Kiefer [2007b], and Lando and Skodeberg [2002]).

The following is the average transition matrix developed with quarterly figures, using the database described in Section II.

$$\hat{P}_{prom} = \begin{pmatrix} & A & B & C & D & E \\ A & 0{,}937 & 0{,}047 & 0{,}011 & 0{,}003 & 0{,}002 \\ B & 0{,}286 & 0{,}508 & 0{,}175 & 0{,}025 & 0{,}006 \\ C & 0{,}122 & 0{,}079 & 0{,}360 & 0{,}414 & 0{,}024 \\ D & 0{,}061 & 0{,}022 & 0{,}022 & 0{,}611 & 0{,}284 \\ E & 0{,}027 & 0{,}006 & 0{,}004 & 0{,}012 & 0{,}952 \end{pmatrix}$$

The mass is concentrated in the diagonal elements of the matrix, particularly for categories A and E. This implies more migration for the best and the worst loans, which seems reasonable. Highly- rated loans rarely migrate to poor ratings, and poorly-rated loans are unlikely to improve over time. The mass is concentrated in the diagonal element for the other loans as well, but less so than for categories A and E.

Migration to a specific category is concentrated in the neighboring categories. This makes sense, as migration is anticipated as being relatively slow. The loan risk profile cannot be expected to correct itself quickly.

The following is an interesting exercise that compares this matrix to the average transition matrix for 2006:

$$\hat{P}_{2006} = \begin{pmatrix} & A & B & C & D & E \\ A & 0,960 & 0,033 & 0,005 & 0,001 & 0,001 \\ B & 0,357 & 0,408 & 0,215 & 0,012 & 0,008 \\ C & 0,103 & 0,068 & 0,278 & 0,517 & 0,034 \\ D & 0,036 & 0,013 & 0,014 & 0,646 & 0,292 \\ E & 0,023 & 0,004 & 0,004 & 0,010 & 0,959 \end{pmatrix}$$

Although both matrices seem quite similar at first glance, matrix $\hat{P}_{2006}$ indicates a greater probability that type-A loans will remain as such. For the other categories, the effect is the opposite: matrix $\hat{P}_{2006}$ shows increased probability of migration to lower categories. This would indicate that, although good quality loans recently appear to be more likely to remain so, poor quality loans are now more likely to migrate to categories with lower ratings. Yet, even without homogeneity, the transition matrices estimated in discrete time have a series of difficulties that can be summed up in two important points. First of all, this method does not ensure the estimated probabilities of migration are strictly positive. Without transition from category *i* to category *j* within a certain period of time, the estimator of maximum verisimilitude for the probability of migration is zero. This problem is evident when migrations are considered unlikely, such as those from high to low ratings. However, it is difficult to imagine the impossibility of migration between categories, even if no migration has occurred within a particular period of time. For example, in the risk-rating system used by Standard & Poor's, direct transitions between investment and junk bond categories are uncommon, and the estimators of maximum verisimilitude in discrete time for such transitions can be zero for a number of periods. Of course, this does not mean the debt of a company with a good rating cannot fall into a poor category during a specific period of time. In short, the first problem with this method is that it tends to underestimate the possibility of migration between extreme categories.

The second problem concerns the fact that discrete-time estimates are subject to arbitrary definition of the migration periods. On the one hand, the choice of migration periodicity might not match the true periodicity of the data generation process. On the other, discrete-time transition matrices cannot be changed to continuous-time transition matrices (Norris, 2005). For that reason, the forecast exercises done with those matrices must be complete multiples of the period for which the matrices were estimated.

Both problems can be resolved by estimating continuous-time transition matrices. The method and the results of the estimate, using the same database, are presented in the next sub-section.

### B. Continuous-time Estimates

The starting point for estimating continuous-time Markov chains is to assume homogeneity over time for a short period. Homogeneity is assumed for one year in the estimates presented in this. The results of the estimates developed without that assumption are presented afterwards.

Let us suppose the rating for $N$ loans is observed between time 0 and time $T$, and the space of the states is finite, with category 1 being the best rating and $z$ the worst. The transition matrix for a specified time period is $P(t)$. The transition matrix can be expressed in terms of transition intensities, which represent the instantaneous probabilities of migration between the different states. In this sense,

$$(4) \qquad P(t) = \exp(\Lambda t), \; t \geq 0$$

where $\Lambda$ is the generator matrix. Its elements are the transition intensities. Using the instantaneous probabilities as input, the respective transition matrix can be obtained for the desired period, by scaling the generator matrix by time. This is the advantage of being able to express the transition matrix in terms of the generator matrix and solves the problem with arbitrary definition of the period for discrete-time estimates.

Because the migration matrix for any t is a monotone function of the generator matrix, estimators of maximum verisimilitude can be obtained for migration possibilities by first finding the maximum verisimilitude estimators for the migration intensities, then scaling according to the appropriate time period.

The estimators of maximum verisimilitude for the generator matrix elements are given by (see Kuchler and Sorensen, 1997):

$$(5) \qquad \hat{\lambda}_{ij} = \frac{N_{ij}(T)}{\int_0^T Y_i(s)\,ds}, \quad \text{para } i \neq j$$

where $N_{ij}(T)$ represents all migrations from state $i$ to state $j$ between time 0 and time $T$, and $Y_i(s)$ is the number of loans rated $i$ during time $s$. The diagonal elements of the generator matrix are given by $\hat{\lambda}_{ii} = -\sum_{j \neq i} \hat{\lambda}_{ij}$. The denominator considers every loan rated $i$ at some point between time 0 and time $T$. One advantage of this method is that it also considers indirect transitions from one state to another, which solves the problem of underestimating the probability of infrequent events. The estimated transition is strictly positive, provided a sequence of migrations between intermediate categories occurred during the period in question, even if there was no direct migration and no loan experienced that sequence of intermediate migrations. For example, if we want to estimate the probability of a rare event, such as migration from category1 to category s during a one-year period, but no loan directly experienced that transition, we still could estimate a positive probability with even one loan that migrated from 1 to 2, another from 2 to 3, and another from *s* - 1 to *s*, during that period.

The following is the annual transition matrix estimated in continuous time with assumed homogeneity, using the database described in Section II:

$$\hat{P}_{cont\_prom} = \begin{pmatrix} & A & B & C & D & E \\ A & 0,849 & 0,075 & 0,029 & 0,026 & 0,021 \\ B & 0,519 & 0,192 & 0,091 & 0,107 & 0,091 \\ C & 0,287 & 0,067 & 0,108 & 0,243 & 0,294 \\ D & 0,176 & 0,035 & 0,024 & 0,238 & 0,526 \\ E & 0,104 & 0,017 & 0,010 & 0,028 & 0,840 \end{pmatrix}$$

A comparison of matrix $\hat{P}_{cont\_prom}$ to matrix $\hat{P}_{prom}$ shows the first has less mass concentrated in the diagonal elements. This is because $\hat{P}_{cont\_prom}$ takes intermediate migrations into account. In other words, when the probability of reaching a certain category through a sequence of indirect migrations is not considered, the probability of that migration is underestimated. For example, when element $p_{AA}$.is taken into account, we see the probability of remaining in category *A* is much higher in $\hat{P}_{prom}$ than in $\hat{P}_{cont\_prom}$ .

When we consider only the year 2006, the annual transition matrix estimated in continuous time, with assumed homogeneity, is given by:

$$\hat{P}_{cont\_2006} = \begin{pmatrix} & A & B & C & D & E \\ A & 0,904 & 0,049 & 0,018 & 0,017 & 0,012 \\ B & 0,570 & 0,130 & 0,076 & 0,119 & 0,105 \\ C & 0,234 & 0,039 & 0,076 & 0,287 & 0,365 \\ D & 0,126 & 0,017 & 0,014 & 0,268 & 0,574 \\ E & 0,094 & 0,010 & 0,007 & 0,025 & 0,863 \end{pmatrix}$$

A comparison of matrix $\hat{P}_{cont\_prom}$ and matrix $\hat{P}_{cont\_2006}$ hows they are different. This suggests the assumption of homogeneity is inappropriate for long periods of time. On the other hand,, the continuous-time transition matrices can be estimated with the assumption of homogeneity. The method for doing so is not parametric and is summarized in the Aalen-Johansen estimator. Transition matrix *P*(*t*) can be estimated as follows:

(6) $$\tilde{P}(T_i) = \prod_{i=1}^{m} (I + \Delta\tilde{A}(T_i))$$

where $I$ is the identity matrix and $T_i$ is a jump time that occurs during the observation period. $\Delta\tilde{A}(T_i)$ is a matrix where the non-diagonal element *ij* is given by the ratio of the number of transitions observed between states *i* and *j* on date $T_i$ and the total number of loans rated i just before the transition occurs. The diagonal elements correspond to the negative of the sum of the non-diagonal elements on the respective line.

The following is the transition matrix for 2006, estimated in continuous time with no assumption of homogeneity and using the database described in Section II.

$$\hat{P}_{cont\_2006\_nh} = \begin{pmatrix} & A & B & C & D & E \\ A & 0,901 & 0,052 & 0,020 & 0,018 & 0,009 \\ B & 0,601 & 0,072 & 0,052 & 0,163 & 0,112 \\ C & 0,241 & 0,034 & 0,029 & 0,284 & 0,412 \\ D & 0,114 & 0,019 & 0,015 & 0,205 & 0,647 \\ E & 0,091 & 0,013 & 0,009 & 0,030 & 0,857 \end{pmatrix}$$

There are differences between matrix $\hat{P}_{cont\_2006\_nh}$ and $\hat{P}_{cont\_2006}$. However, the differences are particularly notorious when compared to matrix $\hat{P}_{2006}$. This indicates that once a continuous-time method is used to estimate transition matrices, the assumption of homogeneity does not appear to be detrimental for short periods of time. Making discrete-time estimates is more problematic, as it can lead to underestimating the probability of migration outside each state, especially in the case of extreme states.

## IV. Duration Model[6]

The estimated transition matrices presented in the previous section were developed under the assumption that Markov chains are adequate to represent the stochastic process that generates the momentum in migration. Migration probabilities are estimated so they can be included in a credit risk management system. For that reason, it is important to verify just how precise they are. This can be done by weighing the validity of the Markov assumption with a duration model that incorporates explanatory variables through the use of survival analysis techniques.

A variation of Cox's semiparametric model (1972)[7] was used in this article:

7) $$\lambda_{ij}^{n}(t) = Y_i^{n}(t)\alpha_{ij}^{n}\left(\beta_{ij}, t, X^{n}(t)\right)$$

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6 The methods outlined in this section are based on Gómez, González and Kiefer (2007)a.

7 Cox's semiparametric model (1972) was selected, because initial statistical tests on the unconditional distribution of duration in time showed the behavior of the function with respect to conditional instantaneous probability of change of state for all the categories under consideration is not a monotonal and does not resemble the behavior expected with commonly used density functions.

where $\lambda_{ij}^{n}(t)$ represents the intensity of migration from category *i* to *j* at the moment; *t*, $Y_i^n(t)$ is an indicator function that is activated when the loan is in state *i* in time *t*; and $\alpha_{ij}^{n}\left(\beta_{ij}, t, X^n(t)\right)$ is a time and vector function of independent variables of loan n in time $t, X^n(t)$. Explicative variables that vary in time are used in this case. The assumption is that function ?$\alpha_{ij}^{n}\left(\beta_{ij}, t, X^n(t)\right)$ admits a multiplicative form:

(8) $$\alpha_{ij}^{n}\left(\beta_{ij}, t, X^n(t)\right) = \alpha_{ij}^{0}(t)\exp\left(\beta_{ij} X^n(t)\right),$$

where $\alpha_{ij}^{0}(t)$ represents the base intensity common to all the loans. If the Markov assumption is valid, $\alpha_{ij}^{n}\left(\beta_{ij}, t, X^n(t)\right) = \alpha_{ij}^{0}(t)$. In other words, all the parameters must be statistically equal to zero. Therefore, the tests of the Markov assumption are statistically significant with respect to the vector of parameters $\beta$. The exponential form was chosen as the transformation function for reasons of convenience; it ensures non-negative intensities, without restricting the value of the parameters. With this specification for intensities, the estimated parameters are interpreted as semi-elasticities. The maximum partial verisimilitude method developed by Cox (1972) is used to estimate the model.

### A. Description of the Explanatory Variables used in the Duration Model

We selected indicators that can be used to determine the probability of deterioration in the quality of a firm's loan portfolio. There are two types: indictors that describe the financial characteristics of the company itself and those that represent the circumstances in the economic environment. The following is a brief description of both types.

1. Liquidity: (current assets + LT investments + LT borrowers)/(current liabilities + LT financial and labor obligations + LT accounts payable + LT estimated bonds and liabilities). This indicator measures long-term liquidity.

2. Debt structure: current liabilities /(current liabilities + LT liabilities). The higher the indicator, the more the firm's liabilities are concentrated in the short term, which is a reflection of less stable financing.

3. Indebtedness: liabilities/equity. Among the accounts that constitute equity, the ones pertaining to surplus and profits in the ongoing accounting period were weighted by 50% to acknowledge their nature as secondary capital. The interpretation of this indicator is controversial. In the case of two firms with equal profits, the one with more debt will be more profitable. However, the one with less debt is more sound (because it has more equity) and,

therefore, is better able to cope with the added possibility of adverse situations, which makes it more profitable in the long run.

4. Profitability: profits before taxes/assets

5. Efficiency: operating expenses /sales. This traditional indicator signals the firm's efficiency, which is becoming an increasingly important component of its cost structure.

7. Size: assets/1,000,000. This variable is controlled by the size of the institution. The expectation is that larger firms are less likely to see their portfolio deteriorate than smaller firms.

8. Type of collateral: This is a dummy variable with a value of one, if collateral is suitable, and zero if it is not.

9. Credit history: The number of quarters when the firm had at least one loan from the financial system.

10. Number of bank relationships: The number of lenders with which the firm has loans.

11. Real GDP growth: Companies are more apt to fulfill their obligations during economic growth cycles.

12. Exchange rate: The quarterly average representative market rate of exchange.

13. Tradables or Non-tradables: Refers to whether the company belongs to a tradable sector (defined as agriculture, fishing, mining and manufacturing) or a non-tradable sector (all others). The non-parametric tests showed the survival function is statistically different for these two sectors. As a result, separate regressions were done for tradables and non-tradables.

**B. Results of the Estimated Model**

The results of the estimates for tradables and non-tradables, and for each of the adjacent transition pairs are shown in Tables 1 and 2, together with the estimated coefficients and the standard errors.[8] The chi-squared test used to identify the global significance of the estimators shows all parameters $\beta$ are jointly and

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[8] See the aforementioned tables for details on the individually significant variables of each transition and the groups of companies. Combined significance tests on groups of variables can be done as well, using the reported coefficients and standard errors.

statistically different from zero. In other words, in every case, the variables of the companies' financial characteristics and the variables of the macroeconomic environment explain the intensity of migration between categories. Accordingly, we can conclude the Markov assumption is not borne out.

Because our objective is to estimate transition matrices as precisely as possible, the invalidity of the Markov assumption has important implications, since the intensity of migration depends not only on elements common to all loans, but also

TABLE 1

## RESULTS OF THE ESTIMATE FOR COMPANIES IN TRADABLE SECTORS

**Transitions to Poorer Categories**

| Variable | AB | | BC | | CD | | DE | |
|---|---|---|---|---|---|---|---|---|
| | **Coef.** | **Est. Err.** | **Coef.** | **Est. Err.** | **Coef.** | **Est. Err.** | **Coef.** | **Est. Err.** |
| Liquidity | -0.004 | 0.000 | -0.006 | 0.001 | -0.003 | 0.001 | -0.000 | 0.001 |
| Indebtedness | 0.006 | 0.005 | -0.004 | 0.078 | 0.004 | 0.007 | -0.009 | 0.012 |
| Size | -0.001 | 0.000 | -0.008 | 0.001 | -0.014 | 0.002 | -0.005 | 0.001 |
| Efficiency | 0.005 | 0.003 | -0.003 | 0.000 | 0.001 | 0.008 | -0.002 | 0.018 |
| Debt Components | -0.005 | 0.000 | -0.000 | 0.001 | 0.002 | 0.001 | 0.004 | 0.002 |
| Number of Relationships | 0.041 | 0.006 | 0.106 | 0.012 | 0.055 | 0.016 | 0.049 | 0.018 |
| Age | -0.006 | 0.002 | -0.025 | 0.004 | -0.018 | 0.005 | -0.025 | 0.005 |
| Collateral | -0.295 | 0.031 | 0.046 | 0.064 | 0.073 | 0.078 | -0.039 | 0.086 |
| GDP Growth | -0.026 | 0.006 | -0.032 | 0.012 | -0.009 | 0.015 | -0.003 | 0.015 |
| Exchange Rate | -0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Profitability | -0.127 | 0.011 | -0.601 | 0.065 | -0.267 | 0.089 | -0.029 | 0.049 |
| Global Significance a/ | 1,038.44 | (0.000) | 495.72 | (0.000) | 189.82 | (0.000) | 69.88 | (0.000) |

**Transiciones hacia mejores categorías**

| Variable | AB | | BC | | CD | | DE | |
|---|---|---|---|---|---|---|---|---|
| | **Coef.** | **Est. Err.** | **Coef.** | **Est. Err.** | **Coef.** | **Est. Err.** | **Coef.** | **Est. Err.** |
| Liquidity | 0.000 | 0.000 | 0.002 | 0.001 | 0.002 | 0.001 | 0.001 | 0.000 |
| Indebtedness | 0.005 | 0.004 | -0.002 | 0.012 | 0.000 | 0.000 | 0.009 | 0.025 |
| Size | -0.005 | 0.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 | 0.001 |
| Efficiency | 0.003 | 0.003 | -0.110 | 0.123 | -0.000 | 0.001 | -0.239 | 0.222 |
| Debt Components | 0.010 | 0.001 | 0.006 | 0.002 | 0.004 | 0.004 | -0.004 | 0.003 |
| Number of Relationships | -0.021 | 0.007 | -0.023 | 0.022 | -0.083 | 0.044 | -0.035 | 0.039 |
| Age | -0.011 | 0.002 | 0.006 | 0.007 | 0.034 | 0.012 | 0.050 | 0.011 |
| Collateral | 0.119 | 0.037 | -0.109 | 0.108 | -0.307 | 0.183 | -0.309 | 0.166 |
| GDP Growth 0.046 | 0.008 | 0.049 | 0.020 | 0.008 | 0.034 | -0.075 | 0.033 | |
| Exchange Rate | 0.000 | 0.000 | -0.001 | 0.000 | -0.001 | 0.000 | -0.000 | 0.000 |
| Profitability | 1.373 | 0.121 | 0.386 | 0.266 | 0.079 | 0.110 | -0.364 | 0.183 |
| Global Significance | 679.63 | (0.000) | 61.28 | (0.000) | 30.14 | (0.002) | 44.07 | (0.000) |

**a/** The chi-square statistic and the respective p-value are listed in this row.
Source: Authors' calculations.

on information inherent in each company and the economic situation at each point in time. Consequently, the next step should be to include that information in estimates of transition matrices, so as to build a more precise early warning system for financial institutions and regulators alike.

## V. Conclusions

Transition matrices are a fundamental tool in credit risk analysis. They can be used to forecast changes in loan portfolio quality during a specific period of time, which

**Table 2**

**Results of the Estimate for Companies in Non-tradable Sectors**

| Variable | Transition to Poorer Categories | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | AB | | BC | | CD | | DE | |
| | Coef. | Est. Err. | Coef. | Est. Err. | Coef. | Est. Err. | Coef. | Est. Err. |
| Liquidity | 0.003 | 0.002 | -0.001 | 0.000 | -0.000 | 0.000 | -0.000 | 0.000 |
| Indebtedness | 0.002 | 0.002 | -0.000 | 0.001 | -0.000 | 0.000 | 0.006 | 0.002 |
| Size | -0.000 | 0.000 | -0.004 | 0.001 | -0.004 | 0.001 | -0.004 | 0.002 |
| Efficiency | 0.000 | 0.000 | 0.000 | 0.001 | -0.001 | 0.001 | 0.000 | 0.001 |
| Debt Components | -0.008 | 0.000 | -0.000 | 0.001 | 0.002 | 0.001 | 0.004 | 0.001 |
| Number of Relationships | 0.040 | 0.005 | 0.089 | 0.012 | 0.040 | 0.014 | 0.031 | 0.019 |
| Age | -0.005 | 0.001 | -0.024 | 0.003 | -0.021 | 0.003 | -0.026 | 0.004 |
| Collateral | -0.262 | 0.026 | 0.025 | 0.059 | 0.099 | 0.066 | 0.058 | 0.080 |
| GDP Growth -0.041 | 0.005 | -0.032 | 0.010 | 0.003 | 0.011 | 0.001 | 0.014 | |
| Exchange Rate | -0.000 | 0.000 | 0.000 | 0.000 | -0.000 | 0.000 | -0.000 | 0.000 |
| Profitability | -0.100 | 0.006 | -0.422 | 0.032 | -0.138 | 0.040 | -0.004 | 0.006 |
| Global Significance | 1158.65 | (0.000) | 275.03 | (0.000) | 104.60 | (0.000) | 83.15 | (0.000) |

| Variable | Transiciones hacia mejores categorías | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | AB | | BC | | CD | | DE | |
| | Coef. | Est. Err. | Coef. | Est. Err. | Coef. | Est. Err. | Coef. | Est. Err. |
| Liquidity | 0.000 | 0.001 | 0.006 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 |
| Indebtedness | -0.000 | 0.002 | -0.001 | 0.011 | -0.004 | 0.022 | 0.008 | 0.002 |
| Size | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 | 0.002 | 0.000 | 0.001 |
| Efficiency | -0.003 | 0.003 | -0.002 | 0.006 | -0.017 | 0.029 | -0.000 | 0.003 |
| Debt Components | 0.005 | 0.000 | 0.000 | 0.002 | -0.000 | 0.002 | 0.001 | 0.003 |
| Number of Relationships | -0.021 | 0.007 | -0.032 | 0.024 | -0.112 | 0.045 | 0.194 | 0.052 |
| Age | -0.009 | 0.001 | 0.003 | 0.006 | 0.029 | 0.009 | -0.023 | 0.013 |
| Collateral | 0.096 | 0.031 | -0.201 | 0.110 | -0.102 | 0.173 | -0.181 | 0.227 |
| CGDP Growth0.063 | 0.006 | -0.023 | 0.021 | 0.073 | 0.032 | -0.045 | 0.043 | |
| Exchange Rate | 0.000 | 0.000 | 0.000 | 0.000 | -0.000 | 0.000 | 0.000 | 0.000 |
| Profitability | 0.533 | 0.046 | 0.936 | 0.286 | 0.255 | 0.302 | 0.121 | 0.198 |
| Global Significance | 585.57 | (0.000) | 31 | (0.001) | 22.71 | (0.019) | 19.94 | (0.046) |

Source: Authors' calculations

makes them an important element for measuring the losses companies can incur with loan default. In this article, different transition matrices were estimated for debtors in the Colombian commercial loan portfolio and, depending on the method used, the results are different.

We show the Markov assumption, under which the transition matrices were estimated, is not borne out. Duration models should be used for a more precise estimate of transition matrices, and they should allow for including explanatory variables as the determinants of migration probabilities. If the powers that be insist on developing estimates under the Markov assumption, those estimates should be done in continuous time to overcome the problems inherent in discrete-time estimates, which were discussed throughout this article.

Given the results, we propose that future estimates of migration probabilities for all categories use the duration model presented in this article as a starting point.

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