Box 1: Adjusting long-term growth forecasting models to address Colombia's demographic shifts

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1. Introduction

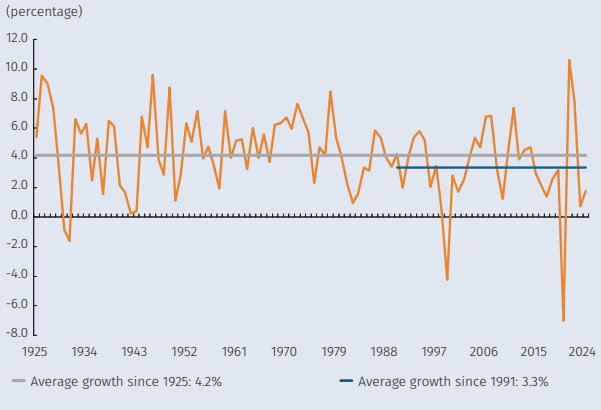
One of the central inputs for *Banco de la República's* macroeconomic forecasting models is the long-term rate of growth of the economy. In technical terms, this is known as steady-state growth, defined as the growth rate of the economy in the absence of new shocks, also understood as the reference convergence in terms of growth of gross domestic product (GDP) in the long run. Occasionally, dependent on the shocks that affected the economy, this convergence may occur beyond the forecast horizon.[1]

The two models currently used by the technical staff—Patacon and 4GM - (reference Gonzalez *et al.*, 2011, and Gonzalez *et al.*, 2020, respectively) —adjust this parameter at 3.3 %, a figure aligned with the average annual growth rate of the Colombian economy over the last three decades (Graph B1.1). However, the country's new demographic outlook, marked by a longer-lasting decline in the national birth rate and a more rapid aging of the population than foreseen a decade ago, suggests a structural slowdown in the buildup of effective labor, one of the main drivers of economic growth.

This phenomenon, which transcends the Colombian case and mirrors a global trend, forces us to reconsider the steady-state growth value, not only for the forecasting models currently in place but also for the new generation of models under construction (Avila-Montealegre *et al.*, 2025; Grajales-Olarte *et al.*, 2025; Ramos-Veloza *et al.*, 2025).

This Box presents the conceptual framework and quantitative estimates that support updating the long-term growth value to 2.65% annually. This quantification is part of a broader study on the macroeconomic effects of demographic shifts to be published by *Banco de la República* in 2026 within the journal "*Ensayos de Política Económica*" (Jaramillo *et al.*, 2026).

Graph B1.1
Annual GDP growth



Source: Calculations by *Banco de la República.*

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1 In general, it is possible that the economy's long-term growth does not coincide with its potential short-term growth, which may be affected by transitory supply shocks that maintain varying degrees of persistence.

2. Conceptual framework

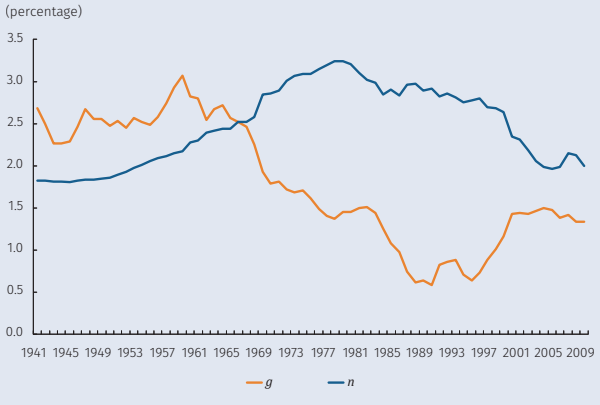
The theory of economic growth postulates that the economy's long-run output is determined by the accrual of factors of production (i.e., the aggregate quantity of capital, labor, etc.), as well as the efficiency with which these combine to generate output (i.e., the productivity of these factors of production). The most stylized growth theory model (Solow, 1956; Swan, 1956) postulates that, even when considering multiple factors of production,[2] the steady-state growth rate of output converges to the sum of two components: the average growth rate of the labor force (n) and the average growth rate of labor productivity, or output per worker(g).[3]

In practice, such average rates are usually measured over time windows (both past and forward-looking) long enough to capture the cyclical patterns of the economy. Using GDP data and hours worked by employed individuals, collected and spliced for the last hundred years,[4] Graph B1.2 illustrates the changes in n and g computed over thirty-year rolling intervals.[5] In the latter, each point on the graph refers to the corresponding average for the thirty-year interval of the specific year.

Graph B1.2 illustrates how n has been steadily decreasing since the early 1980s, in line with the slower population growth rate and a continual decline in the number of hours worked per employed individual. This drop occurred despite an increase in the labor participation rate resulting from the growing inclusion of women in the labor market. In terms of g, three different intervals are seen: the first, until the mid-sixties, with a growth rate nearing 2.5%; a second, exhibiting a downward trend, lasts until the end of the nineties; and a third, present during the whole of the 21st century, wherein labor productivity growth has remained relatively stable, hovering around 1.4% per year.

Demographic change, understood as the transition from a population structure characterized by high birth and mortality rates to one of low rates, implies a transformation in the age make-up of the population, particularly underscoring a process of population aging.[6] While the change in n may affect labor productivity growth—that is, the value of

Graph B1.2
Observed steady-state growth components [a/]



a/ The graph displays the rolling averages centered on 30-year windows of labor factor growth (total hours worked, n), and of labor productivity (g), employing only observed data. Consequently, for example, the 2009 data corresponds to the average of the 1994-2024 window.
Source: Calculations by *Banco de la República*.

2 The original Solow-Swan model considers two productive factors: capital and labor. The model can be extended to include additional productive factors, such as human capital (Mankiw et al., 1991), without altering steady-state growth. This occurs because, for an economy to maintain ensure state growth, productivity must increase the labor factor (in technical terms, be Harrod-neutral; see Uzawa, 1961).

3 Although at first glance this breakdown appears tautological, the determinants that explain each component are different in origin and, therefore, usually studied separately.

4 The analysis uses as its primary source the database constructed by Banco de la Republica for its centennial celebration (available in Spanish at https://www.banrep.gov.co/es/estadisticas-economicas/series-estadisticas-historicas-colombia), as well as own splices based on DANE's ENH (National Household Survey), ECH (Continuous Household Survey) and GEIH (Large Integrated Household Survey) labor statistics, and computation by Greco (1999}, based on Florez (1998}.

5 A literature review finds varying estimates of the average length of business cycles in Colombia, ranging from 6.8 years (Alfonso et al., 2014) to 10.1 years (Arango et al., 2025). Therefore, regardless of the point value of the cycle length, thirty-year periods ensure the averages are not biased by a single economic cycle phase. Moreover, this window size is consistent with the literature: for example, Rao (2010) uses twenty-to-thirty-year periods to measure steady-state variables, placing our windows at their upper bound.

6 For a detailed description of this phenomenon and its implications, see Lee (2003).

g (see, for example, Lee and Mason, 2009; Bloom *et al*., 2010)—its most notorious effect is seen in the dynamics of the effective labor force, namely the expected value of n in future years.[7] Accordingly, we present below two different methodologies to forecast the values of n and g over the next twenty years. For n, demographic shifts have an explicit impact, and it is estimated based on DANE population projections by age group, complemented with assumptions on the evolution of the leading labor market indicators. The values of g are exogenously forecasted using econometric techniques.

3. Forecast methodologies

For n, the average growth in hours worked by those employed (denoted as H) is calculated as the average centered on the thirty-year average rolling window for H_t, where the latter are forecast using:

$$H_t = PET_t \times TGP_t \times (1 - TD_t) \times \overline{h_t}$$

where PET_t corresponds to the working age population, TGP_t to the labor participation rate, and *TD* to the unemployment rate. Consequently, $PET_t \times TGP_t \times (1 - TD_t)$ equals the total employed, and $\overline{h}_t$ the average number of hours worked by the employed in the year. Each of these components is forecast individually. The PET_t is obtained from DANE's demographic projections based on the results of the 2018 National Population and Housing Census (CNPV for its Spanish acronym).[8] The TGP_t and TD_t are modeled based on their configuration by different age groups i derived from the same DANE projections, which take into account population aging, following the formula:

$$x_t = \sum_i s_{t,i} \bar{x}_i, \text{ para } x = TGP \; o \; TD$$

where $\bar{x}_t$ the average rate of age group i during the past ten years (excluding the pandemic[9]), and s_{ti} is age group i's share in the PET_t. Thus, the implicit assumption is that the labor indicators of each age group remain stable at the average values of the last decade.[10] Finally, the variable $\overline{h}_t$, the average number of hours worked for all employed individuals, recorded annual drops in 2023 and 2024, attributable in the formal segment to the regulatory change introduced by Law 2101 of 2021 that reduced the working day by one hour per year. Furthermore, its level should fall again in 2025 and 2026, proportional to the total hour series seen in 2023 and 2024, because this Law includes additional reductions of two hours per year during those years. From 2027 onwards, h_t it is assumed this variable will remain constant.

For g (average growth of labor productivity), we follow the methodology of Ochsner *et al*. (2024), employing statistical filters to isolate cyclical fluctuations from long-term trends in productivity and project them over long horizons. In particular, for productivity point forecasting, we use the econometric model of Chan *et al.* (2018), which has a highly flexible specification (for example, allowing for time-varying parameters and volatility, as well as the use of an external benchmark series), and it is estimated using Bayesian methods. Representing

7 Empirical evidence supports this relationship. For example, Aksoy et al. (2019), using a panel of countries, find that demographic composition has significant and persistent macroeconomic effects. In particular, a higher share of the working-age population is associated with higher rates of GDP per capita growth.

8 These population projections were updated in 2023 to reflect the effect of the Covid-19 pandemic on the various demographic components (fertility, mortality, and migration).

9 The Covid-19 pandemic created a temporary and abrupt disruption in the labor indicators, surpassing those recorded by economic activity (Bonilla *et al*., 2019), which distorts the averages. For a summary of the main margins of labor market adjustment during this period, see Pulido *et al*. (2024).

10 It is worth noting that certain structural forces that have influenced persistent trends in labor indicators in recent decades, such as the integration of women into the labor market and the compositional effects of internal migration to urban areas (which exhibit high employment, participation, and unemployment rates), are losing momentum. The gender disparity in participation has been stable over the past decade, and current unemployment rates are comparable between urban and rural areas. In this framework, it is appropriate to assume stability in labor indicators by age group, utilizing the averages from the past decade (excluding the pandemic).

annual labor productivity growth as *y*, the model of Chan *et al.* (2018) specifies:

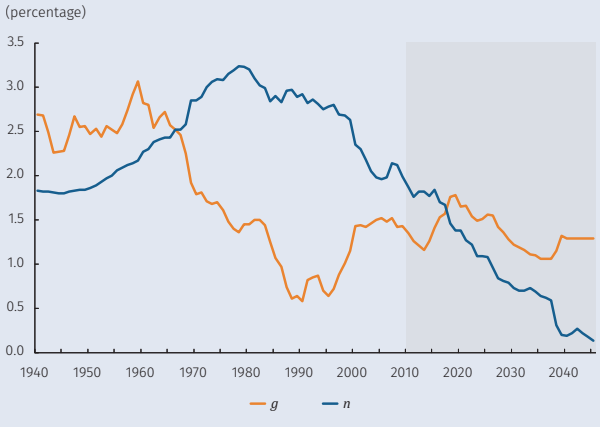
$$
\begin{aligned}
y_t - y_t^* &= b_t(y_{t-1} - y_{t-1}^*) + v_t, \\
z_t &= d_{0t} + d_{1t} y_t^* + \varepsilon_{z,t} + \psi \varepsilon_{z,t-1}, \quad \varepsilon_{z,t} \sim N(0, \sigma_z^2), \\
y_t^* &= y_{t-1}^* + u_t, \\
b_t &= b_{t-1} + \varepsilon_{b,t}, \quad \varepsilon_{b,t} \sim TN(0, \sigma_b^2), \\
d_{it} - \mu_{d_i} &= \rho_{d_i}(d_{i,t-1} - \mu_{d_i}) + \varepsilon_{d_i,t}, \quad \varepsilon_{d_i,t} \sim N(0, \sigma_{d_i}^2), \quad i = 0, 1, \\
v_t &= \lambda_{v,t}^{0.5} \varepsilon_{v,t}, \quad \varepsilon_{v,t} \sim N(0, 1), \\
u_t &= \lambda_{u,t}^{0.5} \varepsilon_{u,t}, \quad \varepsilon_{u,t} \sim N(0, 1), \\
\log(\lambda_{i,t}) &= \log(\lambda_{i,t-1}) + \nu_{i,t}, \quad \nu_{i,t} \sim N(0, \phi_i), \quad i = u, v.
\end{aligned}
$$

where y^* corresponds to the productivity growth trend and z equals the external reference variable. The model is estimated using labor productivity growth series as of 1940 (excluding pandemic-related outliers), a Markov Chain Monte Carlo (MCMC) sampling with 30,000 simulations (discarding the first 1,000), and the thirty-year centered rolling average of *y* as the benchmark *z*-series. The model's specific *y* forecasts are added to the observed values of productivity growth in recent years to compute *g* in year *t* as the average over the 30-year rolling windows, understood as: $g_t = \frac{1}{31}\sum_{i=t-15}^{t+15} y_i$.

4. Results and conclusions

Graph B1.3 shows the results of the *n* and *g* projections for the next twenty years. As in Graph B1.2, each point indicates the average calculated over a thirty-year rolling window centered on the indicated year. The demographic change would prolong the downward trend of *n* over the forecast horizon, explained by both the slowdown in the growth rate of the working-age population and by population aging, thus reducing the overall labor participation rate as the older age groups, which participate less in the labor market, gain relative weight. As for *g*, average labor productivity growth would remain at similar levels to those recorded so far this century, ranging from 1% to 1.5%. It is important to note, however, that the Bayesian forecasting model's results show a large amount of uncertainty, reflecting the difficulties of precisely projecting these types of variables.

Graph B1.3
Forecasts for steady-state growth components [a/]

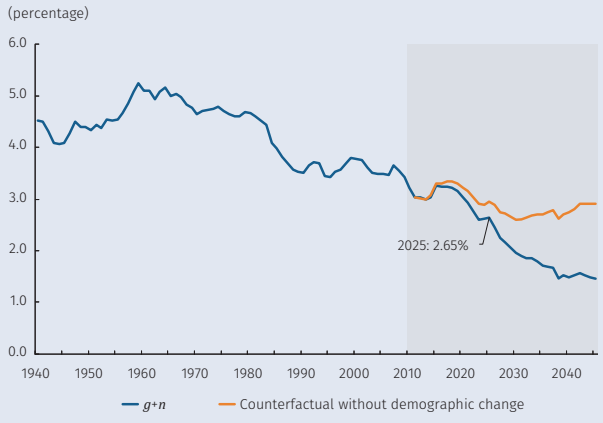


a/ The graph displays the rolling averages centered on 30-year windows of labor factor growth (total hours worked, *n*), and of labor productivity (*g*), employing both observed and forecast data. Consequently, for example, the 2025 data corresponds to the average of the 2010-2040 window, wherein the pre-2024 data is forecast using the aforementioned methodologies. The shaded area indicates that these figures were computed using estimates for one or more years.
Source: Calculations by *Banco de la República*.

Employing the projected values of *n* and *g*, Graph B1.4 offers the predicted steady-state growth for 2025, as well as its projection for the next twenty years. In 2025, such growth would be 2.65% (with a *g* value of 1.57% and *n* of 1.08%); consequently, in the absence of shocks, this would be the growth rate for the Colombian economy. Going forward, as demographic trends in population projections take shape, steady-state growth would continue to slow, reaching levels close to 1.5% at the end of the twenty-year horizon.

Graph B1.4 also illustrates a counterfactual scenario forecast in which demographic shifts would not materialize to visualize the influence of these changes on the estimates obtained. To construct this scenario, the *n* forecasts assume that the working-age population would continue to grow at the rate observed in 2025, and its average age composition would remain constant with the values seen over the past five years. Under this scenario, where demographic change does not occur,

Graph B1.4
Steady-state growth [a/]



a/ The shaded area indicates that these figures were computed using estimates for one or more years.
Source: Calculations by *Banco de la República*.

steady-state growth in 2025 would approach 3%, only slowing to 2.7 % at the end of the contemplated window. Thus, the impact of demographic shifts would imply a reduction of 0.35 percentage points (pp) in the current steady-state growth, and of 1.2 pp by 2045.

The results of this exercise confirm that, based on available information, it is necessary to readjust the steady-state growth values used in the technical staff's forecast models to reflect the new figure of 2.65%. This restatement process, initiated in this *Report*, is expected to be completed in the forecast models this quarter. However, it is essential to highlight that the projections suggest that, in the future, these estimates must once again be revised to reflect future changes in demographic variables, given recent birth-rate data that points to an even faster demographic shift than that projected by DANE (see Parra-Polanla and Ladino, 2025; Jaramillo *et al.,* 2026).

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