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Abstract

We extend the semi-structural model for monetary policy analysis and forecasting in Colombia, the 4GM model (González et al., 2020), by integrating labor market dynamics into its specification. We incorporate four key mechanisms: (1) wage-price feedback and its effects on marginal costs; (2) the impact of the NAIRU on potential output; (3) Okun's law to connect cyclical unemployment to the output gap; and (4) the impact of real wages and unemployment on aggregate demand. Our model adds to the baseline 4GM model interactions between goods and labor markets, which affect the transmission of the monetary policy. Focusing on Colombia, a country with a labor market characterized by rigidities and persistently high unemployment, we demonstrate how wage dynamics and labor market tightness amplify inflationary pressures and delay adjustment. Our model aids in disentangling supply and demand shocks, particularly during crises (e.g., the pandemic), when labor market idiosyncrasies distort wage and inflation trends.

JEL classification: E24, E52, J64

Keywords: Unemployment, wages, inflation, monetary policy model.

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Dinámica Salarial, Desempleo e Inflación: Extendiendo el Modelo Semi-Estructural 4GM

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Resumen

Extendemos el modelo semi-estructural para el análisis y pronóstico de la política monetaria en Colombia, el modelo 4GM (González et al., 2020), integrando la dinámica del mercado laboral en su especificación. Incorporamos cuatro mecanismos clave: (1) la retroalimentación salario-precio y sus efectos en los costos marginales; (2) el impacto de la NAIRU (tasa de desempleo no aceleradora de la inflación) sobre el producto potencial; (3) la ley de Okun para conectar el desempleo cíclico con la brecha del producto; y (4) el impacto de los salarios reales y el desempleo sobre la demanda agregada. Nuestro modelo añade a la línea base del 4GM las interacciones entre los mercados de bienes y de trabajo, las cuales afectan la transmisión de la política monetaria. Centrándonos en Colombia, un país con un mercado laboral caracterizado por rigideces y un desempleo persistentemente alto, demostramos cómo la dinámica salarial y la tensión en el mercado laboral amplifican las presiones inflacionarias y retrasan el ajuste. Nuestro modelo ayuda a distinguir entre choques de oferta y demanda, particularmente durante crisis (por ejemplo, la pandemia), cuando las idiosincrasias del mercado laboral distorsionan las tendencias salariales e inflacionarias.

Clasificación JEL: E24, E52, J64

***Palabras clave**: Desempleo, salarios, inflación, modelo de política monetaria.*

1 Introduction

Monetary policy relies on a continuous assessment of the current state of the economy and the shocks that affect its short-run dynamics. To support this process, central banks develop both structural and semi-structural models that capture the key mechanisms linking external conditions, consumer spending, investment, economic activity, and prices. These models help predict how short-term dynamics affect aggregate demand and inflationary pressures, thus guiding monetary authorities' responses. In addition, they are critical for analyzing various scenarios involving changes in monetary policy. In Colombia, a country operating under an inflation-target regime, González et al. (2020) developed the 4GM, a semi-structural model, building on Benes and N'Diaye (2004) and IMF (2010).[1]

In summary, the 4GM is based on a New-Keynesian rational expectation framework for a small open economy, which consists of four main blocks: an IS curve, four Phillips curves (covering goods, services, food, and regulated items), a Taylor rule, and an uncovered interest parity (UIP) condition. Notably, it does not include explicitly any elements of the labor market, despite the well-established interdependence between the goods and labor markets. For example, it is well known that employment levels and wages directly affect household income and, consequently, consumer spending, which is a major component of aggregate demand. Furthermore, labor market dynamics influence inflation through wage setting mechanisms. As wages increase, firms often transfer higher labor costs to consumers, intensifying inflationary pressures.

In this study, we extend the 4GM model by incorporating four key mechanisms that connect labor and goods markets, following insights from semi-structural models by Alichi et al. (2019), Angelini et al. (2019), and Demidenko et al. (2016). First, we link wage dynamics to price inflation through the marginal cost channel. Second, we incorporate the effect of the evolution of the non-accelerating inflation rate of unemployment (NAIRU) into potential output. Third, we introduce the relationship between cyclical unemployment and output (Okun's law). Finally, we model the influence of real wage and unemployment growth on aggregate demand. The inclusion of all these elements offers two key improvements over the baseline 4GM. First, it enhances macroeconomic analysis by explicitly capturing the relationships among labor market conditions, economic activity, and inflation. In addition, it allows the identification of new sources of shocks from the labor market to explain the volatility of macroeconomic data.

The study of labor and goods market interactions is conducted in the context of Colombia, a country with a notoriously dysfunctional labor market, making it an ideal candidate for enhancing its main semi-structural model for monetary policy analysis by incorporating labor market features. Colombia's labor market is characterized by significant wage rigidity in its formal sector (Iregui, Melo, and Ramirez, 2012; Agudelo and Sala, 2017)[2], which shapes price adjustments and generates notable pass-through effects on inflation. Unemployment remains persistently high, accompanied by widespread informal employment. Structural factors such as a high mandatory Minimum Wage (MW hereafter), costly dismissal procedures, and burdensome payroll taxes contribute to elevated unemployment and informality, although formalization improved during the pre-pandemic

[1]By "semi-structural," we mean that although the model's formulation is theoretically motivated, it does not identify all the primitive parameters of the underlying structural model.

[2]This wage rigidity is both nominal and real, given the extensive presence of time-dependent nominal wage adjustments (Iregui, Melo, and Ramirez, 2012) and large deficits in real wage cuts relative to those observed in the wage-setting distributions in advanced economies (Agudelo and Sala, 2017).

decade, driven by gains in labor productivity and policy reforms (Pulido, Vargas, and Ospina, 2023).

The model is calibrated and estimated using Colombian data from 2003 to 2025. To account for the unprecedented disruptions caused by the COVID-19 pandemic, we adapt the methodology of Ferroni, Fisher, and Melosi (2024) within our semi-structural framework. This approach introduces COVID-specific shocks that capture unusual deviations in the variables, allowing for a clearer distinction between pandemic-related fluctuations and structural economic dynamics.

Our estimated model provides the monetary authority with a tool tailored to the Colombian economy, especially the idiosyncratic characteristics of its labor market described above, which is fully compatible with the technology currently used to generate policy forecasts. In particular, our implementation enables the production of forecasts for the same set of variables currently included in the Quarterly Monetary Policy Report, along with forecasts for unemployment, the NAIRU, wages, and their trends. More importantly, the inclusion of these new variables does not compromise the forecasting performance of the original model and even improves it in several aspects. Moreover, by incorporating labor market dynamics, our specification also enriches the historical shock decomposition of the main macroeconomic variables, allowing the identification of new sources of shocks arising from labor market idiosyncrasies, including regulatory ones (e.g., MW-related). Finally, our model is fully compatible with extensions of the original framework in other dimensions, particularly a recent one that considers the variable credibility of monetary policy (Grajales-Olarte et al., 2025), and can be used to produce quantitative risk balance assessments when used together with predictive density analysis, as in Méndez-Vizcaíno et al. (2021).

Through the lens of our model, we find that labor market dynamics significantly influence inflation trends and monetary policy responses, with nominal wage rigidities (e.g., MW policies) amplifying price pressures or delaying real wage adjustments following economic shocks. Thus, the model serves as a tool for policymakers to better identify supply and demand shocks, particularly during crises (e.g., the pandemic), when labor market idiosyncrasies distort wage and inflation dynamics. Thus, the model provides a more internally consistent assessment of the sources of inflationary pressures, the degree of labor market slack, and the overall state of the economy, resulting in more informed decision-making and better-targeted policy responses to exogenous shocks.

This paper is organized into seven sections, starting with this introduction. Section 2 describes the main characteristics of the Colombian labor market and explains some of the modeling choices made. Section 3 explains the specification of the equations for unemployment and wages, along with their interaction with the key macroeconomic variables already included in the 4GM. Section 4 presents the new data included in the model, particularly the construction of the wage series as a dynamic factor model derived from five representative wage series. Section 5 describes the calibration and estimation procedures, particularly the introduction of COVID-specific shocks. Section 6 presents our main findings on impulse response functions, shock decomposition analysis, the ability to replicate data moments, and predictive accuracy relative to standard benchmarks and models currently in use. Finally, Section 7 concludes.

2 Stylized Facts of the Colombian Labor Market

In this section, we outline the structural characteristics of the Colombian labor market that act as constraints and transmission mechanisms within our macroeconomic, semi-structural model. We focus on the aggregate determination of both prices (wages) and quantities (labor utilization) in the market, emphasizing the empirical regularities that justify the nominal rigidities and shock processes assumed in our policy model.

2.1 Wage Formation and Nominal Rigidities

The wage-setting process in Colombia exhibits two prominent structural features worth considering in a policy model: a high degree of nominal rigidity, a cornerstone of New Keynesian modeling, and a strong dependence on institutional factors, specifically the MW.

First, regarding nominal frictions, survey evidence provides robust micro-foundations for the existence of sticky wages in the formal sector. For instance, Iregui, Melo, and Ramirez (2012) document that Colombian firms adjust wages less frequently than prices, with a clear prevalence of time-dependent adjustment rules over state-dependent ones. The data also reveal significant downward wage rigidity: Agudelo and Sala (2017) compute a measure of downward real wage rigidity four times larger than their aggregate estimate for OECD economies. These findings support the inclusion of wage formation equations in our semi-structural model, parameterized to generate persistence levels comparable to or exceeding those implied by the price Phillips curves.

Second, the wage distribution is heavily censored by the mandatory MW. The ratio of the MW to the median wage in Colombia is the highest among OECD members (Pulido, Vargas, and Ospina, 2023) and ranks second among Latin American economies (Maloney and Mendez, 2004), generating low variance and high skewness in the wage distribution. Furthermore, shifts in the Kaitz index (the ratio of the MW to the 70th percentile of the wage distribution) are associated with shifts across nearly all quantiles of the earnings distribution (Arango et al., 2022; Lasso and Vargas, 2024), implying that the MW also affects the determination of other salaries in the economy. Thus, the relevance of the MW extends beyond the lower tail of the distribution, affecting any aggregate measure of wages used in the model.

Accordingly, our model incorporates the MW not as an explicit friction, but as an exogenous driver affecting the metric of aggregate wages, with a transmission mechanism estimated externally (see Appendix B.). It is worth noting that, since the pervasiveness of the MW in Colombia's wage distribution stems from an institutional rule bounding adjustments by lagged inflation, the MW acts as a structural propagation mechanism for past inflation shocks. This generates higher inflation persistence (and indexation) than would occur in the absence of this rule.

2.2 Labor Market Slack and Business Cycle Synchronization

Regarding quantities, we abstract from the heterogeneity of labor supply across demographic groups to focus on the aggregate utilization rates. The Colombian labor market is characterized by a persistently high unemployment rate (UR). Since the 1980s, the average national UR has hovered around 10.7% (10.4% excluding the pandemic period), never falling below 8% even during episodes of strong output expansion, and remaining near those levels even after major structural shifts such as the large migration inflow of workers from Venezuela into Colombia in the 2017-2020 period.[3] This persistent behavior has resulted in a high Non-Accelerating Inflation Rate of Unemployment (NAIRU). NAIRU estimates suggest that it has fluctuated between 10% and 15% in the last three decades (GAMLA, 2020; GAMLA, 2023), highlighting a significant degree of structural mismatches that are mostly independent of cyclical fluctuations.

While we acknowledge the prevalence of informality —where roughly half the workforce operates outside regulatory scopes, due in part to high non-wage labor costs and rigidities (see Otero-Cortes et al. (2025) for a complete characterization of the phenomenon in Colombia)— our model focuses on the UR as the primary measure of aggregate slack. We exclude the informal margin for two critical reasons.

First, the UR acts as the most robust labor indicator, coherent with cyclical economic activity. As shown in Table 1, the cross-correlation between the cyclical component of the UR and the output gap (0.85) is statistically significant at the 99% level and exceeds that of other labor indicators, including the informality and participation rates. Figure 1 further illustrates that the turning points of the UR are highly synchronized with the standard business cycle chronology defined by Alfonso et al. (2013). Thus, to assess the cyclical position of the economy through the labor market, the UR seems to capture economic fluctuations more effectively.

Table 1: Maximum Correlation of Quarterly Labor Market Indicators Relative to the Output Gap

Ranking	**Variable**	**Max. Correlation**	**Sign**	**Order**
1	Unemployment rate (UR)	0.85***	-	0
2	Employment rate (ER)	0.73***	+	0
3	Informality rate (as non-salaried share)	0.36***	-	1
4	Labor participation rate (LFPR)	0.31***	+	0

Notes: Correlations computed for the period 1984-2023 using standard Hodrick-Prescott filters. Order displays the number of quarters that each series leads (+) or lags (-) the cyclical component of the GDP. Significance levels: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Second, our wage analysis relies mainly on data from formal employment. Labor income in the informal sector is collected solely through household surveys, which are more susceptible to measurement error than data from the formal sector. In contrast, the formal segment benefits from additional sources, such as firm surveys and administrative records, which enable cross-validation and enhance accuracy. Indeed, our aggregated wage metric combines information from these different sources. Since the UR is predominantly determined by the flows into and out of the salaried (formal) sector (Lasso, 2020), using income data exclusively from the formal sector makes the UR a more appropriate indicator for capturing the interaction between the quantities and prices of the labor market in our policy model.[4]

[3]Between 2017 and 2020 there was a significant inflow of migrants from Venezuela, driven by the economic and social crisis in that country. Approximately 2 million working-age people migrated to Colombia, especially to the main cities. The effects of this massive migration have been extensively analyzed in the literature, including the comprehensive study of Tribín-Uribe et al. (2020). Their results suggest that this episode stimulated economic growth in the short run, while having no significant effect on long-run potential growth. Furthermore, given that the vast majority of migrants obtained informal jobs, there were no major impacts on the UR, nor formal employment, inflation, or the output gap.

[4]Lasso (2020) show that the cyclical variations in salaried employment and unemployment exhibit an inverse relationship and are largely driven by flows between these two labor market states; in contrast, transitions between the

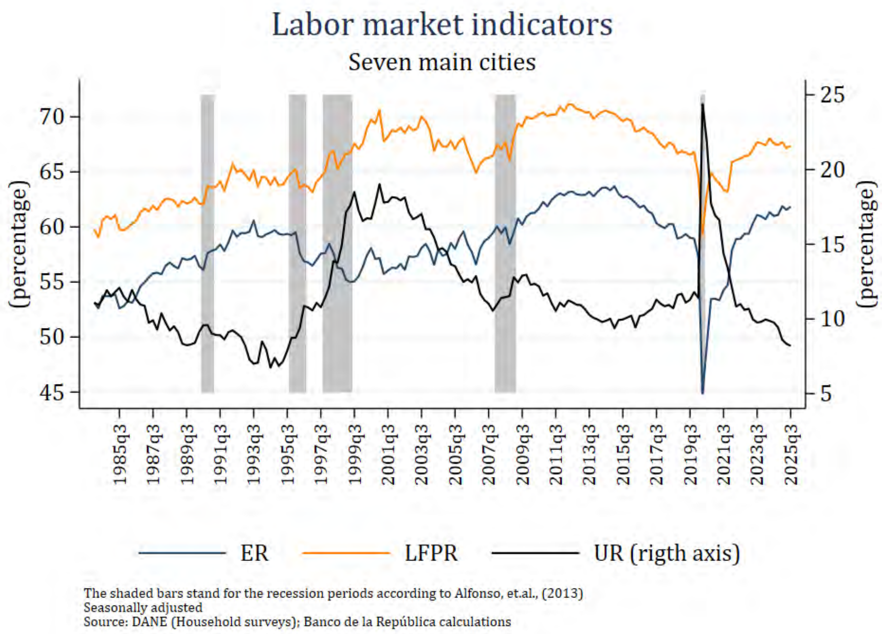

Figure 1: Labor Market Indicators and Recession Periods in Colombia

Finally, the UR is preferred as a measure of labor market slack over alternatives such as the vacancy-to-unemployment ratio (V/U) for both theoretical and practical reasons. Although recent empirical evidence for the U.S. suggests that the V/U ratio outperforms the UR in fitting the (wage) Phillips curve (Barnichon and Shapiro, 2024; Heise, Pearce, and Weber, 2025), applying the V/U ratio in Colombia faces limitations related to vacancy data. First, there are issues of representativeness, as vacancy data is restricted to newspaper postings (Arango, 2013), data from an online public job board with only urban coverage (as in Chaparro et al., 2025), or indirect estimation methods based on hires (Morales, Davalos, and Zapata, 2023). Second, and most importantly, there are constraints regarding the temporal availability of these series within our estimation timeframe. Since the UR remains a parsimonious proxy for labor market slack that can be incorporated as a forcing variable in a micro-founded wage Phillips curve (see Galí (2011) for derivations), we opt to use the UR alone in our model.

3 Labor Market Dynamics Incorporated in the 4GM model

In this section, we outline the channels introduced into the baseline 4GM model that capture the interactions between economic activity, price formation, and the labor market. We begin with the equations describing the dynamics of the UR, followed by those governing the evolution of wages. For a detailed presentation of the full set of equations comprising the entire semi-structural model, see Appendix A. The UR u_t is decomposed into its trend and cyclical component or gap, that is, $u_t = \overline{u}_t + \hat{u}_t$. Following Alichi et al. (2019), this trend is also known as the NAIRU, and its evolution is explained by its own lag, the cyclical position of the economy,

non-salaried and inactive states are the main drivers of fluctuations in non-salaried employment and inactivity rates.

measured by the output gap $\hat{y}_t$ over a rolling window that captures the effect of hysteresis, and a shock $\varepsilon_t^{\overline{u}}$ that may generate persistent deviations:

$$\overline{u}_t = \rho_{\overline{u}}\overline{u}_{t-1} + (1-\rho_{\overline{u}})\overline{u}_{ss} - \lambda_1 \sum_{j=-12}^{12} \frac{E_t\hat{y}_{t-j}}{24} + \varepsilon_t^{\overline{u}} \tag{1}$$

Hysteresis refers to the phenomenon in which certain economic shocks affect economic activity beyond the typical business cycle duration. This effect is supported by a range of empirical evidence indicating that large recessions can have lasting negative impacts across multiple dimensions of the economy. For example, Blanchard and Summers (1986) show that labor force participation declined permanently in 30 European countries after major recessions, while Cerra and Saxena (2017) find that all types of recessions lead to a permanent loss in output. In Colombia, Cardona-Arenas and Sierra-Suarez (2024) and Iregui and Otero (2003) provide evidence of hysteresis in the UR. In our model, sluggish recoveries or prolonged periods of negative output gaps imply that structural unemployment adjusts slowly. Therefore, when the rolling window —spanning the past three years to the next three— of the output gap remains negative, it signals a delayed recovery and leads to an increase in the NAIRU.

With respect to the cyclical component of the UR, we specify an Okun's law relationship in which this component is a function of past slack in the economy $\hat{y}_{t-1}$ and a temporary unemployment cycle shock $\varepsilon_t^{\hat{u}}$.

$$\hat{u}_t = \rho_{\hat{u}}\hat{u}_{t-1} - \lambda_2\hat{y}_{t-1} + \varepsilon_t^{\hat{u}} \tag{2}$$

We include the first lag of GDP to capture the fact that the labor market usually lags behind the behavior of the economy. A negative value of the unemployment gap indicates that labor demand is high and the labor market is tight, which will have repercussions on inflationary pressures given by the Phillips curves.

We also include a wage block to capture the relationship between unemployment, marginal costs, and economic activity. The nominal wage is defined as w_t^n and its change is given by the growth of real wages w_t augmented by inflation expectations $E_t\pi_{t+1}$:

$$\Delta w_t^n \equiv E_t\pi_{t+1} + \Delta w_t \tag{3}$$

Real wage is decomposed into two components, trend and cycle:

$$w_t = \overline{w_t} + \hat{w}_t \tag{4}$$

The key component of the wage block is the wage Phillips curve, which determines the growth of the nominal wage Δw_t^n based on the cyclical components of real wages $\hat{w}_t$ and output $\hat{y}_t$. Following Demidenko et al. (2016), this formulation assumes that nominal wage growth accelerates in booms but decelerates when real wages are already too high relative to equilibrium, which is a standard result in the labor market literature:

$$\Delta w_t^n = \rho_{\Delta w^n}\Delta w_{t-1}^n + (1-\rho_{\Delta w^n})E_t\Delta w_{t+1}^n + \lambda_4\Big[-\lambda_5\hat{w}_t + (1-\lambda_5)(-\hat{u}_t)\Big] + \varepsilon_t^{\Delta w^n} \tag{5}$$

It is worth mentioning that the shock $\varepsilon_t^{\Delta w^n}$ in the wage Phillips curve in Equation (5) can be related to surprises in the annual adjustment of the mandatory MW, following the decomposition technique explained in Appendix B. The growth of the trend $\overline{w}_t$ is modeled as an autoregressive process around a long-term steady-state value of real wage growth, augmented by the deviation of the NAIRU with respect to its steady-state value:

$$\Delta\overline{w}_t = \rho_{\Delta\overline{w}}\Delta\overline{w}_{t-1} + (1-\rho_{\Delta\overline{w}})\Big[\Delta\overline{w}_{ss} - \lambda_3(\Delta\overline{u}_t - \Delta\overline{u}_{ss})\Big] + \varepsilon_t^{\Delta\overline{w}} \tag{6}$$

On the other hand, inflation is influenced by the cyclical component of the real wage through the Phillips curve for each of the four price categories (goods, services, food, and regulated items),[5] because we incorporate the cyclical component of the real wage into marginal costs:

$$\pi_t^j = \alpha_{\pi^j}\pi_{t-1}^j + (1-\alpha_{\pi^j})E_t\pi_{t+1}^j + \alpha_{rmc^j}rmc_t^j + \varepsilon_t^{\pi^j} \tag{7}$$

where rmc_t^j refers to the real marginal cost of basket j which includes the output gap, the real wage gap, and a component that depends on the real exchange rate (RER hereafter) gap $\hat{z}_t$, the relative price gap of its own sector $\widehat{pr}_t^j$ and, in the case of the food basket, the relative price gap of world food $\widehat{pr}_t^{F^\star}$ (to capture shifts in imported food prices):[6]

$$rmc_t^j = \begin{cases} \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + \alpha_{\hat{w}}^{rmc^j}\hat{w}_t + (1-\alpha_{\hat{y}}^{rmc^j} - \alpha_{\hat{w}}^{rmc^j})(\hat{z}_t - \widehat{pr}_t^j) & \text{for } j = g, s, r \\ \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + \alpha_{\hat{w}}^{rmc^j}\hat{w}_t + (1-\alpha_{\hat{y}}^{rmc^j} - \alpha_{\hat{w}}^{rmc^j})(\widehat{pr}_t^{F^\star} + \hat{z}_t - \widehat{pr}_t^j) & \text{for } j = f \end{cases} \tag{8}$$

Finally, on the economic activity side, the IS equation, which describes the evolution of the output gap, is modified to include interactions between aggregate demand and the labor market. Given that aggregate demand may be affected by cyclical movements in the labor market, both in terms of prices and quantities, we include the effects of both cyclical real wages and cyclical unemployment.[7] Thus, following the IS specification in the original 4GM model, the output gap depends, in addition to cyclical real wages and unemployment, on its past and expected future values, the cyclical position of foreign demand ($\hat{y}_t^*$), the index of monetary conditions (MCI) (ϕ_t), which captures both the effects of the real interest rate gap and the RER gap, and a component reflecting the impact on external income arising from fluctuations in foreign prices, namely the terms of trade

[5]Regulated items include products that are subject to government price controls, such as gasoline, electricity, and other utilities. The goods basket includes tradable items whose prices are shaped by international competition, thus reflecting both domestic and exchange rate-related factors. The services category comprises non-tradable items whose pricing is less influenced by exchange rates and, therefore, is more contingent on internal demand dynamics than that of goods. The food category encompasses products that are vulnerable to weather and transportation cost fluctuations. The weights of each basket are as follows: regulated items represent 17.3%; goods 18.7%; services 48.9%; and food 15.1%. For a comprehensive review of this categorization, refer to González-Molano et al. (2020).

[6]For a definition of these relative price gaps see Appendix A. or González et al. (2020).

[7]The intuition for including these two variables in the IS equation is as follows. First, greater labor market tightness, which lowers cyclical unemployment, could increase aggregate demand in the short run. Second, after an unexpected increase in wages, for example due to a MW rise, there could be a short-term increase in aggregate demand that does not reflect a change in unemployment in the short run (Arango et al., 2022).

(TOT hereafter) gap ($\widehat{TOT}_t$):

$$\hat{y}_t = \beta_1 \hat{y}_{t-1} + \beta_2 E_t \hat{y}_{t+1} + \beta_3 \hat{y}^*_t + \beta_4 \widehat{TOT}_t - \beta_5 \phi_t - \beta_6 \hat{u}_t + \beta_7 \hat{w}_t + \eta^{\hat{y}}_t \tag{9}$$

where:

$$\phi_t = \alpha^{\phi}_{\hat{r}r} \hat{r}_t - (1 - \alpha^{\phi}_{\hat{r}r}) \hat{z}_t$$

$$\eta^{\hat{y}}_t = \beta_{\eta^{\hat{y}}} \eta^{\hat{y}}_{t-1} + \varepsilon^{\hat{y}}_t$$

Compared to the original 4GM model, we replace the oil price gap with the TOT gap as a measure of external income derived from changes in foreign prices. This is because, in recent years, the Colombian economy has undergone a structural transformation that has led to a more diversified export basket. Thus, the TOT provides a more comprehensive measure by capturing price fluctuations across a broader range of products and is consistent with modern macroeconomic models that treat the TOT as the fundamental channel for foreign price shocks.[8] This adjustment provides policymakers with a more robust metric for assessing external price shocks than an approach that focuses exclusively on oil prices.

Second, the trend growth rate of potential output is determined by an auto-regressive component, the steady-state potential growth rate, and deviations of the trends of the TOT, real wages, and the NAIRU from their respective steady-state values:

$$\begin{aligned} \Delta\overline{y}_t = {} & \rho_{\Delta\overline{y}} \Delta\overline{y}_{t-1} + (1 - \rho_{\Delta\overline{y}}) \\ & \left[\Delta\overline{y}_{ss} + \kappa_1 (\Delta\overline{TOT}_t - \Delta\overline{TOT}_{ss}) + \kappa_2 (\Delta\overline{w}_t - \Delta\overline{w}_{ss}) - \kappa_3 (\Delta\overline{u}_t - \Delta\overline{u}_{ss})\right] + \varepsilon^{\Delta\overline{y}}_t \end{aligned} \tag{10}$$

Thus, $(\Delta\overline{u}_t - \Delta\overline{u}_{ss})$ captures the effect of changes in the NAIRU, and its impact is isomorphic to a change in the share of labor in a Cobb-Douglas production function.[9]

4 Data

In the setup described in the previous section, we include three new series in the set of observable series included in the original 4GM model. The first is the nominal wage. Given that there are different available series that capture wage dynamics, we estimate the wage series as a latent wage factor w^F_t, using a dynamic factor model with five observable nominal wage indicators: median wages for all employees, median wages for wage workers specifically, and sectoral wages for the commerce, industry, and construction sectors. The first two series are computed using the National Administrative Bureau of Statistics (DANE) household survey data, while the remaining three sectoral wage series are drawn from DANE's monthly establishment surveys. These five wage series are plotted in Figure 2.

We implement several pre-processing steps for all five series: (1) seasonal adjustment using X13 Arima, (2) computation of quarterly growth, and (3) standardization to eliminate scale dependence. The dynamic factor

[8] The TOT is an observed exogenous foreign variable, its dynamics is described in equation A.30 of Appendix A..

[9] For a micro-foundation of the effect of the NAIRU on potential output growth see Appendix B of Alichi et al. (2019).

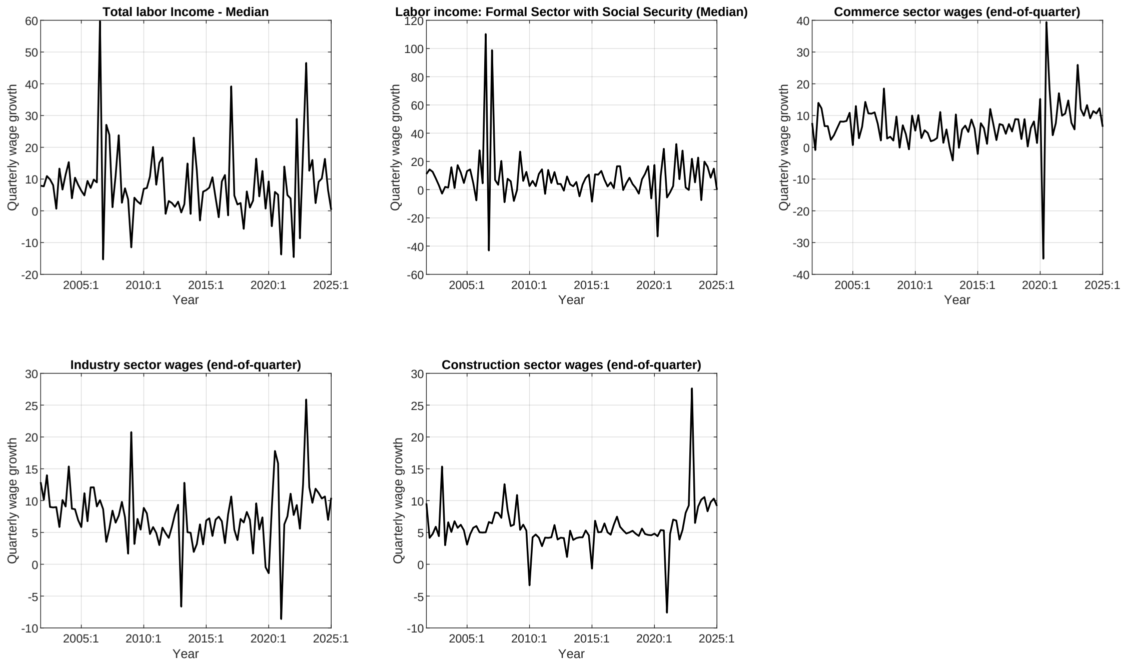

Figure 2: Quarterly Wage Growth Series (Seasonally Adjusted)

model is described by the following equations:

$$\Delta W_t = \Lambda \Delta w_t^F + \varepsilon_t \tag{11}$$

$$\Delta w_t^F = \gamma \Delta w_{t-1}^F + \eta_t \tag{12}$$

where ΔW_t is a 5×1 vector of the quarterly growth of each wage indicator, Λ is a 5×1 vector of the factor loadings. Equation (11) specifies that the dynamics of the vector of quarterly nominal growth of the wage series is explained by a common factor Δw_t^F, given their loadings and idiosyncratic movements captured in a 5X1 vector ε_t We assume that there is no relationship between the idiosyncratic components, that is, the covariance matrix of the idiosyncratic shocks is diagonal. Equation (12) specifies that the factor is an AR(1) process with an autoregressive parameter γ.

The two remaining series include the UR for the 13 main cities in Colombia, extracted from the DANE Household Survey, and the TOT calculated by the Central Bank of Colombia, as proposed by Garavito-Acosta et al. (2011). Figure 3 presents the three new series: the UR, quarterly growth of nominal wages (the estimated dynamic factor), and the TOT. In addition to these three new series, we observe GDP, inflation indexes (total, core, food, regulated, services, and goods), nominal interest rate, exchange rate, foreign inflation and interest rate (EE.UU), foreign growth gap, risk premium, and external food index, as discussed by González et al. (2020).

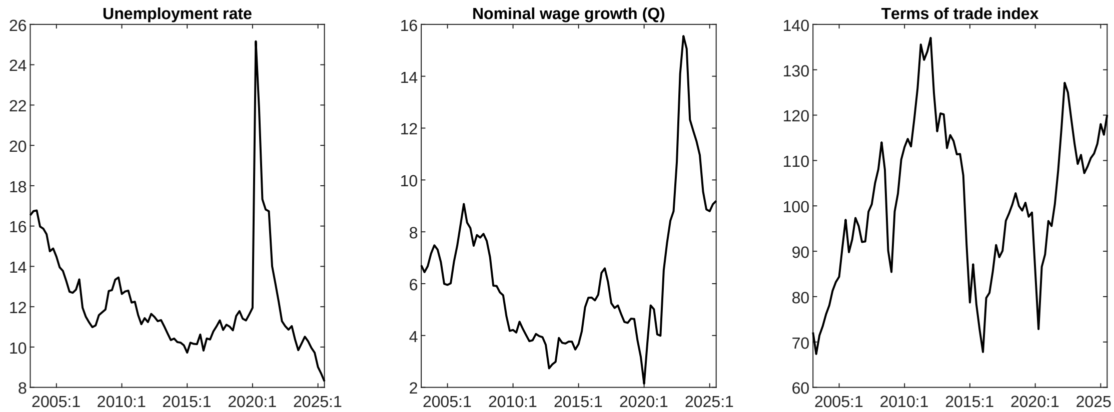

Figure 3: New Series Included in the 4GM+LM

5 Calibration and Bayesian Estimation

To adjust the model to the Colombian economy, we use a mixed strategy to calibrate a subset of the parameters and estimate the remaining ones. Those calibrated primarily relate to steady-state values, including the long-term GDP growth rate and UR, the central bank's inflation target, long-term real wage growth, and the convergence speed of inflation components toward their long-term values, among others. We adopt the estimated persistence parameters for the key exogenous processes of González et al. (2020). Table 2 presents the full set of calibrated parameters.

Table 2: Calibrated Parameters to Steady-State Values

Parameter	Value	Description	Source
$\Delta\overline{y}_{ss}$	2.7%	Long run Potential Output Growth	Ospina and Pulido (2025)
$\overline{u}_{ss}$	10.4%	Long run Unemployment rate	Ospina and Pulido (2025)
$\overline{\pi}$	3.0%	Long run inflation	Central bank target
$\overline{r}$	2.0%	Long run neutral real interest rate	González et al. (2020)
$\overline{\pi}^{\star}$	2.0%	Long run US inflation	González et al. (2020)
$\overline{r}^{\star}$	0.5%	Long run US neutral real interest rate	González et al. (2020)
$\Delta\overline{z}$	0.0%	Long run depreciation	González et al. (2020)
$\overline{\omega}$	1.5%	Long run risk premium	González et al. (2020)
$\Delta\overline{TOT}_t$	0.0%	Long run TOT Trend growth	Own estimates
$\Delta\overline{w}_t$	1.8%	Trend growth, of real wages	Own estimates

The second group of parameters was estimated using Bayesian techniques, following González et al. (2020), Alichi et al. (2019), and Ferroni, Fisher, and Melosi (2024). Our estimation employs quarterly data from the first quarter of 2003 to the second quarter of 2025, covering 21 domestic and nine foreign variables. The domestic dataset includes real GDP (y_t, in 2015 millions of Colombian pesos), monetary policy interest rate (i_t), annual inflation target ($\overline{\pi}_t$), nominal exchange rate (s_t), dynamic factor of nominal wages (w_t), the UR (u_t), along with price indices for headline CPI (p_t), core CPI which includes goods and services prices (p_t^c), and sectoral baskets (goods p_t^b, services p_t^s, food p_t^f, and regulated items p_t^r). We also incorporate trend components of relative prices for goods ($\overline{p_t^b}$), food ($\overline{p_t^f}$), and regulated items ($\overline{p_t^r}$). The set of foreign variables consists of the

US CPI p_t^*, the US monetary policy rate i_t^*, proxied by the 1-Year US FED rate, the Colombian risk premium ψ_t measured through the 5-year CDS spread on sovereign Colombian bonds, and the TOT, TOT_t. We also include estimates of the neutral real interest rate in the US $\overline{r_t^*}$, gaps for foreign output $\hat{y}_t^*$ and the relative price of world food $\widehat{pr}_t^{F*}$, trend components of the risk premium $\overline{\psi}_t$, and TOT $\overline{TOT_t}$. The trends and gaps in external variables correspond to estimates outside the model that combines satellite models and judgments, as defined by González et al. (2020).

To isolate the extraordinary disruption of the COVID-19 pandemic and estimate the parameters using the information from the full sample, we adapted the methodology from Ferroni, Fisher, and Melosi (2024) within our semi-structural framework. In summary, the core idea of the approach is that the pandemic generated substantial deviations of the variables from their steady-state values that cannot be fully captured by standard structural shocks due to the mean-reverting nature of the model's solution. Thus, to account for these unusual fluctuations, tailored specific COVID shocks are needed to absorb the large standard deviations in the variables observed during the pandemic. This methodology offers the advantage of incorporating post-pandemic dynamics into the estimation process, allowing us to use recent data without distorting the parameter estimates.

Our adaptation of Ferroni, Fisher, and Melosi (2024)'s methodology follows a three-step process. First, we estimate the structural parameters using a pre-pandemic sample. Second, conditional on those estimates, we introduce a set of COVID-specific shocks that directly affect the gaps and trends of the variables that were most affected by the health crisis.[10] Finally, after correctly identifying the scale (standard deviations) of the COVID-shocks, we re-estimated the full set of model structural parameters in the full sample. A detailed description of the entire procedure is provided in Appendix C.

Table 3 summarizes the parameters estimated for the labor market and economic activity equations, and Table 4 presents the estimates of Phillips curves and real marginal costs. Details of the prior distributions used and the resulting posterior distributions are provided in Appendix C.

6 Results

In this section, we present the performance of our model (4GM+LM) along several dimensions. First, to understand the channels and effects of domestic and labor market shocks, we analyze the impulse response functions. Second, we evaluate the shock decomposition of the main macroeconomic variables of interest (output gap, inflation, inflation expectations, RER, and interest rates), as well as the main labor market variables (real wage gap, UR, and unemployment gap) over a recent period (2020-2024), to illustrate how the model can help identify the sources of macroeconomic fluctuations. Third, we assess the predictive performance of the model by comparing its forecasts with those generated by the naive benchmarks and the original 4GM model. Finally, we evaluate the ability of the model to replicate the moments of the main variables of interest.

[10] Specifically, these shocks target the dynamics of the output gap and potential output, unemployment gap and NAIRU, Phillips curves, and the monetary policy rule; and are active from the second quarter of 2020 to the fourth quarter of the same year.

Table 3: Estimated Parameters for Labor Market and Economic Activity Equations

Labor Market					
Wage Dynamics		**Unemployment**		**NAIRU**	
Nominal Wage Phillips Curve		Backward component ($\rho_{\hat{u}}$)	0.498	Backward component ($\rho_{\bar{u}}$)	0.701
Backward component ($\rho_{\Delta w^n}$)	0.398	Output elasticity (λ_2)	0.199	SS component ($1-\rho_{\bar{u}}$)	0.298
Forward component (1-$\rho_{\Delta w^n}$)	0.602			Hysteresis (λ_1)	0.099
Labor market adjustment (λ_4)	0.602				
Real wage gap elasticity (λ_5)	0.398				
Okun's elasticity ($1-\lambda_5$)	0.601				
Trend of Real Wage					
Backward component ($\rho_{\Delta \overline{w}}$)	0.801				
SS component ($1-\rho_{\Delta \overline{w}}$)	0.199				
NAIRU elasticity (λ_3)	0.298				

Economic Activity			
IS Curve		**Potential Output**	
Backward component (β_1)	0.549	Backward component ($\rho_{\Delta y^*}$)	0.801
Forward component (β_2)	0.053	SS component ($1-\rho_{\Delta y^*}$)	0.198
Foreign demand elasticity (β_3)	0.089	TOT trend elasticity (κ_1)	0.029
TOT price elasticity (β_4)	0.034	Real wage trend elasticity (κ_2)	0.099
MCI weight (β_5)	0.153	NAIRU elasticity (κ_5)	0.402
Unemployment gap effect (β_6)	0.199		
Wage gap effect (β_7)	0.033		
Interest rate gap elasticity ($\alpha^{\phi}_{\hat{rr}}$)	0.751		
RER gap elasticity ($1-\alpha^{\phi}_{\hat{rr}}$)	0.249		

Table 4: Estimated Parameters for Inflation Equations

Variable	Parameter	Food	Goods	Services	Regulated
Phillips Curves					
Backward component	α_{π^j}	0.302	0.298	0.359	0.348
Marginal cost	α_{rmc^j}	0.169	0.095	0.169	0.049
Real marginal costs					
Output Gap	$\alpha^{rmc^j}_{\hat{y}}$	0.529	0.251	0.393	0.373
Real wage gap	$\alpha^{rmc^j}_{\hat{w}}$	0.168	0.149	0.458	0.547
RER and relatives prices	$1-\alpha^{rmc^j}_{\hat{w}}-\alpha^{rmc^j}_{\hat{y}}$	0.301	0.599	0.149	0.079

6.1 Impulse Response Functions

In this subsection, we present the responses of the main macroeconomic variables to two sets of shocks. First, we examine the responses to demand (IS curve), supply (Phillips curve), and monetary policy (Taylor rule) shocks to assess whether the dynamics of the variables align with standard economic theory. We then focus on the impact of the new shocks introduced in the model—namely, shocks to nominal and real wages (wage Phillips curve and real wage trend), as well as to the UR (Okun's law).

Figure 4 presents the responses to demand (black line), supply (dashed brown line), and monetary policy (dashed blue line) shocks for up to five years.[11] An increase of 100 basis points (bp) in the output gap (a demand shock) affects inflation through two main channels. First, higher demand directly increases prices, which raises inflation expectations. Second, through the labor market channel, stronger aggregate demand reduces the unemployment gap, feeding back into aggregate demand and increasing the cyclical component of real wages, thereby raising marginal costs, demand, and inflation. The observed reduction in the UR aligns with the estimates of Okun's law from Flórez, Pulido-Mahecha, and Ramos-Veloza (2018). In response to these inflationary pressures, the central bank raises interest rates, which helps reduce inflation but also dampens domestic economic activity. Additionally, higher interest rates lead to currency appreciation, thereby reducing the RER gap.

To analyze the supply shock, we examine a 100 bp shock to the CPI service basket. This shock increases headline inflation by 64 bp and raises the short-term inflation expectations. In response, the central bank increases the nominal interest rate, which weakens aggregate demand and reduces the output gap. According to Okun's law, lower economic activity increases the unemployment gap, exerting downward pressure on wages. This reduces the real wage gap and, consequently, lowers inflationary pressure from marginal costs.

Finally, a contractionary monetary policy shock reduces both headline and core inflation as well as inflationary expectations. The interest rate increase reduces aggregate demand, raising the UR and reducing labor market tightness. This, in turn, puts downward pressure on nominal and real wages, reinforcing disinflationary processes. The shock also reduces the RER gap, resulting in currency appreciation.

[11] These IR functions are qualitatively similar to those obtained by González et al. (2020).

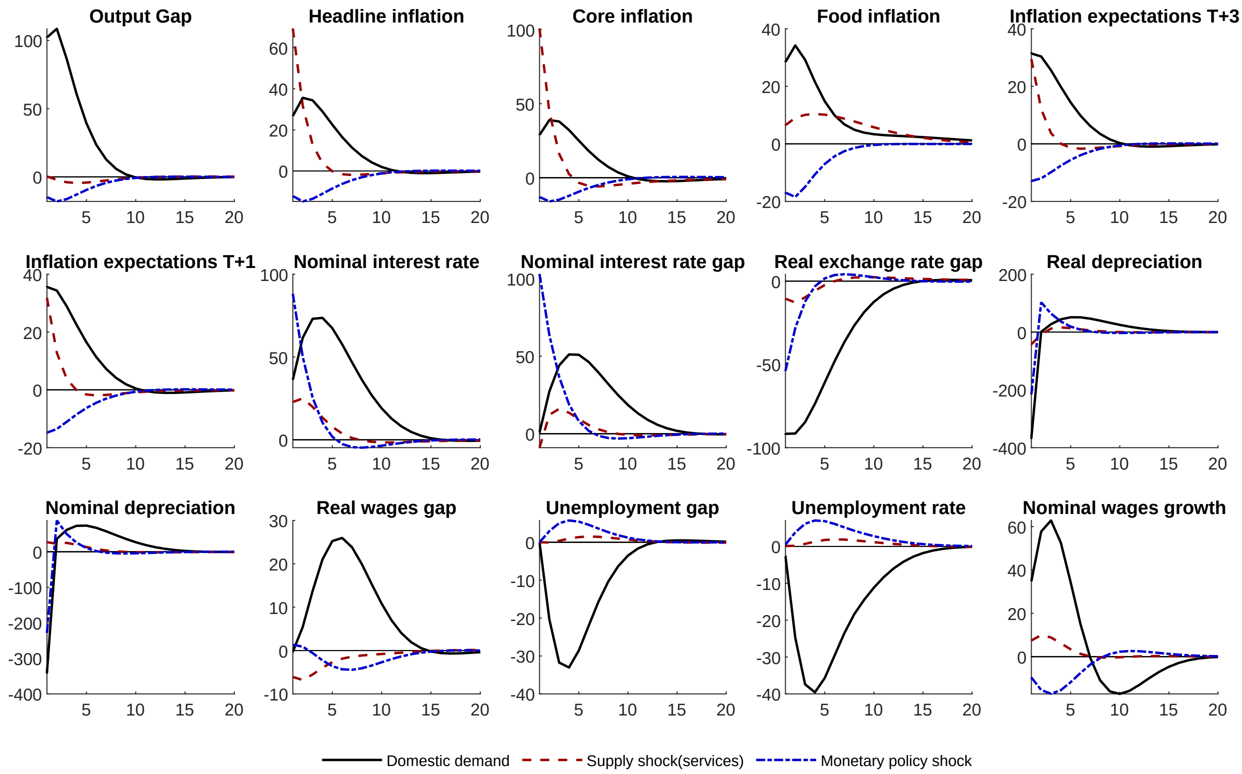

Figure 4: Domestic Shocks

Figure 5 presents the impulse response of the main economic variables to labor market shocks: nominal wages (black line), real trend wages (dashed blue line, a proxy for long term labor productivity shocks), and UR (dashed brown line). An unexpected increase in the nominal wage increases real marginal costs, inflation, and inflation expectations. This shock generates a small increase in aggregate demand in the short run due to the income effect, which temporarily expands the output gap. The resulting increase in marginal costs, demand, inflation, and expectations prompts the central bank to raise interest rates, ultimately leading to a reduction in inflation and economic activity. In the medium term, the effect of monetary policy dominates, and the output gap becomes negative.

The shock to the trend of real wages, akin to a shock in long-term labor productivity, has two primary effects. First, it reduces the real wage gap, which decreases the marginal costs across all CPI baskets and lowers the headline inflation. A decrease in the real wage gap also reduces aggregate demand. In response, the central bank lowers interest rates to stimulate domestic demand, which reduces the unemployment gap and helps close the negative real wage gap. Second, the higher trend in real wages increases potential output, as it reflects improved productivity. Consequently, this shock produces effects on macroeconomic variables similar to those of productivity shocks in structural models.

Finally, an unexpected increase in the unemployment gap (a shock to Okun's law) reduces the cyclical component of real wages and dampens aggregate demand. Both effects lower inflation and inflation expectations, leading to a decrease in the interest rate set by the central bank. This monetary easing helps reduce the negative output gap and marginal costs, further alleviating disinflationary pressures. As domestic demand recovers, it drives a convergence of both unemployment and real wage gaps to zero.

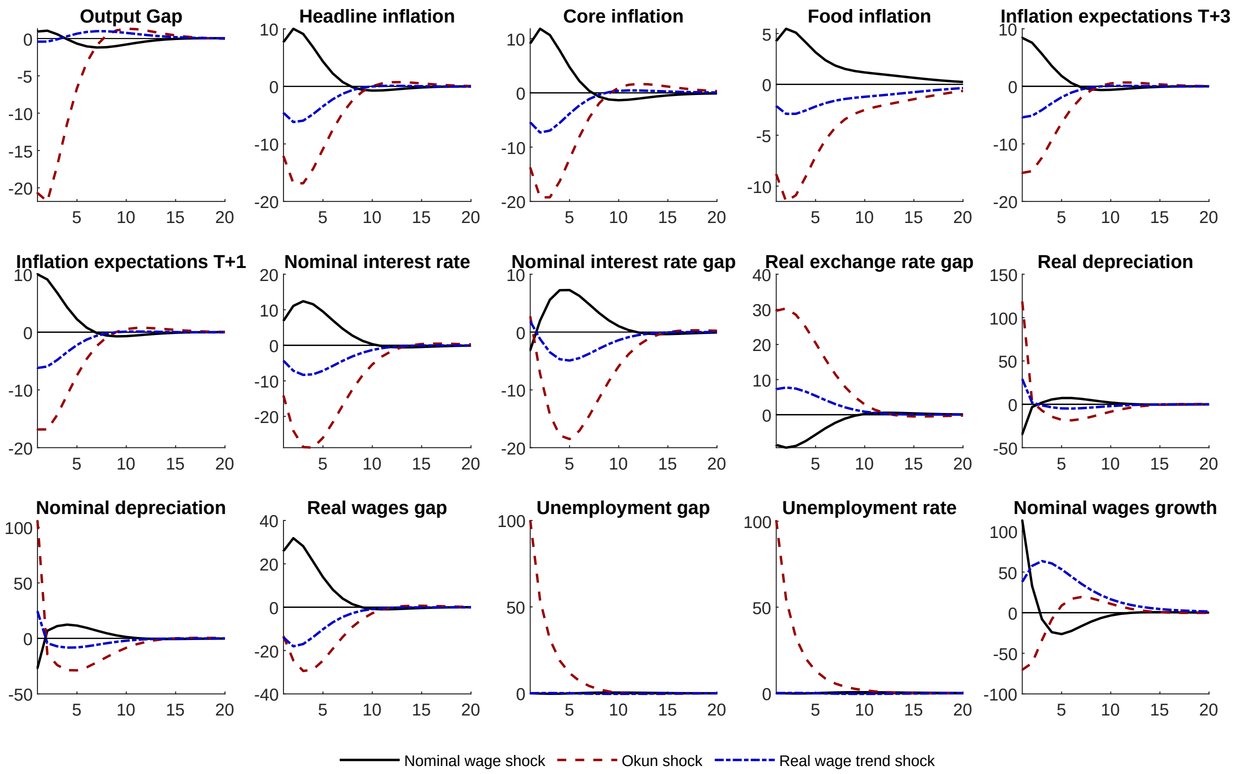

Figure 5: Labor Market Shocks

6.2 Shock Decomposition Analysis

In this subsection, we present shock decompositions to analyze the dynamics of the main macroeconomic variables (GDP gap, inflation, inflation expectations, and the exchange rate; see Figure 6) and labor market variables (UR, UR gap, and real wage gap; see Figure 7) through the lens of the model. This analysis estimates the model's relevant "gap variables" and identifies the proportion of fluctuations in each variable attributable to specific structural shocks, explaining how these shocks influenced the central bank's monetary policy decisions. We focus our analysis on the model's shock decomposition and unobservable estimates for the 2020–2024 period.[12]

The period from 2020 to 2024 was defined by the economic repercussions of two major global shocks —the COVID-19 pandemic in 2020 and the Russian invasion of Ukraine in 2022— as well as two significant domestic shocks: social unrest in 2021 and, more recently, nominal wage shocks.

The 2020 pandemic caused a severe contraction in both supply and demand, both globally and domestically. During that year, both headline inflation and inflation expectations fell nearly 2 percentage points below the 3% target. This prompted the central bank to cut the monetary policy rate to stimulate economic activity and guide inflation back to its target. The model captures these events as a deep negative output gap, a positive RER gap, and a negative real interest rate gap, all driven primarily by negative foreign and domestic demand shocks. Additionally, the model identifies a negative shock to regulated prices, albeit of smaller magnitude, reflecting government price reductions for utilities.

[12]For a similar exercise performed during other periods of interest within the estimation sample, see Appendix D.

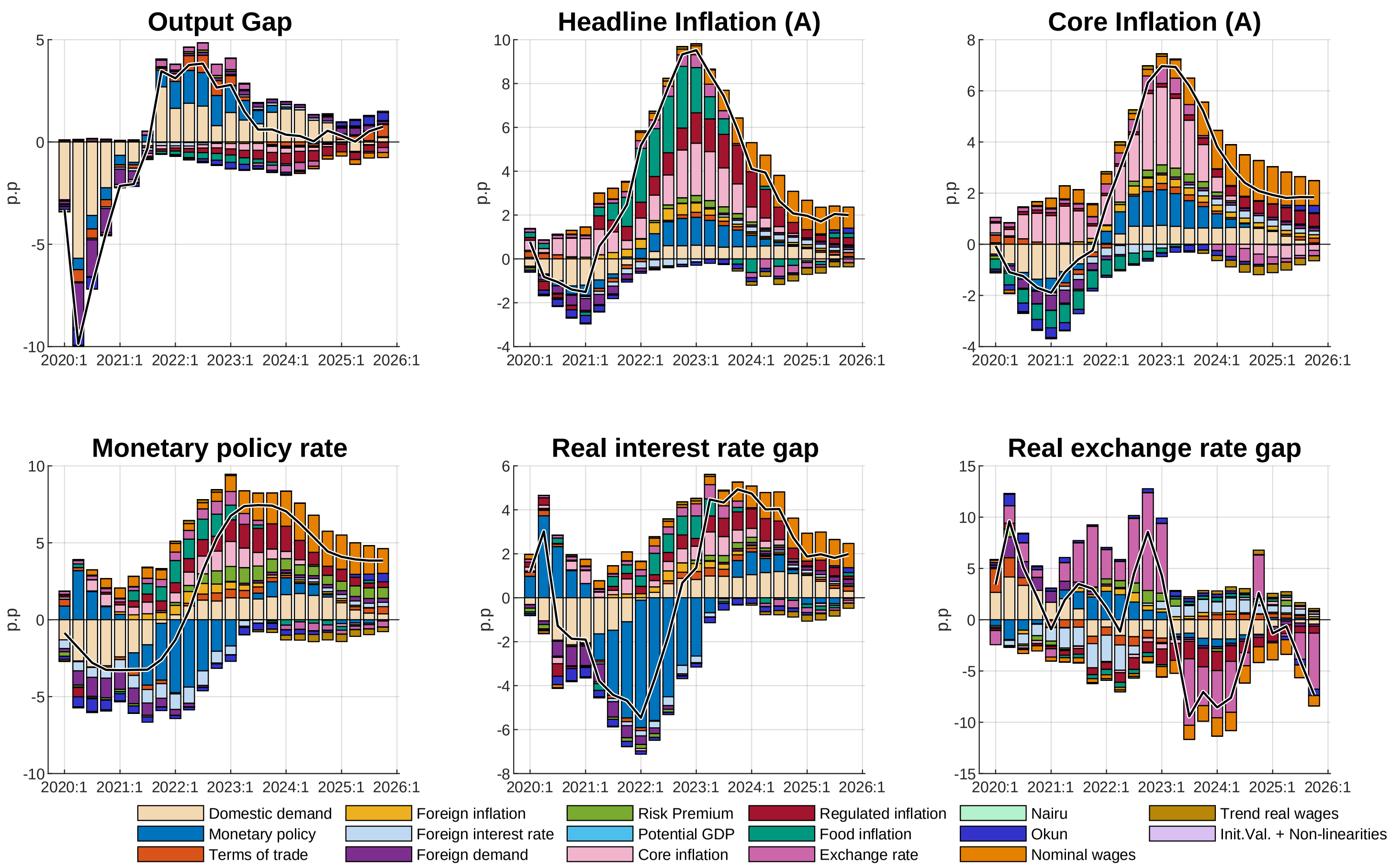

Figure 6: Shock Decomposition of Macroeconomic Variables

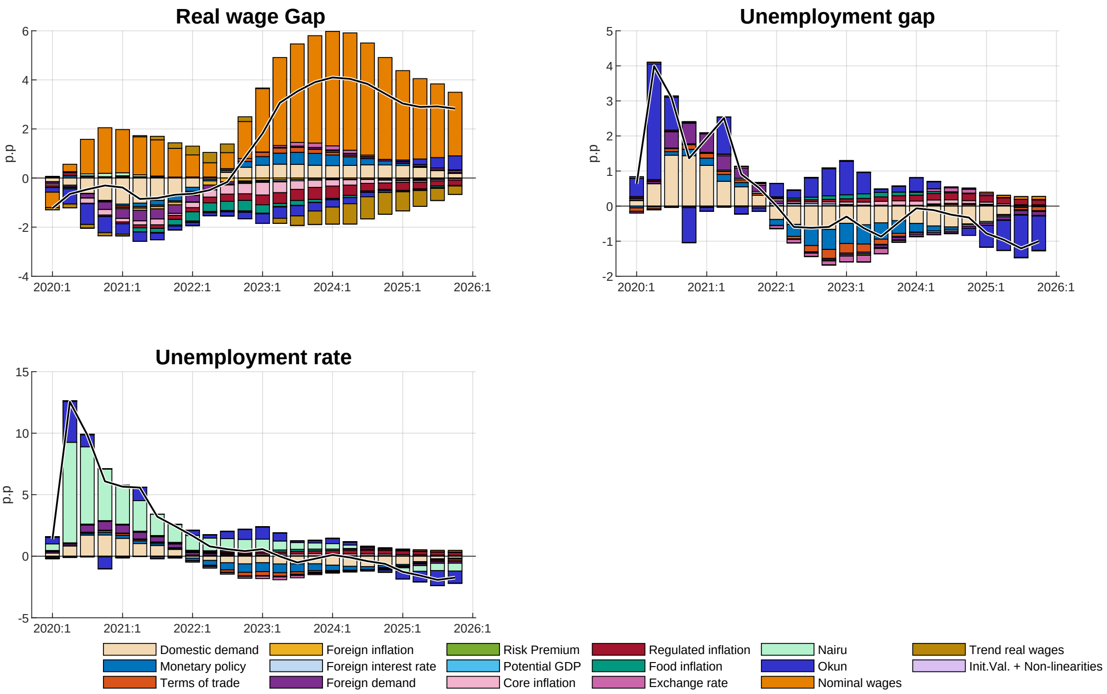

Figure 7: Shock Decomposition of Labor Market Variables

The pandemic also severely impacted the labor market, which had been relatively stable in 2019 with an UR of approximately 11%.[13] The abrupt decline in aggregate demand and widespread business closures at the crisis peak led to an unprecedented rise in the UR, which reached 24%. The model attributes these dynamics to a significant increase in the NAIRU, estimating it as the largest increase in Colombian history (about 7 percentage points in 2020:Q2), as well as idiosyncratic unemployment shocks (approximately 4 percentage points in 2020:Q2). The model also identifies a large negative real wage gap, close to 2 percentage points despite positive nominal wage shocks. By early 2022, both UR and NAIRU had returned to pre-pandemic levels, showing further improvements thereafter. Since then, negative unemployment gaps and a declining NAIRU have supported the economic recovery, but the real wage gap has persisted, driven mainly by even larger positive nominal wage shocks in the following years.

As the economy reopened in 2021 and regulated prices normalized, domestic social unrest erupted at the beginning of that year (2021:Q1), causing sharp increases in food prices. This shock had long-lasting effects on food and headline inflation, as documented by Gáfaro-Gonzalez et al. (2022). These domestic challenges were compounded by another major global shock: in the first quarter of 2022, Russia invaded Ukraine, causing disruptions to global supply chains and driving up transportation costs, energy costs and international food prices. Consequently, the domestic economic recovery of 2021–2022 was accompanied by significant supply chain disruptions, leading to reduced goods supply and increased food prices both internationally and domestically.[14]

[13]For a comprehensive review of the effects of the pandemic on the Colombian labor market, see Pulido, Vargas, and Ospina (2023); for an assessment focusing solely on the effects of mobility restrictions, see Morales, Bonilla, et al. (2022).

[14]The global supply chain disruptions were not only an effect of the Russian invasion of Ukraine, but were also the result of unprecedented logistical and operational challenges faced by the international transportation industry (primarily maritime) during the pandemic, which led to a historically large surge in freight rates that persisted until 2023. In

The model captures all these events as food and regulated price shocks, which explained a significant portion of headline inflation by the end of the first quarter of 2023, when inflation peaked above 13%. Furthermore, the model identifies that from 2021 until the first quarter of 2023, the RER gap contributed to additional inflationary pressures. In that quarter, the model estimates that the cumulative effects of nominal wage shocks up to that quarter, were rather mild (about 50 bp).

In 2021, inflation swung by approximately four percentage points, moving from approximately 200 basis points (bp) below the target at the beginning of the year to 200 bp above the target by year-end, driven mainly by food and regulated price shocks. Inflation expectations drifted away from the target, and by late 2021, the central bank initiated a monetary policy tightening cycle. The pace of the interest rate increase was more gradual than what the model's policy rule would have suggested. The model estimates that the real interest rate gap shifted from negative in the first quarter of 2022 to positive, peaking in the last quarter of 2023.

For 2023 and 2024, the model estimates that the output gap closed and turned slightly negative, while the RER gap quickly turned negative and food inflation shocks dissipated. Combined with a restrictive monetary policy stance, these factors helped reduce inflation from approximately 13% in 2023:Q1 to around 5% in 2024:Q1. However, regulated price shocks persisted during this period, especially due to a necessary adjustment in domestic fuel prices, and rising labor costs exerted significant additional pressure on core inflation, slowing the convergence of headline inflation to the target. With a decreasing UR and positive domestic demand, the model identifies increases in both nominal wages and the real wage gap. This is consistent with the significant real MW adjustments. The model estimates that the positive demand impulse of nominal wage shocks were small and short lived but exerted cost pressures primarily on core inflation (mainly the services basket), which has exhibited persistent behavior in recent years. These accumulated rising labor costs, combined with a restrictive monetary policy stance, led to a slowdown in aggregate demand and a slightly negative output gap. At the end of 2024, inflation closed at 5.2%.

6.3 Forecasting Performance

In this subsection, we evaluate the predictive accuracy of our model (4GM+LM) by analyzing both the conditional Root Mean Squared Error (RMSE) and the conditional Root Median Squared Error (RMdSE) across forecast horizons of four and eight quarters ahead (denoted by 4Q and 8Q respectively), using forecasts of the model in the entire sample (2003-2025).[15] Columns 1-4 of Table 5 display the RMSE and RMdSE for each horizon for the main endogenous variables (GDP growth, headline and core inflation, interest rates, and UR). For the RMSE, we compare these numbers with those from a naive forecast (columns 5-6) and from the model currently used by the central bank staff (4GM model, columns 7-8).[16] The results reveal the following patterns.

First, the forecasting accuracy of the 4GM+LM framework improves substantially relative to the naive specification across all variables and horizons. Notably, although the absolute RMSE values for annual economic growth (3.14 at 4Q and 3.47 at 8Q) are significantly high, reflecting the exceptional forecasting challenges

Colombia, this freight disruption not only had consequences for inflation, but also generated real effects (Pulido, 2024).

[15]The forecasts are conditioned on actual data for external variables —external inflation, the Fed funds rate, TOT, risk premium, and trade partner growth—, 4 quarters ahead.

[16]This model corresponds to the original 4GM model (González et al., 2020) re-estimated after some changes in the composition of the CPI baskets introduced by DANE in 2019.

Table 5: Out-of-sample Forecasting Performance by Horizon

	Model: 4GM+LM				**Model/Naive**		**Model/4GM**	
	RMSE		RMdSE		Relative RMSE		Relative RMSE	
	4Q	8Q	4Q	8Q	4Q	8Q	4Q	8Q
GDP growth (annual)	3.14	3.47	1.24	1.45	0.52	0.53	0.92	0.99
Headline inflation (annual)	1.88	2.91	0.98	1.32	0.69	0.82	0.76	0.98
Core inflation (annual)	1.23	1.95	0.54	0.83	0.61	0.77	0.78	0.94
Monetary policy rate	1.67	2.18	0.70	0.93	0.60	0.56	0.86	0.99
Unemployment rate (UR)	2.19	2.32	0.72	0.82	0.84	0.70	–	–

Notes: This table shows the model's Root Mean Square Error (RMSE) and the Root Median Square Error (RMdSE) for horizons of four quarters (4Q) and eight quarters (8Q). Columns Model/Naive and Model/4GM compare the model's RMSE relative to a naive forecast and the 4GM model, respectively (values below one indicate a better forecasting performance).
Source: Author's calculations based on model results.

posed by the COVID-19 pandemic, the model demonstrates strong relative performance for this variable, with a relative RMSE of 0.52 and 0.53 at 4Q and 8Q, respectively, corresponding to an improvement of more than 45% over the naive benchmark. An assessment of the RMdSE, which is less influenced by atypical forecast errors during the pandemic, yields a similar conclusion.

Similarly, for inflation, the model achieves better performance than the naive forecast (with relative RMSE values below one at both 4Q and 8Q), although to a lesser extent than the forecast gains observed for economic growth. Interest rate forecasts show substantial improvements, with a relative RMSE ratio close to 0.6, indicating gains of 40-44% over the naive benchmark. Unemployment forecasts also exhibit improved performance, with relative RMSE values around 0.8, highlighting the model's ability to capture medium- to long-term labor market dynamics

The comparative analysis against the official 4GM reveals the consistent outperformance of the 4GM+LM framework. Our model achieves lower RMSE values, especially at the four-quarter horizon, for interest rates, inflation, and economic growth, with important gains (of approximately 23%) for both inflation metrics. The inclusion of internally consistent unemployment forecasts, which are not available in the official 4GM, represents one of the key contributions of our model. Accordingly, the new model delivers forecasts that are at least as accurate as those produced by the tools currently in use while offering additional advantages for policymakers through its joint forecasting of prices, economic activity, and labor market variables.

6.4 Moments Simulation

In this section, we compare empirical moments with those generated by simulations of the 4GM+LM model (as well as those of the original 4GM model) as an additional criterion for evaluating model performance. By comparing key statistical moments, such as the correlations and autocorrelations of the main macroeconomic variables of interest, we quantitatively evaluate the model's ability to replicate the salient features of the data. We perform 2,000 simulations of each model for 300 periods, discarding the first 212 periods to ensure that the results are independent of the initial conditions, thereby yielding 22 years of data per simulation, a number comparable to the sample length used to compute the empirical moments (2003-2025).

Table 6 presents the mean estimates and corresponding confidence intervals, computed using the 5th (P_5) and 95th (P_{95}) percentiles of the simulations for the first-order autocorrelations in the upper panel and correlations in the lower panel. Given the uncertainty in the data sample, we also compute confidence intervals for autocorrelations using Bartlett's approximation, and for correlations, we apply the Fisher transformation. The upper panel of Table 6 shows that the model is able to replicate the high degree of autocorrelation observed in annual GDP growth $\Delta^A y_t$, annual inflation π_t^A, core inflation $\pi_t^{A,C}$, nominal wage inflation $\Delta^A w_t^n$, and the unemployment rate (UR) u_t. Despite some discrepancies between the model and data in the autocorrelations of certain variables, most notably for GDP growth, these differences are not statistically significant, as the confidence intervals of the autocorrelations in the data overlap with those generated by the simulated model. More importantly, the point estimates for the model's autocorrelations are closer to those observed in the data than those produced by the 4GM.

Table 6: Moments from the Data and from Model Simulations

	4GM+LM		**4GM**		**Data**	
	Mean	$[P_5, P_{95}]$	Mean	$[P_5, P_{95}]$	Mean	$[P_5, P_{95}]$
Autocorrelations						
$C(\Delta^A y_t, \Delta^A y_{t-1})$	0.79	[0.72, 0.83]	0.82	[0.79, 0.85]	0.69	[0.55, 0.82]
$C(\pi_t^A, \pi_{t-1}^A)$	0.97	[0.96, 0.98]	0.98	[0.98, 0.98]	0.94	[0.80, 1.00]
$C(\pi_t^{A,C}, \pi_{t-1}^{A,C})$	0.98	[0.97, 0.99]	0.98	[0.98, 0.99]	0.95	[0.81, 1.00]
$C(\Delta^A w_t^n, \Delta^A w_{t-1}^n)$	0.96	[0.94, 0.97]	–	–	0.97	[0.83, 1.00]
$C(u_t, u_{t-1})$	0.84	[0.77, 0.89]	–	–	0.78	[0.64, 0.92]
Correlations						
$C(\pi_t, \Delta w_t^n)$	0.73	[0.62, 0.82]	–	–	0.76	[0.65, 0.84]
$C(\Delta^A y_t, u_t)$	-0.09	[-0.24, 0.06]	–	–	-0.29	[-0.47, -0-09]
$C(\widehat{y}_t, \widehat{u}_t)$	-0.76	[-0.84, -0.65]	–	–	-0.90	[-0.93, -0.85]

Note: This table compares the first-order autocorrelations of annual GDP growth ($\Delta^A y_t$), annual headline inflation (π_t^A), annual core inflation ($\pi_t^{A,C}$), annual nominal wage growth ($\Delta^A w_t^n$), and the unemployment rate (u_t), as well as the correlations among a subset of these variables, as simulated by the 4GM and 4GM+LM models, with those observed in the data. Confidence intervals are computed as described in the text.
Source: Author's calculations based on model results.

The lower panel presents the correlations between inflation and nominal wage growth, C(π_t, Δw_t^n), between annual GDP growth and unemployment, C($\Delta^A y_t$, u_t), and between the GDP gap and the UR gap, C($\widehat{y}_t$, $\widehat{u}_t$). The correlations exhibit the expected positive sign between wages and inflation and the expected negative signs between measures of aggregate economic activity and unemployment. In terms of magnitude, the correlations produced by the 4GM+LM model are similar to those observed in the data, with no statistically significant differences between them.

7 Conclusions

This study develops a semi-structural model for monetary policy analysis that enhances Colombia's current 4GM framework by integrating labor market dynamics, both on the quantity side (through unemployment rates) and the price side (via nominal and real wage adjustments). By incorporating wage-price linkages through marginal costs, connecting the NAIRU to potential output, embedding Okun's law, and accounting for the direct impact of labor market variables on aggregate demand, we establish explicit relationships between labor market conditions, economic activity, and price formation. These channels enable policymakers to better quantify labor market slack, inflationary pressures, and the cyclical position of the economy, providing a more refined tool for scenario analysis and short-term forecasting than the one currently in use.

The inclusion of labor market variables highlights their role in shaping inflation dynamics and the responses of monetary policy. Incorporating their transmission mechanisms proves particularly valuable for economies such as Colombia's, which are characterized by persistently high UR and both nominal and real wage rigidities that influence macroeconomic relationships. Through the lens of this model, we demonstrate how labor market dynamics significantly influence inflation trends and policy responses. For instance, nominal wage shocks (i.e., those derived from the MW) amplify price pressures and, in some cases, delay real wage adjustments following various economic shocks. The framework allows policymakers to better distinguish between supply and demand shocks, especially during crises such as the pandemic, when labor market idiosyncrasies distort wage and inflation patterns. Thus, the model provides a tool for a more internally consistent assessment of inflationary pressure sources, labor market slack, and overall economic conditions, leading to better-informed decisions.

In conclusion, our model offers Colombian policymakers a new semi-structural tool for navigating the complex interplay between price stability, employment, and economic growth. The framework maintains theoretical coherence while incorporating Colombia's specific macroeconomic characteristics in the goods and labor markets. Although developed for the Colombian context, it can be adapted to other emerging economies with similar structural labor market conditions. Future research avenues include fully capturing informal sector dynamics and their interaction with aggregate productivity and real wages, further disaggregating labor market segments, particularly those related to structural trends that do not depend on business cycles, and testing the model's robustness when incorporating additional modules currently under development, such as fiscal and financial blocks.

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Appendix

A. Full model specification

The structure of the complete model can be divided into five main blocks: 1) The IS Curve and potential output growth; 2) the Phillips curves for each CPI basket; 3) the monetary policy rule and definitions of other interest rates; 4) the labor market (unemployment and wages); and 5) the uncovered interest parity (UIP) and the process for foreign variables. Shocks are denoted by ε_t and are assumed to be normally distributed.

1) IS Curve and Potential Output Growth

The output level in logarithmic terms y_t is defined in terms of a cyclical component $\hat{y}_t$ (output gap) and a trend $\overline{y}_t$ (potential output):

$$y_t = \overline{y}_t + \hat{y}_t \quad \text{(A.1)}$$

$$\overline{y}_t = \overline{y}_{t-1} + \frac{\Delta\overline{y}_t}{4} \quad \text{(A.2)}$$

$$\Delta\overline{y}_t = \rho_{\Delta\overline{y}}\Delta\overline{y}_{t-1} + (1-\rho_{\Delta\overline{y}})\Big[\Delta\overline{y}_{ss} + \kappa_1(\Delta\overline{TOT}_t - \Delta\overline{TOT}_{ss}) + \kappa_2(\Delta\overline{w}_t - \Delta\overline{w}_{ss}) - \kappa_3(\Delta\overline{u}_t - \Delta\overline{u}_{ss})\Big] + \varepsilon_t^{\Delta\overline{y}} \quad \text{(A.3)}$$

$$\hat{y}_t = \beta_1\widehat{y}_{t-1} + \beta_2 E_t\hat{y}_{t+1} + \beta_3\hat{y}_t^{\star} + \beta_4\widehat{TOT}_t - \beta_5\phi_t - \beta_6\hat{u}_t + \beta_7\hat{w}_t + \eta_t^{\widehat{y}} \quad \text{(A.4)}$$

$$\phi_t = \alpha_{\hat{r}r}^{\phi}\hat{r}_t - (1-\alpha_{\hat{r}r}^{\phi})\hat{z}_t \quad \text{(A.5)}$$

$$\eta_t^{\hat{y}} = \beta_{\eta^{\hat{y}}}\eta_{t-1}^{\hat{y}} + \varepsilon_t^{\hat{y}} \quad \text{(A.6)}$$

where ϕ_t is the real monetary condition index that captures the effect of the real interest rate gap $\hat{r}_t$ and the RER gap $\hat{z}_t$; $\hat{y}_t^*$ is the foreign output gap; $\widehat{TOT}_t$ is the TOT gap; $\hat{w}_t$ is the real wage gap; $\hat{u}_t$ is the unemployment gap; and $\eta_t^{\widehat{y}}$ is a demand shock that follows an AR(1) process. Note that the law of motion of potential growth $\Delta\overline{y}_t$ depends on its lagged value, the long-term growth rate (steady state) $\Delta\overline{y}_{ss}$, deviations of the trend growth of the TOT from its steady state rate ($\Delta\overline{TOT}_t - \Delta\overline{TOT}_{ss}$) and changes in the NAIRU ($\Delta\overline{u}_t$) and the trend real wages ($\Delta\overline{w}_t$) from their steady state deviations.

2) Phillips Curves and CPI Aggregation

The model considers four CPI baskets j: goods (g), services (s), food (f), and regulated goods (r). Each has a hybrid Phillips curve:

$$\pi_t^j = \alpha_{\pi^j}\pi_{t-1}^j + (1-\alpha_{\pi^j})E_t\pi_{t+1}^j + \alpha_{rmc^j}rmc_t^j + \varepsilon_t^{\pi^j} \quad \text{for } j = g,s,f,r \quad \text{(A.7)}$$

where π_t^j is the annualized quarterly inflation and rmc_t^j is the real marginal cost, given by

$$rmc_t^j = \begin{cases} \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + \alpha_{\hat{w}}^{rmc^j}\hat{w}_t + (1-\alpha_{\hat{y}}^{rmc^j} - \alpha_{\hat{w}}^{rmc^j})(\hat{z}_t - \widehat{pr}_t^j) & \text{for } j = g,s,r \\ \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + \alpha_{\hat{w}}^{rmc^j}\hat{w}_t + (1-\alpha_{\hat{y}}^{rmc^j} - \alpha_{\hat{w}}^{rmc^j})(\widehat{pr}_t^{F\star} + \hat{z}_t - \widehat{pr}_t^j) & \text{for } j = f \end{cases} \quad \text{(A.8)}$$

Thus, real marginal costs depend positively on the output gap $\hat{y}$, real wage gap $\hat{w}_t$, RER gap $\hat{z}_t$, and each basket's relative price $\widehat{pr}_t^j$ gap. In addition, the Phillips curves for food items include the real relative price gap of world food prices $\widehat{pr}_t^{F*}$. Relative prices and the aggregation of the CPI are given by

$$\widehat{pr}_t^j = pr_t^j - \overline{pr}_t^j \quad \text{for } j = g, s, f, r \tag{A.9}$$

$$pr_t^j = p_t^j - p_t \tag{A.10}$$

$$\overline{pr}_t^j = \overline{pr}_{t-1}^j + \frac{\Delta \overline{pr}_t^j}{4} \tag{A.11}$$

$$\Delta \overline{pr}_t^j = \rho_{\overline{pr}^j} \Delta \overline{pr}_{t-1}^j + \left(1 - \rho_{\overline{pr}^j}\right) \Delta \overline{pr}_{ss}^j + \varepsilon_t^{\Delta \overline{pr}^j} \quad \text{for } j = s, f, r \tag{A.12}$$

$$p_t = \omega^g p_t^g + \omega^s p_t^s + \omega^f p_t^f + \omega^r p_t^r + \eta_t \tag{A.13}$$

$$0 = \omega^g \widehat{pr}_t^g + \omega^s \widehat{pr}_t^s + \omega^f \widehat{pr}_t^f + \omega^r \widehat{pr}_t^r \tag{A.14}$$

Thus, relative prices pr_t^j are the difference between the price index of sector j, p_t^j, and the consumer price index (CPI) p_t. Each relative price is decomposed into a long-term trend component $\overline{pr}_t^j$ and a gap component $\widehat{pr}_t^j$, which enter into each corresponding Phillips curve.

3) Monetary Policy Rule and Interest Rates

The monetary policy rate i_t is given by the Taylor rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\bar{i}_t + \varphi_\pi (E_t \pi_{t+3}^A - \bar{\pi}_{t+3}^A) + \varphi_{\hat{y}} \hat{y}_t\right) + \varepsilon_t^i \tag{A.15}$$

where $\bar{i}_t$ is the neutral nominal interest rate and $(E_t \pi_{t+3}^A - \bar{\pi}_{t+3}^A)$ captures the deviation of annual inflation expectations from the three-period-ahead target. The neutral nominal interest rate is defined by the Fisher equation $\bar{i}_t = \bar{r} + \pi_{t+1}$, where $\bar{r}$ is the neutral real interest rate determined by:

$$\Delta \bar{z} = \bar{r} - \bar{r}^* + \overline{\psi}_t \tag{A.16}$$

where $\bar{r}^*$ is the neutral real interest rate of the US, $\overline{\psi}_t$ is the country's long-term trend risk premium, and $\Delta \bar{z}$ is the long-term real depreciation in the steady state.

4) Labor Market (Unemployment and Wages)

The UR u_t is decomposed into its trend (NAIRU) $\overline{u}_t$ and cyclical component $\hat{u}_t$ as follows:

$$u_t = \overline{u}_t + \hat{u}_t \tag{A.17}$$

$$\overline{u}_t = \overline{u}_{t-1} + \Delta \overline{u}_t \tag{A.18}$$

$$\overline{u}_t = \rho_{\overline{u}} \overline{u}_{t-1} + (1 - \rho_{\overline{u}}) \overline{u}_{ss} - \lambda_1 \sum_{j=-12}^{12} \frac{E_t \hat{y}_{t-j}}{24} + \varepsilon_t^{\overline{u}} \tag{A.19}$$

$$\hat{u}_t = \rho_{\hat{u}} \hat{u}_{t-1} - \lambda_2 \hat{y}_{t-1} + \varepsilon_t^{\hat{u}} \tag{A.20}$$

The wage block captures nominal wage growth Δw_t^n as the sum of expected inflation π_{t+1}^e and real wage growth Δw_t:

$$\Delta w_t^n = E_t \pi_{t+1} + \Delta w_t \tag{A.21}$$

$$w_t = \overline{w}_t + \hat{w}_t \tag{A.22}$$

$$\Delta \overline{w}_t = \rho_{\Delta \overline{w}} \Delta \overline{w}_{t-1} + (1 - \rho_{\Delta \overline{w}}) \left[\Delta \overline{w}_{ss} - \lambda_3 (\Delta \overline{u}_t - \Delta \overline{u}_{ss})\right] + \varepsilon_t^{\Delta \overline{w}} \tag{A.23}$$

$$\Delta w_t^n = \rho_{\Delta w^n} \Delta w_{t-1}^n + (1 - \rho_{\Delta w^n}) E_t \Delta w_{t+1}^n + \lambda_4 \left[-\lambda_5 \hat{w}_t + (1 - \lambda_5)(-\hat{u}_t)\right] + \varepsilon_t^{\Delta w^n} \tag{A.24}$$

where $\overline{w}_t$ is the trend real wage, $\hat{w}_t$ is the real wage gap, and $\Delta\overline{w}_{ss}$ is the steady-state real wage growth.

5) Uncovered Interest Parity (UIP) and Foreign Variables

One-period-ahead expectations of depreciation $\Delta E_t s_{t+1}$ are determined by the UIP:

$$\Delta E_t s_{t+1} = i_t^* - i_t + \psi_t + \varepsilon_t^{ls} \tag{A.25}$$

$$z_t = s_t + p_t^\star - p_t \tag{A.26}$$

$$\hat{z}_t = z_t - \bar{z}_t \tag{A.27}$$

$$\bar{z}_t = \overline{z}_{t-1} + \frac{\Delta\overline{z}_t}{4} \tag{A.28}$$

$$\Delta\overline{z}_t = \rho_{\Delta\overline{z}}\Delta\overline{z}_{t-1} + (1-\rho_{\Delta\overline{z}})\left(\Delta\overline{z}_{ss} - \nu_{\Delta\overline{z}}(\Delta\overline{TOT}_t - \Delta\overline{TOT}_{ss})\right) + \varepsilon_t^{\Delta\overline{z}} \tag{A.29}$$

where Δs_t is the nominal depreciation, i_t^* is the FED funds rate, and ψ_t is the risk premium. The RER, z_t, is decomposed into a long-term component $\overline{z}_t$ and a cyclical component $\hat{z}_t$.

Finally, the model has a set of exogenous paths for the following foreign variables: the TOT gap $\widehat{TOT}_t$, the world food relative price gap $\hat{pr}_t^{\star f}$, the foreign output gap $\hat{y}_t^\star$, the foreign headline inflation $\pi_t^\star$, the nominal foreign interest rate $i_t^\star$, the foreign real neutral interest rate $\overline{r}_t^\star$, the country risk premium ψ_t, and the long-term trend country risk premium $\overline{\psi}_t$. All follow an AR(1) process:

$$(\cdot)_t = \rho_{(\cdot)}\,(\cdot)_{t-1} + \left(1-\rho_{(\cdot)}\right)(\cdot)_{ss} + \varepsilon_t^{(\cdot)} \tag{A.30}$$

for $(\cdot) = \left\{\widehat{TOT}_t, \widehat{pr}_t^{\star f}, \hat{y}_t^\star, \pi_t^\star, i_t^\star, \overline{r}_t^\star, \psi_t, \overline{\psi}_t\right\}$, where $\rho_{(\cdot)}$ is the persistence and $\left(1-\rho_{(\cdot)}\right)$ represents the speed of adjustment towards the corresponding steady-state value $(\cdot)_{ss}$.

B. Transmission of MW to the Wage Series

As discussed in Section 2, the MW (w_t^{min}) serves as a regulatory mechanism in Colombia's labor market, influencing the determination of other wages and, consequently, prices. This wage floor is adjusted annually through negotiations between labor unions and employers. In practice, the adjustment consists of two components: an *expected* element, which accounts for the previous year's inflation π_{t-1} and productivity ϕ_{t-1}^A, if it is positive, and an *unexpected/surprise* component. Thus, the adjustment rule can be described as follows:

$$\Delta w_t^{min} = \underbrace{\pi_{t-1}^A + max(0, \phi_{t-1}^A)}_{\text{Expected}} + \underbrace{\varepsilon_t^{\phi,\pi}}_{\text{Unexpected}}$$

Surprises may arise when the annual increase deviates from the rule, which could be the result of an agreement in the negotiation process or the government's decision after negotiations fail. Figure B.1 shows the surprise $\varepsilon_t^{\phi,\pi}$ of the MW increment computed using the two measures of productivity. In the left panel, total factor productivity, which was the measure mainly considered up to 2016, and in the right panel, the average labor productivity.

(a) Total Factor Productivity (TFP) (b) Average Labor Productivity (ALP)

Figure B.1: Unexpected component of MW increases

In practice, the model's dynamic relationships and general equilibrium account for the expected component of wage adjustment. However, the unexpected component is not captured by these dynamics and may exhibit distinct behavior compared to idiosyncratic shocks in the nominal wage series. Moreover, as Lasso and Vargas (2024) show, the transmission of increments in the MW to other wages in the economy is not complete.

To determine the pass-through of increments in the MW to our wage measure, we estimate the impulse-response function to a surprise in the MW from a standard Monetary VAR (inflation, output gap, and the monetary policy rate), augmented with the surprise series and the nominal wage series. We analyze quarterly data from 2005 to 2024 and compare the estimation results using the two productivity measures presented in Figure B.1, i.e. total factor productivity and average labor productivity. Table B.1 reports the estimated impulse responses over four quarters of a 100 bp surprise in the MW. As shown in the table, the strongest response occurred in the first quarter, dissipating gradually over the subsequent quarters. The cumulative effect on the nominal wage series is 32 bp when using TFP as the productivity measure, compared to 28 bp when using ALP.

Table B.1: Pass through of surprises in MW to Wage factor series

Quarters	TFP	ALP
I	17.97	16.55
II	9.67	7.30
III	3.16	3.60
IV	1.19	0.55

Finally, to accommodate the effect of an increase in the MW in our framework, we consider that the shock in the nominal wage in Equation 5, $\varepsilon_t^{\Delta w^n}$ can be decomposed into idiosyncratic movements of the wage series $\eta_t^{\Delta w^n}$ and the MW surprise. Thus,

$$\varepsilon_t^{\Delta w^n} = \eta_t^{\Delta w^n} + \varepsilon_t^{\Delta w^{min}}$$

As $\varepsilon_t^{\Delta w^{min}}$ quantifies the pass-through from the unexpected MW adjustment to the wage series, we calibrate the value of this shock as an AR(1) process that has a strong effect in the first quarter and dissipates during the four quarters, with a pass-through to annual wage inflation of 30% of the value of the MW surprise increase, as suggested (on average) by our estimates.

The impulse response of the model to the surprise increment of the MW is presented in Figure B.2 and is qualitatively similar to that of a nominal wage shock presented in Section 6.1, with greater persistence given the AR(1) nature of the shock.

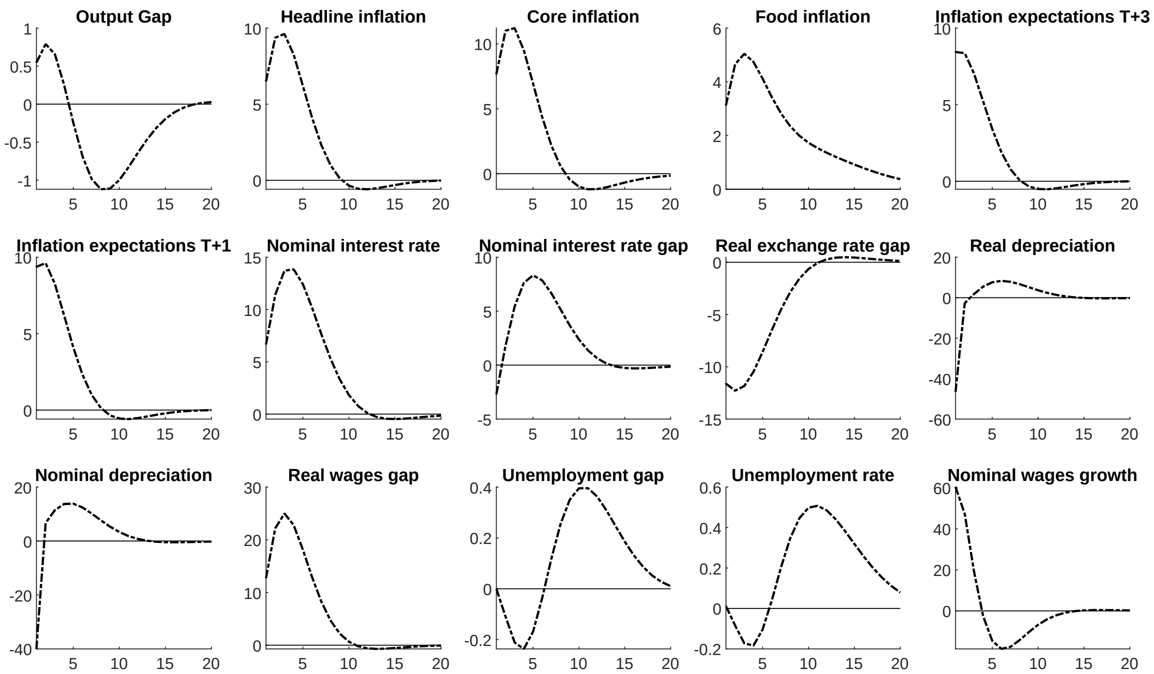

Figure B.2: Impulse Response to a MW Shock

C. Prior and Posterior distributions

The estimation follows a sequential approach designed to isolate the unusual effects of the COVID-19 pandemic while preserving the structural interpretation of the model's parameters. The procedure consists of three steps.

1. **Pre-Pandemic Parameter Estimation**

   The model's structural parameters, including those governing the Phillips curves, economic growth, unemployment, wage dynamics, and the stochastic properties (standard deviations and autoregressive coefficients) of the usual shocks, are estimated using Bayesian methods over the sample period 2003Q1-2019Q4. This step yields a posterior distribution for the parameter vector Θ, which reflects the model dynamics during typical business cycle conditions. The prior distributions were specified with sufficiently wide dispersions to allow the data to inform the estimates. The results are reported in Tables C.1 and C.2.

2. **Estimation of the COVID Shock Volatility**

   Holding the structural parameters Θ fixed at their pre-pandemic estimates from Step 1, the second stage aims to identify the unusual nature of the pandemic period by estimating the volatility of a set of transitory *COVID shocks*. This approach is inspired by the methodology proposed by Ferroni, Fisher, and Melosi (2024), who introduced a *COVID shock* as a parsimonious way to capture the extreme and unprecedented macroeconomic dynamics observed during the pandemic without altering the core structure of the model. The COVID shock in our semi-structural framework is modeled as an exogenous, short-lived disturbance that loads onto a specific set of model equations akin to the *wedge* approach used in structural models. As Ferroni, Fisher, and Melosi (2024) argue, *including* this single new shock goes a long way toward capturing the unusual dynamics while making the usual shocks more normal-sized. In our case, the COVID shocks were active only during the acute phase of the pandemic (2020Q2–2020Q4), a period in which economic activity was dominated by the effects of containment policies, supply disruptions, and shifts in private behavior.

   The standard deviations of these shocks were estimated using data exclusively from the pandemic period. This allows the model to absorb the extreme volatility observed in 2020 without attributing it to the usual structural shocks, which would otherwise be pushed far outside their historical distribution. As noted by Ferroni, Fisher, and Melosi (2024), this approach prevents the model from misinterpreting unusual episodes as permanent changes in the propagation of business cycle shocks.

   The estimation results confirm that COVID shocks exhibit significantly higher volatility than pre-pandemic shock distributions. This is consistent with the view that the pandemic represented an outlier event that required distinct statistical treatment. By isolating these effects in a limited time window, the model retains its ability to capture typical business cycle dynamics before and after the pandemic, while still accounting for the exceptional volatility of the COVID period. The results of this part of the estimation are presented in Table C.3.

3. **Full-Sample Estimation with COVID Shock Recognition**

   Using the COVID shock volatilities estimated in Step 2 as fixed, we re-estimate all structural parameters over the extended sample period of 2003Q1–2025Q1. This step updates the parameter posteriors using both pre- and post-pandemic data, while explicitly accounting for the COVID period via the previously identified shock process. The inclusion of post-2020 data allows the model to absorb potential structural changes in the economy after the pandemic. Importantly, because COVID shocks are treated as unusual and transient, their influence on the estimated structural parameters is minimized. The resulting posteriors reflect the evolving dynamics of the economy over the full sample without being disproportionately distorted by the high-volatility pandemic quarters. The results of the final estimation are presented in Tables C.1 and C.2.

While our semi-structural approach shares the logic of Ferroni, Fisher, and Melosi (2024), there are important differences in its implementation. In their DSGE framework, the COVID shock is explicitly modeled as a factor that loads onto multiple structural wedges (e.g., productivity, discount factor, and markups) with a news component that captures the anticipated effects. In contrast, our semi-structural model introduces COVID shocks directly into reduced-form equations without imposing a full theoretical structure. This provides greater flexibility in capturing the effects of the pandemic while maintaining empirical tractability.

However, both approaches share the key insight that treating the COVID period as a series of unusually large realizations of conventional shocks leads to implausible parameter estimates and state inferences. By explicitly modeling the pandemic as an unusual event, we preserved the integrity of the model for policy analysis and forecasting during normal times.

Table C.1: Prior and Posterior Distributions for Activity and Labor Market Parameters

Parameter	Prior Distribution	Prior Mean	Prior Std Dev	Posterior Step 1	Posterior Step 3
		Nominal Wage Dynamics			
$\rho_{\Delta w^n}$ (Backward component)	Beta	0.300	0.100	0.374	0.398
λ_4 (Labor market adjustment)	Beta	0.700	0.200	0.560	0.602
λ_5 (Real wage gap elasticity)	Beta	0.300	0.200	0.374	0.398
		Trend of real Wage			
$\rho_{\Delta \overline{w}}$ (Backward component)	Beta	0.800	0.300	0.747	0.801
λ_3 (NAIRU elasticity)	Beta	0.300	0.200	0.280	0.299
		Cyclical Unemployment			
$\rho_{\hat{u}}$ (Backward component)	Beta	0.500	0.500	0.467	0.498
λ_2 (Output elasticity)	Beta	0.100	0.100	0.187	0.199
		NAIRU			
$\rho_{\bar{u}}$ (Backward component)	Beta	0.800	0.300	0.654	0.701
λ_1 (Hysteresis)	Beta	0.100	0.100	0.093	0.099
		IS Curve			
β_1 (Backward component)	Beta	0.500	0.10	0.467	0.549
β_2 (Forward component)	Beta	0.050	0.010	0.0465	0.053
β_3 (Foreign demand elasticity)	Beta	0.100	0.100	0.083	0.089
β_4 (TOT elasticity)	Beta	0.010	0.010	0.028	0.034
β_5 (MCI weight)	Beta	0.150	0.010	0.202	0.153
β_6 (Unemployment gap effect)	Beta	0.200	0.100	0.187	0.199
β_7 (Real Wage gap effect)	Beta	0.020	0.010	0.0365	0.034
$\alpha^{\phi}_{\hat{r}r}$ (Interest rate gap elasticity)	Beta	0.750	0.010	0.701	0.751
$\beta_{\eta \hat{y}}$ (Shock persistency)	Beta	0.500	0.100	0.467	0.502
		Potential output			
$\rho_{\Delta \bar{y}}$ (Backward component)	Beta	0.850	0.200	0.794	0.801
κ_1 (TOT elasticity)	Beta	0.030	0.030	0.0897	0.029
κ_2 (Wage elasticity)	Beta	0.100	0.100	0.093	0.099
κ_3 (NAIRU elasticity)	Beta	0.400	0.200	0.374	0.402

Table C.2: Prior and Posterior Distributions for Inflation Parameters

Parameter	Component	Prior Distribution	Prior Mean	Prior Std Dev	Posterior Step 1	Posterior Step 3
Food Sector						
α_{π^f}	Phillips Curve (Backward)	Beta	0.300	0.100	0.280	0.302
α_{rmc^f}	Phillips Curve (Marginal cost)	Beta	0.180	0.100	0.140	0.149
$\alpha_{\hat{y}}^{rmc^f}$	Marginal Cost (Output Gap)	Beta	0.400	0.100	0.439	0.529
$\alpha_{\hat{w}}^{rmc^f}$	Marginal Cost (Real wage gap)	Beta	0.300	0.200	0.280	0.169
Goods Sector						
α_{π^g}	Phillips Curve (Backward)	Beta	0.300	0.100	0.280	0.298
α_{rmc^g}	Phillips Curve (Marginal cost)	Beta	0.150	0.100	0.093	0.095
$\alpha_{\hat{y}}^{rmc^g}$	Marginal Cost (Output Gap)	Beta	0.250	0.100	0.299	0.239
$\alpha_{\hat{w}}^{rmc^g}$	Marginal Cost (Real wage gap)	Beta	0.200	0.200	0.140	0.149
Services Sector						
α_{π^s}	Phillips Curve (Backward)	Beta	0.350	0.100	0.326	0.359
α_{rmc^s}	Phillips Curve (Marginal cost)	Beta	0.150	0.100	0.139	0.169
$\alpha_{\hat{y}}^{rmc^s}$	Marginal Cost (Output Gap)	Beta	0.300	0.100	0.383	0.393
$\alpha_{\hat{w}}^{rmc^s}$	Marginal Cost (Real wage gap)	Beta	0.400	0.200	0.429	0.458
Regulated Sector						
α_{π^r}	Phillips Curve (Backward)	Beta	0.350	0.100	0.326	0.348
α_{rmc^r}	Phillips Curve (Marginal cost)	Beta	0.050	0.050	0.0464	0.049
$\alpha_{\hat{y}}^{rmc^r}$	Marginal Cost (Output Gap)	Beta	0.300	0.100	0.411	0.372
$\alpha_{\hat{w}}^{rmc^r}$	Marginal Cost (Real wage gap)	Beta	0.500	0.500	0.5139	0.547

Table C.3: Prior and Posterior Distributions for COVID Shock Parameters

Parameter	Prior Distribution	Prior Mean	Prior Std Dev	Posterior Estimate
Real Activity COVID Shocks				
$\sigma_{\overline{y}}^C$	Gamma	1	0.500	0.348
$\sigma_{\hat{y}}^C$	Gamma	1	0.500	0.165
$\rho_{\hat{y}}^C$	Beta	0.4	0.300	0.256
Unemployment COVID Shocks				
$\sigma_{\overline{u}}^C$	Gamma	1	0.500	0.305
$\sigma_{\hat{u}}^C$	Gamma	1	0.500	0.174
$\rho_{\overline{u}}^C$	Beta	1	0.500	0.0871
Inflation COVID Shocks				
$\sigma_{\pi^g}^C$	Gamma	1	0.500	0.445
$\sigma_{\pi^s}^C$	Gamma	1	0.500	0.348
$\sigma_{\pi^r}^C$	Gamma	1	0.500	0.722
$\sigma_{\pi^f}^C$	Gamma	1	0.500	0.690
Monetary Policy COVID Shock				
σ_i^C	Gamma	1	0.500	0.306

D. Other Shock Decomposition Episodes

In this Appendix, we replicate the shock decomposition analysis presented in Section 6.2 for additional periods of interest within our estimation sample. We organize the presentation around three distinct periods of interest. Figures D.1 and D.2 present the shock decompositions of the macroeconomic variables and the labor market variables, respectively.

The High Commodity Prices Period (2007 – 2008)
From 2007 to the third quarter of 2008, the economy experienced strong growth driven by rising commodity prices (which generated a notable improvement in the TOT), substantial capital inflows, and, to a lesser extent, impulses from domestic and external demand. This expansion resulted in a positive output gap, creating inflationary pressures, which added to the global inflationary push stemming from high international oil and food prices, as seen in the shock decomposition of headline inflation (particularly in the magnitudes of the food and regulated price shocks). Although the higher TOT generated currency appreciation and lower risk premia, which partially mitigated inflationary pressures, the overall expansion ultimately led to an increase in inflation and inflation expectations.[17] In response to these factors, the monetary policy rate was increased. Strong aggregate demand also led to a decline in the UR, and the real wage gap was positive and increasing during this period.

The Global Financial Crisis and Recovery (2008 – 2014)
This period began with the onset of the global financial crisis in the fourth quarter of 2008 and extended through the third quarter of 2014. The crisis triggered a sharp deterioration in economic conditions, marked by high uncertainty and a contraction in global economic growth. The decline in external demand, coupled with financial market volatility and a deterioration in the TOT, contributed to a negative output gap and a positive unemployment gap. Headline inflation decreased, approaching its long-term target of 3%, driven by the decline in international food and energy prices and, to a lesser extent, by the global slowdown. In response, the central bank implemented an expansionary monetary policy to stimulate economic activity and counteract the effects of the recession.

Following the global financial crisis, from the third quarter of 2009 to the first quarter of 2011, the output gap remained negative due to a significant decline in both domestic and external demand, despite the recovery in TOT and an expansionary monetary policy. Between 2011 and 2014, the economy exhibited signs of recovery, allowing the output gap to return to a positive value. This recovery was largely driven by improved TOT, mainly due to oil prices and low external interest rates, which also contributed to the real appreciation of the currency. This appreciation, in conjunction with a favorable external sector in terms of inflationary conditions and a suitable stance of monetary policy, kept headline inflation close to the target.

During this period, the dynamics of the labor market was significantly influenced by the contraction of economic activity, resulting in an increased UR beginning in early 2008. Consequently, the UR peaked in late 2009 and gradually returned to its pre-crisis level by 2013. The deterioration of the labor market was also driven by fluctuations in the TOT, especially oil prices. In 2009, real wages showed a negative gap but began to increase as inflation decreased despite the UR exerting downward pressure on real wages.

The Oil Price Collapse and a Severe El Niño Event (2014 - 2016)
Between late 2014 and the end of 2016, the Colombian economy was affected by two major shocks. The first was the sustained decline in international oil prices, which reduced the TOT and severely affected the country's exports. This situation led to currency depreciation, resulting in a positive RER gap, and a negative income shock for the country that hurt demand (as seen in the shock decomposition of the output gap and the real wage gap). This shock was later compounded by a second large shock. An atypically strong *El Niño* phenomenon

[17] Additionally, several measures were implemented during this period to regulate capital flows and mitigate the impact of currency appreciation on the external sector.

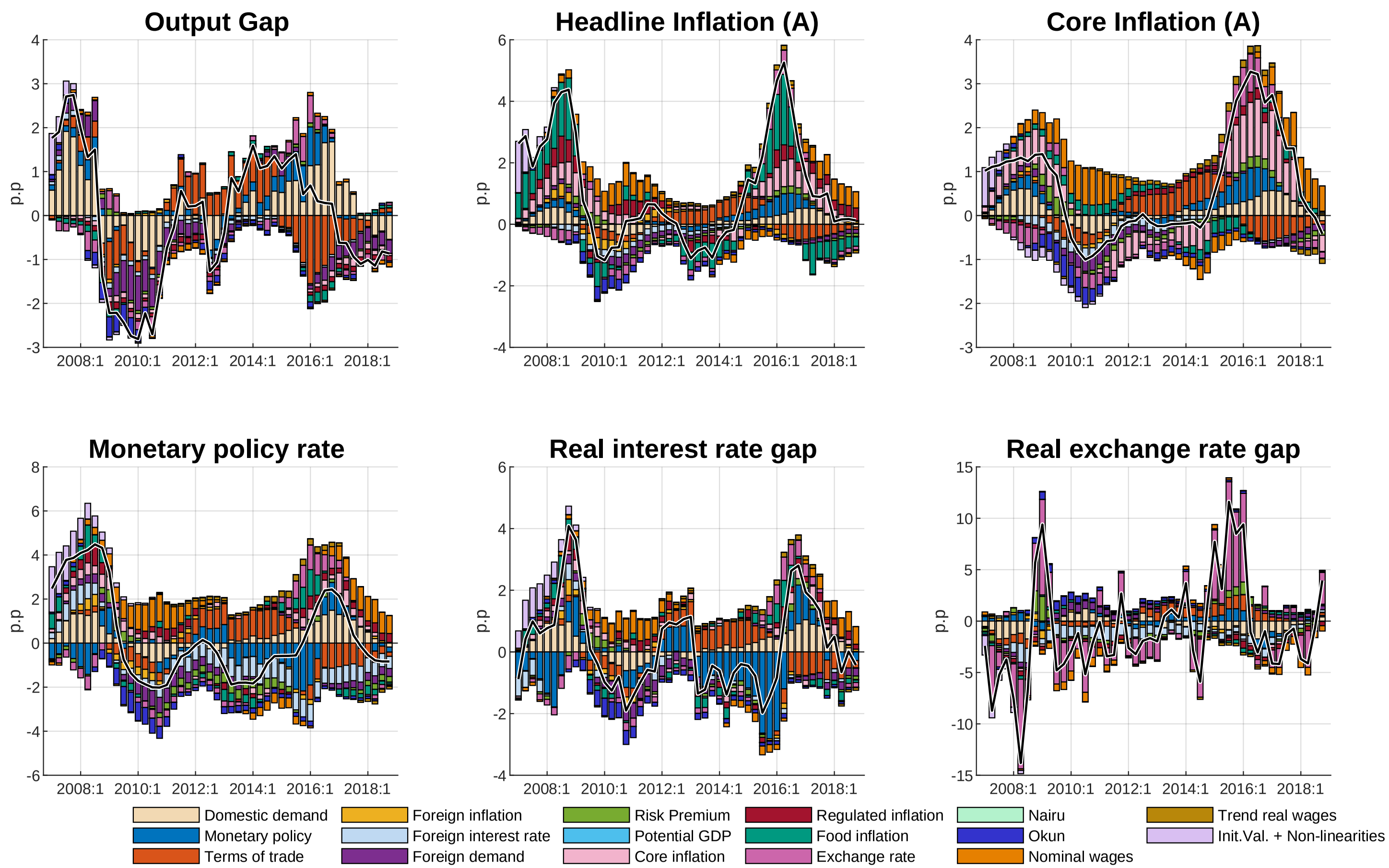

Figure D.1: Shock Decomposition of Macroeconomic Variables

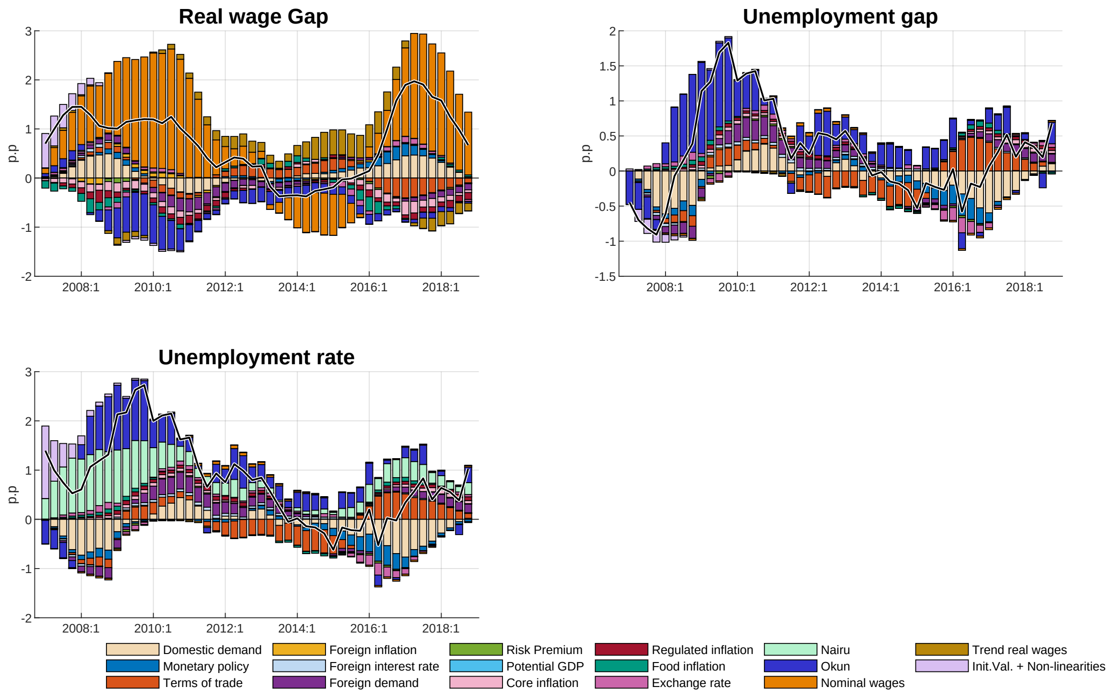

Figure D.2: Shock Decomposition of Labor Market Variables

significantly reduced the supply of agricultural goods and hydroelectric power, increasing prices in the food and regulated CPI baskets.[18]

These two shocks combined to increase both annual headline inflation and inflation expectations. In response, the central bank adopted a contractionary monetary policy by raising the interest rate in September 2015, nearly a year after the initial oil price shock. This relatively delayed response meant that monetary policy remained expansionary until the first quarter of 2016, contributing to a positive output gap until the end of that year. However, it is important to note that the oil price collapse alone generated policy trade-offs that rendered the appropriate monetary policy response less clear, since the inflationary effects of currency depreciation were partially offset by weaker aggregate demand, and, in addition, the persistence of the TOT shock implied a shift in the trend of the RER that was not inflationary. Finally, with respect to wages, the real wage gap increased and turned positive, despite the downward pressure from the TOT and inflation shocks, driven instead by nominal wage and productivity shocks.

[18] *El Niño* is a natural climatic phenomenon characterized by the abnormal increase in sea surface temperatures in the central and eastern Pacific Ocean. This temperature rise is a component of the broader El Niño-Southern Oscillation (ENSO) cycle, which exerts a significant influence on global weather patterns and can result in extreme events such as droughts, floods, and other environmental challenges.