Securities cross-holding in the Colombian financial system: A topological approach

By: Carlos León
Javier Miguélez

Borradores de ECONOMÍA

No. 1134
2020

BANCO DE LA REPÚBLICA
COLOMBIA

Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia

Securities cross-holding in the Colombian financial system:

A topological approach[α]

Carlos León[β]

Javier Miguélez[δ]

The opinions contained in this document are the sole responsibility of the authors and do not commit Banco de la República or its Board of Directors.

Abstract

Cross-holding of securities in financial systems occurs when i) two financial institutions hold securities issued by each other or ii) more than two financial institutions hold securities issued by each other in a circular structure. Securities cross-holding is relevant for financial stability because they may further propagate and amplify shocks, therefore increasing the potential for contagion among interconnected financial institutions. We use a unique dataset of securities issued and held by financial institutions to measure the extent of securities cross-holding in Colombia. The dataset comprises bonds, certificates of deposit, and equity issued and held (in proprietary position) by financial institutions from 2016 to 2019. Results show that the extent of securities cross-holding in the Colombian financial system is particularly low—even when cross-holding across different types of securities is considered. The network topology suggests that potential contagion from securities cross-holding is rather limited.

JEL Classification: D85, L14, G32, G2

Keywords: financial stability, contagion, networks, reciprocity, transitivity, securities.

[α] Opinions and statements in this article are the sole responsibility of the authors, and do not represent neither those of Banco de la República nor of its Board of Directors. We thank Clara Machado, Serafín Martínez, and Wilmar Cabrera for their comments and suggestions. Any remaining errors are our own.

[β] Financial Infrastructure Oversight Department, Banco de la República; Finance Department, Tilburg University. E-mail: cleonrin@banrep.gov.co / carlosleonr@hotmail.com. [corresponding author]

[δ] Financial Infrastructure Oversight Department, Banco de la República. E-mail: jmiguema@banrep.gov.co.

Las posiciones cruzadas de títulos valores en el sistema financiero colombiano: una aproximación topológica[α]

Carlos León[β]

Javier Miguélez[δ]

Las opiniones contenidas en el presente documento son responsabilidad exclusiva de los autores y no comprometen al Banco de la República ni a su Junta Directiva

Resumen

Las posiciones cruzadas en los sistemas financieros suceden cuando i) dos instituciones financieras tienen títulos valores emitidos por la otra o ii) dos o más instituciones financieras tienen títulos valores emitidos por las otras en una estructura circular. Las posiciones cruzadas en títulos valores son importantes para la estabilidad financiera porque pueden contribuir a la propagación y amplificación de choques, y así incrementar el potencial de contagio entre instituciones financieras interconectadas. Para medir la presencia de posiciones cruzadas de títulos valores en Colombia utilizamos una base de datos de características excepcionales. Esta base de datos contiene los bonos, certificados de depósito y acciones emitidos y en posición propia de las instituciones financieras desde 2016 hasta 2019. Los resultados muestran que las posiciones cruzadas de títulos valores en el sistema financiero colombiano son particularmente escasas—incluso cuando se considera la posibilidad de posiciones cruzadas entre títulos valores de diferente tipo. La topología de red sugiere que el potencial de contagio por posiciones cruzadas en títulos valores es limitado.

Clasificación JEL: D85, L14, G32, G2

Palabras clave: estabilidad financiera, contagio, redes, reciprocidad, transitividad, títulos valores.

[α] Las opiniones contenidas en este documento son responsabilidad exclusiva de los autores y no comprometen al Banco de la República ni a su Junta Directiva. Agradecemos a Clara Machado, Serafín Martínez y Wilmar Cabrera por sus comentarios y sugerencias. Todos los errores son de los autores.

[β] Departamento de Seguimiento a la Infraestructura Financiera, Banco de la República; Departamento de Finanzas, Universidad de Tilburg. E-mail: cleonrin@banrep.gov.co / carlosleonr@hotmail.com. [autor para correspondencia]

[δ] Departamento de Seguimiento a la Infraestructura Financiera, Banco de la República. E-mail: jmiguema@banrep.gov.co.

1 Introduction

The global financial crisis that erupted in 2007-2008 exposed the fact that financial authorities and market participants had very limited information about the network of exposures and the potential of financial contagion (Glasserman & Young, 2016). Consequently, interest in exposures networks spurred after the global financial crisis. However, exposures are typically estimated due to a lack of detailed counterparty data (Hüser & Kok, 2019).

Most research on exposures networks and financial contagion is based on secured and unsecured lending. Exposures arising from institutions holding securities issued by other financial institutions (i.e. cross-holding of securities) are often neglected despite their demonstrated contribution to systemic risk (see Poledna, et al., 2015). Further, existing literature on financial networks suggest that securities cross-holding may be particularly important for financial stability because of potential contagion and cascades arising from *cyclical interdependencies* and feedback effects in the connective architecture of financial systems (see Eisenberg & Noe, 2001, Elsinger, 2009, Gouriéroux, et al. 2012, Elliot, et al., 2014).

Concurrent with the literature on exposures networks, most literature on networks of securities cross-holding does not work on observed data. Some noteworthy exceptions are Poledna, et al. (2015) with Mexican data, and Hüser and Kok (2019) with European data. In our case, we use a unique dataset of securities issued and held by financial institutions (in proprietary position) to measure the extent of securities cross-holding in the Colombian financial system, weekly from 2016 to 2019. The dataset enables us to work with a time-series of multi-layer networks, in which bonds, certificates of deposit, and equity issued and held by financial institutions are analyzed separately and as a whole, along 193 weeks. Therefore, our work is closely related to Poledna, et al. and Hüser and Kok.

We add to Poledna, et al. (2015) and Hüser and Kok (2019) in several ways. First, we focus on the topology of securities cross-holding networks by measuring the extent to which reciprocal and transitive relations exist. That is, based on the two main types of cross-holding (see Adams, 1999), we employ network analysis basics to measure the presence of *cyclical interdependencies* and potential feedback effects. Second, we study how cross-holding evolve through time. Third, our dataset comprises banking and non-banking institutions. However, unlike Poledna, et al (2015), we do not measure overall expected systemic losses from securities cross-holding and other types of exposures (e.g. deposits, lending, derivatives, foreign exchange). Also, unlike Hüser and Kok (2019), we neither breakdown the data by seniority, nor maturity nor securitization.

We find that exposures in the securities network are scarce. Furthermore, reciprocal and transitive exposures are uncommon. Results are stable during the period under analysis. This suggests that the incidence of cyclical interdependencies and potential feedback effects in networks of securities issued and held by Colombian financial institutions are minor. Therefore, results suggest that potential contagion effects related to the network topology of the securities cross-holding in the Colombian financial system are rather low.

Our work contributes to related literature in several ways. First, to our knowledge, this is the first time that the extent of securities cross-holding is measured based on the quantification of reciprocal and transitive holding relations, which are those creating cyclical interdependencies and feedback effects that augment potential contagion and cascades in financial networks. Second, based on evidence that relates reciprocal and transitive relations to risk build-up before financial crises (see Squartini, et al., 2013, Squartini & Garlaschelli, 2014, Kawada, 2016, Sahabat, et al., 2017, Alamsyah, et al., 2019, and Cimini, et al., 2019), our work highlights the importance of monitoring the level and dynamics of cyclical interdependencies for early-warning models and other financial stability purposes. Third, this is the first time the network topology of securities cross-holding is studied in the Colombian case. With financial stability in view, this is particularly important because the outstanding of securities issued and held by Colombian financial institutions is about 1.5 times that of secured and unsecured lending among them.[1]

Our work is a first step towards measuring the expected systemic losses arising from reductions in the individual and aggregate value of financial institutions due to securities cross-holding. As in Battiston, et al. (2012b) and Poledna, et al. (2015), the next step involves

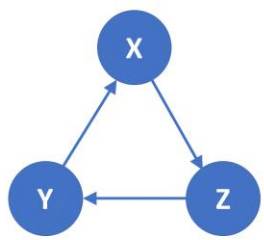
[1] Weekly average, based on data from January 2016 to September 2019, from Financial Superintendence of Colombia and Banco de la República (i.e. the Central Bank of Colombia).

incorporating financial institutions' solvency to calculate aggregate and individual contribution to systemic risk. Further, other exposures (e.g. deposits, lending, derivatives, foreign exchange) should be considered to attain a comprehensive measure of systemic risk.

2 Cross-holding and financial stability

Cross-holding occurs when firms own securities issued by other firms (Fedenia, et al., 1994). Two types of cross-holding exist (see Adams, 1999). First, direct cross-holding occurs when two firms hold securities issued by each other. In this case, firm A owns securities issued by firm B, and firm B owns securities issued by firm A. Second, indirect cross-holding occurs when more than two firms hold securities issued by each other in a circular structure. In this case, for example, firm X owns securities issued by firm Y, firm Y owns securities issued by firm Z, and Z owns securities issued by X.[2] Both types of cross-holding are portrayed in Figure 1, panel a. and b., respectively; each firm is represented as a node and the arrow represents the holding of a security, departing from the issuer and arriving at the holder.[3]

a. Direct cross-holding



b. Indirect cross-holding

Figure 1. Direct and indirect cross-holding. Source: authors' design.

[2] It is feasible to study cross-holding among more than three financial institutions holding securities in circular structures. However, to the best of our knowledge, this is not considered in literature. Further, as highlighted by Squartini et al. (2013), cyclical structures involving more than three financial institutions imply smaller probabilities of contagion.

[3] In the case of indirect cross-holding, the example in panel b. of Figure 1 is not the only feasible configuration –as is the case with direct cross-holding (in panel a.). For instance, a circular structure is preserved if firm Z owns securities issued by firm Y. Following Squartini, et al. (2013), there are seven different configurations of circular structures involving three firms.

When direct cross-holding exists (panel a., Figure 1), a shock affecting the value of firm A (e.g. a credit event) affects the value of securities issued by A. As firm B holds securities issued by firm A in its portfolio, the value of firm B is affected, and—in turn—the value of securities issued by B may be affected as well, creating a feedback effect that may reach firm A's value—and so on. Likewise, when indirect cross-holding (panel b., Figure 1) exists, a shock affecting the value of firm X affects the value of firm Z and Y recursively through changes in the value of securities issued by X and Z. The feedback effect arises when the change in the value of firm Y affects firm X—and the recursion continues.

Literature about cross-holding in corporate finance is protracted and abundant (see Fedenia, et al., 1994, Adams, 1999, Suzuki, 2002, Clayton & Jorgensen, 2005, Trivedi & Young 2006, Elliott, et al, 2014, He, et al., 2019). Cross-holding of equity shares among firms, also called cross-shareholding, is a typical practice in Japan, Korea, the United Kingdom, and Germany, but it is less common in the United States. Most of this literature addresses questions about how cross-holding may induce biases in corporate valuation, asset pricing, portfolio management, credit risk, monitoring, and governance.

From a financial stability perspective, literature acknowledges that cross-holding is particularly important to explain contagion. The key is the potential feedback effect created by firms' payoffs and value being dependent on claims on other firms. According to Eisenberg & Noe (2001), this feedback effect is enabled by the existence of cyclical linkages among firms, which they refer to as *cyclical interdependence*. This cyclical interdependence creates a channel that amplifies shocks and has the potential to cause cascades through interconnected firms and the wider macroeconomy (see Trivedi & Young, 2006, Elliott, et al., 2014). This cyclical interdependence is a general feature of financial system architectures (Eisenberg & Noe, 2001) that was revealed in the global financial crisis (Gouriéroux, et al. 2012). Furthermore, as reported in Squartini, et al. (2013), Squartini & Garlaschelli (2014), Kawada (2016), Sahabat, et al. (2017), Alamsyah, et al. (2019), and Cimini, et al. (2019), the evolution of reciprocal and transitive relations among financial institutions is potentially useful in early-warning models of financial turmoil.[4]

Nevertheless, it is well-known that the relation between interconnectedness in financial networks and financial instability is non-monotonic (see Battiston, et al., 2012, Caccioli, et al., 2014, Elliot, et al., 2014, Glasserman & Young, 2016, Caccioli, et al., 2018, Roncoroni et al., 2019). All in all, Glasserman and Young (2016) highlight that network connections can have a positive effect by diversifying risk exposures for individual banks, but they can also have a negative effect by creating channels through which shocks can spread. Elliot, et al. (2014) point out that low levels of cross-holding circumscribe contagion through a weakly connected system (see Simon, 1962) that limits the interdependencies among firms. As the number of firms held by others increases the system becomes more connected and interdependent, and contagion occurs. But, as the number of firms held by others approaches the maximum (i.e. as in a fully connected network), Elliot, et al. expect firms to be well-insured against the failure of any other firm. Furthermore, not only the relation between interconnectedness and financial stability has been reported as non-monotonic, but also as dependent on several features, such as network architecture, size of shocks, and financial institutions' health (e.g. solvency) and homogeneity. Thus, the relation between the extent of cross-holding and financial stability is a complex one—unless cross-holding is inexistent or extremely rare.

To our knowledge, the empirical research work of Poledna, et al. (2015) and Hüser and Kok (2019) are the only using observed data on cross-holding in financial systems. Poledna, et al. (2015) use Mexican data and DebtRank methodology (see Battiston, et al., 2012b) to measure systemic risk arising from different types of exposures among banking institutions. Securities cross-holding is one of the types of exposures included in their multi-layer network of exposures. They find that the contribution of securities cross-holding to systemic risk is crucial in the Mexican

[4] Squartini, et al. (2013), Squartini & Garlaschelli (2014), and Cimini, et al. (2019) study reciprocal and transitive relations in the Dutch interbank market, whereas Kawada (2016) does in the Japanese interbank market. In both cases, authors report significant changes in the extent of reciprocal and transitive relations around the global financial crisis. Sahabat, et al. (2017) report that reciprocity in Indonesian large-value and retail payments has potential as early warning signal for liquidity crisis conditions. Alamsyah, et al. (2019) report that some cases of transitive relations in the Indonesian interbank funding market may be used as predictors of financial crisis. Also, Fricke and Lux (2014) report that reciprocity in the Italian interbank market decreased due to the global financial crisis—but they neither study nor claim any early-warning potential.

case from 2007 to 2013. However, Poledna, et al. (2015) do not focus on the cross-holding network's structure; particularly, they do not measure the extent to which cyclical interdependencies (i.e. direct and indirect cross-holding) exist. On the other hand, Hüser and Kok (2019) study securities cross-holding in the euro area. Akin to our case, they employ a multi-layer network in which different types of cross-holding networks are studied, individually and in aggregate. They study the structure of the networks but focus on measures that do not reveal whether cyclical interdependencies exist or not. Further, they work on a single observation corresponding to the fourth quarter of 2015.

3 Measuring cross-holding

Network analysis is dedicated to describing and understanding an underlying system, focused on capturing its structure or topology (Börner, et al., 2007). Network analysis basics comprise two measures that correspond to both types of cross-holding documented by Adams (1999). The reciprocity coefficient corresponds to the frequency of direct cross-holding (panel a., Figure 1). The transitivity coefficient corresponds to the frequency of indirect cross-holding (panel b., Figure 1). Together, reciprocity and transitivity, along with other measures from network analysis, enable us to measure the extent of cross-holding in financial systems.[5,6]

3.1 The network of securities issued and held by financial institutions

A traditional representation of a network is the adjacency matrix. Let n represent the number of financial institutions in the network at time t, A is an adjacency matrix of dimensions $n \times n \times t$, with elements A_{ijt} such that

[5] There are numerous measures to describe a network. Our aim is to focus on those measures that contribute to the study of securities cross-holding. Some traditional network measures are not analyzed but are reported in Table 3 (in the Appendix). All in all, networks display a topology that is consistent with what is expected from financial networks, such as sparseness, low mean geodesic distances, and an inhomogeneous distribution of linkages and their weights, i.e. with right-skewed frequency distributions (see figures 11 and 12, in Appendix) and the corresponding distributions exhibiting power-law coefficients approximately in the 2-3 range (see Bollobás, et al., 2007, Newman, 2010). Measures related to the centrality (i.e. network importance) of financial institutions are beyond the scope of this article.

[6] *Motifs* are patterns of interconnections involving a small set of nodes (Cimini, et al., 2019). Therefore, our case pertains to literature related to motifs in financial networks.

$$A_{ijt} = \left\{ \begin{array}{l} 1 \text{ if there is a link from } i \text{ to } j \text{ at time } t, \\ 0 \text{ otherwise.} \end{array} \right\} \quad (1)$$

The adjacency matrix is binary, with a 1 when there is a link from i to j, corresponding to j holding a security issued by i, and 0 otherwise. In our case, the adjacency matrix is directed, where a linkage from i to j does not imply the existence of a linkage from j to i. Also, as the dataset contains weekly data from 2016 to 2019, each link from i to j is observed at a time t.

Further, as the dataset allows to differentiate among three different types of securities held by financial institutions, namely bonds, certificates of deposit, and equity, it is possible to work under a multi-layer approach. In our case, the network of securities issued and held by financial institutions may be built as the aggregation of three different layers at which financial institutions issue and hold securities among them. Following Baxter, et al. (2014), it is a multiplex network because constituent (i.e. bonds, certificates of deposit, equity) and resultant (i.e. all securities) networks contain participants of one sort (i.e. financial institutions) but with several kinds of connections between them (i.e. different types of securities issued and held). Therefore, in our case, the adjacency matrix contains four dimensions, A_{ijtp}, with p corresponding to the type of security issued and held by financial institutions i and j, respectively, in period t.[7] Figure 2 portrays an illustration of the multiplex of securities issued and held by financial institutions at some time t.

[7] It is possible to make the adjacency matrix weighted according to the monetary value of the holdings; in that case, W is an adjacency with each element W_{ijtp} corresponding to the monetary value of securities p issued by i that are held by j at time t. However, based on the informational content of adjacency matrices (see Squartini, et al., 2013), we focus on the topological properties of the networks, namely by analyzing A.

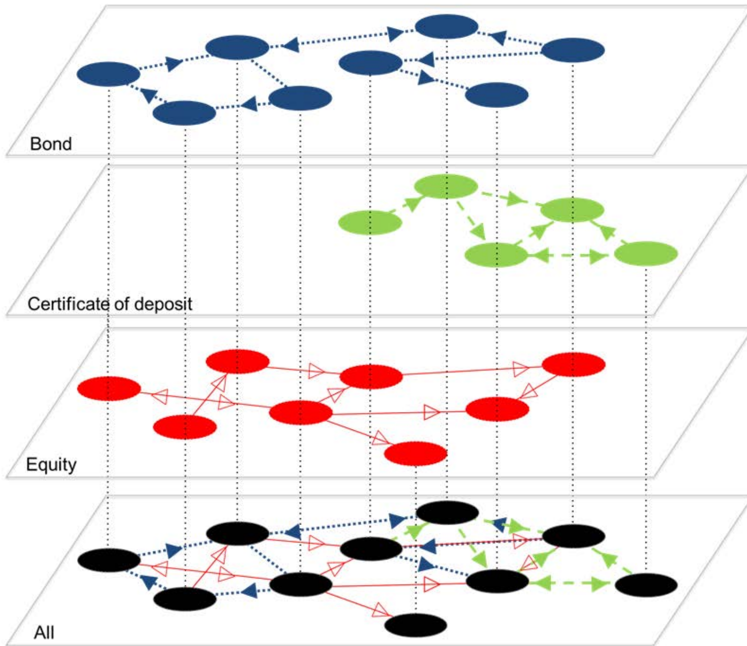


Figure 2. Illustration of a multiplex network of securities issued and held by financial institutions. Arrows are directed from the issuer to the holder. Vertical lines connecting superimposed nodes are financial institutions, whereas each node is a role (i.e. issuer or holder) in the corresponding layer. Source: authors' design.

Embracing a multi-layer network approach is convenient from an analytical viewpoint. As suggested by Cardillo, et al. (2013), the multi-layer approach enables us to study whether the topological properties of the aggregated network of securities issued and held by financial institutions can be traced to those of its constituent layers—or if they emerge from the aggregation of the layers.

3.2 Network measures

Based on adjacency matrix A, several measures are available for analyzing the topology of the cross-holding network. First, density (d) measures the cohesion of the network. It is calculated as the ratio of the number of actual linkages (m) to the maximum possible number of linkages.[8]

[8] In our case, the maximum possible number of linkages is n^2. Thus, $d = m/n^2$. From now on, t is omitted when unnecessary.

Density is bounded to the $0 < d \leq 1$ range, with networks approximating the upper limit labeled as *dense*, whereas those with a much smaller density ($d \ll 1$) labeled as *sparse*. In our case, density reflects the overall extent of holdings of securities issued by financial institutions among financial institutions—without focusing on cyclical linkages that create cross-holding.

Reciprocity (r) measures the frequency with which a linkage from i to j is complemented by the reciprocal linkage, i.e. from j to i. Reciprocity is calculated as the fraction of links for which there is a link in the opposite direction in the network.[9] If $r = 1$ then the network is purely bidirectional (i.e. reciprocal), while if $r = 0$ the network is purely unidirectional. In our case, one relation is reciprocal if two financial institutions hold securities issued by each other. That is, reciprocity exists if the first type of cross-holding of securities exists—as in panel a. of Figure 1. In financial networks, reciprocity is a signature of trust between financial institutions (Cimini, et al., 2019). Correspondingly, interbank reciprocity has been reported to be higher than expected during the build-up of the 2008 crisis and to decrease as the crisis became imminent, therefore with potential to be used in early-warning models (see Squartini, et al., 2013, Squartini & Garlaschelli, 2014, Sahabat, et al., 2017, Cimini, et al., 2019).

Transitivity, commonly referred to as clustering coefficient (c), measures the frequency with which loops of length three appear in the network; that is, it measures the density of triangles in a network (Newman, 2010). In our case, it is calculated as the fraction of pairs of financial institutions that hold securities from a common third financial institution that also holds securities from each other.[10] Findings reported by Squartini, et al. (2013), Squartini and Garlaschelli (2014), Kawada (2016), Alamsyah, et al. (2019), and Cimini, et al. (2019) suggest that loops involving three banks are particularly important for systemic risk and that their dynamics contain key information for early-warning models.

---

[9] Reciprocity of adjacency matrix A is calculated as $r = \left(\sum_{ij} A_{ij}A_{ji}\right)/m$, where $0 \leq r \leq 1$.

[10] There is no closed-form formula for calculating transitivity. It is the ratio of the number of triangles (i.e. three nodes interlinked by three connections) to the number of connected triples (i.e. three nodes linked by two or three connections). That is, $c = (number\ of\ triangles \times 3)/(number\ of\ connected\ triples)$, where the factor of three arises because each connected triple is counted three times (see Newman, 2010).

From a temporal perspective, it is interesting to assess how stable linkages are over time. Measuring the persistence or survival of linkages between financial intuitions enables us to check whether the extent of reciprocal and transitive relations results from stable relations between financial institutions. If linkages persist over time, reciprocal and transitive relations likely correspond to stable relations between pairs or triples of financial institutions; on the other hand, if linkages do not tend to survive from one period to another, then it is likely that reciprocal and transitive relations emerge from new relations in each period. In our case, we implement the *single-step survival ratio* (see Onnela, et al., 2003, León & Miguélez, 2020), which is calculated as the fraction of linkages found common in two consecutive networks.[11]

From a cross-section perspective, it is also interesting to assess how similar linkages are across the four networks, corresponding to the three types of securities and their aggregate. The Jaccard index (see Jaccard, 1912), also known as the *coefficient of community,* is commonly used to compare how similar networks are.[12] Cross-section similarity is particularly important in our case. As in Hüser and Kok (2019), when layers display *point-wise similarity* in the form of a high Jaccard index, the same financial institutions tend to be linked in different layers. In our case, this means that the same connections tend to coexist across the different types of securities, and that distress will propagate across different markets.[13]

4 The dataset

There are two central securities depositories in Colombia, namely DCV and Deceval. DCV (*Depósito Central de Valores*) is the securities depository and settlement system for sovereign

[11] Let E_t be the set of linkages in A at time t, $\cap$ the intersection operator, and $|\cdots|$ the number of elements in the set, the single step survival ratio at time t is denoted s_t, and is calculated as $s_t = |E_t \cap E_{t-1}|/|E_{t-1}|$, where $0 \leq s_t \leq 1$ (see León & Miguélez, 2020).

[12] Let j_{xy} be the Jaccard index that measures the similarity of networks x and y, and $\cup$ the union operator, $j_{xy} = |E_x \cap E_y|/|E_x \cup E_y|$, where $0 \leq j_{xy} \leq 1$.

[13] As in Hüser and Kok (2019), *point-wise similarity* refers to common connective patterns between nodes across layers, which corresponds to what extent a layer is representative of the other; on the other hand, *topological similarity* refers to overall connective properties of the network, such as density. Point-wise similarity is related to *correlated multiplexity* (see Lee et al., 2014), which refers to a non-random pattern of network multiplexity, in which nodes tend to have a similar connective pattern across layers. In such case, some network dynamics, such as contagion, may be facilitated by similarity across layers in a multiplex network.

securities exclusively; it is owned and managed by the Central Bank of Colombia. Deceval (*Depósito Centralizado de Valores de Colombia*) is the securities depository and settlement system for corporate and non-sovereign government securities, and the securities depository for the equity market; it is owned by the Colombian Stock Exchange - BVC (*Bolsa de Valores de Colombia*). As we focus on securities issued by financial institutions, we work with data from Deceval alone.

As the Colombian financial market infrastructure overseer[14], the Central Bank of Colombia requires a granular and periodical report of all securities deposited in Deceval. From those reports, we extract the dataset comprising all securities issued and held (in proprietary position) by institutions supervised by the Financial Superintendence of Colombia (i.e. financial institutions).[15] Each register in the securities dataset includes the date, issuer, holder, outstanding value, and type of security. The dataset contains 193 weekly observations (i.e. each Friday[16]), from January 8th, 2016 to September 6th, 2019.

The dataset comprises three types of securities: certificates of deposit, bonds, and equity.[17] The dataset has 801,619 registries (i.e. rows) and five columns. About 77 percent of the registries correspond to certificates of deposit, 19 percent to bonds, and the remaining four percent to equity. Regarding the contribution of each type to the value of all securities issued and held by financial institutions, certificates of deposit represent about 59 percent, bonds 19 percent, and equity the remaining 22 percent. Figure 3 displays how the contribution to the total outstanding evolves in the period under study.

[14] In Colombia, the oversight of the financial market infrastructure is about monitoring the payment systems, their participants and their interconnections, with the main objective of pursuing the safe and efficient functioning of payments in the economy, identifying risks, and suggesting mitigation procedures.
[15] The dataset comprising all securities issued and held (in proprietary position) by financial institutions accounts for about 57 per cent of all securities issued and held (in proprietary position) reported by Deceval. The remainder corresponds to securities issued or held by non-financial institutions.
[16] When Friday is not a business day, we use the previous business day.
[17] There are two other types that contribute to about five per cent of the total outstanding of securities in the dataset, and they are issued by four financial institutions only. Therefore, they were excluded from the dataset.

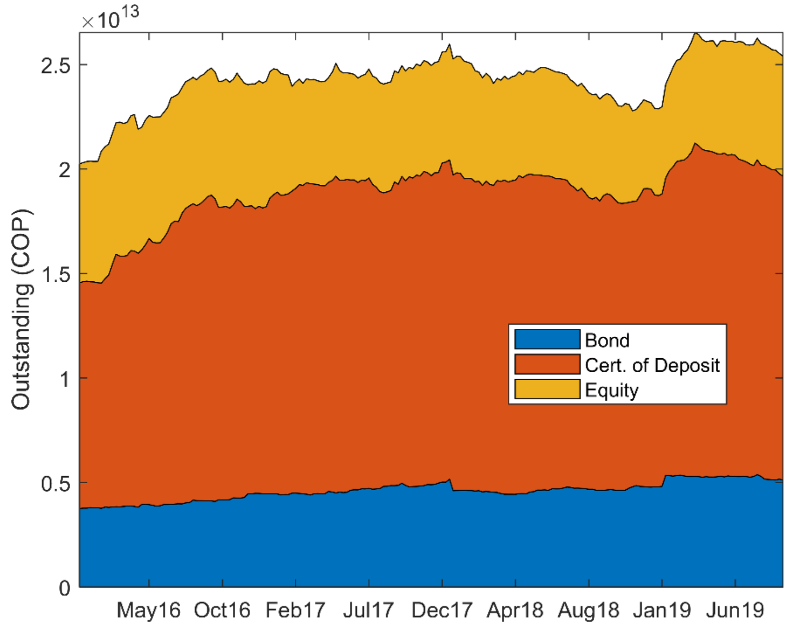


Figure 3. Evolution of securities issued and held by financial institutions, by type of security. Source: authors' calculations, based on data from Banco de la República and Deceval.

After processing, the dataset is transformed into a 4-dimensional adjacency matrix with dimensions 137 × 137 × 193 × 4. The first two dimensions correspond to the number of financial institutions in the sample, 137. The third dimension corresponds to the number of observations, 193 (i.e. all Fridays, from January 8th, 2016 to September 6th, 2019). The fourth dimension corresponds to the three types of securities and their aggregate.

5 Main results

Based on the dataset, we study four different networks. They correspond to bonds, certificates of deposit, equity, and their aggregation (i.e. *all*). Figure 4 exhibits the four networks as of the first week of September 2019. In all networks, nodes represent financial institutions, whereas the arrows represent holdings of securities, pointing from the issuer to the holder. For visualization purposes, the number and position of financial institutions are preserved across the four networks in a circular layout; the identity of financial institutions is concealed for confidentiality reasons.

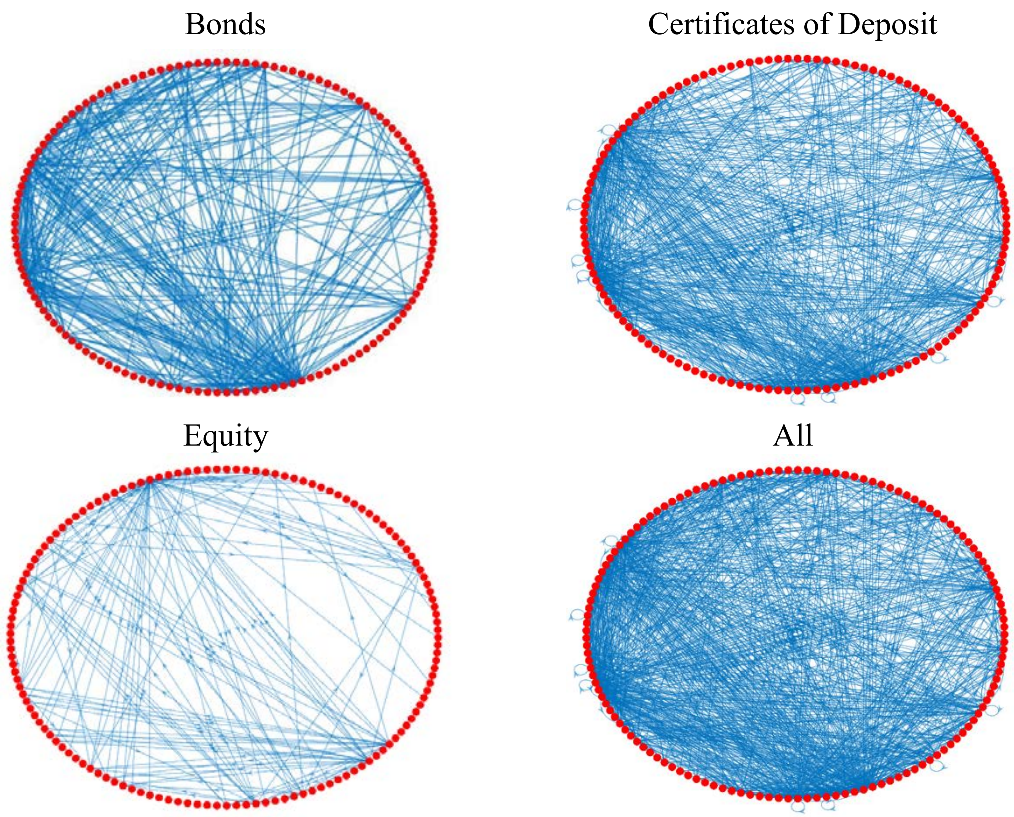


Figure 4. Network graphs of securities issued and held by financial institutions, as of the first week of September 2019. Source: authors' calculations, based on data from Banco de la República and Deceval.

As expected, the fourth network, corresponding to the aggregation of the three other networks is the densest in visual inspection. Individually, the certificates of deposit network is the densest, followed by bonds and—lastly—the equity network. Table 1 exhibits the main average topological features of the four networks calculated on 193 weeks from January 2016 to September 2019; percentiles 5 and 95 are reported in brackets.

Statistic	Bond	Cert. of Dep.	Equity	All
Contribution to All	18.94	58.75	22.31	100.00
(× 100)	[16.91; 20.93]	[53.60; 61.46]	[19.05; 28.50]	[100; 100]
Number of participants [a]	75.19	103.73	71.30	124.65
	[71; 79]	[98; 108]	[63; 78]	[120; 129]
Density	1.76	4.08	0.88	5.53
(× 100) [b]	[1.67; 1.87]	[3.82; 4.47]	[0.74; 1.08]	[5.27; 5.87]
Reciprocity	1.06	6.75	0.00	5.89
(× 100) [c]	[0.04; 1.85]	[5.69; 7.83]	[0.00; 0.00]	[4.66; 7.08]
Transitivity	0.01	0.27	0.00	0.30
(× 100) [d]	[0.00; 0.04]	[0.19; 0.36]	[0.00; 0.00]	[0.18; 0.39]
Survival ratio	96.74	97.39	97.25	97.55
(× 100) [e]	[94.40; 98.76]	[96.12; 98.42]	[93.93; 99.40]	[96.36; 98.62]

Table 1. Networks' average statistics calculated on 193 weeks from January 2016 to September 2019; percentiles 5 and 95 are reported in brackets. [a] Number of nodes with at least one linkage in the network; [b] fraction of possible linkages observed in the network; [c] fraction of linkages that are reciprocated; [d] fraction of transitive relations observed in the network; [e] fraction of linkages that survived in two consecutive periods. Source: authors' calculations, based on data from Banco de la República and Deceval.

As shown in Figure 3, the most contributive network (by value) is the one corresponding to certificates of deposit (58.75 percent), followed by equity (22.31 percent) and bonds (18.94 percent). The certificates of deposit network is also the one displaying the highest average number of participants (i.e. nodes with at least one linkage), with an average of about 103 financial institutions per week; bonds and equity networks have an average of 75 and 71 participants. When aggregated, the number of participants reaches an average of about 124 financial institutions. As exhibited in Table 1 (i.e. percentiles 5 and 95) and Figure 5, the number of participants of each network is somewhat stable throughout the period under analysis, except for the equity network, which displays a clear decreasing trend.

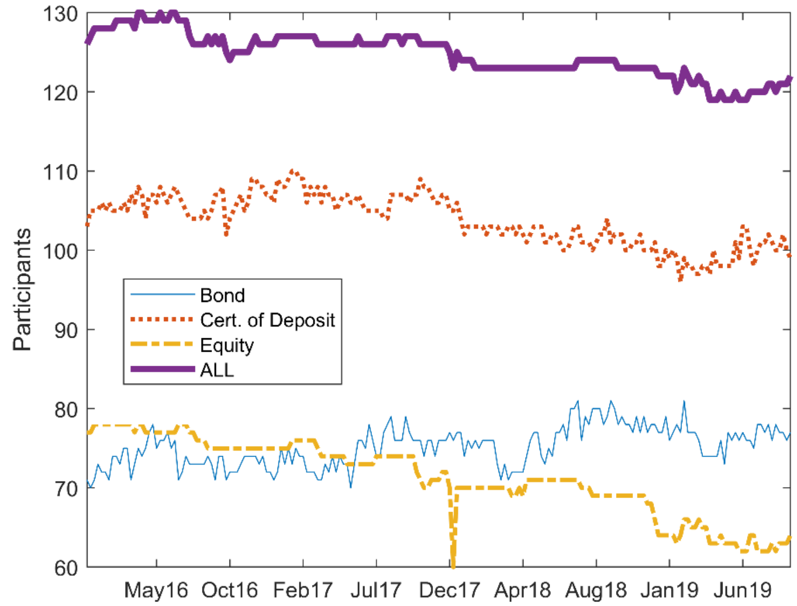


Figure 5. The number of participants. Source: authors' calculations, based on data from Banco de la República and Deceval.

Figure 6 shows the evolution of density for the four networks. As is the case with most financial networks reported in the literature, the density is particularly low.[18] The density of the network that aggregates all types of securities is on average 5.53 percent; that is, from all possible linkages, only about a twentieth is observed. Individually, as expected from visual inspection of Figure 6, the certificates of deposit network exhibits the highest density, on average 4.08 percent. The bond and equity networks display particularly low average densities, about 1.76 and 0.88 percent. In the case of equity, high sparseness is expected because there are some legal restrictions to financial institutions holding equity issued by themselves.[19] From an analytical viewpoint, the sparseness of the four networks suggests that financial institutions tend to hold securities issued by a few of their peers, mostly in the form of certificates of deposit, rarely in the form of equity.

[18] Interbank networks are typically sparse, i.e. low density (see Hüser, 2016). Density has been reported below 1 per cent in several interbank networks, such as those from the United States (see Furfine, 1999, Bech & Atalay, 2010), Germany (see Craig & von Peter, 2014), and Mexico (see Molina-Borboa, et al., 2015). In the Colombian case, most financial networks have been reported as sparse too, with some notable exceptions (see Berndsen, et al., 2018, León, et al., 2018, Ortega & León, 2018, León & Miguélez, 2020).

[19] For instance, banking institutions are not allowed to be the real beneficiaries of equity shares issued by other banking institutions. Also, banking institutions are not allowed to hold equity shares as collateral in excess of 10% of total shares of the other banking institution (see Decree-Law 666, April 5, 1993 and Law 510, August 4, 1999).

Compared to results reported by Hüser and Kok (2019) for the euro area case, securities cross-holding networks in Colombia are particularly sparse; only networks classified by Hüser and Kok as short-term debt cross-holding display similar levels of sparseness.

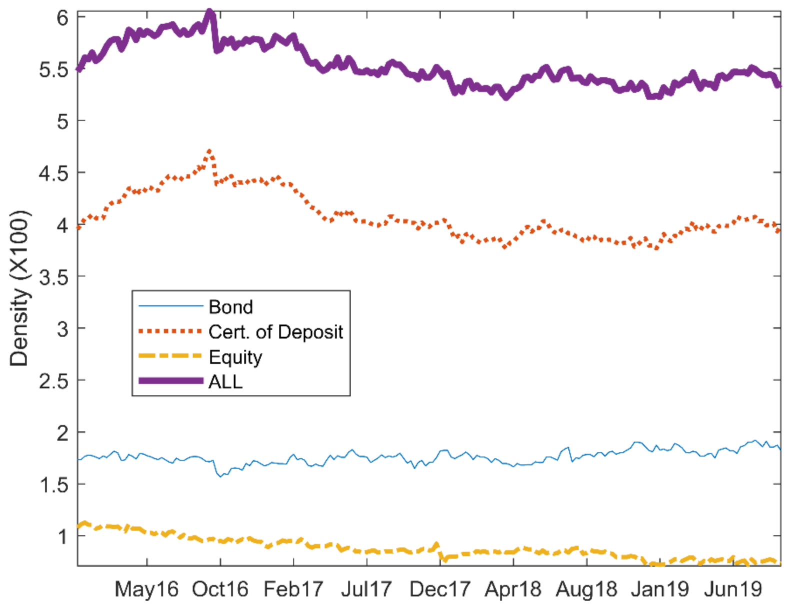


Figure 6. Density. Source: authors' calculations, based on data from Banco de la República and Deceval.

Regarding the direct securities cross-holding, consisting of reciprocal relations between financial institutions, Table 1 shows it is scarce. When all securities are aggregated into a single network, reciprocal relations are on average 5.89 percent of all relations. That is, about 94 percent of the holding of securities by financial institutions is not complemented by the reciprocal relation. Individually, the certificates of deposit network shows the highest average reciprocity (6.75 percent). Bonds and equity show particularly low levels of reciprocity, about 1 and 0 percent, respectively. Again, legal restrictions to financial institutions holding equity issued by their peers may explain the null reciprocity in the corresponding network. Consequently, it is fair to say that direct cross-holding of securities in the Colombian financial system is low and stable throughout the period under analysis (see Figure 7). Also, it is fair to say that cyclical interdependencies arising from reciprocal holdings of securities among financial institutions are uncommon.

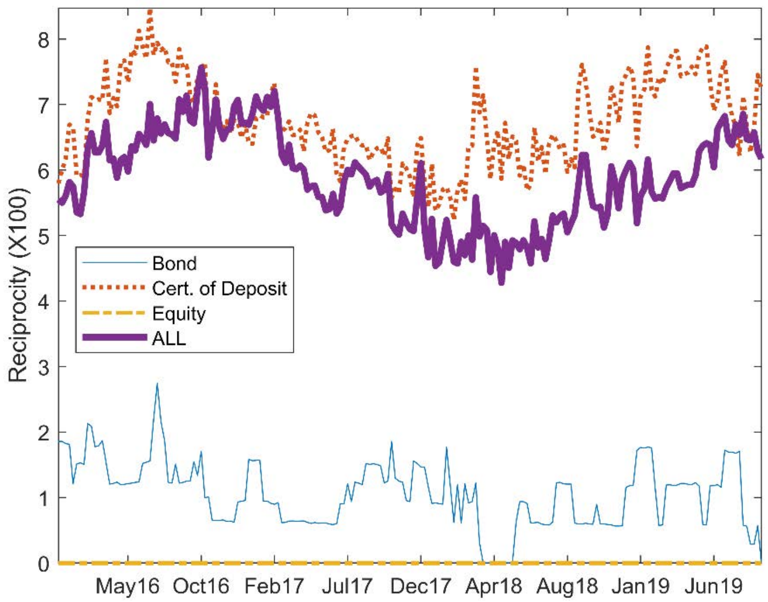


Figure 7. Reciprocity. Source: authors' calculations, based on data from Banco de la República and Deceval.

Transitivity is lower than reciprocity. The network that aggregates all types of securities has average transitivity of 0.30 percent, with a percentile 95 that reaches 0.39 percent. The three individual networks display lower levels of transitivity, with bonds and equity being zero or close to zero. Akin to reciprocity, transitivity levels suggest that indirect cross-holding is uncommon and stable throughout the period under analysis (see Figure 8). Together, the low levels of reciprocity and transitivity suggest that the incidence of cyclical interdependencies is particularly low. Consequently, the potential feedback contagion arising from securities cross-holding is small in the Colombian financial system during the period under analysis. Compared to results reported by Hüser and Kok (2019) for the euro area case, securities cross-holding networks in Colombia display low levels of transitivity.[20]

[20] Low levels of transitivity in Colombian securities cross-holding networks are at odds with stylized facts of interbank networks, which are typically transitive, i.e. clustered (see Hüser, 2016). Also, they are at odds with other financial networks in the Colombian case, which have been reported to display high or mild transitivity (see Berndsen, et al., 2018, León, et al., 2018, Ortega & León, 2018, León & Miguélez, 2020).

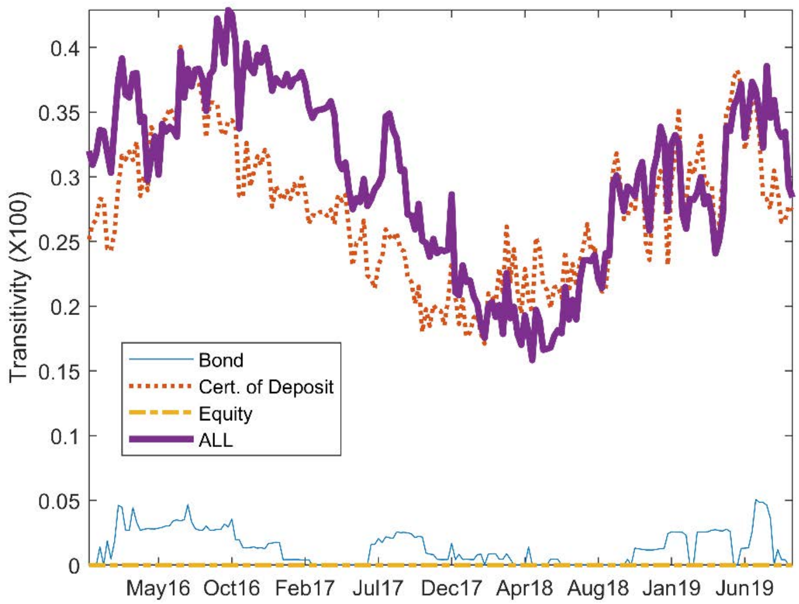


Figure 8. Transitivity. Source: authors' calculations, based on data from Banco de la República and Deceval.

Regarding the stability of the linkages in the networks, the survival ratio shows that they are rather stable from one week to the next throughout the period (see Figure 9). For all types of securities, the survival ratio is higher than 96 percent. This means that, on average, less than 4 percent of linkages disappear from one week to the next. Hence, as both the main topological features of the networks (i.e. low density, reciprocity and transitivity) and the relations between financial institutions are stable over time, it is fair to say that the low incidence of cyclical interdependencies arising from securities cross-holding is a structural trait of the Colombian financial system.

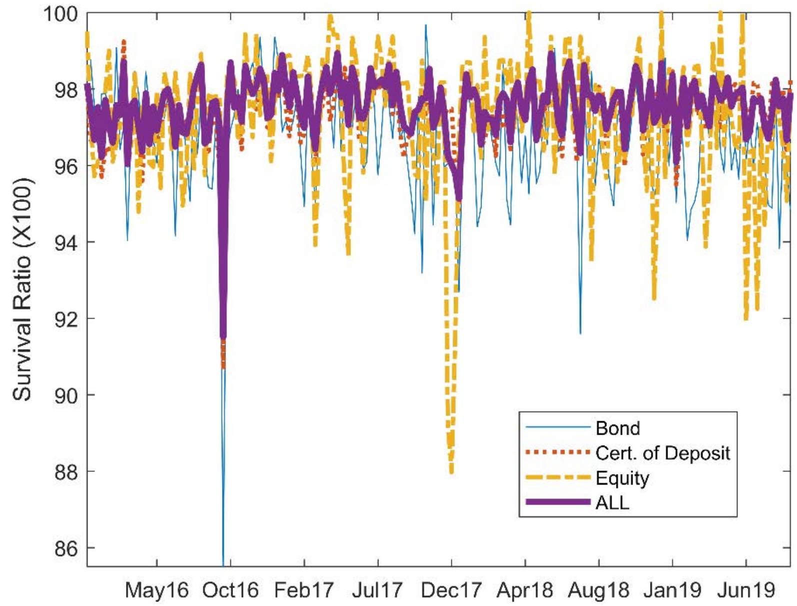


Figure 9. Survival ratio. Source: authors' calculations, based on data from Banco de la República and Deceval.

Table 2 exhibits the average Jaccard index, which measures how similar networks are in cross-section during the period under analysis. Interestingly, concurrent with results reported by Hüser and Kok (2019), as indexes corresponding to the similarity between individual networks are rather far from the upper limit (i.e. when the index is 1, networks are identical), it is clear that they are not very similar. The most similar pair of individual networks are those corresponding to bonds and certificates of deposit, with an average Jaccard index of about 0.21; that is, on average, circa one-fifth of the linkages is shared by both networks. The other two available pairs of networks are very dissimilar, with average Jaccard indexes close to zero. Following Hüser and Kok (2019), dissimilarity across individual networks suggests that it is somewhat unlikely that distress in one layer will spread to others through common connective patterns among nodes (i.e. point-wise similarity). With respect to the aggregation of individual networks, the overall securities network is similar to the certificates of deposit network, with an average Jaccard index of about 0.74. This suggests that most of the connective patterns among nodes in the aggregated network is inherited from the certificates of deposit network. However, akin to Hüser and Kok (2019), the overall dissimilarity shows that individual networks yield additional information, which reinforces the case of working under a multi-layer network approach.

	Bond	Cert. of Dep.	Equity	All
Bond	1.00			
Cert. of Dep.	0.21	1.00		
Equity	0.05	0.03	1.00	
All	0.32	0.74	0.16	1.00

Table 2. Average Jaccard index. Source: authors' calculations, based on data from Banco de la República and Deceval.

As displayed in Figure 10, similarity across networks is stable over time. The low and stable similarity between individual networks suggests that holdings of securities issued by other financial institutions are dependent on the type of security. That is, financial institutions do not hold securities based on the identity of the issuer alone but depending on other features such as the type of security. From a financial stability viewpoint, the rather dissimilar topology of individual networks suggests that contagion arising from common connective patterns across layers is limited. Also, topological dissimilarity supports studying each network and their aggregate to attain a comprehensive view of cross-holding in the securities market.

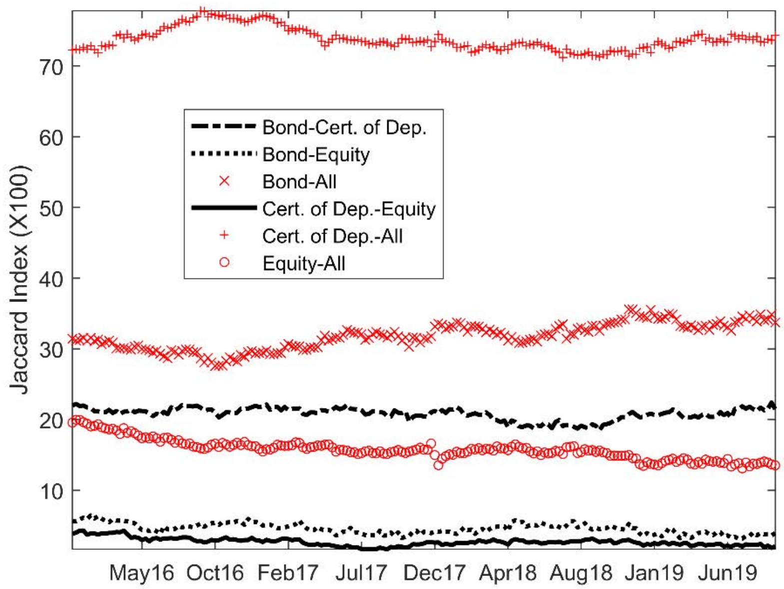


Figure 10. Jaccard index. Source: authors' calculations, based on data from Banco de la República and Deceval.

6 Test

Reciprocity and transitivity are particularly low. It is safe to say that securities cross-holding in the Colombian financial sector is negligible. Hence, from a financial stability viewpoint, the extent of cyclical interdependencies with the potential to cause feedback and reinforce contagion is small.

However, a comparison with a benchmark or null model is advisable. We perform limited randomization of the networks by randomly reallocating linkages from issuer to holder financial institutions in each observation while preserving the exact number of linkages of each network.[21] This randomization is performed 100 times for each of the 579 individual networks (i.e. 193 days, three types), whereas the aggregated network is constructed based on those randomized individual

[21] This is randomly shuffling observed linkages among actual issuer and holder institutions until the observed number of linkages is reached. Randomizing linkages within the entire network is not a sound choice because the existence of a small and stable group of issuers and holders would be disrupted. An alternative to the randomizing procedure herein implemented could be based on network modeling. However, not only it is a more complex and demanding procedure, but it is uncertain to what extent it could replicate over time the small and stable set of issuer and holder institutions that is observed in the network.

networks. The distributions of reciprocity and transitivity attained from randomized networks are compared with that from observed networks using a two-sample Kolmogorov-Smirnov test (at 99 percent confidence), which is a standard non-parametric test of the equality of probability distributions (see Table 3).

	Density ($\times$ 100) [a]		Reciprocity ($\times$ 100) [b]		Transitivity ($\times$ 100) [c]	
	Observed	Randomized	Observed	Randomized	Observed	Randomized
Bond	1.76	1.75	1.06†	1.15	0.01†	0.00
Cert. of Dep.	4.08	4.08	6.75†	4.74	0.27†	0.29
Equity	0.88	0.87	0.00†	2.26	0.00†	0.00
All	5.53	5.53	5.89	5.23	0.30	0.36

Table 3. Observed average density, reciprocity, and transitivity against a null randomized model. [a] Fraction of possible linkages observed in the network; [b] fraction of linkages that are reciprocated; [c] fraction of transitive relations observed in the network. † Null hypothesis of equal distribution of observed and randomized data is rejected at 99 percent confidence (i.e. p-value < 0.01) with Kolmogorov-Smirnov non-parametric two-sample test. Source: authors' calculations, based on data from Banco de la República and Deceval.

As expected, the average density is almost identical, and the Kolmogorov-Smirnov test is not rejected at the chosen confidence level for any of the networks. Regarding reciprocity, the Kolmogorov-Smirnov test is rejected in all three individual networks but is not rejected in the aggregate network. In the bond and equity networks, the observed average reciprocity is lower than that of the randomized network, whereas it is higher in the case of certificates of deposit and the aggregate networks. Although differences are not statistically negligible according to the distributional test, it is evident that average reciprocity in observed and randomized networks are particularly low, and not distant from zero in the bond and equity networks. Regarding transitivity, the Kolmogorov-Smirnov test is rejected in all three individual networks too, but differences concerning randomized networks are smaller than in reciprocity. Average observed and randomized transitivity levels are low and close to zero for all networks.

All in all, observed reciprocity and transitivity levels are particularly low. They are close (far) to (from) the lower (upper) limit of both measures, which are bounded to the [0,1] range. Moreover, observed reciprocity and transitivity levels are of the same order of magnitude (i.e. hundredths) as those calculated for randomized versions of the networks, which are also close to the lower limit of both measures. Rejecting the Kolmogorov-Smirnov test shows that observed networks significantly differ from the null model herein selected but does not challenge the main message from numerical outcomes: reciprocity and transitivity are particularly low, sometimes close to nil. In turn, this suggests that cyclical interdependencies and potential feedback effects prompted by reciprocal and transitive relations among financial institutions holding each other's securities are uncommon in the Colombian case. Furthermore, the observed cyclical interdependencies and potential feedback effects are not too different from what would result from a random allocation of linkages among issuers and holders of securities.

7 Final remarks

We study the network of exposures arising from institutions holding securities issued by other financial institutions—an often-neglected source of contagion in financial networks. We study the network topology to measure the extent to which reciprocal and transitive relations exist. Reciprocal and transitive relations result in what is known as securities cross-holding. Securities cross-holding among financial institutions is a latent source of contagion and instability as they prompt cyclical interdependencies and potential feedback effects in financial networks. Our dataset encompasses holdings of bonds, certificates of deposit, and equity issued and held by financial institutions in Colombia, from 2016 to 2019. The value of securities issued and held by financial institutions is sizeable: about 1.5 times that of secured and unsecured lending among them.

Evidence suggests that not only exposures in the securities network are scarce but also that reciprocal and transitive exposures are uncommon through the period under analysis—even when cross-holding across different types of securities is considered. On average, the frequency of reciprocal relations is below seven percent, with bonds and equity networks exhibiting frequencies below two percent. On average, the frequency of transitive relations is even lower, below one percent. Such low frequencies are stable over time. This suggests that the incidence of cyclical interdependencies and potential feedback effects in networks of securities issued and held by Colombian financial institutions are minor, rather close to nil. Consequently, potential contagion effects related to the network topology of the securities cross-holding in the Colombian financial system are rather low. That is, the low density, reciprocity, and transitivity circumscribe contagion by means of a weakly connected system (see Simon, 1962) that limits the interdependence among firms holding each other's securities. Consequently, from the financial stability viewpoint, results are optimistic.

Nevertheless, there is a major caveat. As we focus on the topology of the network, we disregard the monetary value of exposures and their size with respect to financial institutions' balance sheets. Therefore, results are conditional on the solvency of financial institutions. Studying how exposures caused by securities issued and held by financial institutions may impact the financial system's solvency (as in Battiston, et al., 2012b, Poledna, et al., 2015) is a compulsory research path from the financial stability viewpoint.[22]

Other challenges are pending for future research. First, our results regarding the limited extent of potential contagion are restricted to the exposures herein studied and should not be interpreted in isolation. All exposure networks should be studied—individually and in an aggregated manner—to achieve a comprehensive view of exposures in the financial system. Second, we focused on measuring the extent to which reciprocal and transitive relations exist in the networks under analysis. Other connective features of the networks, along with the importance (i.e. centrality) of financial institutions in the networks, should be studied to further understand the exposures created from financial institutions issuing and holding securities among them. Third, studying securities cross-holding within and between financial conglomerates is an interesting research path. Finally, as financial stability involves the real sector, it is important to replicate this study on a dataset comprising all agents that issue and hold securities in the economy.

[22] We expect counterparty concentration risk limits in Colombian financial regulation to curb the impact of securities crossholding on solvency For instance, there is an unsecured counterparty concentration limit for banking institutions equivalent to ten per cent of the regulatory capital (i.e. tier 1 and tier 2 capital); when the counterparty is a financial institution, the counterparty concentration limit is thirty per cent. A preliminary attempt (unpublished) to implement DebtRank (see Battiston, et al., 2012b) delivered rather low impact on solvency in the Colombian case. However, that attempt focused on interbank lending, thus a comprehensive implementation that includes several types of exposures (as in Poledna, et al., 2015) is still pending.

8 References

Adams, M. (1999). Cross holdings in Germany. *Journal of Institutional and Theoretical Economics, 155(1)*, 90-109.

Battiston, S., Delli Gatti, D.D., Gallegati, M., Greenwald, B., & Stiglitz, J.E. (2012). Default cascades: When does risk diversification increase stability? *Journal of Financial Stability, 8*, 138-149.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., & Caldarelli, G. (2012b). DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk? *Scientific Reports, 2:541*.

Baxter, G.J. Dorogovtsev, A. Glotsec, V., & Mendes, J.F.F. (2014). Avalanches in Multiplex and Interdependent Networks. In D'Agostino, G. & Scala, A. (Eds.), *Networks of Networks: the Last Frontier of Complexity* (37-52). Springer: Cham.

Bech, M.L. & Atalay, E. (2010). The topology of the federal funds market, *Physica A, 389*, 5223-5246.

Berndsen, R.J., León, C., & Renneboog, L. (2018). Financial stability in networks of financial institutions and market infrastructures. *Journal of Financial Stability, 35*, 120-135.

Bollobás, B., Janson, S., & Riordan, O. (2007). The phase transition in inhomogeneous random graphs. *Random Structures & Algorithms 31(1)*, 3–122.

Börner, K., Sanyal, S., & Vespignani, A. (2007). Network science. *Annual Review of Information Science and Technology, 41(1)*, 537–607.

Caccioli, F. Barucca, P., & Kobayashi, T. (2018). Network models of financial systemic risk: a review. *Journal of Computational Social Science, 1*, 81-114.

Caccioli, F., Shrestha, M., Moore, C., & Farmer, J.D. (2014). Stability analysis of financial contagion due to overlapping portfolios. *Journal of Banking & Finance, 46*, 233-245.

Cardillo, A., Gómez-Gardeñes, J., Zanin, M., Romance, M., Papo, D., del Pozo, F., & Boccaletti, S. (2013). Emergence of network features from multiplexity. *Scientific Reports, 3:1344*, 1-6.

Cimini, G., Squartini, T., Saracco, F., Garlaschelli, D., Gabrielli, A. & Caldarelli, G. (2019). The statistical physics of real-world networks. *Nature Reviews Physics, 1*, 58-71.

Clayton, M.J. & Jorgensen, B.N. (2005). Optimal cross holding with externalities and strategic interactions. *The Journal of Business, 78(4)*, 1505-1522.

Craig, B. & von Peter, G. (2014). Interbank tiering and money center banks. *Journal of Financial Intermediation*, *23*, 322-347.

Eisenberg, L. & Noe, T.H. (2001). Systemic risk in financial systems. *Management Science, 47(2)*, 236-249.

Elliott, M., Golub, B., & Jackson, M.O. (2014). Financial networks and contagion. *American Economic Review, 104(10)*, 3115-3153.

Elsinger, H. (2009). Financial networks, cross holdings, and limited liability. *Working Papers, 156*, Austrian Central Bank.

Fedenia, M., Hodder, J.E., & Triantis, A.J. (1994). Cross-Holdings: Estimation issues, biases, and distortions. *The Review of Financial Studies, 7(1)*, 61-96.

Fricke, D. & Lux, T. (2014). Core–periphery structure in the overnight money market: evidence from the e-MID trading platform. *Computational Economics, 45*, 359-395.

Furfine, C. (1999). The microstructure of the federal funds market, *Financial Markets, Institutions and Instruments*, *8*, 24-44.

Glasserman, P. & Young, P. (2016). Contagion in financial networks. *Journal of Economic Literature, 54(3)*, 779-831.

Gouriéroux, C., Héam, J.-C., & Monfort, A. (2012). Bilateral exposures and systemic solvency risk. *The Canadian Journal of Economics, 45(4)*, 1273-1309.

He, J., Huang, J., & Zhao, S. (2019). Internalizing governance externalities: The role of institutional cross-ownership. *Journal of Financial Economics, 134*, 400-418.

Hüser, A.-C. (2016). Too interconnected to fail: A survey of the interbank networks literature. *SAFE Working Paper, 91*, Goethe University Frankfurt-SAFE.

Hüser, A.-C. & Kok, C. (2019). Mapping bank securities across euro area sectors: comparing funding and exposure networks. *Staff Working Paper, 795*, Bank of England.

Jaccard, P. (1912). The distribution of the flora in the Alpine zone. *The New Phytologist, XI(2)*, 37-50.

Kawada, Y. (2016). Early-warning signals in interbank transactions: Evidence from Japan. *Bank of Finland Simulator Seminar,* Helsinki, August 25-26.

Lee, K-M., Kim, J.Y., Lee, S., & Goh, K.-I.. (2014). Multiplex Networks. In D'Agostino, G. & Scala, A. (Eds.), *Networks of Networks: The Last Frontier of Complexity* (53-72). Springer: Cham.

León, C. & Miguélez, J. (2020). Interbank relationship lending in Colombia. (*mimeo*)

León, C., Machado, C., & Sarmiento, M. (2018). Identifying central bank liquidity super-spreaders in interbank funds networks. *Journal of Financial Stability, 35*, 75-92.

Molina-Borboa, J.-L., Martínez-Jaramillo, S., López-Gallo, F., & van der Leij, M. (2014). A multiplex network analysis of the Mexican banking system: Link persistence, overlap and waiting times. *Journal of Network Theory in Finance, 1(1)*, 99-138.

Newman, M.E.J. (2010). *Networks: An Introduction*. Oxford University Press: New York.

Onnela, J.P., Chakraborti, A., & Kaski, K. (2003). Dynamics of market correlations: Taxonomy and portfolio analysis. *Physical Review E, 68*.

Ortega, F. & León, C. (2018). Las transferencias procesadas por ACH Colombia: Un análisis desde la perspectiva de topología de redes. *Lecturas de Economía, 88*, 109-153.

Poledna, S., Molina-Borboa, J.L., Martínez-Jaramillo, S., van der Leij, M. &, Thurner, S. (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability, 20*, 70-81.

Roncoroni, A., Battiston, S., D'Errico, M., Halaj, G., & Kok, C. (2019). Interconnected banks and systemically important exposures. *Bank of Canada Staff Working Paper, 2019-44*, Bank of Canada.

Sahabat, I., Silalahi, T., Indrastuti, R. & Herlina, M. (2017). Network motif of interbank payment as early warning signal of liquidity crisis: directed random graph and directed configuration model approaches. *Working Paper - LHP 2017*, Bank Indonesia.

Simon, H.A. (1962). The Architecture of Complexity. *Proceedings of the American Philosophical Society, 6(106)*, 467-482.

Squartini, T., van Lelyveld, I., & Garlaschelli, D. (2013). Early-warning signals of topological collapse in interbank networks. *Scientific Reports, 3:3357*.

Squartini, T. & Garlaschelli, D. (2014). Stationarity, non-stationarity and early warning signals in economic networks. arXiv:1403.4460v2 [q-fin.GN].

Suzuki, T. (2002). Valuing corporate debt: the effect of cross-holdings of stock and debt. *Journal of the Operations Research Society of Japan, 45(2)*, 123-144.

Trivedi, K. & Young, G. (2006). Corporate cross-holdings of equity, leverage and pensions: Simulation and empirical evidence from the UK. *The Economic Journal, 116(510)*, 190-208.

9 Appendix

Statistic	Bond	Cert. of Dep.	Equity	All
Contribution to All (× 100)	18.94 [16.91; 20.93]	58.75 [53.60; 61.46]	22.31 [19.05; 28.50]	100.00 [100; 100]
Number of participants [a]	75.19 [71; 79]	103.73 [98; 108]	71.30 [63; 78]	124.65 [120; 129]
Mean geodesic distance [b]	1.54 [1.28; 1.94]	2.22 [1.94; 2.52]	1.52 [1.47; 1.58]	2.36 [2.07; 2.60]
Density (× 100) [c]	1.76 [1.67; 1.87]	4.08 [3.82; 4.47]	0.88 [0.74; 1.08]	5.53 [5.27; 5.87]
Reciprocity (× 100) [d]	1.06 [0.04; 1.85]	6.75 [5.69; 7.83]	0.00 [0.00; 0.00]	5.89 [4.66; 7.08]
Transitivity (× 100) [e]	0.01 [0.00; 0.04]	0.27 [0.19; 0.36]	0.00 [0.00; 0.00]	0.30 [0.18; 0.39]
Survival ratio (× 100) [f]	96.74 [94.40; 98.76]	97.39 [96.12; 98.42]	97.25 [93.93; 99.40]	97.55 [96.36; 98.62]
Degree power-law coefficient [g]	3.31 [2.97; 3.58]	2.65 [2.32; 3.01]	2.54 [2.28; 3.14]	2.78 [2.63; 2.93]
Strength power-law coefficient [h]	2.20 [1.40; 3.87]	2.10 [1.46; 3.11]	1.45 [1.42; 1.52]	1.75 [1.45; 2.42]

Table 3. Networks' average statistics calculated on 193 weeks from January 2016 to September 2019; percentiles 5 and 95 are reported in brackets. [a] Number of nodes with at least one linkage in the network; [b] average minimal distance (i.e. measured in number of links) between all pairs of reachable nodes; [c] fraction of possible linkages observed in the network, under the assumption of no self-connections; [d] fraction of linkages that are reciprocated; [e] fraction of transitive relations observed in the network; [f] fraction of linkages that survived in two consecutive periods. [g] values in the [2,3] range suggest that the distribution of linkages (i.e. degree) approximates a power-law distribution (see Newman, 2010). [h] values in the [2,3] range suggest that the distribution of exposures values (i.e. strength) approximates a power-law distribution (see Newman, 2010). Source: authors' calculations, based on data from Banco de la República.

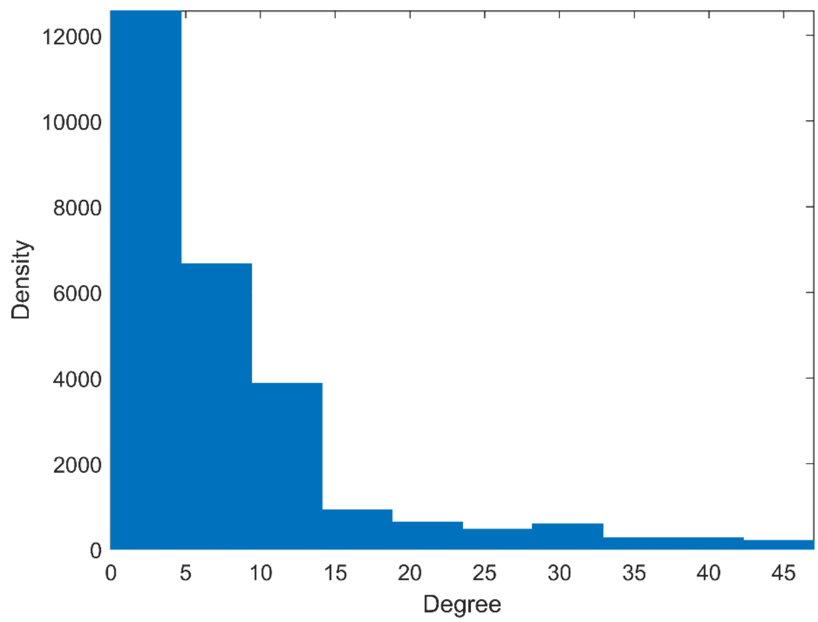


Figure 11. Distribution of the number of linkages (i.e. degree). Source: authors' calculations, based on data from Banco de la República and Deceval.

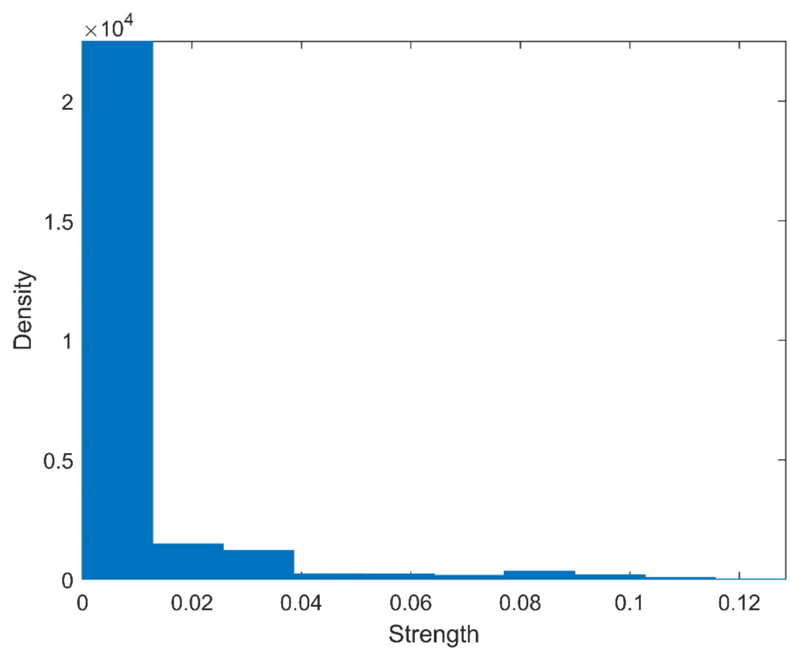


Figure 12. Distribution of linkages' contribution to the total value of exposures (i.e. strength). Source: authors' calculations, based on data from Banco de la República and Deceval.


BANCO DE LA REPÚBLICA
LIBERTAD
COLOMBIA