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4GM: A New Model for the Monetary Policy Analysis in Colombia[*]

Andres Gonzalez[†]   Alexander Guarin[‡]   Diego A. Rodriguez-Guzman[†]   Hernando Vargas-Herrera[§]
agonzalezgomez@imf.org   aguarilo@banrep.gov.co   drodriguezguzman@imf.org   hvargahe@banrep.gov.co

Banco de la República.

Abstract

This paper introduces 4GM, a semi-structural model for monetary policy analysis and macroeconomic forecasting in Colombia. This model is based on a New-Keynesian rational expectation framework for an oil-exporting small open economy. In this paper, we present the model structure and examine the response of its variables to domestic, foreign and oil-price shocks. Further, we assess 4GM in terms of its historical shock decomposition and its out-of-sample forecasting.

Keywords: semi-structural model, monetary policy, macroeconomic forecasting.

JEL Codes: E17, E37, E47, E52, E58

[†]Senior Economist of International Monetary Fund.

[‡]Head of the Models Development Section of Banco de la República.

[§]Deputy Technical Governor of Banco de la República.

4GM: Un Nuevo Modelo para el Análisis de Política Monetaria en Colombia[*]

Andres Gonzalez[†] Alexander Guarin[‡] Diego A. Rodriguez-Guzman[†] Hernando Vargas-Herrera[§]
agonzalezgomez@imf.org aguarilo@banrep.gov.co drodriguezguzman@imf.org hvargahe@banrep.gov.co

Banco de la República.

Resumen

En este documento se presenta el 4GM, un modelo semi-estructural para el análisis de política monetaria y el pronóstico macroeconómico en Colombia. El modelo sigue un enfoque Neo-Keynesiano con expectativas racionales para una economía pequeña y abierta exportadora de petróleo. En el artículo, presentamos la estructura del modelo, y examinamos la respuesta de sus variables a choques domésticos, externos y al precio del petróleo. Además, evaluamos el 4GM en términos de su descomposición histórica de choques y su pronóstico fuera de muestra.

Palabras clave: modelo semi-estructural, política monetaria, pronóstico macroeconómico.

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1   Introduction

Monetary policy operates in an uncertain environment and any central bank faces challenges when assessing the state of the economy. First, data are not perfect, there are significant revisions and some relevant information is only available with delays. Second, it is difficult to identify the source of shocks, their permanent or transitory nature, or how persistent they are. Third, central banks have no complete understanding of the structure of the economy and the transmission mechanism of monetary policy. The lag between the monetary policy actions and the response of the economy, as well as the strength of the different transmission mechanisms vary over time. In this environment, forecasting and risk assessment become an essential ingredient of the policy decision-making process. At Banco de la República the macroeconomic forecast is the outcome of substantial economic judgment from the Technical Staff (TS) and the Board of Directors. In fact, the forecast results of an iterative process involving meetings of the staff as well as the interaction between this team and the Board. In this process, macroeconomic models are essential because they provide a common language to discuss economic conditions and their implications on inflation and monetary policy, bring consistency to judgments and allow for risk analysis.

Banco de la República has undertaken several changes to enhance its policy decision-making process. Some of them aim at better integrating the analysis of the labor market, the financial conditions, and the fiscal policy into the quarterly macroeconomic assessment. Other changes seek to improve the quality of the economic analysis. Indeed, since 2018 the Bank reduced the number of monetary policy meetings from twelve to eight and their dates were aligned with major data releases. This arrangement enhances the predictability of the policy decision, facilitates its communication and contributes to the quality of the analysis since there is more available time to evaluate the economic issues that arise during the forecasting exercise, and to discuss them internally with the Board.

The TS evaluated the structure and statistical properties of the main macroeconomic models used by the Bank, namely *MMT* (Gómez et al. (2002); Bejarano (2002); Vávra (2003); Hamann (2005)) and *PATACON* (Bonaldi et al. (2011a); Bonaldi et al. (2011b); González et al. (2011)). After assessment, a new model (known as 4GM) is introduced to replace the MMT and that will be used simultaneously with PATACON in the monetary policy exercise.

The structure of this new model is presented in this document. 4GM is a semi-structural New-Keynesian rational expectation model for an oil-exporting small open economy. The 4GM is similar to the IMF's Global Projection Model (Carabenciov et al. (2008a,b); Carabenciov et al. (2013)), and other models used at several policy institutions (Coats et al. (2003); Andrle et al. (2013); Andrle et al. (2014); Charry et al. (2014); Obstfeld et al. (2016); Benes et al. (2017); Musil et al. (2018); Benlamine et al. (2018)). This similarity facilitates improvements through international cooperation and peer-to-peer information sharing.

The 4GM consists of four main behavioral equations: an IS curve, a set of four Phillips curves, an UIP condition, and a monetary policy rule. The Phillips curves characterize the inflation of the baskets of tradable, non-tradable, food and regulated goods[1]. Breaking down the Consumer Price Index (CPI) into these baskets adds flexibility to the model, enhances the monetary policy analysis, and captures the different sensitivities of each component to the output gap and the real exchange rate (RER) gap.

The oil price plays a fundamental role within 4GM, reflecting its importance in the Colombian economy. Changes in this price have direct implications on the main macroeconomic variables.

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[1]Following a long tradition at Banco de la República, the core inflation is splitted into the inflation of the tradable and non-tradable baskets.

Nevertheless, the transmission channels within the model depend on how persistent these changes are. Transitory changes affect the domestic demand while permanent changes have a direct impact on the potential output level and the trend component of the RER.

Since the Bank follows an IT regime, the main monetary policy instrument is the short-term interest rate which, in the model, follows a reaction function that responds to deviations of output and inflation expectations with respect to their long-term values. 4GM considers both the UIP condition and a flexible exchange rate regime. The UIP condition is standard for economies with an increasingly open financial account like Colombia. Likewise, the exchange regime assumption is also appropriate given the high-degree of flexibility of the exchange rate.

The 4GM's calibration reflects a set of stylized facts of the Colombian economy. It captures the role of external factors in explaining output dynamics, the high impact that food prices have on inflation, the partial pass-through from world oil prices to inflation explained by the offsetting role of the foreign exchange rate. The calibration also reflects a low pass-through of the exchange rate to domestic prices that results from the combination of a credible inflation targeting regime, exchange rate flexibility and the relatively low trade openness (IMF (2016); Carriere-Swallow et al. (2016)).

This paper is organized as follows. Section 2 presents some stylized facts of the Colombian economy and describes the monetary policy regime. Section 3 outlines the structure of the model, the data set and the estimation results. In Section 4, we illustrates the estimated impact of the most relevant shocks to the economy. Section 5 presents the historical shock decomposition recovered by the model and assesses the out-of-sample forecasting. Finally, Section 6 provides some conclusions.

2 Overview of the Colombian Economy

In this Section, we provide a brief illustration of the current monetary policy and foreign exchange rate regimes in Colombia, as well as some recent shocks faced by this economy.

2.1 Monetary Policy Regime and Foreign Exchange Rate

In 1999, after 32 years of managing the exchange rate in different ways, Banco de la República changed its monetary policy framework to inflation targeting (IT) and paired it with a flexible exchange rate. Under the full-fledged IT regime, a 10-year disinflation period followed during which the Bank set decreasing annual targets aiming to achieve a long-term inflation target of 3%. The IT strategy has been successful. Inflation has fluctuated around the target, its mean is well below the historical average values (Panel A in Figure 1), while inflation expectations are relatively well anchored (Panel B in Figure 1).

The fact that inflation came down along with the decreasing trend of annual targets, despite strong supply and external shocks, signaled a good understanding of the inflationary process, boosting the credibility of the Bank and its policy regime. This credibility was crucial to deal with two recent macroeconomic shocks. Firstly, the significant drop of the oil price at the end of 2014, leading to a nominal depreciation of almost 80%, and secondly, "El Niño" phenomenon from 2015 to 2016 that pushed food inflation up to 16%. On the back of these shocks, headline inflation rose from 2.13% in January 2014 to almost 9% in July 2016, but inflation expectations remained close to the target, allowing a gradual adjustment of the monetary policy stance.

Figure 1. Inflation and its Expectations

A. Headline Inflation

B. Inflation Expectations
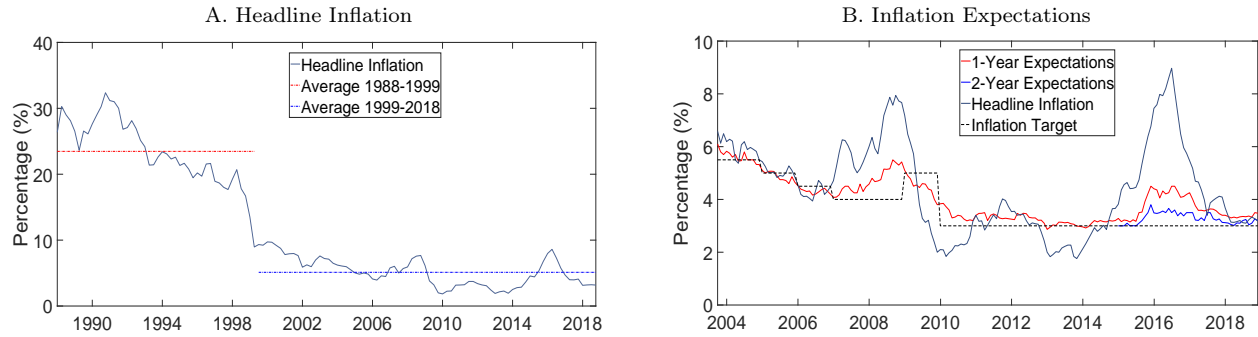
In Colombia, the flexibility of the exchange rate that accompanies the IT regime allows for an independent monetary policy. As expected for a small open economy, there should be some synchronization in the medium-term between the US Effective Federal Fund Rate (EFFR) and the Colombia's overnight interest rate (Figure 2). However, in the short-run this synchrony is weak. A simple correlation coefficient between the US and Colombian policy rates in the period 2009-2018 is nil (barely positive, 0.05%), and in a variance decomposition of the Colombia's monetary policy rate, shocks to the US FFR only explain around 1,5% of the total variance.

Under the current regime, the flexibility of the exchange rate has served as a shock absorber and has cushioned the effect that large external shocks could have had on the Colombian economy. For instance, the *2016 Latin America and Caribbean Macroeconomic IADB Report* (Powell (2016)) finds that the oil shock at the end of 2014 had a subtler macroeconomic impact on Colombia than on Ecuador, and it attributes the difference to the exchange rate regime. The comparison is meaningful since the oil revenue as a share of GDP is similar in both counties, but Ecuador is a fully dollarized economy.

The Colombian economy has benefited from the flexibility of the exchange rate in part because there have not been no large currency mismatches. Firstly, the dollar denominated debt of the non-financial sector in Colombia is small as proportion to the total debt and it is hedged to a significant extent (Figure 3). Secondly, a strong and careful regulation prevents currency mismatches in the financial sector.

Figure 2. Short-Term Interest Rates
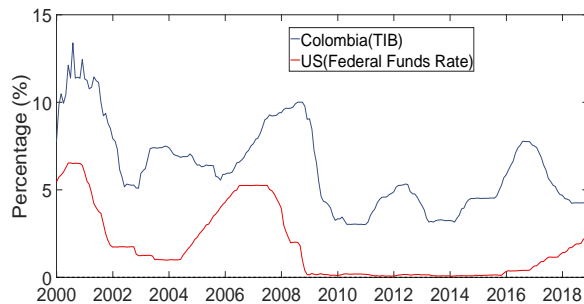
Figure 3. Colombian Debt
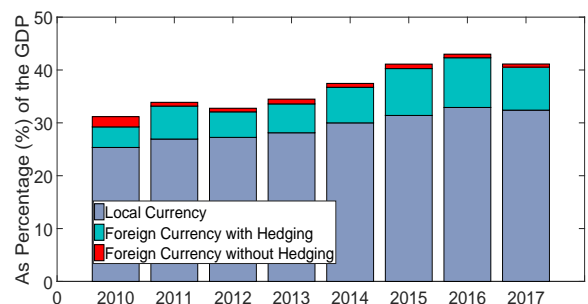
Within the IT framework, Banco de la República uses as an operational instrument the overnight interest rate and signals its policy stance through changes in the monetary policy rate. The current operational scheme ensures that commercial banks can place their surplus liquidity and obtain short-

term funding at stable and predictable rates. When setting the policy rate, the Bank aims to guide the overnight interbank rate (TIB), and the interest rates of deposits and loans of the banking system toward levels consistent with the achievement of the inflation target.

The scheme works remarkably well. The overnight interest rate departs from the policy rate by few basis points and the Bank provides short-term liquidity at a stable rate. Before IT regime, the interbank rates fluctuated widely since the Bank tried to set paths for the monetary aggregates or defended the exchange rate. The immediate effect of the change of the operational target was the stabilization of the overnight interbank rate (Panel A in Figure 4) and the release of a clear signal on the stance of monetary policy. In fact, in Colombia the transmission of the policy rate to market short- and long-term interest rates has been strong (Panel B in Figure 4).

Figure 4. Colombian Interest Rates
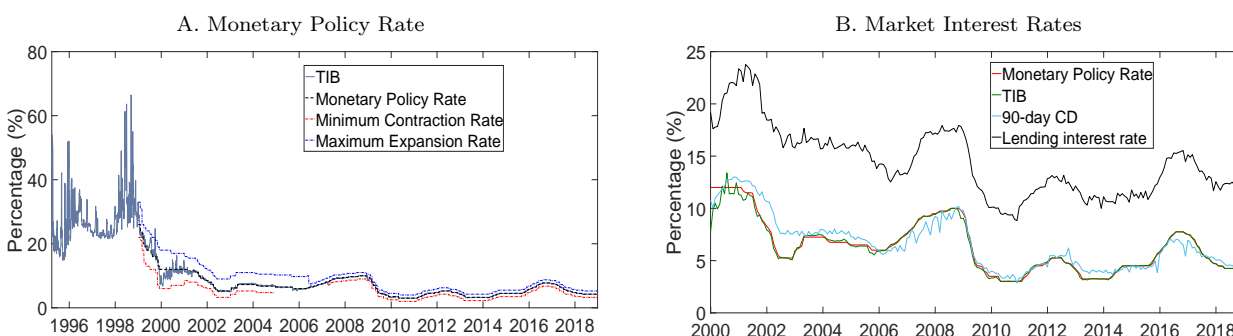
Within the flexible exchange rate regime, Banco de la República intervenes in the foreign exchange (FX) market following rules-based procedures and clear objectives (Cardozo (2019)). All purchases and sales by the Bank are sterilized so that the short-term interest rate does not deviate from the policy rate. FX purchases are mostly aimed at keeping an adequate level of international reserves, but they may be occasionally used to curb excessive volatility or short-term movements of the exchange rate that are not backed by its fundamentals. FX sales also may be used to meet these goals and solve temporary liquidity shortfalls in the FX market.

Intervention is usually announced and explained. Further, their different modalities are suited to its specific objectives. Intervention with the purpose of international reserve accumulation or smoothing volatility has been done through mechanisms that minimize any signal about an exchange rate target to maintain the consistency and credibility of monetary policy regime.

2.2   Some Recent Shocks Faced by Colombian Economy

The conduct of monetary policy in Colombia is a challenging task since the economy is subject to both extensive external shocks and frequent domestic supply shocks. In particular, shocks to the terms of trade, the growth rate of the trading partners, sovereign risk premia and foreign interest rates explain to a large extent business cycle fluctuations in Colombia and shocks to food and regulated prices drive the short-term fluctuations of the headline inflation.

For example, from 2006 to 2008, the increase in commodity prices, the strength of external demand, and a surge of capital inflows together with a strong shift in banks' asset portfolios generated an appreciation of the exchange rate, an expansion of domestic credit and an increase in the price of real assets that, coupled with an adverse shock to domestic food prices, pushed up inflation

and output. Similarly, between 2008 and 2009, Colombian economy suffered the consequences of the Global Financial Crisis (GFC) from which it recovered relatively quickly in part thanks to the recovery of international oil prices and ample external financial conditions.

Likewise, between 2014 and 2016 two significant shocks affected the Colombian economy. First, at the end of 2014 the oil price drop 50% from 120 USD to 60 USD per barrel, and second, between 2015 and 2016 "El Niño" phenomenon affected food and regulated prices, as well as the agricultural output.
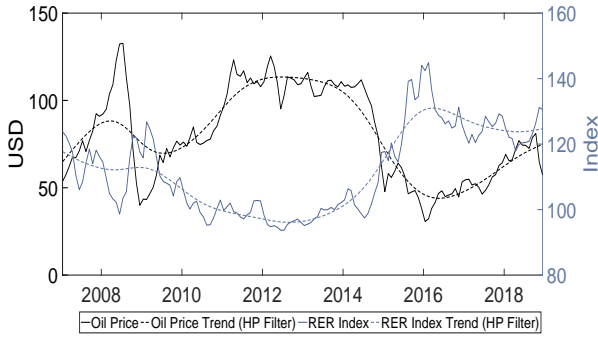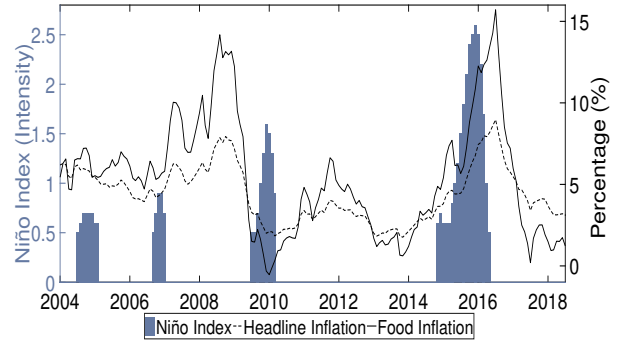
Figure 5. Oil Price and RER	**Figure 6.** "El Niño" Intensity

These shocks affected the economy through different channels. For example, an oil price shock affected both the output gap and the potential output level, but had a relatively limited impact on the inflation rate. The permanent drop in the oil price impacted the trend component of the RER (Figure 5) and the potential level of output, while the transitory component of oil affected mostly the output gap (Fernández et al. (2018)). The direct impact of oil price shocks on inflation was small, and occurred mostly through its effect on energy and regulated prices. Finally, the "El Niño" shock had transitory impacts on food and regulated prices, and on headline inflation with limited effects on inflation expectations (Figure 6).

3 Model, Data and Estimation

4GM is a semi-structural New-Keynesian model for an oil-exporting small open economy, with 4-CPI components and their relative prices, whose structure facilitates the forecasting process and the inclusion of off-model judgments. For instance, the endogenous variables in the model are decomposed between trend and cyclical components. This decomposition adds flexibility and allows shocks to have permanent and transitory effects.

The model has a well defined long-run equilibrium in which a) inflation converges to its target; b) the nominal interest rate converges to its neutral level; c) trends components are consistent with economic relations such as the UIP and the PPP; d) the trends of growth rates converge towards their steady state values; e) gaps are closed.

3.1 Structure of the Model

The model is divided into four blocks and a set of foreign variables. The first block considers the IS Curve and the potential output growth. The second block shows a Phillips curve for each CPI basket with their relative prices and carries out the CPI aggregation. The third block describes

the monetary policy rule. The fourth block explains how the foreign exchange rate is determined. 4GM's full structure is stated in Appendix A. In the following, we illustrate each of these blocks.

IS Curve and Potential GDP Growth

The output level y_t[2] is decomposed into its cyclical component $\hat{y}_t$ (i.e. output gap), which is considered an indicator of the business cycle, and its trend component[3] $\overline{y}_t$ (i.e. potential output). The cyclical component is modeled through an IS curve, in which the output gap $\widehat{y}_t$ is defined as

$$\hat{y}_t = \beta_1\widehat{y}_{t-1} + \beta_2\mathbb{E}_t\hat{y}_{t+1} - \beta_\Phi\Phi_t + \beta_{\hat{y}^\star}\hat{y}_t^\star + \beta_{\hat{r}p_t^{oil}}\widehat{rp}_t^{oil} + \eta_t^{\widehat{y}} \tag{1}$$

where $\widehat{y}_{t-1}$ captures the persistence of the economic cycle, and $\mathbb{E}_t\hat{y}_{t+1}$ is the forward looking component.

The output gap $\widehat{y}_t$ depends on a real monetary condition index $\Phi_t = \beta_{\hat{r}}\hat{r}_t - (1-\beta_{\hat{r}})\hat{z}_t$ which collects changes in business cycle derived from both the real interest rate gap $\hat{r}_t$ and the RER gap $\hat{z}_t$. The gap $\hat{r}_t$ captures the effects of the monetary policy on aggregate demand, while $\hat{z}_t$ captures the expenditure switching of changes in the exchange rate.

The IS curve also depends of a foreign output gap $\hat{y}_t^\star$ that captures external demand pressures arising from abroad, and a real oil price gap $\widehat{rp}_t^{oil}$ to reflect effects that transitory variations in this price have on the domestic demand. The output gap $\widehat{y}_t$ includes a demand shock $\eta_t^{\widehat{y}}$ that follows an AR(1) process given by $\eta_t^{\widehat{y}} = \beta_{\eta\widehat{y}}\eta_{t-1}^{\widehat{y}} + \varepsilon_t^{\widehat{y}}$ where $\beta_{\eta\widehat{y}}$ is the persistence, and $\varepsilon_t^{\widehat{y}}$ is a white noise demand shock.

The potential output grows at a rate[4]

$$\Delta\overline{y}_t = \rho_{\Delta\overline{y}}\Delta\overline{y}_{t-1} + (1 - \rho_{\Delta\overline{y}})\left(\Delta\overline{y}_{ss} + \kappa_{\Delta\overline{y}}\left(\Delta\overline{rp}_t^{oil} - \Delta\overline{rp}_{ss}^{oil}\right)\right) + \varepsilon_t^{\Delta\overline{y}} \tag{2}$$

which depends on its past $\Delta\overline{y}_{t-1}$, the long-term growth rate, and shocks to the potential growth $\varepsilon_t^{\Delta\overline{y}}$. Following Demidenko et al. (2016), the long-term component of output is function of the steady-state rate $\Delta\overline{y}_{ss}$ and the growth driven by the commodity sector, which is approximated with deviations of the trend growth of the real oil price from its steady-state rate $\left(\Delta\overline{rp}_t^{oil} - \Delta\overline{rp}_{ss}^{oil}\right)$ in a scale $\kappa_{\Delta\overline{y}}$[5].

Phillips Curves, Relative Prices and CPI aggregation

The short-term aggregate supply is modeled through Phillips curves that link inflation rates of 4-CPI baskets with proxies of the real marginal costs of each sector, namely *Tradable T*, *Non-Tradable NT*, *Food F* and *Regulated goods* R[6]. The CPI decomposition into its distinct baskets allows capturing the heterogeneity implicit in inflation rates, in terms of their long-term mean, their volatility, and their time-varying contribution to the headline inflation (Panels A and B in Figure 7).

This breakdown allows to capture the different elasticities of the inflation rate of each sector to the output gap $\hat{y}_t$ and the RER gap $\hat{z}_t$. For instance, the elasticity of non-tradable inflation with respect

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[2]Model variables are defined in logarithmic terms such that $y_t = ln(Y_t)$.

[3]This is the output level, that in the absence of shocks, do not generate inflationary pressures.

[4]Growth rates $\Delta(\cdot)_t$ correspond to annualized quarterly rates.

[5]From now on, parameters $\rho_{\Delta(\cdot)}$ and $1 - \rho_{\Delta(\cdot)}$ denote the persistence and the speed of adjustment towards the long-term value of growth rate $\Delta(\cdot)$, respectively.

[6]4GM works with baskets of tradable T and non-tradable NT goods following a traditional decomposition used by the TS within the Bank. However, 4GM is not a tradable/non-tradable T/NT goods model, and therefore, a different allocation could be considered (e.g. rigid and flexible prices).

to the RER is lower than in the tradable sector (Panel C in Figure 7), and the sensitivity of food inflation to these gaps is high compared to the other sectors. This occurs because more than 85% of the food basket includes goods that respond to the economic cycle (e.g. food outside of the home) and movements of the exchange rate (e.g. processed food).

Figure 7. Sectoral Inflation
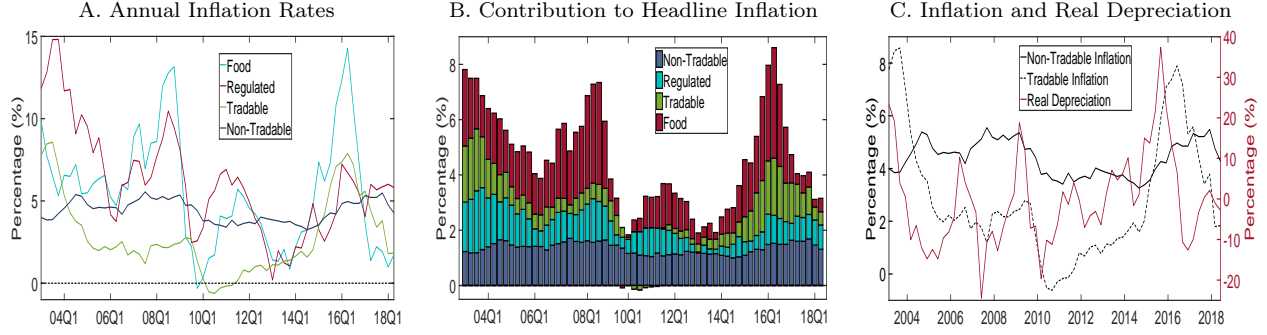
The inflation rate of each CPI component is modeled through a Phillips curve of the form

$$\pi_t^j = \alpha_{\pi^j}\pi_{t-1}^j + (1 - \alpha_{\pi^j})\mathbb{E}_t\pi_{t+1}^j + \alpha_{rmc^j}^{\pi^j}rmc_t^j + \varepsilon_t^{\pi^j} \quad for \; j = T, NT, F, R \tag{3}$$

where π_t^j is the annualized quarterly inflation, π_{t-1}^j reflects the inflationary inertia, $\mathbb{E}_t\pi_{t+1}^j$ represents the forward-looking component, and $\varepsilon_t^{\pi^j}$ is a supply shock of the sector j.

The real marginal cost rmc_t^j is given by

$$rmc_t^j = \begin{cases} \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + (1 - \alpha_{\hat{y}}^{rmc^j})(\hat{z}_t - \hat{rp}_t^j) & for \; j = T, NT \\ \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + (1 - \alpha_{\hat{y}}^{rmc^j})(\hat{rp}_t^{F^\star} + \hat{z}_t - \hat{rp}_t^j) & for \; j = F \\ \alpha_{\hat{y}}^{rmc^j}\hat{y}_t + (1 - \alpha_{\hat{y}}^{rmc^j})(\hat{rp}_t^{oil} + \hat{z}_t - \hat{rp}_t^j) & for \; j = R \end{cases} \tag{4}$$

which depends positively on the output gap $\hat{y}_t$ and the RER gap $\hat{z}_t$, and negatively on the relative price gap of its own sector $\hat{rp}_t^j$[7]. Further, rmc_t^j for food and regulated goods include the relative price gap of world food $\hat{rp}_t^{F^\star}$ and world oil price $\hat{rp}_t^{oil}$, respectively. These gaps capture changes in marginal costs linked to shifts in imported food prices, or changes in world oil prices. The adjustment mechanism of prices within the model makes that a positive (negative) relative price gap, reflecting deviations of it above (below) its trend, pressures real marginal costs, inflation and prices of the j-sector down (up), until the gap closes.

The aggregation of the headline price level p_t is given by

$$p_t = \omega^T p_t^T + \omega^{NT} p_t^{NT} + \omega^F p_t^F + \omega^R p_t^R + \eta_t^{p_t} \tag{5}$$

which is a weighted sum of p_t^j, the j-basket price index, using ω^j, weights for each basket within the CPI, plus a persistent shock η_t that follows a random walk[8]. Further, the equation

$$0 = \omega^T \hat{rp}_t^T + \omega^{NT} \hat{rp}_t^{NT} + \omega^F \hat{rp}_t^F + \omega^R \hat{rp}_t^R \tag{6}$$

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[7]This gap is defined as the difference between the relative price rp_t^j and its long-term trend component $\overline{rp}_t^j$. The former is computed as the difference between the j-sector price index p_t^j, and the CPI p_t, while the latter is defined as $\overline{rp}_t^j = \overline{rp}_{t-1}^j + \frac{\Delta\overline{rp}_t^j}{4}$. For NT, F and R sectors, the rate $\Delta\overline{rp}_t^j$ follows an AR(1) process, while for the T sector, 4GM sets up $\Delta\overline{rp}_t^T$ and the steady state $\Delta\overline{rp}_{ss}^T$ such that close the model by adjusting all prices, their gaps and trends

[8]This is a transitory shock that captures the approximation error in the CPI aggregation.

guarantees that the weighted sum of the relative price gaps is zero, and that the price level equals its trend.

Monetary Policy Rule and Interest Rates

The monetary policy rate i_t is set through a reaction function of the form

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\bar{i}_t + \varphi_\pi (\mathbb{E}_t \pi^A_{t+3} - \mathbb{E}_t \bar{\pi}^A_{t+3}) + \varphi_{\hat{y}} \hat{y}_t \right) + \varepsilon^i_t \tag{7}$$

where i_{t-1} is its lagged level, $\bar{i}_t$ is the neutral nominal interest rate and ε^i_t are monetary policy shocks. The reaction function also depends on the output gap $\hat{y}_t$ and the deviation of annual inflation expectations from its target three periods[9] ahead $(\mathbb{E}_t \pi^A_{t+3} - \mathbb{E}_t \bar{\pi}^A_{t+3})$. This formulation allows that shocks to headline inflation affect the policy rate.

The parameter ρ_i is the smoothing coefficient, φ_π and $\varphi_{\hat{y}}$ weight the expectations deviation $(\mathbb{E}_t \pi^A_{t+3} - \mathbb{E}_t \bar{\pi}^A_{t+3})$ and output gap $\hat{y}_t$ within the reaction function. The neutral rate $\bar{i}_t$ follows a Fisher equation $\bar{i}_t = \bar{r}_t + \mathbb{E}_t \pi_{t+1}$ where $\bar{r}_t$ is the neutral real interest rate, which is determined by the trend component of the real *Uncovered Interest Parity* (UIP) condition $\bar{r}_t = \bar{r}^\star_t + \bar{\vartheta}_t + \Delta \bar{z}_{ss}$, and $\mathbb{E}_t \pi_{t+1}$ denotes the model's quarterly inflation expectations at time $t + 1$. The variables $\bar{r}^\star_t$, $\bar{\vartheta}_t$ and $\Delta \bar{z}_{ss}$ are the US neutral real interest rate, the trend component of the risk premium, and the steady-state depreciation of the RER trend, respectively. The variable $\bar{\pi}^A_t$ stands for the annual inflation target. The real interest rate gap $\hat{r}_t$ is computed as $\hat{r}_t = r_t - \bar{r}_t$, where $r_t = i_t - \mathbb{E}_t \pi_{t+1}$ is the real interest rate .

Determination of the Nominal and Real Exchange Rates

The UIP condition

$$i_t = i^\star_t + \vartheta_t + \Delta \mathbb{E}_t s_{t+1} + \varepsilon^s_t \tag{8}$$

links the interest rate differential with the expected nominal depreciation $\Delta \mathbb{E}_t s_{t+1}$, stated as the difference between the expected value of the exchange rate at time $t + 1$, $\mathbb{E}_t s_{t+1}$, and its current value at time t, s_t. The RER gap $\hat{z}_t$ is defined as $\hat{z}_t = z_t - \bar{z}_t$, where z_t is the RER, and $\bar{z}_t$ is its trend component which grows at an annualized quarterly rate given by

$$\Delta \bar{z}_t = \rho_{\Delta \bar{z}} \Delta \bar{z}_{t-1} + (1 - \rho_{\Delta \bar{z}}) \left(\Delta \bar{z}_{ss} - \nu_{\Delta \bar{z}} \left(\Delta \overline{rp}^{oil}_t - \Delta \overline{rp}^{oil}_{ss} \right) \right) + \varepsilon^{\Delta \bar{z}}_t \tag{9}$$

which depends on its lagged value, its long-term growth, and shocks to the rate $\varepsilon^{\Delta \bar{z}}_t$. The RER z_t is defined through a PPP condition $z_t = s_t + p^\star_t - p_t$ and $p^\star_t$ stands for the US price level. Following Demidenko et al. (2016), the long-term component of the RER for an oil-exporting country evolves around a steady-state growth $\Delta \bar{z}_{ss}$ and fluctuates around on it in accordance with the productivity and technology improvements driven by the commodity sector. This fluctuation is approximated by deviations of the trend growth of the real oil price[10] from its steady-state $\left(\Delta \overline{rp}^{oil}_t - \Delta \overline{rp}^{oil}_{ss} \right)$ in a proportion $\nu_{\Delta \bar{z}}$.

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[9]This representation of inflation expectations allows the Bank to react to both the effects that shocks have on the current quarterly inflation π_t and those that are transmitted to quarterly inflation expectations $\mathbb{E}_t \pi_{t+(\cdot)}$ over the next three periods. Even if shocks disappear without affecting inflation expectations, the Bank reacts to its effects on contemporaneous inflation.

[10]Positive (negative) shocks to the trend of the oil price will appreciate (depreciate) the trend RER.

Foreign variables

The real price of oil is computed as $rp_t^{oil} = p_t^{oil} - p_t^\star$ where p_t^{oil} is the oil price and $p_t^\star$ is the US price level. Its cyclical component $\hat{rp}_t^{oil}$ is defined as the difference between the real price rp_t^{oil} and its long-term trend component $\bar{rp}_t^{oil}$, which increases at an annualized quarterly rate $\Delta\bar{rp}_t^{oil}$. The foreign variables, specifically output gap of social partners $\hat{y}_t^\star$, US headline inflation $\pi_t^\star$ and the US nominal interest rate $i_t^\star$, follow AR(1) processes. The dynamics of the latter is driven by the US neutral real rate of interest $\bar{r}_t^\star$ together with the US inflation expectations $\mathbb{E}_t\pi_{t+1}^\star$ derived from a satellite model for the US economy. The dynamics of the country risk premium ϑ_t and its trend component $\bar{\vartheta}_t$, as well as the real price gap of world food $\hat{rp}_t^{F\star}$ are characterized by AR(1) processes. The gaps $\hat{rp}_t^{oil}$, $\hat{rp}_t^{F\star}$ and $\hat{y}_t^\star$ are computed off-model. All shocks are normally distributed $\varepsilon_t \sim N(0, \sigma^2)$.

3.2 Data, Steady-State Values and Model Estimation

4GM is estimated using 13 domestic variables and 9 foreign variables. The first set includes the real GDP y_t, in millions of Colombian Pesos, the monetary policy rate i_t, the annual inflation target $\bar{\pi}_t^A$, the nominal exchange rate s_t defined as COP/USD, the CPI p_t, the core CPI p_t^C, as well as the price index for the basket of tradable goods p_t^T, non-tradable goods p_t^{NT}, food goods p_t^F and regulated goods p_t^R. This data set also includes the trend components of relative prices of the non-tradable basket $\bar{pr}_t^{NT}$, the food basket $\bar{pr}_t^F$, and the regulated goods basket $\bar{pr}_t^R$.

The set of foreign variables comprises the US CPI p_t^*, the US monetary policy rate i_t^* proxied by the 1-Year US FED rate, the Colombian risk premium ϑ_t measured through the 5-year CDS spread on sovereign Colombian bonds, and the real oil price pr_t^{Oil} in US dollars. We also include estimates of the US neutral real interest rate[11] $\bar{r}_t^*$, the gaps for the foreign output $\hat{y}_t^*$ and the relative price of world food $\hat{pr}_t^{F*}$ [12], as well as the trend components of the risk premium $\bar{\vartheta}_t$ and the real oil price $\bar{pr}_t^{Oil}$. Trends and gaps of external variables correspond to off-model estimates that combine satellite models and judgments from the TS[13].

Steady-state values largely influence the medium- to long-term forecast of the model and capture some stylized facts of the Colombian economy. For example, the long-term inflation target $\bar{\pi}_{ss}^A$ is set at 3%, while the steady-state growth rate of the potential output $\hat{y}_{ss}$ is defined as 3.3%. The neutral real interest rate $\bar{r}_{ss}$ equals 2%, and the steady-state real depreciation $\Delta\bar{z}_{ss}$ is assumed equal to zero. We also assumed a steady-state value for the trend component of the risk premium $\bar{\vartheta}_{ss}$ equal to 1.5%, which is the average of the 5-Year Colombian CDS between 2006 and 2017. We also calibrated steady-state values for some foreign variables. The long-term US inflation $\bar{\pi}_{ss}^{*A}$ equals 2%, the US neutral real interest rate $\bar{r}_{ss}^*$ is set at 0.5%, which is consistent with an average of its estimates between 2004 and 2017. We also assumed that in the long-term, both oil and world food prices grow at the same rate that the US inflation.

In line with the data, we assume different long-term inflation rates of the core, non-tradable, food and regulated goods baskets. We set these steady-state values to match the means of observed inflation rates for specific periods within the sample 2003Q1-2017Q4, such that there were no large

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[11]This rate is estimated following the methodology by Laubach & Williams (2003)

[12]The foreign GDP is computed as a weighted average of GDP growth rates of Colombia's trading partners and the price index of world food is provided by the World Bank

[13]The trend- and cycle-decomposition is carried out using the Hodrick-Prescott filter with priors to reflect TS judgments.

shocks that could bias the long-term values[14]. Steady-state values for inflation rates of non-tradable, food and regulated goods baskets are set at 3.7%, 2.8% and 3.6%, respectively. For the tradable basket, its steady-state value is estimated at 2.2%, so that the weighted average of the long-term inflation rates equals the inflation target. From these data, we derive the steady-state values of the trend growth of relative prices for each CPI basket. Table 1 summarizes all steady-state values considered in 4GM.

Table 1. Steady-State Values

Variable		%	Variable		%
Trend GDP growth	$\Delta \overline{y}_{ss}$	3.30	US neutral real interest rate	$\overline{r}^*_{ss}$	0.50
Annual Inflation target	$\overline{\pi}^y_{ss}$	3.00	Country risk premium	ϑ_{ss}	1.50
Annual Foreign inflation	π^*_{ss}	2.00	Depreciation of the trend RER	$\Delta \overline{z}_{ss}$	0.00
Trend growth of relative prices of F	$\Delta \overline{pr}^F_{ss}$	-0.20	Neutral real interest rate	$\overline{r}_{ss}$	2.00
Trend growth of relative prices of R	$\Delta \overline{pr}^R_{ss}$	0.60	Trend growth of real prices of oil	$\Delta \overline{pr}^{oil}_{ss}$	0.00
Trend growth of relative prices of NT	$\Delta \overline{pr}^{NT}_{ss}$	0.70	Trend growth of real prices of world food	$\Delta \overline{pr}^{F*}_{ss}$	0.00

We carry out the estimation of model parameters using a Bayesian approach, which approximates the posterior distribution of the estimates using the MCMC Metropolis-Hastings algorithm. However, the model estimation does not rely only on data relationships inherited from the past, but also it should capture expected transmission channels (Demidenko et al. (2016)). In this context, we elicit prior distributions for parameters using a calibration exercise that makes the model's impulse-response functions (IRFs) consistent with macroeconomic theory, international experience and the TS judgment. In this line, the posterior estimation was conducted with our tight priors to reflect the findings of the calibration exercise. Table 2 reports the estimated parameters for the 4GM.

Table 2. Estimated Parameters

Parameter		Value	Parameter		Value	Parameter		Value
IS Curve			***Phillips' Curves***			***Phillips' Curves***		
Backward component	β_1	0.470	**Tradable**			**Regulated**		
Forward component	β_2	0.048	Backward component	α_{π^T}	0.405	Backward component	α_{π^R}	0.296
Monetary condition	β_Φ	0.140	Real marginal cost	$\alpha^{\pi^T}_{rmc^T}$	0.153	Real marginal cost	$\alpha^{\pi^R}_{rmc^R}$	0.020
Foreign output gap	$\beta_{\hat{y}^\star}$	0.102	Output gap weight	$\alpha^{rmc^T}_{\hat{y}}$	0.307	Output gap weight	$\alpha^{rmc^R}_{\hat{y}}$	0.848
Oil relative price gap	$\beta_{\hat{p}^{roil}_t}$	0.018	**Non-Tradable**					
Real interest rate gap	$\beta_{\hat{r}}$	0.750	Backward component	$\alpha_{\pi^{NT}}$	0.296	**Inflation Weights**		
Shock persistence	$\beta_{\eta \hat{y}}$	0.500	Real marginal cost	$\alpha^{\pi^{NT}}_{rmc^{NT}}$	0.074	Food basket	ω^F	0.28
			Output gap weight	$\alpha^{rmc^{NT}}_{\hat{y}}$	0.576	Regulated basket	ω^R	0.15
Taylor Rule			**Food**			Tradable basket	ω^T	0.26
Backward component	ρ_i	0.700	Backward component	α_{π^F}	0.312	Non-Tradable basket	ω^{NT}	0.31
Inflation weight	φ_π	1.500	Real marginal cost	$\alpha^{\pi^F}_{rmc^F}$	0.175			
Output gap weight	$\varphi_{\hat{y}}$	0.375	Output gap weight	$\alpha^{rmc^F}_{\hat{y}}$	0.641			

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[14]For example, climate phenomena like "El Niño", abrupt changes in world oil prices, excesses of international liquidity and persistent nominal appreciations.

4   Transmission Mechanisms

In this section, we present 4GM's impulse-response function (IRF) and describe the transmission channel of the most crucial shocks. The transmission mechanisms and the corresponding monetary policy response are characterized qualitatively. For this illustration, shocks are defined as positive, transitory, and equal to 100 basis points unless otherwise indicated.

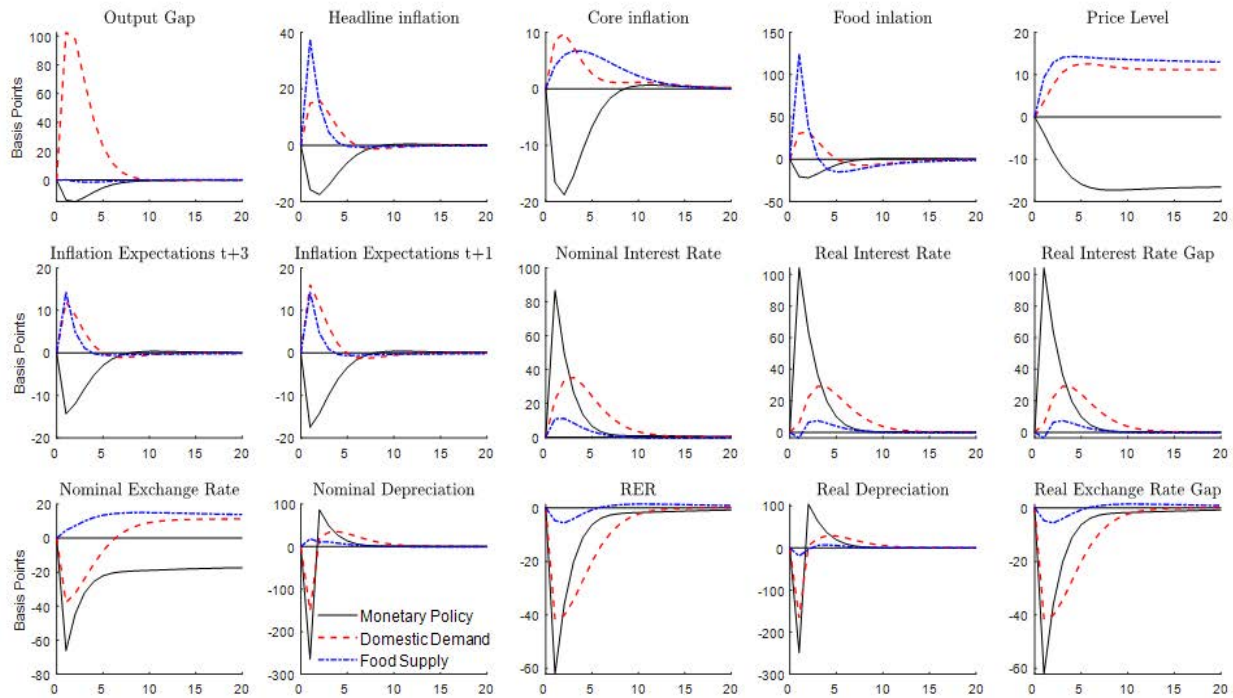### 4.1   IRFs: Domestic Shocks

Figure 8. IRFs: Domestic Shocks

Figure 8 shows the IRFs to the monetary policy shock (Black line), the domestic demand shock (red line), and the food supply shock (blue line). The *Monetary Policy Shock* implies an increase in the market rate along with a fall in inflation expectations that raises the real interest rate and opens a positive gap regarding to its neutral rate. Additionally, the rise of the interest rate leads to an immediate appreciation of the nominal exchange rate. The currency also appreciates in real terms and produces a negative gap with respect to its long-term non-inflationary trend. The contractionary monetary policy stance together with the negative RER gap, put downward pressure on the aggregate demand inducing a negative output gap, and in turn, a reduction of the headline inflation. As the monetary policy shock vanishes, gaps close while the price level and the nominal exchange rate get to a lower value than its initial point. The remaining variables return to their steady-state values.

The *Domestic Demand Shock* involves a positive output gap that pressures inflation upward and induces a deviation of its expectations above the long-term target. In response to this shock, the Central Bank raises the interest rate, the policy stance becomes contractionary, and the RER gap turns negative. These two forces push aggregate demand downward, closing the output gap and

leading the inflation as well as its expectations to their long-term equilibrium. Eventually, gaps disappear while both the price level and the nominal exchange rate reach higher values permanently consistent with a constant long-run RER. Qualitatively similar responses to the demand shock would be observed when facing either a foreign demand shock or an oil price gap shock.

The *Food Supply Shock* raises food inflation, headline inflation, inflation expectations, and the corresponding price levels. The monetary authority reacts by raising the interest rate. However, on impact, this reaction does not offset the increase in inflation expectations, and the real interest rate falls, creating a small negative gap with respect to its neutral level. Moreover, the currency depreciates in nominal terms, but appreciates in real terms, opening a small negative gap. In impact, the combined net effect of these two gaps results in a slightly positive output gap. The resulting sluggish reaction of monetary policy rate leads to downward pressures on inflation and inflation expectations that finally imply a monetary contractionary policy stance. The latter along with a negative RER gap lead to a negative output gap. Following the normalization of the policy stance, both real interest rate and RER gaps close, output returns to its potential level and headline inflation converges towards the inflation target. In the end, all gaps are closed, but the nominal exchange rate and the headline price level end permanently at higher values.

4.2   IRFs: Foreign Shocks
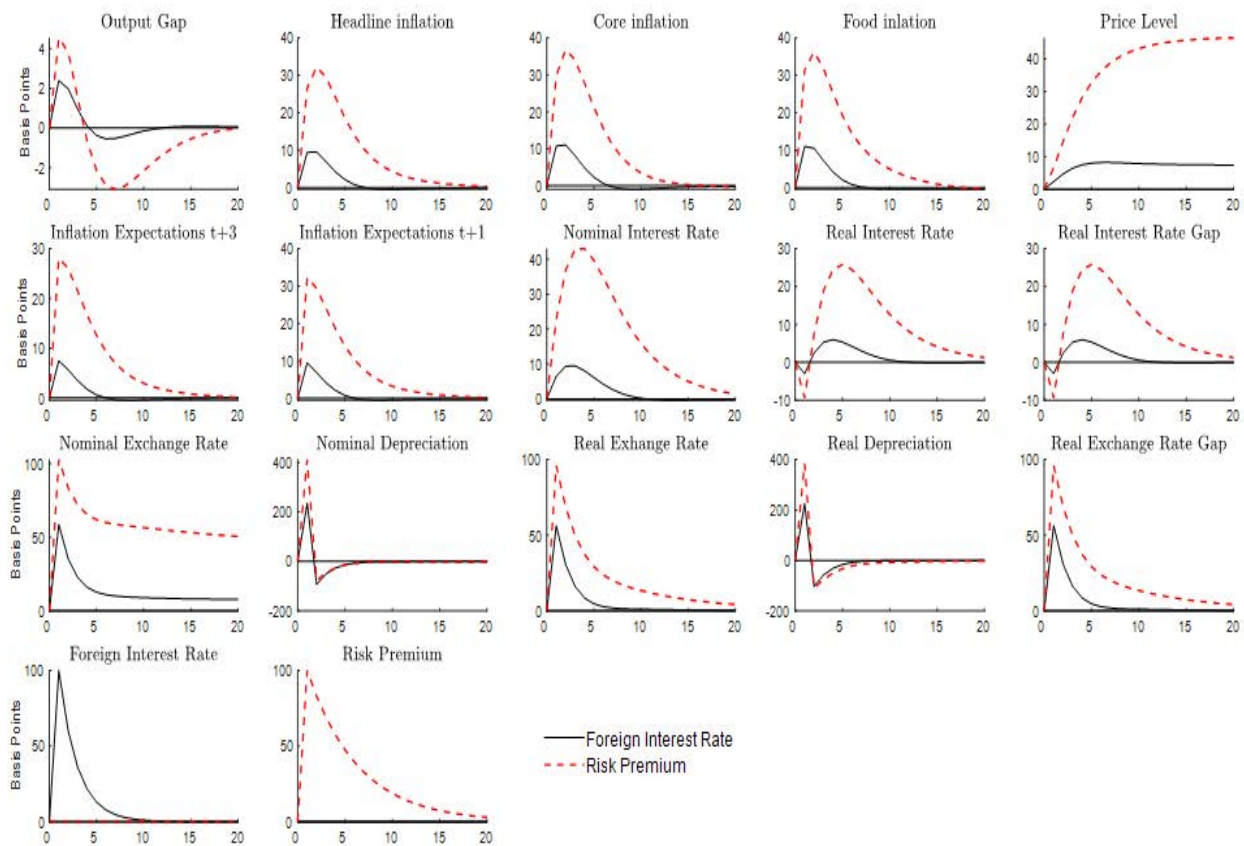
Figure 9. IRFs: Foreign Shocks

Figure 9 shows the IRFs to shocks to the foreign interest rate (Black line) and the risk premium (red line). These two shocks provide a qualitatively identical response, and their quantitative differences depend on the degree of persistence of each shock. The increase of either the foreign interest rate or the risk premium causes a real and nominal depreciation of the currency on impact. Accordingly, it produces a positive RER gap, generating demand pressures and a positive output gap[15]. Both the depreciated currency and the positive output gap push inflation up. On impact, the real interest rate falls because the initial response of monetary authority is not strong enough to offset the increase of inflation expectations. The persistent response of the central bank and the reduction of inflation expectations end up tightening the monetary policy, and the output gap becomes negative. The latter effect compensates the positive pressures of the RER gap and drives both inflation and its expectations towards the inflation target.

4.3   IRFs: Oil Price Shocks

Figure 10. IRFs: Oil Price Shocks
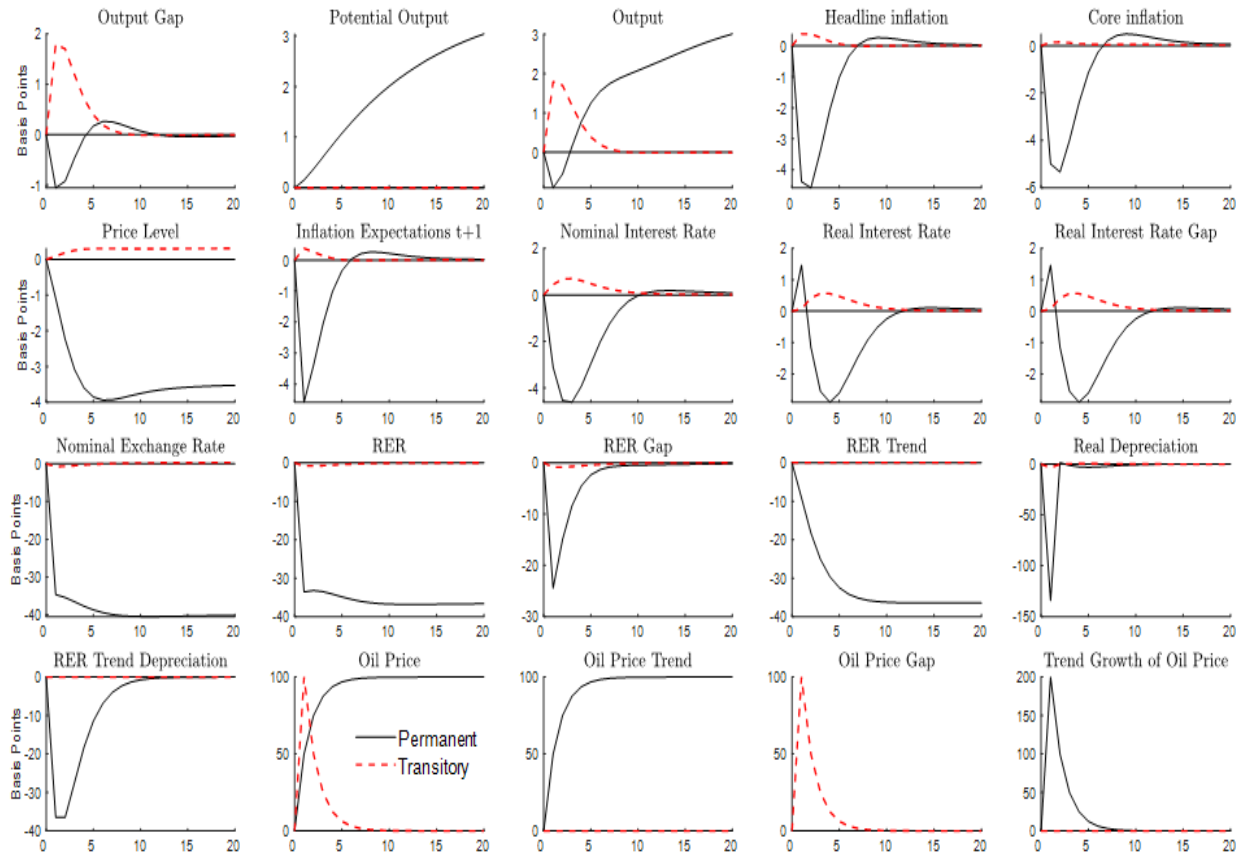
The effects of an oil price shock are different depending on its persistence. To illustrate this, Figure 10 exhibits the IRFs of both a permanent shock (Black line) and a transitory shock (Red line) to

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[15]This result reflects a relatively strong "Mundell-Fleming" effect in the model and the absence of other channels in which a tightening of external conditions may negatively affect output (e.g. Balance-sheet effects).

the oil price. Firstly, we assumed a shock to the growth rate of its trend component, reflecting a permanent but not immediate shock of 100 basis point to its level.

A *Permanent Shock to the Oil Price* implies a real appreciation of the level and trend component of the currency, though the level falls below its trend, resulting in a negative gap[16]. This shock also implies an increase in the potential output. However, the output level initially drops because of the negative effect of the real appreciation of the currency on aggregate demand, and thus, a negative output gap arises. The combined effect of these gaps drives inflation and its expectations down. Following a reaction function, the monetary authority lowers the policy rate. Initially, the real interest rate gap is contractionary because inflation expectations drop faster than the interest rate, but as long as the policy rate keeps going down and inflation expectations start increasing, the monetary policy stance turns out expansionary, closing the output gap. The prolonged appreciation of the exchange rate together with a persistent monetary easing bring the output level to its potential and inflation back to the target.

In a transitory shock to the oil price, the potential output and the long-term trend of the RER are not affected. Instead, this shock causes to a positive oil price gap that increases regulated inflation and pushes aggregate demand upward, opening a positive output gap. As a consequence, both core and headline inflation and their expectations move above the target. In response, the Central Bank raises the policy rate to counteract the inflationary pressures, so that the currency appreciates and aggregate demand is reduced. The contractionary policy stance along with the negative RER gap close the positive output gap and lead inflation and its expectations back to the long-term target.

5   Model Evaluation

In this section, we assess 4GM in terms of both its historical shock decomposition[17], and its out-of-sample conditional forecasting.

5.1   Historical shock decomposition (HSD)

Panels A through F in Figure 11 illustrate, for the period 2007-2018, the HSD of six main macroeconomic variables: Output gap, headline inflation, inflation expectations, monetary policy rate, real interest rate gap and RER gap. Three periods deserve particular attention. The first one goes between 2007 and the third quarter of 2008. During this time, the economy was booming on the back of a vigorous external demand, high commodity prices, and substantial capital inflows. The positive output gap and the rise mainly of food and regulated goods prices pushed headline inflation and its expectations above the inflation target, despite an appreciated currency due to a low risk premium and persistently high oil prices. At that time, Banco de la República raised the policy rate to counteract the inflationary pressures and implemented transitory capital flows management measures to curb portfolio inflows.

The second period started with the GFC episode in the last quarter of 2008 and lasted until the third quarter of 2014. With the onset of the GFC ($2008Q4 - 2009Q2$), the output gap became slightly negative, headline inflation fell to levels close to its long-term target and the currency

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[16]The trend component of the RER falls gradually, following the deviation of the growth rate of oil price trend from its steady-state, while the RER level adjusts quickly to its new lower long-term level on impact, despite expected positive differentials between real foreign and domestic interest rates.

[17]It shows the historical contribution of each shock to deviations of endogenous variables from their corresponding steady state values.

depreciated on the back of high uncertainty in the international capital market and the low oil prices. Further, the Bank lowered its policy rate to a neutral stance. Nevertheless, over the next two years ($2009Q3 - 2011Q2$), the output gap turned out persistently negative due to the abrupt fall of the foreign and domestic demand, despite the raise of oil prices and a loosening monetary policy stance. During the following three years ($2011Q3 - 2014Q3$), the output gap was again positive due to high oil prices, that together with a low foreign interest rate led to an appreciated currency in real terms. The latter contributed to keep headline inflation close to target and allowed to continue with a looser monetary policy stance, except by 2012.

Figure 11. Historical Shock Decomposition

A. Output Gap (%) B. Headline Inflation (A, %) C. Inflation Expectations t+3 (A, %)

D. Monetary Policy Rate (%) E. Real Interest Rate Gap (%) F. RER Gap (%)

Domestic Demand | Food Inflation | Monetary Policy | Oil Prices (Gap) | Foreign Demand | Foreign Interest Rate | Others
Core inflation | Regulated Inflation | Exchange Rate | Oil Prices (Trend) | Foreign Inflation | Risk Premium
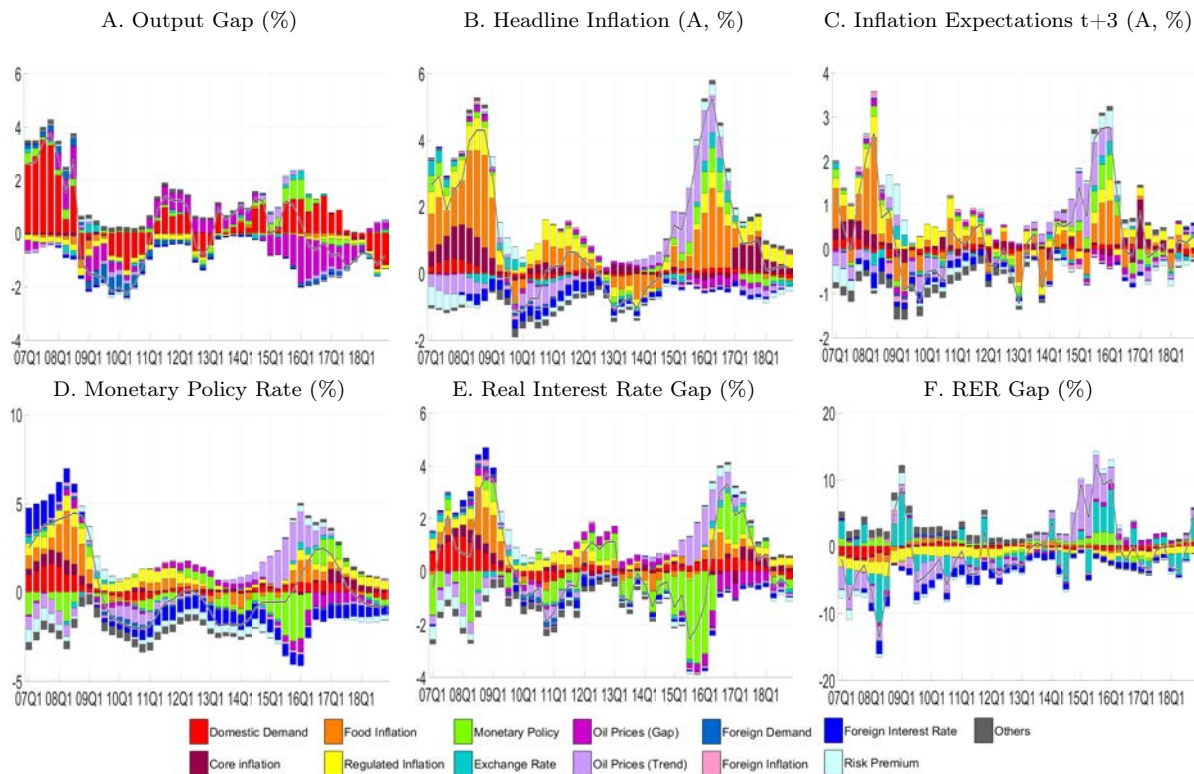
Between the fourth quarter of 2014 and the end of 2016, the economy sustained two large shocks. First, the international oil prices fell persistently causing a permanent depreciation of the currency and opening a positive gap with respect to its non-inflationary level. Second, "El Niño" phenomenon struck the economy, rising food and energy prices. The combined effect of these shocks was an increase in the headline inflation, and to a lesser extent, in the inflation expectations. The Bank raised the policy rate in September 2015 almost a year after of the initial shock, and after inflation expectations deviated significantly from their long-term target. This sluggish response of the monetary authority implied an expansionary policy stance until the first quarter of 2016. The loose monetary policy, together with some positive domestic demand shocks kept the output gap positive until third quarter of 2016.

Figure 12. OoS Conditional Forecasts: Standard Exogenous Paths
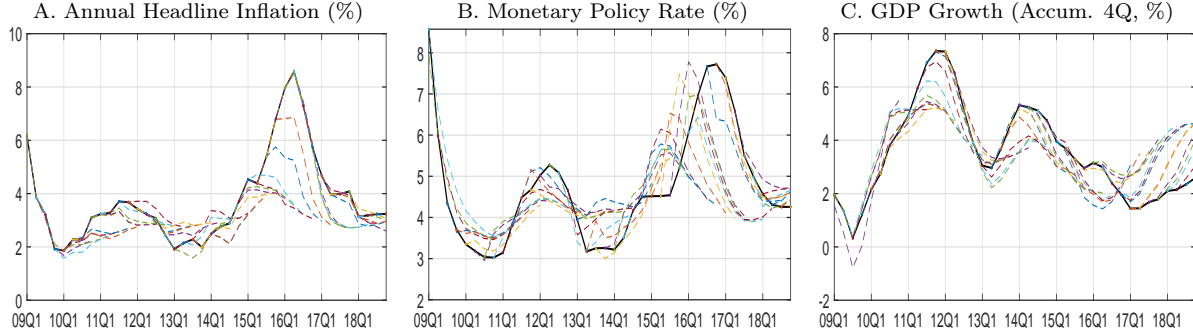
5.2 Out-of-Sample (OoS) Conditional Forecasts

We assess forecasts for each quarter t between 2009 and 2018, and a forecasting horizon h up to 8-quarter ahead. We carry out two exercises. The first one conditions forecasts on the exogenous paths of 8-quarter ahead foreign variables, 2-quarter ahead domestic inflations, and the nowcast of output gap[18]. The second exercise takes out the volatility introduced by food and regulated goods supply shocks by conditioning, additionally, on 8-quarter ahead inflation paths of these two sectors. Both exercises follow explicitly the 4GM's structure and parameters stated in Section 3.2, which differs from regular practice adopted by TS that considers subjective judgements in the forecasting process[19].

Figure 12 plots, at each time t, 8-quarter ahead conditional forecasts (color dashed lines) and their corresponding observed time series (black line) for the annual headline inflation (Panel A), the monetary policy rate (Panel B) and the GDP growth rate[20] (Panel C). For each variable plotted, forecasts do not exhibit any systemic bias, and show a suitable fitting regarding their observed values, especially before the fourth quarter of 2014. Nevertheless, between 2015 and 2017 forecasts deviated persistently due to both oil price and "El Niño" shocks[21]. For this latter period, forecasted paths wrongly over-anticipated the dynamics of actually observed paths, which is explained by the high uncertainty on parameters that reflect the persistence of these shocks.

Table 3 reports, for exercise one (Panel A) and exercise two (Panel B), the root mean squared forecasting errors (RMSFE)[22] for horizons up to 8-quarter ahead, and two periods: First, $2009Q1 - 2014Q3$ (Upper Panel), and second, $2014Q4 - 2018Q4$ (Lower Panel). Panel A supports results shown in Figure 12. Before the fourth quarter of 2014, forecasting errors were relatively small and not very volatile. However, once the oil price, "El Niño", and the truckers strike shocks appeared

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[18]The set of foreign variables include $\bar{rp}_t^{oil}$, $\hat{rp}_t^{oil}$, $\hat{rp}_t^{\star F}$, $\hat{y}_t^\star$, $\pi_t^\star$, $i_t^\star$, $\bar{r}_t^\star$, ϑ_t and $\bar{\vartheta}_t$, and are obtained from specialized entities. The set of domestic inflations considers short-term forecasts of π_t, π_t^C, π_t^T, π_t^{NT}, π_t^F and π_t^R, and the output gap nowcast, which are produced by TS. However, as these historical time series are not available, we run exercises assuming as forecasts for time $t + h$ their corresponding observed values.

[19]e.g. the nature of shocks, an specific stance on the trends of relative prices, the persistence of shocks and their characterization as anticipated or non-anticipated.

[20]This rate corresponds to the annual variation of GDP accumulated for 4 quarters.

[21]These shocks affected directly tradable, food and energy prices, and through the indexation process the non-tradable prices, the headline inflation and its expectations, as well as the corresponding monetary policy response.

[22]$RMSFE_h = \left(\sum_{i=0}^{T-(t+h)} \left(x_{t+h+i}^o - x_{t+h+i}^f \right)^2 / (T - (t+h)) \right)^{1/2}$, $i = 0, \ldots, T - (t+h)$ where x_{t+h+i}^o and x_{t+h+i}^f are the observation and its forecast for horizon h.

Table 3. Conditional RMSFE (%)

	A. Exercise 1								B. Exercise 2							
Variable (%)	Forecasting Horizon								Forecasting Horizon							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
2009Q1-2014Q3																
Headline Inflat.	0,00	0,00	0,29	0,49	0,52	0,61	0,66	0,68	0,00	0,00	0,07	0,12	0,15	0,20	0,25	0,29
Core Inflation	0,00	0,00	0,12	0,19	0,26	0,30	0,32	0,31	0,00	0,00	0,12	0,18	0,28	0,38	0,47	0,53
Policy Rate	0,42	0,51	0,50	0,52	0,55	0,57	0,59	0,62	0,42	0,51	0,54	0,62	0,70	0,71	0,72	0,72
GDP Growth	0,17	0,40	0,69	1,00	1,12	1,17	1,19	1,23	0,17	0,40	0,69	1,00	1,12	1,18	1,20	1,26
2014Q4-2018Q4																
Headline Inflat.	0,00	0,00	0,53	1,10	1,68	2,15	2,12	1,71	0,00	0,00	0,21	0,29	0,39	0,49	0,47	0,40
Core Inflation	0,00	0,00	0,37	0,58	0,85	1,17	1,33	1,36	0,00	0,00	0,36	0,50	0,65	0,82	0,81	0,71
Policy Rate	0,79	1,21	1,20	1,24	1,42	1,69	1,90	1,98	0,79	1,21	1,37	1,41	1,35	1,26	1,18	1,20
GDP Growth	0,16	0,41	0,75	1,13	1,38	1,56	1,68	1,82	0,16	0,41	0,75	1,13	1,39	1,56	1,67	1,76

between the end of 2014 and the third quarter of 2016, these errors became bigger and increased faster with horizon h. In fact, for the $2014Q4 - 2018Q4$ period, RMSFE for the headline inflation, the core inflation[23], the policy rate and the GDP growth rate, became in average 2.9, 3.8, 2.7 and 1.3 times higher, respectively, than those reported for the $2009Q1 - 2014Q3$ period.

Panel B illustrates the reduction of forecasting errors after removing the uncertainty linked to food and regulated goods prices. This difference is particularly notorious for the $2014Q4 - 2018Q4$ period, which includes the effects of "El Niño" shock.

6   Conclusions

The 4GM is a semi-structural model for an oil-exporting small open economy, which captures the heterogeneity of prices involved in the different CPI baskets. The model also captures movements on relative prices to affect inflation forecasts. The results in terms of impulse-response function, historical shock decomposition and conditional forecasting performance illustrate the properties of the model, allows us to tell a coherent economic story and evidence the accuracy of its forecasts, and the convenience of its use for making policy decisions at the Central Bank.

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[23]Headline and core inflations RMSFE for $h = \{1, 2\}$ are equal to zero because short-term forecasts match observed values.

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Appendix A.   4GM: Model Structure

IS Curve and Potential GDP Growth

$$y_t = \overline{y}_t + \hat{y}_t$$

$$\overline{y}_t = \overline{y}_{t-1} + \frac{\Delta \overline{y}_t}{4}$$

$$\Delta \overline{y}_t = \rho_{\Delta \overline{y}} \Delta \overline{y}_{t-1} + (1 - \rho_{\Delta \overline{y}}) \left(\Delta \overline{y}_{ss} + \kappa_{\Delta \overline{y}} \left(\Delta \overline{rp}_t^{oil} - \Delta \overline{rp}_{ss}^{oil} \right) \right) + \varepsilon_t^{\Delta \overline{y}}$$

$$\hat{y}_t = \beta_1 \widehat{y}_{t-1} + \beta_2 \mathbb{E}_t \hat{y}_{t+1} - \beta_\Phi \Phi_t + \beta_{\hat{y}^\star} \hat{y}_t^\star + \beta_{\hat{rp}_t^{oil}} \widehat{rp}_t^{oil} + \eta_t^{\widehat{y}}$$

$$\Phi_t = \beta_{\hat{r}} \hat{r}_t - (1 - \beta_{\hat{r}}) \hat{z}_t$$

$$\eta_t^{\widehat{y}} = \beta_{\eta^{\hat{y}}} \eta_{t-1}^{\widehat{y}} + \varepsilon_t^{\widehat{y}}$$

Phillips Curves, Relative Prices and CPI aggregation

$$\pi_t^j = \alpha_{\pi^j} \pi_{t-1}^j + (1 - \alpha_{\pi^j}) \mathbb{E}_t \pi_{t+1}^j + \alpha_{rmc^j}^{\pi^j} rmc_t^j + \varepsilon_t^{\pi^j} \quad for\ j = T, NT, F, R$$

$$rmc_t^j = \begin{cases} \alpha_{\hat{y}}^{rmc^j} \hat{y}_t + (1 - \alpha_{\hat{y}}^{rmc^j})(\hat{z}_t - \hat{rp}_t^j) & for\ j = T, NT \\ \alpha_{\hat{y}}^{rmc^j} \hat{y}_t + (1 - \alpha_{\hat{y}}^{rmc^j})(\hat{rp}_t^{F\star} + \hat{z}_t - \hat{rp}_t^j) & for\ j = F \\ \alpha_{\hat{y}}^{rmc^j} \hat{y}_t + (1 - \alpha_{\hat{y}}^{rmc^j})(\hat{rp}_t^{oil} + \hat{z}_t - \hat{rp}_t^j) & for\ j = R \end{cases}$$

$$\hat{rp}_t^j = rp_t^j - \overline{rp}_t^j \quad for\ j = T, NT, F, R$$

$$rp_t^j = p_t^j - p_t$$

$$\overline{rp}_t^j = \overline{rp}_{t-1}^j + \frac{\Delta \overline{rp}_t^j}{4}$$

$$\Delta \overline{rp}_t^j = \rho_{\overline{rp}^j} \Delta \overline{rp}_{t-1}^j + \left(1 - \rho_{\overline{rp}^j} \right) \left(\Delta \overline{rp}_{ss}^j \right) + \varepsilon_t^{\Delta \overline{rp}^j} \quad for\ j = NT, F, R$$

$$p_t = \omega^T p_t^T + \omega^{NT} p_t^{NT} + \omega^F p_t^F + \omega^R p_t^R + \eta_t^{p_t}$$

$$\eta_t^{p_t} = \eta_{t-1}^{p_t} + \varepsilon_t^{p_t}$$

$$0 = \omega^T \hat{rp}_t^T + \omega^{NT} \hat{rp}_t^{NT} + \omega^F \hat{rp}_t^F + \omega^R \hat{rp}_t^R$$

Monetary Policy Rule and Interest Rates

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\overline{i}_t + \varphi_\pi (\mathbb{E}_t \pi_{t+3}^A - \mathbb{E}_t \overline{\pi}_{t+3}^A) + \varphi_{\hat{y}} \hat{y}_t \right) + \varepsilon_t^i$$

$$\overline{i}_t = \overline{r}_t + \mathbb{E}_t \pi_{t+1}$$

$$\overline{r}_t = \overline{r}_t^\star + \overline{\vartheta}_t + \Delta \overline{z}_{ss}$$

$$\overline{\pi}_t^A = \rho_{\overline{\pi}^y} \overline{\pi}_{t-1}^A + (1 - \rho_{\overline{\pi}^y}) \overline{\pi}_{ss}^A + \varepsilon_t^{\overline{\pi}^A}$$

$$\hat{r}_t = r_t - \overline{r}_t$$

$$r_t = i_t - \mathbb{E}_t \pi_{t+1}$$

Determination of the Foreign Exchange Rate

$$i_t = i_t^\star + \vartheta_t + \Delta \mathbb{E}_t s_{t+1} + \varepsilon_t^s$$

$$\Delta \mathbb{E}_t s_{t+1} = 4 \left(\mathbb{E}_t s_{t+1} - s_t \right)$$

$$\hat{z}_t = z_t - \bar{z}_t$$

$$z_t = s_t + p_t^\star - p_t$$

$$\bar{z}_t = \bar{z}_{t-1} + \frac{\Delta \bar{z}_t}{4}$$

$$\Delta \bar{z}_t = \rho_{\Delta \bar{z}} \Delta \bar{z}_{t-1} + (1 - \rho_{\Delta \bar{z}}) \left(\Delta \bar{z}_{ss} - \nu_{\Delta \bar{z}} \left(\Delta \overline{rp}_t^{oil} - \Delta \overline{rp}_{ss}^{oil} \right) \right) + \varepsilon_t^{\Delta \bar{z}}$$

Foreign variables

$$\hat{rp}_t^{oil} = rp_t^{oil} - \bar{rp}_t^{oil}$$

$$rp_t^{oil} = p_t^{oil} - p_t^\star$$

$$\bar{rp}_t^{oil} = \bar{rp}_{t-1}^{oil} + \frac{\Delta \bar{rp}_t^{oil}}{4}$$

$$\Delta \bar{rp}_t^{oil} = \rho_{\Delta \bar{rp}^{oil}} \Delta \bar{rp}_{t-1}^{oil} + (1 - \rho_{\Delta \bar{rp}^{oil}}) \Delta \bar{rp}_{ss}^{oil} + \varepsilon_t^{\Delta \bar{rp}^{oil}}$$

$$\hat{rp}_t^{oil} = \rho_{\hat{rp}^{oil}} \hat{rp}_{t-1}^{oil} + \varepsilon_t^{\hat{rp}^{oil}}$$

$$\hat{y}_t^\star = \rho_{\hat{y}^\star} \hat{y}_{t-1}^\star + \varepsilon_t^{\hat{y}^\star}$$

$$p_t^\star = p_{t-1}^\star + \frac{\pi_t^\star}{4}$$

$$\pi_t^\star = \rho_{\pi^\star} \pi_{t-1}^\star + (1 - \rho_{\pi^\star}) \pi_{ss}^\star + \varepsilon_t^{\pi^\star}$$

$$i_t^\star = \rho_{i^\star} i_{t-1}^\star + (1 - \rho_{i^\star}) \bar{i}_t^\star + \varepsilon_t^{i^\star}$$

$$\bar{i}_t^\star = \bar{r}_t^\star + \mathbb{E}_t \pi_{t+1}^\star$$

$$\bar{r}_t^\star = \rho_{\bar{r}^\star} \bar{r}_{t-1}^\star + (1 - \rho_{\bar{r}^\star}) \bar{r}_{ss}^\star + \varepsilon_t^{\bar{r}^\star}$$

$$\vartheta_t = \rho_\vartheta \vartheta_{t-1} + (1 - \rho_\vartheta) \bar{\vartheta}_t + \varepsilon_t^\vartheta$$

$$\bar{\vartheta}_t = \rho_{\bar{\vartheta}} \bar{\vartheta}_{t-1} + (1 - \rho_{\bar{\vartheta}}) \bar{\vartheta}_{ss} + \varepsilon_t^{\bar{\vartheta}}$$

$$\hat{rp}_t^{F\star} = \rho_{\hat{rp}^{F\star}} \hat{rp}_{t-1}^{F\star} + \varepsilon_t^{\hat{rp}^{F\star}}$$