A Model of Rule-of-Thumb Consumers With Nominal Price and Wage Rigidities

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A Model of Rule-of-Thumb Consumers With Nominal Price and Wage Rigidities*

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Abstract

This document presents a dynamic stochastic general equilibrium model with rule of thumb (Non-Ricardian) agents and both nominal price and wage rigidities. The model follows closely that of Galí et al. (2004) and expands it to include a second way form of heterogeneity (besides the Non-Ricardian agents), namely the nominal wage stickiness á la Calvo, as in Erceg et al. (2000). Special attention is given to the algebraic details of the model. The model is calibrated and its dynamics are explored trough the analysis of impulse response functions.

Keywords: DSGE, nominal rigidities, rule of thumb consumers.
JEL Classification: E32, E37, C68.

The model discussed in this paper is a modified version of that of Galí et al. (2004) and represents an economy with two types of agents. The first is the usual agent known to the literature. The second type of agent doesn't have access to any mechanism to smooth consumption over time, both the capital and the bond markets are closed to them, for such reason it is known as Non-Ricardian or Rule-of-Thumb agent. Following Erceg et al. (2000) the individuals (both Ricardian and Non-Ricardian) are assumed to offer a differentiated type of labor, the wage that the agents charge for their labor is adjusted in an staggered way,[1] in this sense this work is similar to that of Colciago (2011), the difference lies in the way nominal rigidities are introduced, allowing them to affect only Ricardian or Rule-of-Thumb consumers, thus generalizing the findings of Galí et al. (2004) and Colciago (2011) by presenting both of theirs models as special cases of the one developed here. As shown in Colciago (2011) the introduction of nominal wage rigidities proves to be crucial when determining the properties of the rational expectations equilibrium (existence, uniqueness and stability).

The economy also consists of labor agencies (firms that hire the labor from the agents and sell it to the intermediate goods producers), intermediate goods producers (operating in a monopolistic environment and subject to Calvo (1983) rigidities in price setting), a final good aggregator, and a monetary authority.

The markets operate as follows: there is a competitive capital market between the Ricardian agents and the intermediate goods firms. The labor supplied by the Ricardian agents is bought by a labor agency and packed in an index of Ricardian work, the same happens to the Rule-of-Thumb agents; afterward both indexes are bought by a third labor agency that generates a labor index sold to the intermediate goods firms in a competitive market. With capital and labor the firms produce intermediate goods that are sold in a market in monopolistic competition to the final good aggregator, then the aggregator sells the final good to the agents in a competitive market, the final good is used for consumption by both types of agents and for investment in capital.

As for the composition of the households it is assumed that a fraction Γ of it is composed by Rule-of-Thumb agents, this fraction is exogenously determined and held constant over time. In the following the subindex "a" will identify variables of the Non-Ricardian agents and the subindex "b" will be used for the Ricardian agents.

Special attention is given to the algebraic details of the model, Sections 1 to 9. Later the model parametrization and steady state is discussed in Section 10. Finally the model is simulated and its impulse response functions

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[1] The nominal wage rigidity is introduced with the mechanism of Calvo (1983).

to a technological and a monetary shock are presented in Section 11, although it is not the main concern of this document some of the economic implications of the model are discussed.

1 Aggregate Labor Agency

The aggregate labor agency buys a Ricardian labor index and a Non-Ricardian labor index in order to produce an aggregate labor index that is sold to the intermediate goods producers in a competitive market.

Their problem is to maximize their profits subject to their aggregation technology:

$$\begin{aligned} \underset{h_{a,t}h_{b,t}}{\text{Max}} \quad & w_t h_t - w_{at}h_{at} - w_{bt}h_{bt} \\ \text{S.T} & \\ h_t \quad =& \quad h_{at}^{\Gamma} h_{bt}^{1-\Gamma} \end{aligned}$$

The first order conditions for the aggregate labor agency are then:

$$h_{at} = \Gamma \frac{w_t}{w_{at}} h_t\,, \qquad h_{bt} = (1-\Gamma)\frac{w_t}{w_{bt}} h_t \qquad \text{and} \qquad h_t = h_{at}^{\Gamma} h_{bt}^{1-\Gamma}$$

The log-linear approximation is given by:

$$\tilde{h}_{at} = (\tilde{w}_t - \tilde{w}_{at}) + \tilde{h}_t\,, \qquad \tilde{h}_{bt} = (\tilde{w}_t - \tilde{w}_{bt}) + \tilde{h}_t \qquad \text{and} \qquad \tilde{h}_t = \Gamma\tilde{h}_{at} + (1-\Gamma)\,\tilde{h}_{bt}$$

2 Non-Ricardian Labor Agency

The Non-Ricardian labor agency buys labor from the Non-Ricardian agents, and aggregates it in a Non-Ricardian labor index. The technology is characterized by the following function:

$$h_{at}^{s} = \left[\int_0^1 h_{zt}^{\frac{\eta-1}{\eta}} dz\right]^{\frac{\eta}{\eta-1}}$$

Note that the integral is done over the space of Non-Ricardian agents, then it holds that, in equilibrium, the demand for Non-Ricardian labor index satisfies: $h_{a,t} = \Gamma h_{a,t}^{s}$.

The problem of the Non-Ricardian labor agency is then given by:

$$\begin{aligned} \underset{\forall_{z\epsilon(0,1)}h_{zt}}{\text{Max}} \quad & w_{at}h_{at}^{s} - \int_0^1 w_{zt}h_{zt}dz \\ \text{S.T} & \\ h_{at}^{s} \quad =& \quad \left[\int_0^1 h_{zt}^{\frac{\eta-1}{\eta}} dz\right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

The optimal demand for the labor of the Non-Ricardian agent "z" is:

$$h_{zt}^{s} = \left(\frac{w_{zt}}{w_{at}}\right)^{-\eta} h_{at}^{s}$$

Using the optimal demand for labor and the technology it is possible to obtain an expression for the Non-Ricardian wage w_{at}:

$$w_{at} = \left[\int_0^1 w_{zt}^{1-\eta} dz\right]^{\frac{1}{1-\eta}}$$

From the last equation and knowing that in equilibrium a portion ξ_a of the wages cannot be adjusted optimally one gets:

$$\begin{aligned} w_{a,t} &= \left[\xi_a \left(w_{a,t-1}\right)^{1-\eta} + \left(1-\xi_a\right)\left(w_{a,t}^*\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ \text{Linearizing} & \\ \tilde{w}_{a,t} &= \xi_a \tilde{w}_{a,t-1} + \left(1-\xi_a\right)\tilde{w}_{a,t}^* \end{aligned}$$

3 Ricardian Labor Agency

The problem and the technology of the Ricardian labor agency are the same as the described above for the Non-Ricardian labor agency. The result of the optimization is then the same and the optimality conditions are given by:

$$h_{zt}^s = \left(\frac{w_{zt}}{w_{bt}}\right)^{-\eta} h_{bt} \qquad \text{and} \qquad w_{bt} = \left[\int_0^1 w_{zt}^{1-\eta} dz\right]^{\frac{1}{1-\eta}}$$

The linearized equation for the average Ricardian wage is given by:

$$\tilde{w}_{b,t} = \xi_b \tilde{w}_{b,t-1} + \left(1-\xi_b\right)\tilde{w}_{b,t}^*$$

where ξ_b is probability that an agent adjusts its wage in a given period. As before in equilibrium the aggregate demand for Ricardian labor index satisfies: $h_{b,t} = \Gamma h_{b,t}^s$.

4 Final Good Aggregator

There is a firm that buys intermediate goods and aggregates them into a final good that is sold to the households in a competitive market. The aggregation technology is given by:

$$Y_t = \left[\int_0^1 v_{j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$$

Then the conditional demand for each type of intermediate good and the price of the consumption bundle are:

$$v_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t \qquad \text{and} \qquad P_t = \left[\int_0^1 \left(P_{j,t}\right)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$$

5 Household

5.1 Non-Ricardian Agents

Non-Ricardian agents seek to maximize their utility by choosing their consumption, the wage to charge for their labor and the acquisition of an Arrow-Debreau security that allows income to be the same among Non-Ricardian agents in the household independently of the wage they charge for their labor.

5.1.1 Consumption

The problem faced by a Non-Ricardian agent when maximizing consumption is:

$$\begin{aligned} \underset{c_{j,t},a_{jt+1}}{\text{Máx}} & \quad \sum_{i=0}^{\infty} \beta^i \left[\frac{c_{j,t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{\left(h_{j,t+i}^s\right)^{1+\vartheta}}{1+\vartheta}\right] \\ \text{S.A.} & \\ 0 &= w_{j,t} h_{j,t}^s + a_{j,t} - c_{j,t} - \int q_{j,t+1,t} a_{j,t+1} d\zeta_{j,t+1,t} \end{aligned}$$

The first order conditions are then:

$$
\begin{aligned}
c_{j,t}^{-\sigma} - \gamma_{j,t} &= 0 \\
w_{j,t} h_{j,t}^{s} + a_{j,t} - c_{j,t} - \int q_{j,t+1,t} a_{j,t+1} d\zeta_{j,t+1,t} &= 0
\end{aligned}
$$

Thanks to the Arrow-Debreau Securities, the first order conditions can be expressed in terms of the average variable of a type a household:

$$
\begin{aligned}
c_{a,t}^{-\sigma} - \gamma_t &= 0 \\
w_{a,t} h_{a,t}^{s} - c_{a,t} &= 0
\end{aligned}
$$

The log linear approximation is:

$$
\begin{aligned}
\tilde{\gamma}_t &= -\sigma \tilde{c}_{a,t} \\
\tilde{c}_{a,t} &= \tilde{w}_{a,t} + \tilde{h}_{a,t}^{s}
\end{aligned}
$$

5.1.2 Wage and Labor

The problem of choosing the optimal wage is solved as in Erceg et al. (2000), the details are in Appendix A. The solution to the problem is given by a wage Phillips curve, the definition of the wage markup and the dynamics of the real wage:

$$
\begin{aligned}
\pi_t^{w_a} &= \beta \pi_{t+1}^{w_a} - \frac{(1-\xi_a)(1-\beta\xi_a)}{\xi_a (1+\eta\vartheta)} \tilde{\mu}_{w_a,t} \\
\tilde{\mu}_{w_a,t} &= \tilde{w}_{a,t} - \tilde{\text{mrs}}_{a,t} \\
\tilde{\text{mrs}}_{a,t} &= \vartheta \tilde{h}_{a,t}^{s} + \sigma \tilde{c}_{a,t} \\
\tilde{w}_{a,t} &= \tilde{w}_{a,t-1} + \pi_t^{w_a} - \pi_t
\end{aligned}
$$

5.2 Ricardian Agents

The problem of the Ricardian agents is similar to the problem already presented for the Non-Ricardian agents, the difference lies in the extra decisions of the former, they have to optimize over their bond holdings, investment and capital accumulation.

5.2.1 Consumption, Bond Holdings, Investment and Capital

The problem of the Ricardian agent is given by:

$$
\begin{aligned}
\underset{c_{j,t}, b_{jt}, x_{jt} k_{jt+1}, a_{jt+1}}{\text{Max}} \quad & \sum_{i=0}^{\infty} \beta^i \left[\frac{c_{j,t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{\left(h_{j,t+i}^{s}\right)^{1+\vartheta}}{1+\vartheta} \right] \\
\text{S.A.} \quad & \\
0 &= r_t^k k_{j,t} + w_{j,t} h_{j,t}^{s} + b_{j,t-1} \frac{i_{t-1}}{\pi_t} + \frac{1}{1-\Gamma} \text{Pr}_t + a_{j,t} - c_{j,t} - x_{j,t} - b_{j,t} - \int q_{j,t+1,t} a_{j,t+1} d\zeta_{j,t+1,t} \\
0 &= \phi\left(\frac{x_{j,t}}{k_{j,t}}\right) k_{j,t} + (1-\delta) k_{j,t} - k_{j,t+1}
\end{aligned}
$$

Where $\phi\left(\frac{x_t}{k_t}\right) k_{j,t}$ represents the capital adjustment costs, it is assumed that $\phi'(\delta) = 1$ and $\phi(\delta) = \delta$.
The first order conditions are then:

$$
\begin{aligned}
c_{j,t}^{-\sigma}-\lambda_{j,t} &= 0\\
-\lambda_{jt}+\beta\lambda_{jt+1}\frac{i_t}{\pi_{t+1}} &= 0\\
-\mu_{jt}+\beta\left[\lambda_{jt+1}r_{t+1}^k+\mu_{jt+1}(1-\delta)+\mu_{jt+1}\left(\phi\left(\frac{x_{j,t+1}}{k_{j,t+1}}\right)-\phi'\left(\frac{x_{j,t+1}}{k_{j,t+1}}\right)\frac{x_{j,t+1}}{k_{j,t+1}}\right)\right] &= 0\\
-\lambda_{jt}+\mu_{jt}\phi'\left(\frac{x_{j,t}}{k_{j,t}}\right) &= 0\\
r_t^k k_{j,t}+w_{j,t}h_{j,t}^s+b_{j,t-1}\frac{i_{t-1}}{\pi_t}+\frac{1}{1-\Gamma}\mathrm{Pr}_t+a_{j,t}-c_{j,t}-x_{j,t}-b_{j,t}-\int q_{j,t+1,t}a_{j,t+1}d\zeta_{j,t+1,t} &= 0\\
\phi\left(\frac{x_{j,t}}{k_{j,t}}\right)k_{j,t}+(1-\delta)k_{j,t}-k_{j,t+1} &= 0
\end{aligned}
$$

Defining Tobin's Q as the ratio between the Lagrange multiplier of the capital dynamics and the Lagrange multiplier of the budget constraint one gets:

$$
\begin{aligned}
c_{j,t}^{-\sigma}-\lambda_{j,t} &= 0\\
-\frac{c_{j,t}^{-\sigma}}{\beta c_{j,t+1}^{-\sigma}}+\frac{i_t}{\pi_{t+1}} &= 0\\
-Q_{jt}+\frac{\beta c_{j,t+1}^{-\sigma}}{c_{j,t}^{-\sigma}}\left[r_{t+1}^k+Q_{jt+1}(1-\delta)+Q_{jt+1}\left(\phi\phi\left(\frac{x_{j,t+1}}{k_{j,t+1}}\right)-\phi'\left(\frac{x_{j,t+1}}{k_{j,t+1}}\right)\frac{x_{j,t+1}}{k_{j,t+1}}\right)\right] &= 0\\
Q_{jt}-\frac{1}{\phi'\left(\frac{x_{j,t}}{k_{j,t}}\right)} &= 0\\
r_t^k k_{j,t}+w_{j,t}h_{j,t}^s+b_{j,t-1}\frac{i_{t-1}}{\pi_t}+\frac{1}{1-\Gamma}\mathrm{Pr}_t+a_{j,t}-c_{j,t}-x_{j,t}-b_{j,t}-\int q_{j,t+1,t}a_{j,t+1}d\zeta_{j,t+1,t} &= 0\\
\phi\left(\frac{x_{j,t}}{k_{j,t}}\right)k_{j,t}+(1-\delta)k_{j,t}-k_{j,t+1} &= 0
\end{aligned}
$$

Thanks to the Arrow-Debreau Securities, the first order conditions can be expressed in terms of the average variable of a type b household, also in equilibrium the bonds "b" are equal to zero. The system is then given by:

$$
\begin{aligned}
c_{b,t}^{-\sigma}-\lambda_t &= 0\\
-\frac{c_{b,t}^{-\sigma}}{\beta c_{b,t+1}^{-\sigma}}+\frac{i_t}{\pi_{t+1}} &= 0\\
-Q_t+\frac{\beta c_{b,t+1}^{-\sigma}}{c_{b,t}^{-\sigma}}\left[r_{t+1}^k+Q_{t+1}(1-\delta)+Q_{t+1}\left(\phi\left(\frac{x_{b,t+1}}{k_{b,t+1}}\right)-\phi'\left(\frac{x_{b,t+1}}{k_{b,t+1}}\right)\frac{x_{b,t+1}}{k_{b,t+1}}\right)\right] &= 0\\
Q_t-\frac{1}{\phi'\left(\frac{x_{b,t}}{k_{b,t}}\right)} &= 0\\
r_t^k k_{b,t}+w_{b,t}h_{b,t}^s+\frac{1}{1-\Gamma}\mathrm{Pr}_t-c_{b,t}-x_{b,t} &= 0\\
\phi\left(\frac{x_{b,t}}{k_{b,t}}\right)k_{b,t}+(1-\delta)k_{b,t}-k_{b,t+1} &= 0
\end{aligned}
$$

Combining the second and third equations:

$$\begin{aligned}
c_{b,t}^{-\sigma} - \lambda_t &= 0 \\
-\frac{c_{b,t}^{-\sigma}}{\beta c_{b,t+1}^{-\sigma}} + \frac{i_t}{\pi_{t+1}} &= 0 \\
-Q_t + \frac{\pi_{t+1}}{i_t}\left[r_{t+1}^k + Q_{t+1}(1-\delta) + Q_{t+1}\left(\phi\left(\frac{x_{b,t+1}}{k_{b,t+1}}\right) - \phi'\left(\frac{x_{b,t+1}}{k_{b,t+1}}\right)\frac{x_{b,t+1}}{k_{b,t+1}}\right)\right] &= 0 \\
Q_t - \frac{1}{\phi'\left(\frac{x_{b,t}}{k_{b,t}}\right)} &= 0 \\
r_t^k k_{b,t} + w_{b,t} h_{b,t}^s + \frac{1}{1-\Gamma}\mathrm{Pr}_t - c_{b,t} - x_{b,t} &= 0 \\
\phi\left(\frac{x_{b,t}}{k_{b,t}}\right) k_{b,t} + (1-\delta) k_{b,t} - k_{b,t+1} &= 0
\end{aligned}$$

The log linear approximation is then:

$$\begin{aligned}
i_t - \pi_{t+1} &= \sigma\left(\tilde{c}_{b,t+1} - \tilde{c}_{b,t}\right) \\
\tilde{q}_t &= -\left(i_t - \pi_{t+1}\right) + \beta\tilde{q}_{t+1} + \left[1 - \beta(1-\delta)\right]\tilde{r}_{t+1}^k \\
\tilde{x}_{b,t} - \tilde{k}_{b,t} &= \left(\frac{-1}{\phi''(\delta)\,\delta}\right)\tilde{q}_t \\
\tilde{k}_{b,t+1} &= \delta\tilde{x}_{b,t} + (1-\delta)\tilde{k}_{b,t}
\end{aligned}$$

The first equation is not necessary for characterizing the equilibrium, and the budget constraint is used later when deriving the aggregate resource constraint. The term $\frac{-1}{\phi''(\delta)\delta}$ is the elasticity of the investment-capital ratio to the Tobin's Q, it will be noted as ι from now on.

5.2.2 Wage and Labor

The decision of the optimal wage is taken in the same way as the one of the Non-Ricardian agents, the result is then:

$$\begin{aligned}
\pi_t^{w_b} &= \beta\pi_{t+1}^{w_b} - \frac{(1-\xi_b)(1-\beta\xi_b)}{\xi_b(1+\eta\vartheta)}\tilde{\mu}_{w_b,t} \\
\tilde{\mu}_{w_b,t} &= \tilde{w}_{b,t} - \tilde{\mathrm{mrs}}_{b,t} \\
\tilde{\mathrm{mrs}}_{b,t} &= \vartheta\tilde{h}_{b,t}^s + \sigma\tilde{c}_{b,t} \\
\tilde{w}_{b,t} &= \tilde{w}_{b,t-1} + \pi_t^{w_b} - \pi_t
\end{aligned}$$

6 Intermediate Goods Producers

There is a continuum of firms in monopolistic competition that produce differentiated goods, each of the firms has the same technology and uses capital and aggregate labor in its production process. The firms face Calvo rigidities in their price setting decisions. If not allowed to optimize over its price a firm must keep the same price of the last period.

The firms technology is characterized by the following production function:

$$v_{j,t} = z_t k_{j,t}^{\alpha} h_{j,t}^{1-\alpha}$$

From the cost minimization process the conditional demands are obtained:

$$r_t^k = \alpha\varphi_{jt} z_t k_{j,t}^{\alpha-1} h_{j,t}^{1-\alpha} \qquad \text{and} \qquad w_t = (1-\alpha)\varphi_{jt} z_t k_{j,t}^{\alpha} h_{j,t}^{-\alpha}$$

From the demands and the production function it can be shown that the real marginal cost φ_t is common to all firms, then one gets:

$$r_t^k = \alpha\varphi_t z_t k_{j,t}^{\alpha-1} h_{j,t}^{1-\alpha} \qquad \text{and} \qquad w_t = (1-\alpha)\,\varphi_t z_t k_{j,t}^{\alpha} h_{j,t}^{-\alpha}$$

linearizing one gets:

$$\begin{aligned}\tilde{r}_t^k &= \tilde{\varphi}_t - (1-\alpha)\left(\tilde{k}_{j,t} - \tilde{h}_{j,t}\right)\\ \tilde{w}_t &= \tilde{\varphi}_t + \alpha\left(\tilde{k}_{j,t} - \tilde{h}_{j,t}\right)\end{aligned}$$

also, for the production function:

$$\tilde{v}_{j,t} = \tilde{z}_t + \alpha\tilde{k}_{j,t} + (1-\alpha)\,\tilde{h}_{j,t}$$

Defining $k_t = \int_0^1 k_{j,t}dj$, $h_t = \int_0^1 h_{j,t}dj$ and integrating:

$$\tilde{r}_t^k = \tilde{\varphi}_t - (1-\alpha)\left(\tilde{k}_t - \tilde{h}_t\right), \qquad \tilde{w}_t = \tilde{\varphi}_t + \alpha\left(\tilde{k}_t - \tilde{h}_t\right) \qquad \text{and} \qquad \tilde{v}_t = \tilde{z}_t + \alpha\tilde{k}_t + (1-\alpha)\,\tilde{h}_t$$

The details for the price decision problem can be found in Appendix B, the result of the problem is:

$$\pi_t = \beta\pi_{t+1} + \frac{(1-\epsilon)(1-\epsilon\beta)}{\epsilon}\tilde{\varphi}_t$$

Where $1-\epsilon$ is the probability of optimally adjusting prices.

7 Aggregation

7.1 Aggregate demand

The aggregate demand is given by:

$$v_t = \int_0^1 v_{jt}dj = \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\theta} y_t dj = \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\theta} dj y_t$$

Then de aggregate demand is:

$$v_t = \nu_t^p y_t$$

where:

$$\nu_t^p = \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\theta} dj$$

From the price definition:

$$\begin{aligned}P_t &= \left[\int_0^1 p_{jt}^{1-\theta}\,dj\right]^{\frac{1}{1-\theta}}\\ 1 &= \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{1-\theta} dj = \int_0^1 e^{(1-\theta)(p_{jt}-p_t)}dj\\ 1 &\simeq 1 + (1-\theta)\int_0^1 p_{jt} - p_t dj\\ 0 &\simeq \int_0^1 p_{jt} - p_t dj\end{aligned}$$

Now, the definition of the price distortion states:

$$\nu_t^p = \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\theta} dj \simeq -\theta \int_0^1 p_{jt} - p_t dj \simeq 0$$

Then one gets that up to a first order approximation the following result holds:

$$\tilde{v}_t = \tilde{y}_t$$

7.2 Demand for Non Ricardian Labor

The demand is given by:

$$h_{at}^s = \int_0^1 h_{zt} dz = \int_0^1 \left(\frac{w_{zt}}{w_{at}}\right)^{-\eta} h_{at} dz = \int_0^1 \left(\frac{w_{zt}}{w_{at}}\right)^{-\eta} dz h_{at}$$
$$h_{at}^s = \nu_{at}^w h_{at}$$

Where:

$$\nu_{at}^w = \int_0^1 \left(\frac{w_{zt}}{w_{at}}\right)^{-\eta} dz$$

By a process similar to that done for the aggregate demand one gets:

$$\tilde{\nu}_{at}^w = 0 \qquad \text{and} \qquad \tilde{h}_{at}^s = \tilde{h}_{at}$$

7.3 Demand for Ricardian Labor

The demand for Ricardian labor is obtained is a similar way, the optimality conditions are:

$$\tilde{\nu}_{bt}^w = 0 \qquad \text{and} \qquad \tilde{h}_{bt}^s = \tilde{h}_{bt}$$

7.4 Capital and Investment

Because the Ricardian agents are the only that invest and accumulate capital one has that the aggregate capital and aggregate investment are given by:

$$k_t = (1 - \Gamma) k_{bt} \qquad \text{and} \qquad x_t = (1 - \Gamma) x_{bt}$$

Linearizing:

$$\tilde{k}_t = \tilde{k}_{bt} \qquad \text{and} \qquad \tilde{x}_t = \tilde{x}_{bt}$$

7.5 Aggregate Consumption

The aggregate consumption is given by:

$$c_t = \Gamma c_{at} + (1 - \Gamma) c_{bt}$$

Linearizing:

$$\tilde{c}_t = \frac{c_a}{c} \Gamma \tilde{c}_{a,t} + \frac{c_b}{c} (1 - \Gamma) \tilde{c}_{b,t}$$

7.6 Aggregate Profits

The profits of the intermediate goods producers are given by:

$$
\begin{aligned}
\mathrm{Pr}_t &= \int_0^1 \left(\frac{p_{j,t}}{P_t} v_{jt} - r_t^k k_{j,t} - w_t h_{j,t} \right) di \\
\mathrm{Pr}_t &= \int_0^1 \left(\left(\frac{p_{jt}}{P_t} \right)^{1-\theta} y_t - r_t^k k_{j,t} - w_t h_{j,t} \right) di \\
\mathrm{Pr}_t &= \int_0^1 \left(\frac{p_{jt}}{P_t} \right)^{1-\theta} di y_t - r_t^k \int_0^1 k_{j,t} di - w_t \int_0^1 h_{j,t} di \\
\mathrm{Pr}_t &= y_t - r_t^k k_t - w_t h_t
\end{aligned}
$$

The integral over the prices in the first term is equal to one by the definition of the price index.

Although not necessary for the characterizing the equilibrium, the log-linearized equation for the aggregate profits is:

$$
\begin{aligned}
\mathrm{Pr}\tilde{\mathrm{pr}}_t &= y\tilde{y}_t - r^k k \left(\tilde{r}_t^k + \tilde{k}_t \right) - wh \left(\tilde{w} + \tilde{h} \right) \\
(1-\varphi) y \tilde{\mathrm{pr}}_t &= y\tilde{y}_t - \alpha\varphi y \left(\tilde{r}_t^k + \tilde{k}_t \right) - (1-\alpha) \varphi y \left(\tilde{w} + \tilde{h} \right) \\
\tilde{\mathrm{pr}}_t &= \frac{1}{1-\varphi} \left(\tilde{y}_t - \alpha\varphi \left(\tilde{r}_t^k + \tilde{k}_t \right) - (1-\alpha) \varphi \left(\tilde{w} + \tilde{h} \right) \right)
\end{aligned}
$$

7.7 Aggregate Resource Constraint

The aggregate resource constraint can be derived from the resource constraints of both type of households. Aggregating over the Ricardian households one gets:

$$
\begin{aligned}
\int_\Gamma^1 \left(r_t^k k_{b,t} + w_{b,t} h_{b,t}^s + \frac{1}{1-\Gamma} \mathrm{Pr}_t - c_{b,t} - x_{b,t} \right) di &= 0 \\
r_t^k (1-\Gamma) k_{b,t} + (1-\Gamma) w_{b,t} h_{b,t}^s + \mathrm{Pr}_t - (1-\Gamma) c_{b,t} - (1-\Gamma) x_{b,t} &= 0
\end{aligned}
$$

From the equilibrium in the capital and investment markets:

$$
r_t^k k_t + (1-\Gamma) w_{b,t} h_{b,t}^s + \mathrm{Pr}_t - (1-\Gamma) c_{b,t} - x_t = 0
$$

From the definition of aggregate consumption one gets:

$$
r_t^k k_t + (1-\Gamma) w_{b,t} h_{b,t}^s + \mathrm{Pr}_t - c_t + \Gamma c_{a,t} - x_t = 0
$$

From the resource constraint of the Non-Ricardian agents:

$$
r_t^k k_t + (1-\Gamma) w_{b,t} h_{b,t}^s + \mathrm{Pr}_t - c_t + \Gamma w_{a,t} h_{a,t}^s - x_t = 0
$$

Replacing the aggregate profits:

$$
\begin{aligned}
r_t^k k_t + (1-\Gamma) w_{b,t} h_{b,t} + y_t - r_t^k k_t - w_t h_t - c_t + \Gamma w_{a,t} h_{a,t} - x_t &= 0 \\
y_t - c_t - x_t + (1-\Gamma) w_{b,t} h_{b,t}^s + \Gamma w_{a,t} h_{a,t}^s - w_t h_t &= 0
\end{aligned}
$$

Finally the equilibrium conditions of the labor indexes markets imply that: $h_{a,t} = \Gamma h^s_{a,t}$ and $h_{b,t} = \Gamma h^s_{b,t}$, this along with the non-profit condition of the aggregate labor agency ($w_t h_t - w_{at} h_{at} - w_{bt} h_{bt} = 0$) gives the aggregate resource constraint:

$$y_t = c_t + x_t$$

Linearizing:

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{x}{y}\tilde{x}_t$$

8 Monetary Policy

The monetary authority acts according to the following rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_y \tilde{y}_t) + \epsilon_{i,t}$$

9 Technology

The technology evolves according to the following process:

$$\tilde{z}_{t+1} = \rho_z \tilde{z}_t + \epsilon_{t+1}$$

10 Parametrization and Steady State

The linear approximation of the model equilibrium conditions (Appendix C) characterizes, along with the parameter values and the steady state of the non-linear model, a linear rational expectations model of difference equations, the solution of such model can be computed by means of the Klein (2000) algorithm (a generalization of the Blanchard and Kahn (1980) algorithm). The next two sections discuss the parametrization of the model and the steady state.

10.1 Parametrization

The parametrization of the model follows closely the one proposed by Galí et al. (2004). In this way the elasticity of output to capital (α) is set to $1/3$, a value taken for consistency with the US labor income share. The capital depreciation rate (δ) is set to 0.025, implying a 10% annual rate. The elasticity of the investment-capital ratio with respect to Tobin's Q (ι) is set to unity. The elasticity of substitution among types of labor (η) is set to 6 in order to get a steady state markup of 2 on the wages. As is shown below the elasticity of substitution between intermediate goods (θ) is chosen to obtain a steady state consumption to output ratio of 80% under the baseline parametrization.

As for the utility parameters, the subjective discount factor (β) is set to 0.99, implying a steady state real annual interest rate of 4%. The relative risk aversion coefficient (σ) and the Frisch elasticity coefficient (ϑ) are both set to unity. The scaling parameter χ is chosen to obtain a steady state value for aggregate labor equal to $1/2$, note that this parameter has no direct impact over the linearized equilibrium conditions (Appendix C) and thus its exact value only affects the models solution by changing the steady state.

The value for the share of Non-Ricardian households (Γ) is not an easy pick, and the interesting thing to do is explore the implications of different values for the model dynamics, yet the baseline value for the parameter will imply that 30% of the households are Non-Ricardian.

Lastly, values for the parameters that only affect the dynamic properties of the model, and not the steady value of the variables, are chosen. The probabilities for price and wage adjustment (ϵ, ξ_a, ξ_b) are all set to $3/4$, corresponding to an average price and wage duration of 4 quarters. The persistence of the technology shock ρ_z is set to 0.8. The persistence of the nominal interest rate in the Taylor rule (ρ_i) is set to 0 in the baseline parametrization, the other parameters of the Taylor rule are set in a way that the Taylor principle is satisfied (Woodford, 2001). The response to changes in inflation (ϕ_π) is set to 1.5 and the response to output deviations from its steady state value (ϕ_y) to 0. Since the only simulation exercises that are going to be carried on with the model are impulse response functions the standard deviations of the two shocks (monetary and technological) are set to unity.

Table 1 summarizes this Section.

Tab. 1: Baseline Parameters Values

Parameter	Value	Description
α	$1/3$	Output to capital elasticity
δ	0.025	Capital depreciation rate
ι	1	Investment-Capital ratio to Tobin's Q elasticity
η	6	Elasticity of substitution between differentiated labor
θ	-	Elasticity of substitution between intermediate goods
β	0.99	Subjective discount factor
σ	1	Relative risk aversion
ϑ	1	Inverse of the Frisch elasticity
χ	-	Labor disutility scaling parameter
Γ	0.3	Share of Non-Ricardian households
ϵ	$3/4$	Probability of price adjustment
ξ_a	$3/4$	Probability of wage adjustment for Non-Ricardian households
ξ_a	$3/4$	Probability of wage adjustment for Ricardian households
ρ_z	0.8	Persistence of the technological shock
ρ_i	0	Persistence of the Taylor rule
ϕ_π	1.5	Nominal interest response to inflation
ϕ_y	0	Nominal interest response to output
σ_{ϵ_z}	1	Technological shock standard deviation
σ_{ϵ_i}	1	Monetary shock standard deviation

10.2 Steady State Non-Linear Equilibrium

It is assumed that the steady state inflation rate is zero and the level of the technological process is 1.

$$\begin{aligned} \pi &= 1 \\ z &= 1 \end{aligned}$$

The optimality conditions in steady state are:

$$\begin{aligned}
&\text{Labor Demand} \\
h_a &= \Gamma \frac{w}{w_a} h \\
h_b &= (1-\Gamma) \frac{w}{w_b} h \\
h &= h_a^{\Gamma} h_b^{1-\Gamma} \\
&\text{Non-Ricardian FOC} \\
c_a &= w_a h_a \\
w_a &= \frac{\eta}{\eta - 1} \frac{\chi h_a^{\vartheta}}{c_a^{-\sigma}} \\
&\text{Ricardian FOC} \\
\frac{i}{\pi} &= \frac{c_b^{-\sigma}}{\beta c_b^{-\sigma}} \\
Q &= \frac{\pi}{i} \left[r^k + Q(1-\delta) + Q\left(\phi\left(\frac{x_b}{k_b}\right) - \phi'\left(\frac{x_b}{k_b}\right)\frac{x_b}{k_b} \right) \right] \\
Q &= \frac{1}{\phi'\left(\frac{x_b}{k_b}\right)} \\
k_b &= \phi\left(\frac{x_b}{k_b}\right) k_b + (1-\delta) k_b \\
w_b &= \frac{\eta}{\eta - 1} \frac{\chi h_b^{\vartheta}}{c_b^{-\sigma}}
\end{aligned}$$

$$\begin{aligned}
&\text{Firms FOC} \\
y &= zk^{\alpha}h^{1-\alpha} \\
r^{k} &= \alpha\varphi zk^{\alpha-1}h^{1-\alpha} \\
w &= (1-\alpha)\,\varphi zk^{\alpha}h^{-\alpha} \\
\varphi &= \frac{\theta-1}{\theta} \\
&\text{Aggregate Conditions} \\
y &= c+x \\
x &= (1-\Gamma)\,x_b \\
k &= (1-\Gamma)\,k_b \\
c &= \Gamma c_a + (1-\Gamma)\,c_b \\
\Pr &= y - r^{k}k - wh
\end{aligned}$$

From the Euler condition of the Ricardian household the steady state gross nominal interest rate is known:

$$i = \frac{1}{\beta}$$

From the capital accumulation equation one gets that in steady state the following condition must hold:

$$x_b = \delta k_b$$

Then from the optimality condition of the capital and the definition of Tobin's Q one gets (Ricardian FOC):

$$\begin{aligned}
Q &= 1 \\
r^{k} &= \frac{1}{\beta}+\delta-1
\end{aligned}$$

Using the demand for capital (firms FOC):

$$k = \left(\frac{r^{k}}{\alpha\varphi z}\right)^{\frac{1}{\alpha-1}} h$$

Replacing on the demand for labor and the production function in the firms FOC one arrives at:

$$\begin{aligned}
w &= (1-\alpha)\,\varphi z\left(\frac{r^{k}}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} \\
y &= z\left(\frac{r^{k}}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} h
\end{aligned}$$

From the definition of the aggregate capital and the aggregate investment:

$$\begin{aligned}
k_b &= \frac{k}{1-\Gamma} \\
x &= (1-\Gamma)\,x_b
\end{aligned}$$

From the optimal wage condition of the Non-Ricardian household and that households budget constraint, one gets the following results given the parametrization $\sigma = 1$ and $\vartheta = 1$:

$$
\begin{aligned}
w_a &= \frac{\eta}{\eta-1}\chi h_a c_a \\
w_a &= \frac{\eta}{\eta-1}\chi h_a w_a h_a \\
h_a &= \left(\frac{\eta}{\eta-1}\chi\right)^{\frac{-1}{2}} \\
c_a &= w_a h_a
\end{aligned}
$$

Using the definition of the aggregate labor index one has the Non-Ricardian labor in terms of the total labor:

$$
h_b = h_a^{\frac{-\Gamma}{1-\Gamma}} h^{\frac{1}{1-\Gamma}}
$$

Then from the demands for each type of labor the respective wages can be obtained:

$$
\begin{aligned}
w_a &= \Gamma \frac{w}{h_a} h \\
w_b &= (1-\Gamma)\, w h_a^{\frac{\Gamma}{1-\Gamma}} h^{\frac{-\Gamma}{1-\Gamma}}
\end{aligned}
$$

From the optimal Ricardian wage the Ricardian consumption is:

$$
c_b = \frac{\eta-1}{\chi\eta}\frac{w_b}{h_b}
$$

With the firms FOC and the definition of the aggregate profits one has:

$$
\Pr = (1-\varphi)\, y
$$

All the variables depend now on the level of the aggregate labor. Replacing the previous results on the aggregate resource constraint one solves the steady state by getting to a closed expression for the aggregate labor as a function of the model parameters:

$$
h = \left[\frac{(1-\Gamma)^2 \frac{\eta-1}{\chi\eta} w h_a^{\frac{2\Gamma}{1-\Gamma}}}{z\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{\alpha}{\alpha-1}} - \delta\left(\frac{r^k}{\alpha\varphi z}\right)^{\frac{1}{\alpha-1}} - \Gamma^2 w}\right]^{\frac{1-\Gamma}{2}}
$$

It can be shown using the previous results that the aggregate consumption to output ratio is given by:

$$
\frac{c}{y} = 1 - \frac{\delta\alpha\left(\frac{\theta-1}{\theta}\right)}{\frac{1}{\beta}+\delta-1}
$$

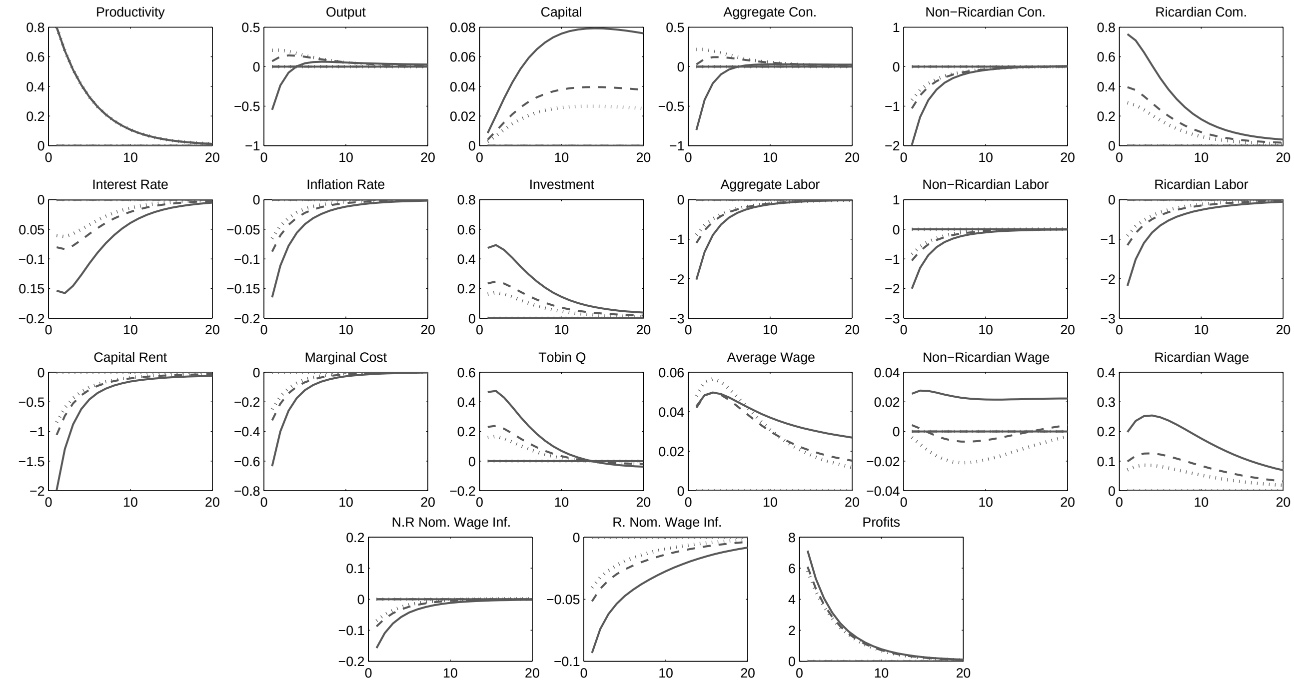
If a value $\overline{\frac{c}{y}}$ is desired for that ratio a value for θ can be found that guarantees that result given values for β, α and δ. θ is chosen for calibration because it is not directly involved in the set of conditions that characterize the linear equilibrium.[2] The value for theta is given by:

$$
\theta = \frac{1}{1 - \frac{\frac{1}{\beta}+\delta-1}{\delta\alpha}\left(1-\frac{c}{y}\right)}
$$

Appendix D summarizes the steady state values of the model variables under the baseline parametrization.

[2] The only equation that θ affects is the log-linear approximation of the aggregate profits, nevertheless that equation enters the model as a definition, and has no impact over the dynamics of variables other than the aggregate profits.

Fig. 1: Responses to a unit shock to productivity



The response to the productivity shock is computed for $\Gamma = 0.3$ *(dotted line),* $\Gamma = 0.6$ *(dashed line) and* $\Gamma = 0.9$ *(solid line).*

11 Model Solution and Impulse Response Functions

As mentioned before the solution to the linearized model can be obtained by means of the Klein (2000) algorithm. With the solution the impulse response functions -IRF- (and other characteristics of the model) can be computed. Here the IRF to a unit shock to productivity (z_t) and to a unit shock to the Taylor rule ($\epsilon_{i,t}$) are presented.[3] Both are computed under the baseline parametrization but varying the composition of the households, this is done in order to determine the effect of the parameter Γ over the model dynamics. Three values for Γ are considered: 30%, 60% and 90%.

Figure 1 presents the responses to the productivity shock and Figure 2 the responses to the monetary shock.

11.1 Productivity Shock

For most of the variables the composition of the households in the model has no effect over the direction of the response to the productivity shock but only over the magnitude. For those variables the direction is the expected from the usual New-Keynesian model without Non-Ricardian agents (see Galí (2008, Ch. 3, Ch. 6)). In response to the productivity shock Tobin's Q raises, then both the investment and the capital stock go above their steady state values. The productivity lowers the marginal costs which leads to lower inflation, the monetary authority reacts to this by lowering the interest rate as indicated by the Taylor rule.

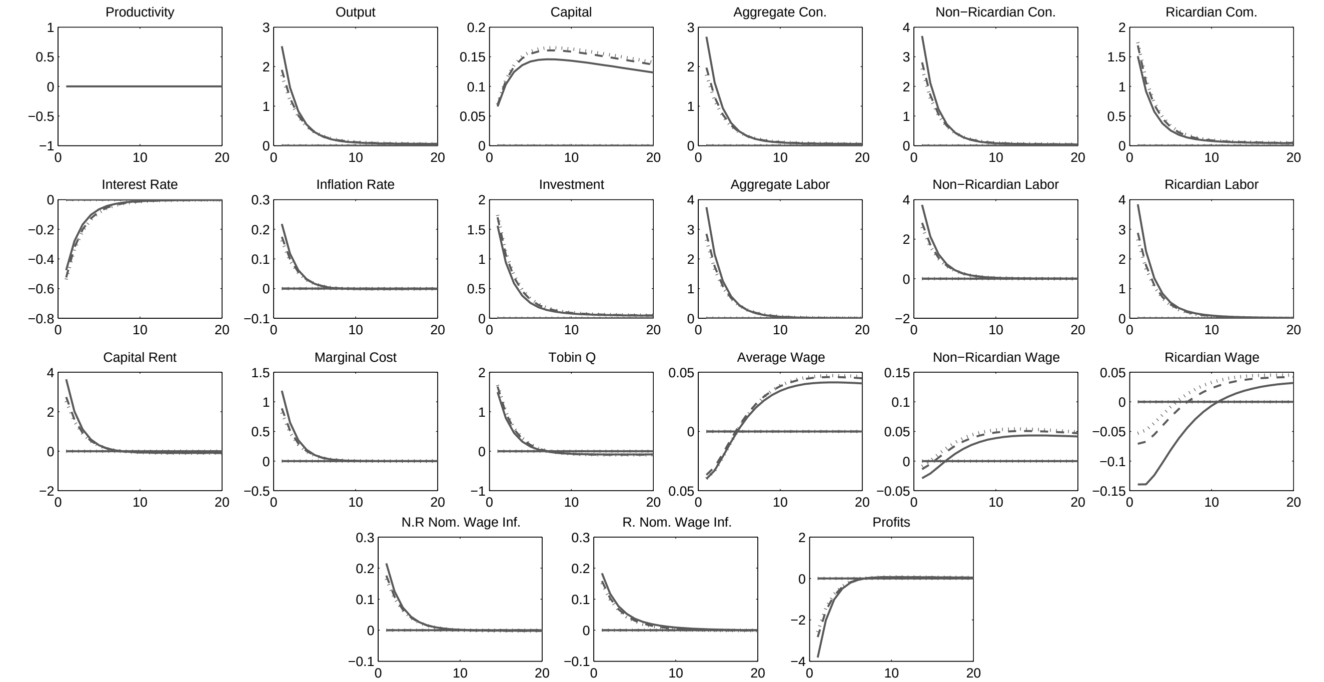
The model also predicts a decrease in labor (both Ricardian and Non-Ricardian) in response to the productivity shock. This goes in line with the evidence of Galí (1999) for the US economy.[4] The intuition behind the decrease in labor rests in the nominal rigidities of the model, unlike the classic RBC model (Kydland and Prescott, 1982) the shock cannot be transmitted immediately to the aggregate demand, this occurs mainly because of the price rigidities that prevent the prices from falling enough to create the aggregate demand necessary to increase the firms labor demand.[5] The response of labor to the productivity shock is then independent of the composition of the households and occurs for both types of agents. As for capital rent, it goes below the steady state value despite the increase in productivity because of the decrease in labor and the increase in capital referenced before.

[3] The shock to the Taylor rule is intended as an expansionary monetary shock, then it implies a negative shock to $\epsilon_{i,t}$.

[4] Other evidence for the response of labor to technological shocks can be found for industrialized economies in Francis and Ramey (2005), Galí and Rabanal (2005), Basu et al. (2006), Smets and Wouters (2007) and Galí (2010) among others. González et al. (2011) document this fact for the Colombian economy.

[5] Smets and Wouters (2007) find that real rigidities as habit formation in consumption and capital adjustment costs also play an important role in generating this result.

Fig. 2: Responses to a unit shock to the Taylor rule



The response to the monetary shock is computed for $\Gamma = 0.3$ *(dotted line),* $\Gamma = 0.6$ *(dashed line) and* $\Gamma = 0.9$ *(solid line).*

The behavior of labor when facing the productivity shock is shown to have a different effect over consumption depending on whether or not the agent is Ricardian. For Non-Ricardian agents the decrease in the labor demand they face implies a decrease in consumption, which follows directly from the lower labor income. On the other hand Ricardian consumption goes above its steady state value, as in the classical RBC model or the standard New-Keynesian model without Non-Ricardian agents. What happens is that the increase in productivity, along with the decrease in the interest rate generates incentives for increasing current consumption, this is financed with the profits that are transferred to the Ricardian agents, recall that the Non-Ricardian agents don't have access to this source of income.

The real wages behave differently because of the decision of the households over the nominal wages. Since the shock induces a decrease in labor demand, the households, that behave as monopolists in the labor market, will lower the nominal wage trying to increase the demand. This is true for both types of households as can be seen in the responses of the nominal wage inflation. The difference is that the decrease in consumption makes the Non-Ricardian households to induce a nominal wage inflation lower that the one of the the Ricardian agents, this explains the behavior of the real wages.

It is important to note that inflation responds much more strongly to the shock as the share of Non-Ricardian agents raises, this is explained by the change in the relative importance of their consumption for aggregate demand. Because Non-Ricardian consumption falls in response to the productivity shock, the aggregates demand response to the shock is affected when the share of those agents is altered. When $\Gamma = 0.9$ aggregate demand gets to fall in response to the shock (output and aggregate consumption fall), this induces a downward pressure over prices, in addition to that of the improvement in technology, this is why inflation ends up decreasing more as the proportion of Non-Ricardian households raises. As a secondary effect the interest rate also reacts more strongly and the real wages increase.

11.2 Monetary Shock

In response to an expansionary monetary shock the interest rate falls below its steady state value independently of the households composition. The decrease in the nominal interest rate creates incentives for the Ricardian households to increase their demand in both consumption and investment, this extra demand has various effects: the firms face upward pressures in the demand for their goods and then proceed to increase their prices, this causes inflation. Firms are also willing to produce more so that the extra demand can be matched, this extra production makes necessary to rent more capital and hire more labor.

The extra labor demand is supplied by the households at a higher nominal wage (as is expected because of their monopolistic behavior), and more importantly it allows the Non-Ricardian households to increase their consumption, their labor income is then increasing. The extra labor affects capitals marginal productivity, raising the capital rent rate and Tobin's Q. Although the real wage falls at impact in response to the shock (because of the rise in inflation), the increase in the capital rent rate is enough to drive upward the marginal costs, this confirms the inflationary pressures originated in the Ricardian households desire to increase consumption and investment.

The response of most of the variables to the monetary shock is virtually independent of the composition of the households, besides some minor changes in the magnitude of the response. The responses obtained are also qualitatively very similar to the ones presented in the New-Keynesian model without rule of thumb consumers (Galí, 2008), in contrast to the technology shock in which some of the responses depended strongly in the value of the parameter Γ.

A New Keynesian Wage Phillips Curve

Following Erceg et al. (2000) and Galí (2008) the following is the problem of sticky wages a la Calvo (1983).

A.1 Labor Demand

Recall from the problem of the labor agency the following results:

The demand for each type of differentiated labor and the wage index are given by:

$$h_{j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\eta} h_t \qquad \text{and} \qquad w_t = \left[\int_0^1 (w_{j,t})^{1-\eta}\, dj\right]^{\frac{1}{1-\eta}}$$

Finally, given that in equilibrium a portion ξ of the wages cannot be adjusted optimally one gets:

$$w_t = \left[\xi\,(w_{t-1})^{1-\eta} + (1-\xi)\,(w_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Linearizing:

$$\tilde{w}_t = \xi \tilde{w}_{t-1} + (1-\xi)\,\tilde{w}_t^*$$

A.2 Household problem and wage setting

There is a continuum of households indexed by $j\epsilon\,(0,1)$, each household supplies one of the varieties of labor $h_{j,t}$, the problem of the household is then to choose a nominal wage that maximizes its utility subject to its resource constraint and the labor demand for its variety of labor. Nevertheless, a household can only decide over its wage optimally with probability ξ, if the household cannot choose optimally its wage then it is kept unaltered until the signal for optimal adjustment is received by the household.

The problem to solve in a given period t in which optimization is allowed is:

$$\begin{aligned}
\underset{w_{j,t}}{\text{Max}} \quad & \sum_{i=0}^{\infty} (\beta\xi)^i\, \mathcal{U}(c_{j,t+i}, h_{j,t+i}) \\
\text{S.T} \quad & \\
0 \quad &= \quad w_{j,t}h_{j,t+i} - P_{t+i}c_{t+i} \\
h_{j,t+i} \quad &= \quad \left(\frac{w_{j,t}}{w_{t+i}}\right)^{-\eta} h_{t+i}
\end{aligned}$$

The firs order condition for the problem is given by:

$$\begin{aligned}
\sum_{i=0}^{\infty} (\beta\xi)^i \left[\lambda_{j,t+i}\left(h_{j,t+i} + w_{j,t}\frac{\partial h_{j,t+i}}{\partial w_{j,t}}\right) + \mathcal{U}_h(c_{j,t+i}, h_{j,t+i})\,\frac{\partial h_{j,t+i}}{\partial w_{j,t}}\right] &= 0 \\
\sum_{i=0}^{\infty} (\beta\xi)^i \left[\lambda_{j,t+i}\left(h_{j,t+i} - \eta w_{j,t}\,(w_{j,t})^{-\eta-1}\left(\frac{1}{w_{t+i}}\right)^{-\eta} h_{t+i}\right)\right] & \\
-\sum_{i=0}^{\infty} (\beta\xi)^i \left[\eta\mathcal{U}_h(c_{j,t+i}, h_{j,t+i})\,(w_{j,t})^{-\eta-1}\left(\frac{1}{h_{t+i}}\right)^{-\eta} h_{t+i}\right] &= 0 \\
\sum_{i=0}^{\infty} (\beta\xi)^i\, h_{j,t+i}\left[\lambda_{j,t+i}w_{j,t} + \frac{\eta}{\eta-1}\mathcal{U}_h(c_{j,t+i}, h_{j,t+i})\right] &= 0
\end{aligned}$$

In the last expression $\lambda_{j,t+i}$ is the Lagrange multiplier of the resource constraint, from other optimality conditions one gets that $\lambda_{j,t+i} = \frac{\mathcal{U}_c(c_{j,t+i},h_{j,t+i})}{P_{t+i}}$. One also knows that all the households that choose over their wages in period "t" face the same problem, then for all of them one gets: $w_{j,t} = w_t^*$. The first order condition is now expressed as:

$$\begin{aligned}\sum_{i=0}^{\infty}(\beta\xi)^i h_{j,t+i}\left[\frac{\mathcal{U}_c\left(c_{j,t+i},n_{j,t+i}\right)}{P_{t+i}}w_t^*+\frac{\eta}{\eta-1}\mathcal{U}_h\left(c_{j,t+i},h_{j,t+i}\right)\right] &= 0\\ \sum_{i=0}^{\infty}(\beta\xi)^i h_{j,t+i}\mathcal{U}_c\left(c_{j,t+i},n_{j,t+i}\right)\left[\frac{w_t^*}{P_{t+i}}-\frac{\eta}{\eta-1}\mathrm{MRS}_{j,t+i}\right] &= 0\end{aligned}$$

where $\mathrm{MRS}_{j,t}=-\frac{\mathcal{U}_h(c_{j,t+i},h_{j,t+i})}{\mathcal{U}_c(c_{j,t+i},n_{j,t+i})}$.

In the last expression the ratio between the marginal disutility of labor and the marginal utility of consumption is defined as the marginal rate of substitution between labor and consumption and denoted as $\mathrm{MRS}_{j,t+i}$. Note that if the household could choose its wage every period (i.e. $\xi=0$) then the real wage $\left(\frac{w_t}{P_t}\right)$ would be equal to $\frac{\eta}{\eta-1}\mathrm{MRS}_{j,t}$. The last result indicates that in the absence of nominal rigidities the wage would be a markup over the cost for the household (its marginal rate of substitution), that markup would be constant and equal to $\frac{\eta}{\eta-1}$, thus the desired frictionless markup is denoted as $\mathcal{M}_w=\frac{\eta}{\eta-1}$.

Below the first order condition is linearized around a zero inflation steady state (note that in steady state $\frac{w^*}{P}=\frac{w}{P}=\mathcal{M}_w\mathrm{MRS}$:

$$\begin{aligned}\sum_{i=0}^{\infty}(\beta\xi)^i h_{j,t+i}\mathcal{U}_c\left(c_{j,t+i},n_{j,t+i}\right)\left[\frac{w_t^*}{P_{t+i}}-\mathcal{M}_w\mathrm{MRS}_{j,t+i}\right] &= 0\\ \sum_{i=0}^{\infty}(\beta\xi)^i h\mathcal{U}_c\left[\frac{w}{P}-\mathcal{M}_w\mathrm{MRS}\right]\left(\tilde{h}_{j,t+i}+\tilde{\mathcal{U}}_{c,t+i}\right)+\sum_{i=0}^{\infty}(\beta\xi)^i h\mathcal{U}_c\left[\frac{w}{P}\left(\tilde{w}_t^*-\tilde{p}_{t+i}\right)-\mathcal{M}_w\mathrm{MRS}\,\tilde{\mathrm{mrs}}_{j,t+i}\right] &= 0\\ \sum_{i=0}^{\infty}(\beta\xi)^i\left[\tilde{w}_t^*-\tilde{p}_{t+i}-\tilde{\mathrm{mrs}}_{j,t+i}\right] &= 0\end{aligned}$$

Noting that $\tilde{w}_t^*$ doesn't depend in the subindex i one gets:

$$\tilde{w}_t^* = (1-\beta\xi)\sum_{i=0}^{\infty}(\beta\xi)^i\left[\tilde{p}_{t+i}+\tilde{\mathrm{mrs}}_{j,t+i}\right]$$

It is now necessary to assume a functional form for the utility:

$$\mathcal{U}_{j,t} = \frac{c_{j,t}^{1-\sigma}}{1-\sigma}-\chi\frac{h_{j,t}^{1+\vartheta}}{1+\vartheta}$$

The functional form assumed above is separable between consumption and labor, in that way, and thanks to Arrow-Debreau securities available to the household, the level of consumption (and the marginal utility of consumption) of a particular household is independent of the wage prevailing for its type of labor. In equilibrium the aggregate level of consumption is equal to the consumption level of each household (i.e. $c_t=c_{j,t}$).

The marginal rate of substitution is then:

$$\mathrm{MRS}_{j,t} = -\frac{\mathcal{U}_h}{\mathcal{U}_c}=\frac{\chi h_{j,t}^{\vartheta}}{c_t^{-\sigma}}$$

The log linear approximation for the MRS_t is:

$$\tilde{\mathrm{mrs}}_{j,t} = \vartheta\tilde{h}_{j,t}+\sigma\tilde{c}_t$$

The average MRS is defined as:

$$\tilde{\mathrm{mrs}}_t = \vartheta\tilde{h}_t+\sigma\tilde{c}_t$$

Expressing in terms of the average MRS:

$$\tilde{\mathrm{mrs}}_{j,t} = \tilde{\mathrm{mrs}}_t+\vartheta\left(\tilde{h}_{j,t}-\tilde{h}_t\right)$$

Recalling the labor demand and linearizing:

$$h_{j,t} = \left(\frac{w_{j,t}}{w_t}\right)^{-\eta} h_t \qquad \text{and} \qquad \tilde{h}_{j,t} - \tilde{h}_t = -\eta\left(\tilde{w}_t^* - \tilde{w}_t\right)$$

Substituting in the MRS:

$$\tilde{\text{mrs}}_{j,t} = \tilde{\text{mrs}}_t - \eta\vartheta\left(\tilde{w}_t^* - \tilde{w}_t\right)$$

Substituting household's "j" marginal rate of substitution in the optimal wage equation:

$$\begin{aligned}
\tilde{w}_t^* &= (1-\beta\xi)\sum_{i=0}^{\infty}(\beta\xi)^i\left[\tilde{p}_{t+i} + \tilde{\text{mrs}}_{j,t+i}\right] \\
\tilde{w}_t^* &= (1-\beta\xi)\sum_{i=0}^{\infty}(\beta\xi)^i\left[\tilde{p}_{t+i} + \tilde{\text{mrs}}_{t+i} - \eta\vartheta\left(\tilde{w}_t^* - \tilde{w}_{t+i}\right)\right] \\
\tilde{w}_t^* &= (1-\beta\xi)\sum_{i=0}^{\infty}(\beta\xi)^i\left[\tilde{p}_{t+i} + \tilde{\text{mrs}}_{t+i} + \eta\vartheta\tilde{w}_{t+i}\right] - (1-\beta\xi)\sum_{i=0}^{\infty}(\beta\xi)^i\eta\vartheta\tilde{w}_t^* \\
\tilde{w}_t^* &= (1-\beta\xi)\sum_{i=0}^{\infty}(\beta\xi)^i\left[\tilde{p}_{t+i} + \tilde{\text{mrs}}_{t+i} + \eta\vartheta\tilde{w}_{t+i}\right] - (1-\beta\xi)\frac{\eta\vartheta}{1-\beta\xi}\tilde{w}_t^* \\
\tilde{w}_t^* &= \left(\frac{1-\beta\xi}{1+\eta\vartheta}\right)\sum_{i=0}^{\infty}(\beta\xi)^i\left[(1+\eta\nu)\,\tilde{w}_{t+i} - \left[\left(\tilde{w}_{t+i} - \tilde{p}_{t+i}\right) - \tilde{\text{mrs}}_{t+i}\right]\right]
\end{aligned}$$

Defining the gross average markup as the ratio between the real wage and the average marginal rate of substitution and linearizing one gets the following expression:

$$\begin{aligned}
\mathcal{M}_{w,t} &= \frac{W_t/P_t}{\text{MRS}_t} \\
\tilde{\mu}_{w,t} &= \left(\tilde{w}_t - \tilde{p}_t\right) - \tilde{\text{mrs}}_t
\end{aligned}$$

Substituting in the optimal wage equation:

$$\tilde{w}_t^* = \left(\frac{1-\beta\xi}{1+\eta\vartheta}\right)\sum_{i=0}^{\infty}(\beta\xi)^i\left[(1+\eta\vartheta)\,\tilde{w}_{t+i} - \tilde{\mu}_{w,t}\right]$$

The last expression is rewritten recursively as:

$$\tilde{w}_t^* = \beta\xi\tilde{w}_{t+1}^* + (1-\beta\xi)\left[\tilde{w}_t - \frac{1}{1+\eta\vartheta}\tilde{\mu}_{w,t}\right]$$

Now recalling the definition of the average wage and substituting the optimal wage:

$$\begin{aligned}
\tilde{w}_t &= \xi\tilde{w}_{t-1} + (1-\xi)\,\tilde{w}_t^* \\
\tilde{w}_t &= \xi\tilde{w}_{t-1} + (1-\xi)\left[\beta\xi\tilde{w}_{t+1}^* + (1-\beta\xi)\left[\tilde{w}_t - \frac{1}{1+\eta\vartheta}\tilde{\mu}_{w,t}\right]\right] \\
\left(\tilde{w}_t - \tilde{w}_{t-1}\right) &= (1-\xi)\,\beta\left(\tilde{w}_{t+1}^* - \tilde{w}_t\right) - \frac{(1-\xi)(1-\beta\xi)}{\xi(1+\eta\vartheta)}\tilde{\mu}_{w,t} \\
\left(\tilde{w}_t - \tilde{w}_{t-1}\right) &= (1-\xi)\,\beta\left(\frac{1}{(1-\xi)}\left(\tilde{w}_{t+1} - \xi\tilde{w}_t\right) - \tilde{w}_t\right) - \frac{(1-\xi)(1-\beta\xi)}{\xi(1+\eta\vartheta)}\tilde{\mu}_{w,t} \\
\left(\tilde{w}_t - \tilde{w}_{t-1}\right) &= \beta\left(\tilde{w}_{t+1} - \tilde{w}_t\right) - \frac{(1-\xi)(1-\beta\xi)}{\xi(1+\eta\vartheta)}\tilde{\mu}_{w,t} \\
\pi_t^w &= \beta\pi_{t+1}^w - \frac{(1-\xi)(1-\beta\xi)}{\xi(1+\eta\vartheta)}\tilde{\mu}_{w,t}
\end{aligned}$$

The last equations is the New Keynesian Wage Phillips Curve, where the wage inflation is given by the log difference of the nominal wage.

B New Keynesian Phillips Curve

Following Galí (2008) the following is the problem of sticky wages a la Calvo (1983).

B.1 Demand for differentiated goods

Recall from the problem of the final good aggregator that the demand for each type of intermediate good is given by

$$v_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t$$

and the aggregate price index by:

$$P_t = \left[\int_0^1 (P_t(j))^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$$

B.2 Firm price setting problem

The intermediate goods producers must decide the price and the demand for factors that maximize their profits subject to the demand for its type of product, Calvo rigidities on prices and its production function.

Then the profit maximization problem in nominal terms for a firm that is allowed to decide optimally its price in period "t" is the following, defining CMG_t as the nominal marginal cost of the firm, and $\beta^i \frac{\lambda_{t+i}}{\lambda_t}$ as the relevant discount factor for the firm:

$$\begin{aligned}
\underset{P_t(j)}{\mathrm{Max}} \quad & \sum \epsilon^i \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left(Pt+i(j)\, v_{t+i}(j) - \mathrm{CMG}_{t+i} v_{t+i}\right) \\
S.t & \\
v_{t+i}(j) &= \left(\frac{P_{t+i}(j)}{P_{t+i}}\right)^{-\theta} Y_{t+i} \\
P_{t+i}(j) &= P_t^o
\end{aligned}$$

$$\underset{p_t(j)}{\mathrm{Max}} \quad \sum (\epsilon\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \left((P_t^o)^{1-\theta} \left(\frac{1}{P_{t+i}}\right)^{-\theta} Y_{t+i} - \mathrm{CMG}_{t+i} \left(\frac{P_t^o}{P_{t+i}}\right)^{-\theta} Y_{t+i} \right)$$

The first order condition for the optimal price is given by:

$$\begin{aligned}
0 &= \sum (\epsilon\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \left((1-\theta)(P_t^o)^{-\theta} \left(\frac{1}{P_{t+i}}\right)^{-\theta} Y_{t+i} + \theta \mathrm{CMG}_{t+i} (P_t^o)^{-\theta-1} \left(\frac{1}{P_{t+i}}\right)^{-\theta} Y_{t+i} \right) \\
0 &= \sum (\epsilon\beta)^i \frac{\lambda_{t+i}}{\lambda_t} \left(\frac{P_t^o}{P_{t+i}}\right)^{-\theta} Y_{t+i} \left(\frac{P_t^o}{P_{t-1}} - \mathcal{M}\mathrm{MC}_{t+i} \frac{P_{t+i}}{P_{t-1}} \right)
\end{aligned}$$

In the previous expression $\mathcal{M} = \frac{\theta}{\theta-1}$ and $\mathrm{MC}_t = \frac{\mathrm{CMG}_t}{P_t}$.
The log-lineal approximation around a zero inflation steady state is:

$$\begin{aligned}
0 &= \sum (\epsilon\beta)^i Y \left(\frac{p^o}{P} - \mathcal{M}\mathrm{MC}\frac{P}{P}\right) \left[\tilde{\lambda}_{t+i} - \tilde{\lambda}_t + \tilde{y}_{t,t+i}\right] \\
&\quad + \sum (\epsilon\beta)^i Y \left(\frac{p^o}{P} \left[\tilde{p^o}_t - \tilde{p}_{t-1}\right] - \mathcal{M}\mathrm{MC}\frac{P}{P} \left[\tilde{mc}_{t+i} + \tilde{p}_{t+i} - \tilde{p}_{t-1}\right] \right) \\
0 &= \sum (\epsilon\beta)^i \left(\left[\tilde{p^o}_t - \tilde{p}_{t-1}\right] - \left[\tilde{mc}_{t+i} + \tilde{p}_{t+i} - \tilde{p}_{t-1}\right] \right)
\end{aligned}$$

$$
\begin{aligned}
0 &= \frac{1}{1-\epsilon\beta}\left[\tilde{p^o}_t - \tilde{p}_{t-1}\right] - \sum_{i=0} (\epsilon\beta)^i \left[\tilde{mc}_{t+i} + \tilde{p}_{t+i} - \tilde{p}_{t-1}\right] \\
0 &= \frac{1}{1-\epsilon\beta}\left[p^o_t - \ln p^o - (p_{t-1} - \ln P)\right] - \sum (\epsilon\beta)^i \left[mc_{t+i} - \ln \mathrm{MC} + (p_{t+i} - \ln P) - (p_{t-1} - \ln P)\right] \\
0 &= \frac{1}{1-\epsilon\beta}\left[p^o_t - p_{t-1}\right] - \sum (\epsilon\beta)^i \left[mc_{t+i} + \mu + p_{t+i} - p_{t-1}\right]
\end{aligned}
$$

It will be useful to express the later equation in terms of the difference between the optimal and lagged price level:

$$
\begin{aligned}
p^o_t - p_{t-1} &= (1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i \left[mc_{t+i} + \mu + p_{t+i} - p_{t-1}\right] \\
p^o_t - p_{t-1} &= (1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i \left[mc_{t+i} + \mu\right] + \left[(1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i p_{t+i}\right] - p_{t-1} \\
p^o_t - p_{t-1} &= (1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i \left[mc_{t+i} + \mu\right] + \left[\sum_{i=0} (\epsilon\beta)^i p_{t+i} - \sum_{i=0} (\epsilon\beta)^{1+i} p_{t+i}\right] - p_{t-1} \\
p^o_t - p_{t-1} &= (1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i \left[mc_{t+i} + \mu\right] + \sum_{i=1} (\epsilon\beta)^i \left[p_{t+i} - p_{t-1+i}\right] + p_t - p_{t-1} \\
p^o_t - p_{t-1} &= (1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i \left[mc_{t+i} + \mu\right] + \sum_{i=0} (\epsilon\beta)^i \left[p_{t+i} - p_{t-1+i}\right] \\
p^o_t - p_{t-1} &= (1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^i \left[mc_{t+i} + \mu\right] + \sum_{i=0} (\epsilon\beta)^i \pi_{t+i} \\
p^o_t - p_{t-1} &= (1-\epsilon\beta)(mc_t + \mu) + \pi_t + \epsilon\beta \left[(1-\epsilon\beta) \sum_{i=0} (\epsilon\beta)^{i+1} \left[mc_{t+1+i} + \mu\right] + \sum_{i=0} (\epsilon\beta)^{i+1} \pi_{t+1+i}\right] \\
p^o_t - p_{t-1} &= (1-\epsilon\beta)(mc_t + \mu) + \pi_t + \epsilon\beta \left[p^o_{t+1} - p_{t-1}\right]
\end{aligned}
$$

From the expression for the aggregate price one gets in log-linear form:

$$
\begin{aligned}
p_t &= \epsilon p_{t-1} + (1-\epsilon) p^o_t \\
\pi_t &= (1-\epsilon)(p^o_t - p_{t-1}) \\
p^o_t - p_{t-1} &= \frac{1}{1-\epsilon}\pi_t
\end{aligned}
$$

Combining the last two results it is possible to obtain the Phillips Curve:

$$
\begin{aligned}
\frac{1}{1-\epsilon}\pi_t &= (1-\epsilon\beta)(mc_t + \mu) + \pi_t + \frac{\epsilon\beta}{1-\epsilon}\pi_{t+1} \\
\epsilon\pi_t &= (1-\epsilon)(1-\epsilon\beta)(mc_t + \mu) + \epsilon\beta\pi_{t+1} \\
\pi_t &= \frac{(1-\epsilon)(1-\epsilon\beta)}{\epsilon}\tilde{mc}_t + \beta\pi_{t+1}
\end{aligned}
$$

C Linear Equilibrium

The following is a list of the models variables and linearized equilibrium conditions:
Variables: $\left\{i\,,\,z\,,\,\pi\,,\,y\,,\,c\,,\,x\,,\,k\,,\,h\,,\,c_a\,,\,c_b\,,\,h_a\,,\,h_b\,,\,\varphi\,,\,r^k\,,\,w\,,\,w_a\,,\,w_b\,,\,\pi^{w_a}\,,\,\pi^{w_b}\,,\,\mu_{w_a}\,,\,\mu_{w_b}\,,\,q\,,\,\mathrm{pr}\right\}$

Exogenous Processes

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t\right) + \epsilon_{i,t}$$

$$\tilde{z}_{t+1} = \rho_z \tilde{z}_t + \epsilon_{t+1}$$

Aggregation

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{x}{y}\tilde{x}_t$$

$$\tilde{c}_{a,t} = \tilde{w}_{a,t} + \tilde{h}_{a,t}$$

$$\tilde{c}_t = \frac{c_a}{c}\Gamma\tilde{c}_{a,t} + \frac{c_b}{c}(1-\Gamma)\,\tilde{c}_{b,t}$$

$$\tilde{\mathrm{pr}}_t = \frac{1}{1-\varphi}\left(\tilde{y}_t - \alpha\varphi\left(\tilde{r}^k_t + \tilde{k}_t\right) - (1-\alpha)\,\varphi\left(\tilde{w} + \tilde{h}\right)\right)$$

Production

$$\tilde{r}^k_t = \tilde{\varphi}_t - (1-\alpha)\left(\tilde{k}_t - \tilde{h}_t\right)$$

$$\tilde{w}_t = \tilde{\varphi}_t + \alpha\left(\tilde{k}_t - \tilde{h}_t\right)$$

$$\tilde{y}_t = \tilde{z}_t + \alpha\tilde{k}_t + (1-\alpha)\,\tilde{h}_t$$

Prices

$$\pi_t = \beta\pi_{t+1} + \frac{(1-\epsilon)(1-\epsilon\beta)}{\epsilon}\tilde{\varphi}_t$$

Labor

$$\tilde{h}_{at} = (\tilde{w}_t - \tilde{w}_{at}) + \tilde{h}_t$$

$$\tilde{h}_{bt} = (\tilde{w}_t - \tilde{w}_{bt}) + \tilde{h}_t$$

$$\tilde{h}_t = \Gamma\tilde{h}_{at} + (1-\Gamma)\,\tilde{h}_{bt}$$

Non-Ricardian Wage

$$\pi^{w_a}_t = \beta\pi^{w_a}_{t+1} - \frac{(1-\xi_a)(1-\beta\xi_a)}{\xi_a(1+\eta\vartheta)}\tilde{\mu}_{w_a,t}$$

$$\tilde{\mu}_{w_a,t} = \tilde{w}_{a,t} - \left(\vartheta\tilde{h}^s_{a,t} + \sigma\tilde{c}_{a,t}\right)$$

$$\tilde{w}_{a,t} = \tilde{w}_{a,t-1} + \pi^{w_a}_t - \pi_t$$

Ricardian Wage

$$\pi^{w_b}_t = \beta\pi^{w_b}_{t+1} - \frac{(1-\xi_b)(1-\beta\xi_b)}{\xi_b(1+\eta\vartheta)}\tilde{\mu}_{w_b,t}$$

$$\tilde{\mu}_{w_b,t} = \tilde{w}_{b,t} - \left(\vartheta\tilde{h}^s_{b,t} + \sigma\tilde{c}_{b,t}\right)$$

$$\tilde{w}_{b,t} = \tilde{w}_{b,t-1} + \pi^{w_b}_t - \pi_t$$

Capital

$$\tilde{q}_t = -(i_t - \pi_{t+1}) + \beta\tilde{q}_{t+1} + [1-\beta(1-\delta)]\,\tilde{r}^k_{t+1}$$

$$\tilde{x}_t - \tilde{k}_t = \iota\tilde{q}_t$$

$$\tilde{k}_{t+1} = \delta\tilde{x}_t + (1-\delta)\,\tilde{k}_t$$

Euler Equation

$$i_t - \pi_{t+1} = \sigma\left(\tilde{c}_{b,t+1} - \tilde{c}_{b,t}\right)$$

D Steady State

Given the parametrization used in the simulation of the model ($\sigma = 1$ and $\vartheta = 1$) the values of the steady state variables is given by the following equations:

$$
\begin{aligned}
\pi &= 1 \\
z &= 1 \\
\varphi &= \frac{\theta - 1}{\theta} \\
i &= \frac{1}{\beta} \\
Q &= 1 \\
r^k &= \frac{1}{\beta} + \delta - 1 \\
h_a &= \left(\frac{\eta}{\eta - 1} \chi \right)^{\frac{-1}{2}} \\
w &= (1 - \alpha) \varphi z \left(\frac{r^k}{\alpha \varphi z} \right)^{\frac{\alpha}{\alpha - 1}} \\
h &= \left[\frac{(1 - \Gamma)^2 \frac{\eta - 1}{\chi \eta} w h_a^{\frac{2\Gamma}{1 - \Gamma}}}{z \left(\frac{r^k}{\alpha \varphi z} \right)^{\frac{\alpha}{\alpha - 1}} - \delta \left(\frac{r^k}{\alpha \varphi z} \right)^{\frac{1}{\alpha - 1}} - \Gamma^2 w} \right]^{\frac{1 - \Gamma}{2}} \\
h_b &= h_a^{\frac{-\Gamma}{1 - \Gamma}} h^{\frac{1}{1 - \Gamma}} \\
k &= \left(\frac{r^k}{\alpha \varphi z} \right)^{\frac{1}{\alpha - 1}} h \\
y &= z \left(\frac{r^k}{\alpha \varphi z} \right)^{\frac{\alpha}{\alpha - 1}} h \\
w_a &= \Gamma \frac{w}{h_a} h \\
w_b &= (1 - \Gamma) w h_a^{\frac{\Gamma}{1 - \Gamma}} h^{\frac{-\Gamma}{1 - \Gamma}} \\
c_a &= w_a h_a \\
c_b &= \frac{\eta - 1}{\chi \eta} \frac{w_b}{h_b} \\
c &= \Gamma c_a + (1 - \Gamma) c_b \\
x &= \delta k \\
\mathrm{Pr} &= (1 - \varphi) y
\end{aligned}
$$

References

Basu, S., Fernald, J. G., and Kimball, M. S. (2006). Are technology improvements contractionary? *American Economic Review*, 96(5):1418–1448.

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5):1305–11.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Colciago, A. (2011). Rule-of-Thumb consumers meet sticky wages. *Journal of Money, Credit and Banking*, 43(2-3):325–353.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2):281–313.

Francis, N. and Ramey, V. A. (2005). Is the technology-driven real business cycle hypothesis dead? shocks and aggregate fluctuations revisited. *Journal of Monetary Economics*, 52(8):1379–1399.

Galí, J. (1999). Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 89(1):249–271.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle*. Princeton University Press.

Galí, J. (2010). Monetary policy and unemployment. In Friedman, B. M. and Woodford, M., editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 10. Elsevier.

Galí, J., López-Salido, J. D., and Vallés, J. (2004). Rule-of-Thumb consumers and the design of interest rate rules. *Journal of Money, Credit and Banking*, 36(4):739–763.

Galí, J. and Rabanal, P. (2005). Technology shocks and aggregate fluctuations: How well does the real business cycle model fit postwar u.s. data? In *NBER Macroeconomics Annual 2004, Volume 19*. National Bureau of Economic Research, Inc.

González, A., Ocampo, S., Rodríguez, D., and Rodríguez, N. (2011). Asimetrías del empleo y el producto, una aproximación de equilibrio general. Borradores de Economia 663, Banco de la República de Colombia.

Klein, P. (2000). Using the generalized Schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, 24(10):1405–1423.

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6):1345–70.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Woodford, M. (2001). The Taylor rule and optimal monetary policy. *American Economic Review*, 91(2):232–237.