Data Revisions and the Output Gap

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Juan Manuel Julio†

Abstract

Preliminary and delayed Colombian GDP reports are replaced with optimal in-sample now-casts of "true" GDP figures derived from a model for data revisions. The new GDP series is augmented with five years of optimal out-of-sample forecasts and back-casts of "true" GDP figures derived from the same model. The trend-cycle component of the augmented GDP series is then filtered. The resulting gap is more resistant than the ordinary HP filter to the end of sample optimal filtering problem as well as to GDP revisions and delays. The short term noise of the final output gap estimate is also reduced.

Adjusting for data revisions and delays reduce the uncertainty of estimated gaps. The extended and further extended HP estimates of the output gap show an impressive efficiency gain with respect to the ordinary HP gap, 43% and 47% respectively, on average. The new extension increases the efficiency in 7.4%, on average, with respect to extended HP estimates. These results constitute a benchmark to future work on real time estimation of the output gap under GDP revisions and delays in Colombia.

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Revisiones de Datos y la Brecha del PIB*

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Resumen

Se reemplazan los datos preliminares y demorados del PIB Colombiano con now-casts óptimos de los datos definitivos derivados del modelo de revisiones propuesto por Julio [11]. La serie resultante se extiende con cinco años de pronósticos y back-casts que se obtienen del mismo modelo. Se aplica el filtro ordinario de Hodrick & Prescott [9] al componente de tendencia-ciclo de la serie extendida. La brecha resultante es más resistente al problema del final de la muestra del filtro de Hodrick & Prescott así como a las revisiones y demoras. El ruido de corto plazo de la brecha estimada también se reduce.

El reemplazo de los datos preliminares y demorados por now-casts reduce la incertidumbre de la brecha estimada. El filtro extendido y la extensión adicional producen ganancias importantes en eficiencia, 43% y 47% en promedio respectivamente, con respecto al filtro ordinario. La extensión adicional incrementa la eficiencia en 7.4% con respecto al filtro extendido. Estos resultados constituyen un punto de referencia para trabajos futuros sobre la estimaci ón de la brecha bajo revisiones y demoras.

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1 Introduction

The output gap plays a key role in the design and analysis of monetary policy. In fact, the output gap not only conveys important information about underlying inflationary pressures, but is also an extensively used input to forecast the short to medium term condition of the economy. Therefore, the output gap plays an essential role in determining the current state of the economy and its short to medium term outlook, the most important element in monetary policy design and analysis.

However, the output gap is a potentially misleading input for monetary policy because of its estimation uncertainty. There are several reasons for this. First, the "true" gap is not observable and therefore its estimates are uncertain at every period of time. Second, optimal filtering techniques are two sided symmetric weighted averages of infinite length over the "true" GDP values. Since the sample size is finite, symmetric two sided averages are feasible only at the center of the sample. At both ends, however, only one-sided averages are possible. Third, there is a significant degree of uncertainty in the relationship between the GDP and the output gap, model uncertainty. As a result, the output gap uncertainty band narrows down at the center of the sample and widens up symmetrically at both ends.

Moreover, GDP data revisions and delays increase the uncertainty of the output gap at the end of the sample. Therefore, the uncertainty band of the output gap is wider at the end of the sample than at the beginning. See Garrat et al [7] and Bank of Iceland [17] for instance.

An additional drawback of standard HP filter output gap estimates is the excess noise of cycle component estimates. This noise, according to Kaiser & Maravall [12], arises from the use of seasonally adjusted series instead of the trend-cycle component. According to these authors this practice compromises the interpretation of output gap estimates also.

However, the HP filter is known to be optimal in several ways. In the middle of the sample the HP filter is a two sided weighted symmetric average of the data, and if the sample is large enough, the HP filter produces an optimal estimate of the cycle at the middle of the sample. Moreover, the HP filter yields an optimal orthogonal decomposition of a time series into trend and cycle under general I(1) and I(2) data generating processes. Mise et al [16] show that this optimality holds for any value of the smoothness parameter.

Therefore, if measures to reduce the end of sample excess variability (due to non symmetric filtering and GDP delays and revisions) are available, HP filter estimates of the output gap might still be useful for monetary policy.

Several procedures to solve the problems mentioned above have been proposed. The EU Commission [5] proposed, for instance, to augment the original time series with out-of-sample forecasts and back-casts from an appropriate model. The rationale behind this procedure is that by augmenting the series the optimality at the center of the sample extends outwards, thus increasing the efficiency of output gap estimates. See Mise et al [16], Kaiser & Maravall [12] and Kaiser & Maravall [13] also.

In order to reduce the optimal filtering problem at both ends of the sample Baxter & King [3] proposed a small sample approximation to the optimal band pass filter. However, for all practical matters these estimates do not differ from standard HP filter estimates. See EU Commission [5], for instance.

It has also been suggested to filter the trend-cycle component of the augmented series instead of its seasonal adjustment. The seasonal adjustment of a time series contains the irregular component of the series in addition to its trend-cycle component. When the trend-cycle component of the GDP is filtered, excess noise is reduced and the resulting gap gains interpretation. See Kaiser & Maravall [12] and [13], for instance.

Preliminary and delayed Colombian GDP reports are replaced with optimal in-sample now-casts of "true" GDP figures derived from the model for data revisions proposed by Julio [11]. The new GDP series is augmented with five years of optimal out-of-sample forecasts and back-casts of "true" GDP figures derived from the same model. The trend-cycle component of the augmented GDP series is filtered. The resulting gap is more resistant than the ordinary HP filter to the end of sample optimal filtering problem as well as to GDP revisions and delays. The short term noise of the final output gap estimate is also reduced.

Adjusting for data revisions and delays reduce the uncertainty of estimated gaps. The extended and further extended HP estimates of the output gap show an impressive efficiency gain with respect to the ordinary HP gap, 43% and 47% respectively, on average. The new extension increases the efficiency in 7.4%, on average, with respect to extended HP estimates. These results constitute a benchmark to future work on real time estimation of the output gap under GDP revisions and delays in Colombia.

A similar approach was suggested by Aastveit & Trovik [1] in the context of a dynamic factor model for real time now-casting of the GDP.

2 Literature Review

Hodrick & Prescott [9] proposed a procedure to decompose macroeconomic time series in a long run trend g_t and the business cycle c_t, as follows

$$y_t = g_t + c_t \qquad \text{for } t = 1, 2, \ldots, T$$

which results from the following minimization problem

$$\min_{\{g_t\}_{t=1}^{T}} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} \left[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2}) \right]^2 \right\}$$

where $c_t = y_t - g_t$. The term to the right of this equation is a growth variability penalizing quantity which may be adjusted through an exogenously provided constant, $\lambda > 0$. Whenever $\lambda \to 0$, $c_t \to 0$ and $g_t \to y_t$, while as $\lambda \to \infty$, g_t approaches a linear trend $\alpha_0 + \alpha_1 t$.

The HP filter has been subject to important criticism. King & Rebelo [14], for instance, report that the HP filter can change the persistence, variability and co-movement of economic time series, but also found that it is optimal, under the minimum square error criteria, for a limited set of the ARIMA family of models. Ehlgen [6] showed that these distortions also arise even when the conditions for optimality are met. According to this author "Such distortions are simply a characteristic of optimal signal extraction filters.".

However, it is also known that the HP filter is the optimal decomposition of a time series into orthogonal "trend" and "cycle" components when the data generating process of the series falls into a particular family. This optimality result arises in an infinite data setting, and thus relate to the estimated cycle at the center of the sample. At the end of the sample, however, the HP filter is sub-optimal. See Baxter & King [3] for instance.

In order to extend this optimality outwards from the middle of the sample it has been suggested to augment the time series with out-of-sample backcasts and forecasts from an appropriate model. See the EU Comission [5], Kaiser & Maravall [12] and [13], for instance.

Moreover, Mise et al [16] show that "the HP filter provides optimal estimators of orthogonal components that can be viewed as growth and cyclical for any I(1) or I(2) generating model". This optimality holds no matter the smoothing parameter used in the HP filter. These authors also show that the HP filter is sub optimal at both ends of the sample and confirm that the use of forecast-augmented series in HP filtering reduces the end of sample problem of the HP filter. See Kaiser & Maravall [12] also.

An additional drawback of the standard HP filter derives from the use of seasonally adjusted series. In addition to the trend-cycle component of the GDP series, its seasonal adjustment contains the irregular component too. This component, according to Kaiser & Maravall [12], is the source of excess noise in HP filtered output gaps. In order to reduce short run excess noise, and improve the interpretation of the output gap, these authors propose to filter the trend-cycle component of the series instead.

GDP revisions and delays increase the uncertainty of the output gap at the end of the sample. GDP delays hinder the calculation of the output gap for the current quarter, which thus becomes a forecast subject to its own forecasting error. Under data revisions, only preliminary GDP figures of the GDP are available at the end of the sample. Therefore the uncertainty band at the end of the sample is wider than at the beginning. See, for instance, by Bank of Iceland [17] and Garrat et al [7].

There are several ways to deal with the effect of GDP data revisions and delays on the output gap. If a model for data revisions and delays is available, its results may be used to replace preliminary GDP figures in production function estimates of the gap. Moreover, if the data revisions model permits the inclusion of a proper formulation of the growth and cycle components, it may provide both results at once, the now casts of the GDP and the corresponding output gap. See Jacobs & Van Norden [10], Julio [11] and Bank of Iceland [17] for instance.

Preliminary and delayed GDP reports are proposed to be replaced with optimal in-sample back-casts and now-casts of the "true" GDP derived from a data revisions model. The resulting series is then augmented with forecasts of the "true" GDP figures derived from the same model. Finally, the trend-cycle component of the augmented GDP series is filtered. The resulting gap is more resistant than ordinary HP estimates to the end of sample filtering problem and to GDP revisions and delays.

The model for data revisions used in this paper comes from Julio [11]. That model extends Jacobs & Van Norden's [10] in two ways. First, the "true" data series is observable up to a fixed period of time M, and second, preliminary figures might be biased estimates of the true series. This extension not only improves the overall identification of the model, but also represents the Colombian GDP data release process more realistically. As the remaining features of Jacobs and Van Norden's model are preserved, so do their gains in the new model.

3 Results

3.1 Data

Two data sets are required to meet the objectives of this paper. The first contains "true" GDP figures from which the true GDP dynamics may be inferred. The second contains the set of preliminary and "true" figures from which their joint dynamics is determined.

Prior to 1994Q1 Colombian end of year GDP figures were reported by the Colombian national planning department, DNP, and further back by Banco de la República, the central bank of Colombia. Valderrama [18] provides a quarterly decomposition of yearly figures which starts in 1978. This quarterly decomposition is popular in local research circles and may be considered "true"[1].

Colombian quarterly real GDP figures result from linking back the latest available figures (those belonging to the last vintage), through previous year to year growths, back to 1978Q1. This procedure avoids GDP level jumps resulting from methodology changes. Although this procedure is not "official", it is popular in research circles in this country and other countries as well. See Jacobs & Van Norden [10] and Arouba [2] for instance.

The data set analyzed in this paper contains Colombian GDP growth vintages from 2002Q2 to 2010Q1 released by DANE, the Colombian statistics bureau. These DGP releases exhibit a delay of one quarter, thus the 2002Q2 vintage, for instance, contains GDP growth reports from 1995Q1 to 2002Q1. The data set comprises two different methodologies. The first, called "base-1994" methodology, contains vintages from 2002Q2 to 2008Q1, whose reports start at 1995Q1, while the second, named "base-2000" methodology, contains vintages from 2008Q2 to 2009Q4, whose reports start at 2001Q1.

In order to characterize the dynamics of the "true" GDP process, linked back GDP level figures from 1978Q1 to 2009Q4 are considered.

3.2 Results

Three sets of real time output gap estimates are presented in this section. The first consists ordinary HP filter estimates of the output gap. In this case the real time GDP seasonally adjusted series are filtered. The second contains extended HP filter estimates. Real time GDP vintages are augmented with five years of optimal forecasts and back-casts derived from the ARIMA model describe in appendix A. The trend-cycle component of the augmented real time series is HP filtered. The third set contains the further

[1] GDP reports before 1977 are not considered because of methodological issues. See López et at [15].

extended HP filter estimates. In this case preliminary and delayed GDP reports are replaced by optimal now-casts derived from the data revisions model proposed by Julio [11]. The resulting series are then augmented with five years of optimal back-casts obtained from the ARIMA model described in appendix A, and five years of optimal forecasts derived from the data revisions model. The trend cycle components of the augmented real-time series are fed into the ordinary HP filter.

Increasing estimation windows of real time data start at 1978Q1 and add one quarter at a time from 2002Q1 up to 2009Q4.Ordinary HP filtering is carried out with $\lambda = 1600$.

The first set of estimates consists of ordinary HP filtering results. In this case seasonally adjusted log-GDP series were fed to the ordinary HP filter, and seasonal adjustment was carried out through the X11 time series decomposition procedure.

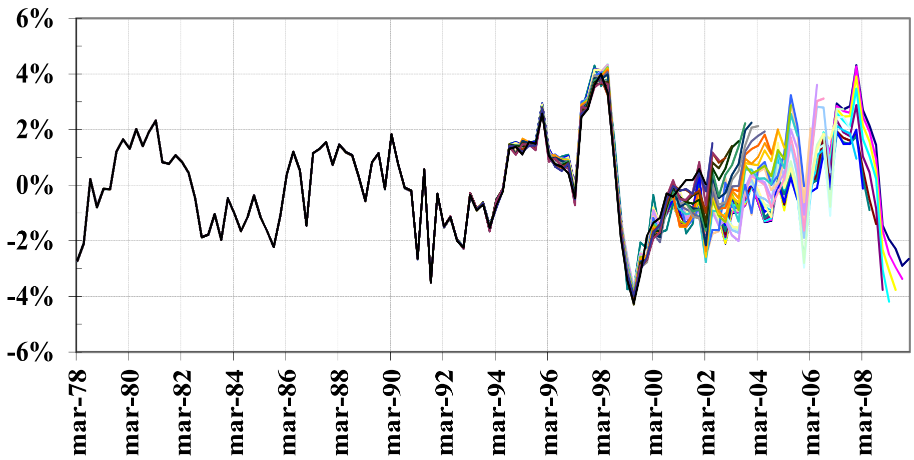

Figure 1: Increasing Windows Output Gap Estimates Simple HP Filter

Real time ordinary HP filter estimates of the output gap are shown in Figure 1. This figure displays distinctive features of the ordinary HP filtration. First, output gap estimates are stable from the start to the middle of the sample, 1994 to 1995. Second, end of sample filtering variability builds up slowly along the following sixth of the sample, 1995 to 1999. Third, end of sample filtering variability increases strongly during the last third of the sample, 2000 to 2009. Fourth, estimated output gaps display excess short run noise. And fifth, the excess variability of HP filter for the start of the sample remains hidden as no information update is performed at that start of the sample.

End of sample excess variability arises from the approximation error of one sided finite filters to the optimal two sided symmetric filters of infinite length. A simple calculation shows the extent of end of sample variability. The root mean square error of the estimated gap for 2002Q1 is 0.64% which, under the assumption of unbiasedness, gives a 95% confidence interval 2.69% wide!. Therefore, the HP estimate of the output gap for the most recent period of time is highly uncertain as an indicator of underlying inflation pressures.

The second set of estimates consists of extended HP filters. Each window of real time GDP figures is augmented with out-of-sample back-casts and forecasts from the ARIMA model described in appendix A. The forecasting and back-casting horizon for data augmentation is five years each as suggested by Kaiser & Maravall [12] and [13]. Real time X11 trend-cycle component of the augmented log-GDP series are fed to the HP filter.

Real time extended HP filter estimates of the output gap are depicted in Figure 2. A dramatic improvement with respect to the results of Figure 1 is observed. First, extended HP output gap estimates are smoother and display little short run noise. Therefore, spurious short lived cycles are eliminated by this filter. Second, end of sample filtering variability reduces dramatically.

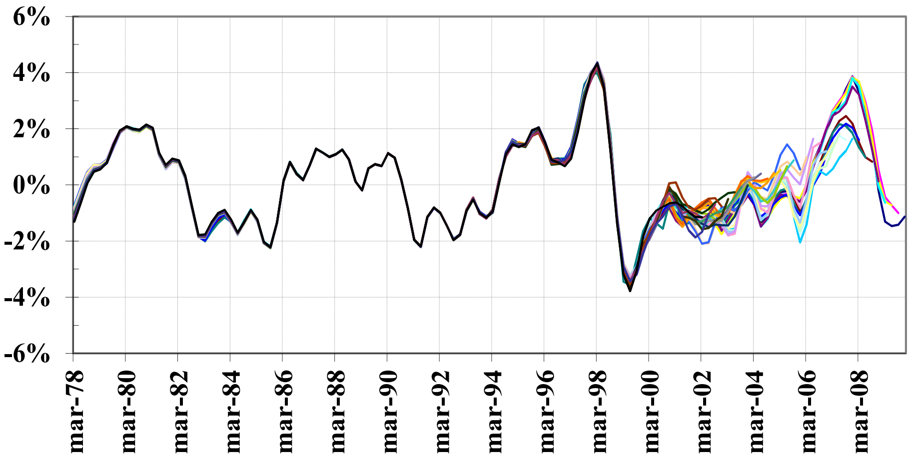

Figure 2: Increasing Windows Output Gap Estimates Extended HP Filter

The root mean square error of the gap for 2002Q1 is 0.36% which, under the assumption of unbiasedness, gives a 95% confidence interval 1.45% wide, a reduction of almost 50% in the width of the interval with respect to the corresponding interval for the ordinary HP filter gap.

The last set of estimates consists of further extended HP filters. Preliminary and delayed GDP reports are replaced with optimal now-casts derived from the model for data revisions proposed by Julio [11]. Each window of GDP data is augmented with five years of out-of-sample back-casts from the ARIMA model described in appendix A and five years of forecasts from Julio's [11] data revisions model. Real time X11 trend cycle components of the augmented series are fed to the ordinary HP filter to obtain the results.

The third set of estimates is displayed in Figure 3. Slight improvement with respect to previous results is observed.

To show the improvement of these results, a comparison of the efficiency of the three sets of estimates is shown in Figure 4. This figure displays the standard deviations of the estimated gaps for the periods of time containing

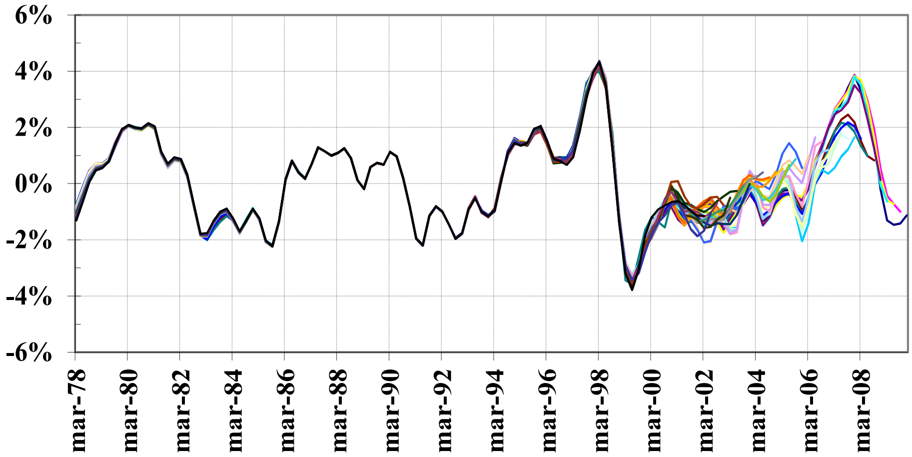

Figure 3: Increasing Windows Output Gap Estimates Further Extended HP Filter

at least 15 estimates. The efficiency gain of the extended HP filter with respect to the ordinary HP filter ranges from 11% to 70% depending on the output gap. This result might suggest that the uncertainty of output gap estimates depends on the gap.

The extended and further extended HP estimates of the output gap show an impressive efficiency gain with respect to the ordinary HP gap, 43% and 47%, on average respectively. By replacing preliminary and delayed GDP figures the efficiency increases in 7.4%, on average, with respect to extended HP estimates.

In order to check for output gap level effects of filtering, mean gap estimates are shown in Figure 5. Standard HP filter estimates differ dramatically from extended and further extended ones which may suggest that end of sample standard HP filter estimates are biased. However, no noticeable differences are found in the mean gaps derived from the extended and further extended filters.

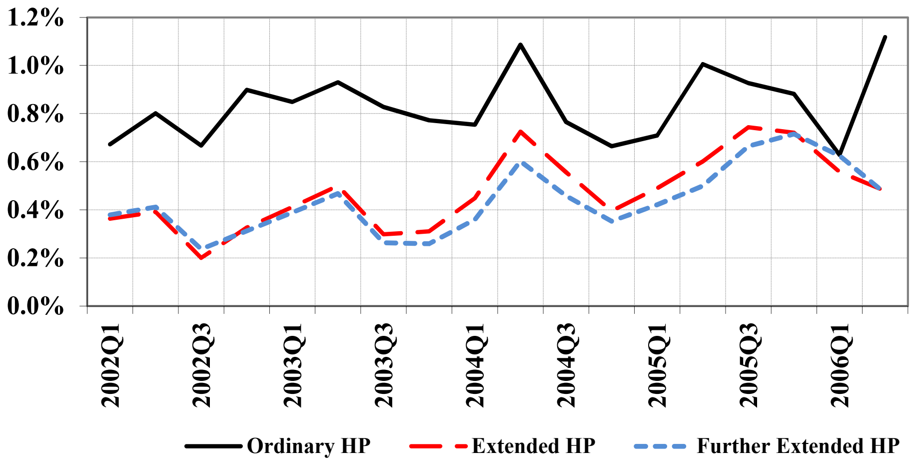

Figure 4: Estimated Standard Deviation of Output Gap Estimates

4 Conclusion and Final Remarks

The output gap plays a key role in the design and analysis of monetary policy. However, the output gap is a potentially misleading input for monetary policy because of its estimation uncertainty. This uncertainty has to do with the end of sample optimal filtering problem and model uncertainty mostly. However, GDP data revisions and delays increase the uncertainty of the output gap at the end of the sample also.

However, the standard HP filter is optimal.

Preliminary and delayed GDP reports were replaced by optimal in-sample now-casts of the “true” GDP figures derived from the model for data revisions and delays proposed by Julio [11]. The sample is then extended with out-of-sample optimal forecasts of the “true” GDP figures derived from the same model, and with out-of-sample optimal back-casts derived from an ARIMA model.

An application to Colombian GDP data reveals that adjusting for data

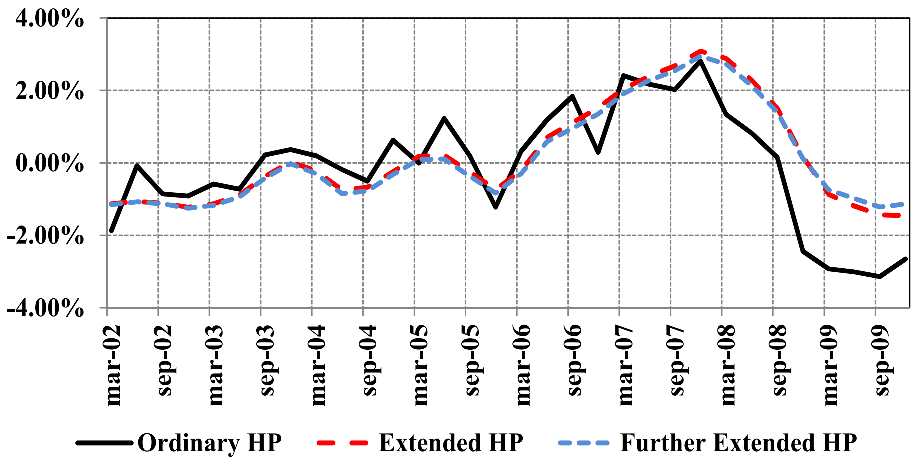

Figure 5: Mean Output Gap Estimates

revisions and delays reduce the uncertainty of estimated gaps. The extended and further extended HP estimates of the output gap show an impressive efficiency gain with respect to the ordinary HP gap, of 43% and 47%, on average, respectively. The new extension increases the efficiency in 7.4%, on average, with respect to extended HP estimates. These results constitute a benchmark to future work on real time estimation of the output gap under GDP revisions and delays.

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A ARMA Model for the Year to Year Growth of the Colombian Quarterly GDP

In order to estimate the dynamics of the true GDP process, GDP data releases up to 2006Q1 are considered as "true". Thus, the dynamics of the true GDP process is estimated based on a sample of 117 quarterly GDP observations from 1977Q1.

Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx** $Pr > \|t\|$	**Lag**
α	3.64	0.58	6.25	$< .0001$	0
θ_2	-0.32	0.11	-2.94	0.0033	2
θ_4	0.26	0.10	2.53	0.0113	4
ϕ	0.72	0.08	9.13	$< .0001$	1

Table 1: Maximum Likelihood Estimation Results

The log GDP series has a unit root. The KPSS test statistics for mean and trend deterministic components are 2.46 and 0.26 respectively, which correspond to p-values well below 0.01. Therefore, stationarity is strongly rejected in favor of a unit root in the log GDP series, showing the existence of a stochastic trend component.

Moreover, the year to year growth of the quarterly GDP seems to be stationary. The KPSS test statistic for mean deterministic component is 0.22 which corresponds to a p-value higher than 0.10. Thus, the unit root contained in the log GDP series reduces by computing the growth over four quarters of the GDP. Consequently, an ARMA representation for the yearly growth of the Colombian quarterly GDP might be appropriate.

The dynamic behavior of the GDP growth series might be appropriately approximated by an $ARMA(p = 1, q = (2, 4))$ model. Table 1 contains the maximum likelihood estimation results for this process. Parameters range

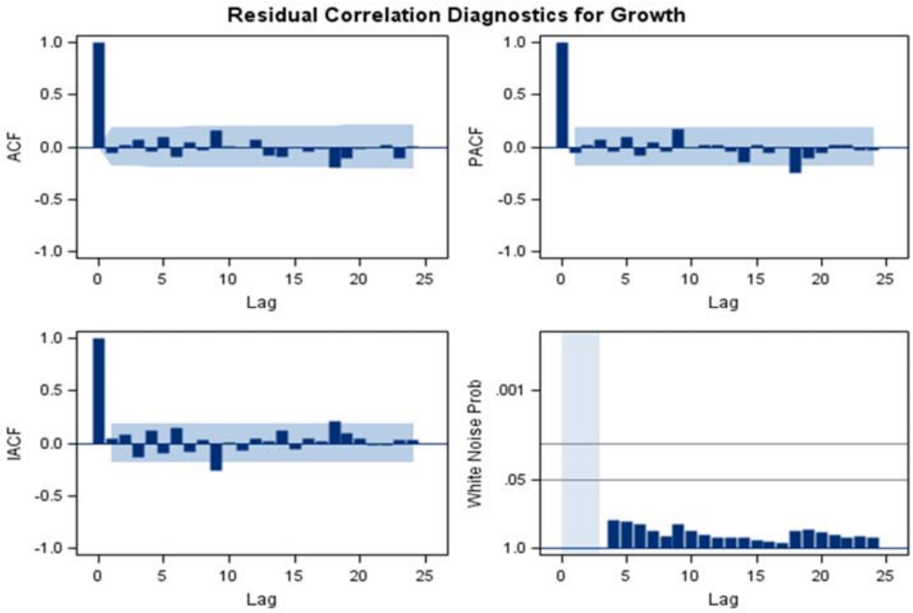

Figure 6: Residual Auto-Correlation Diagnostic Tests for the Yearly Growth of Quarterly GDP Model

from very to highly significant as the p-values are below 0.015.

The estimated model is stationary and invertible and contain MA seasonality. The root of the AR polynomial, $(1-0.72)\Delta^4 Y_t$, is 1.375, outside the unit circle, showing that this process is stationary. In turn, the roots of the MA polynomial, $(1+0.32B^2-0.26B^4)\varepsilon_t$ are ± 1.6521 and $\pm 1.1892i$, which lie outside the unit circle, showing that the process is invertible. Moreover, these roots differ from the root of the AR polynomial, which discards further simplification of the model.

Additionally, no significant autocorrelation is present in the residuals. Figure 6 shows that the estimated autocorrelation, partial and inverse autocorrelation functions are not significant as they lie within the shaded areas, except perhaps for the inverse auto-correlations at lag 9, and the autocorre-

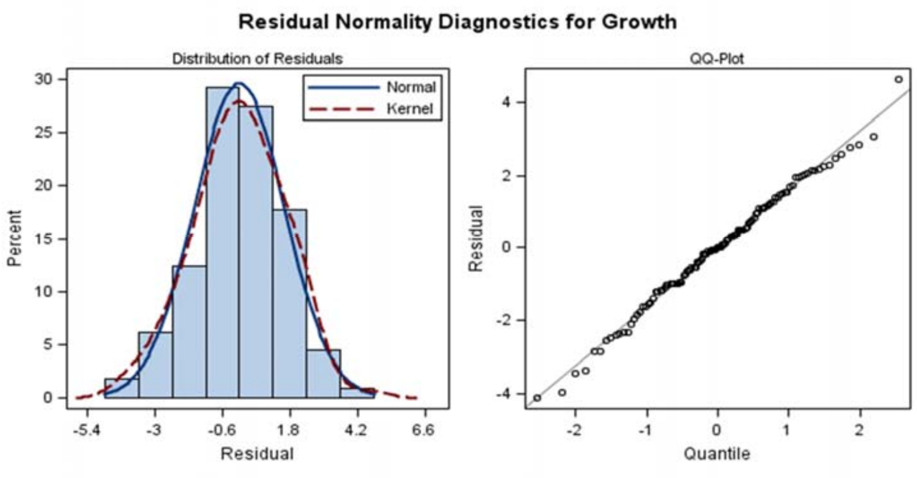

Figure 7: Residual Normality Diagnostics for the Yearly Growth of Quarterly GDP Model

lation and partial autocorrelation at lag 16. These correlations are not considered relevant as the white noise probability graph shows p-values higher than 0.05, not rejecting the null of no autocorrelation. See Chatfield [4].

The residuals show no important deviations from normality. The left panel of Figure 7 shows the histogram and kernel estimate of the residual density along with the normal density, showing important similarities between them. This result is further supported by the QQ-plot on the right panel. However, the left tail of the residual distribution is slightly higher than the normal's, suggesting that Colombian business cycles might not be symmetric as falls tend to be faster than recoveries (see Hamilton [8] for a model of nonlinear cycles). Finally, the Jarque-Bera normality test statistic has a value of 0.43 with a p-value of 0.81, thus confirming residual normality.

Business cycles relate to innovation persistence after reducing the stochastic trend. In this model it summarizes in the estimated $AR(1)$ parameter, 0.74, and the impulse response function of Figure 8. After an unexpected

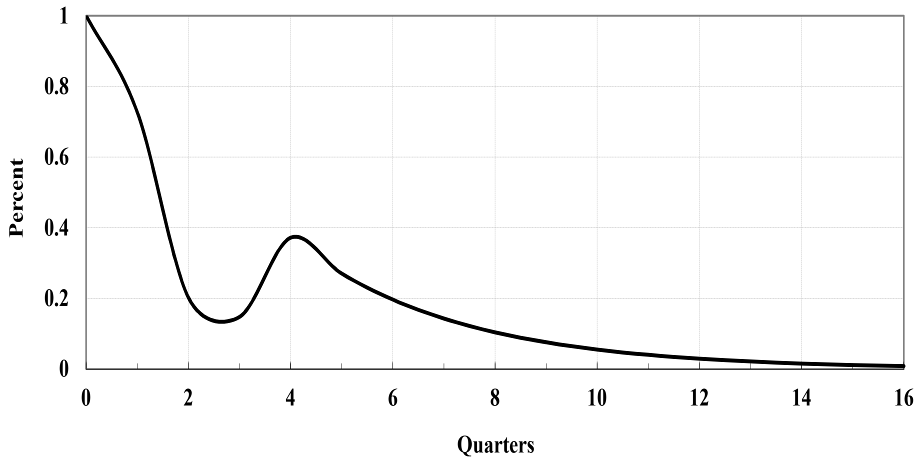

Figure 8: Response of the Yearly Growth of Quarterly GDP Series to a One Time Innovation of One Percent

increase of 1% in the growth of the GDP, it reduces to 0.72% a quarter later. After a year it reduces to 0.27%, and after two years it is just 0.08%. Three years later the effect has mostly vanished because of the stationarity of the business cycle.