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The Factor-Portfolios Approach to Asset Management using Genetic Algorithms [*]

Alejandro Reveiz[♣]

Abstract

We present an investment process that: (i) decomposes securities into risk factors; (ii) allows for the construction of portfolios of assets that would selectively expose the manager to desired risk factors; (iii) perform a risk allocation between these portfolios, allowing for tracking error restrictions in the optimization process and (iv) give the flexibility to manage dinamically the transfer coeffficient (TC).

The contribution of this article is to present an investment process that allows the asset manager to limit risk exposure to macro-factors – including expectations on correlation dynamics - whilst allowing for selective exposure to risk factors using mimicking portfolios that emulate the behaviour of given specific. An Artificial Intelligence (AI) optimisation technique is used for risk-budget allocation to factor-portfolios.

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The Factor-Portfolios Approach to Asset Management using Genetic Algorithms.

Both the original (Grinold, 1989) and the generalized (Clarke et al., 2002) Fundamental Law of Active Management present a compelling argument to maximize both the information and the transfer coefficients (TC) in order to generate alpha returns. It is however difficult to construct orthogonal positions when managing an actual fixed income portfolio. Thus, the transfer coefficient's management is of utmost importance in order to succesfully obtain excess returns.

The contribution of this article is to present an investment process that allows the asset manager to limit risk exposure to macro-factors – including expectations on correlation dynamics - whilst allowing for selective exposure to risk factors using mimicking portfolios (Zangari, 2003) that emulate the behaviour of given specific (*factor-portfolios*). An Artificial Intelligence (AI) optimisation technique is used for risk-budget allocation to factor-portfolios. This investment process gives the asset manager the ability to actively manage the transfer coefficient in order to take advantage, say, of correlation dynamics between factor-portfolios because the tracking error allocated to factor portfolios can readily be modified without the need of a new optimization has underlying assets and their realtive weights are known in advance.

Active management using a multifactor model

To preclude arbitrage (Ross, 1976), financial asset prices must embed the same price per unit of risk for every risk factor they are exposed to. Thus, assuming a 3 factor model, returns per unit of risk obtained through the exposure to these factors (given by

λ_k, the return per unit of risk for factor k) are constant through all the assets and the product $\lambda_k b_{ik}$ are "factor risk premiums" for each of the k factors. Thus, the return for each asset is a function of the price per unit of risk, λ_k, of the surprises from each factor and the exposure of the asset i to each of them:

$$\begin{aligned} E\left[r_i\right] &= \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \lambda_3 b_{i3} \\ &= r_f + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \lambda_3 b_{i3} \end{aligned} \qquad (1)$$

Returns r_p and risks σ_p for an asset or a portfolio can be obtained from the estimation of the sensitivity to the factors and their price of risk.

Risk-factor decomposition is useful for portfolio management, risk management and control and also for performance attribution. In portfolio management, an approach generally used consists in deciding the specific exposure to the set of factors and then performing a portfolio optimization in order to construct a combination of assets that will, in aggregate, generate the most approximate exposures. That is, given a desired vector $\mathbf{B}^*$ of factor exposures for a portfolio P, weights should be allocated between assets such that:

$$\min(\mathbf{B}^p - \mathbf{B}^*) \qquad (2)$$

Subject to (for example):

$$-1 \le w_i \le 1 \text{ with } i \in \text{ universe of assets} \qquad (3)$$

$$\mathbf{b}_k^p - \mathbf{b}_k^* \le \gamma_k \text{ with } k \in \text{ universe of factors} \qquad (4)$$

$$\mathbf{B}^p = \mathbf{W}_p \cdot \mathbf{E} \qquad (5)$$

$\mathbf{B}^p$ is the exposure obtained by combining the n assets with weights $\mathbf{W}_p$.

Matrix $\mathbf{E}$ includes the asset exposures to the k factors for the n assets

$$\mathbf{E} = \begin{bmatrix} b_{1,1}, b_{1,g}, \ldots, b_{1,k} \\ b_{2,1}, b_{j,g}, \ldots, b_{2,k} \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ b_{n,1}, b_{n,2}, \ldots, b_{n,k} \end{bmatrix}$$ and $\mathbf{W}_p = [w_1, w_i, \ldots, w_n]$ is the vector of asset weights.

$\mathbf{b}_g^p = \mathbf{W}.\mathbf{e}_g$ is the aggregate exposure to factor g where $\mathbf{e}_g \begin{bmatrix} b_{1,g} \\ b_{2,g} \\ \vdots \\ b_{n,g} \end{bmatrix}$ and $\mathbf{b}_k^*$ is the desired aggregate exposure to a single factor. γ_k is a small deviation tolerated in the exposure to each risk factor.

In order to determine the proper exposures, the manager normally would forecast spread levels and spot rates (against the forward curve) and then, through an initial optimization process, the factor exposure that maximizes return (including roll down, carry and coupon payments) and minimizes risk would be computed. After including an adequate universe of bonds the second optimization presented above is executed in order to determine the proper bond mix that would result in the desired factor exposures and the resulting tracking error is computed.

When a risk factor has performed as expected the manager must reassess the returns expected for each factor, determine the new desired exposures and re-optimize the portfolio. It must be noted that this approach can include other risk factors, such as credit or prepayment risks.

The Factor-Portfolios Approach.

Direct exposure to the factors included in the model may be difficult to construct, specially for portfolios where restrictive guidelines result in a low transfer coefficient or for mathematically defined factors – say, a curvature factor from a principal components analysis or a term spread slope factor of the form $F_2 = k_2 * \left(1 - e^{\frac{-t}{\tau}}\right)$ where variable t is the term and k_2 and τ are parameters. Moreover, the resolution level at which the factors of the model are defined may be inconsistent with actual position implementation. For instance, a Term Spread that accounts for changes in the slope of the swap curve modeled with an exponential function similar to the one shown above is difficult to construct with financial assets without gaining unwanted exposure to other risk factors.

An alternative approach is to construct active portfolios exposed to specific factors using the first optimization process presented above and allocate a desired tracking error exposure optimally across these factor-mimicking (Zangari, 2003) or *factor-portfolios* - using a risk budgeting approach - by maximizing the information ratio and limiting the tracking error by subsets of factor-portfolios; i.e. macro-factors. The latter could include all duration or slope positions across markets, a basket of currencies or exposure to various corporate risk positions (Corporate spread duration, credit quality barbells). Historical returns (from factor returns) and covariance matrices can be computed from the constructed risk exposures for factor-portfolios and, independently, for macro-factors.

Maximum tracking error exposures to macro-factors can be determined from views on their information ratios and expected correlations using, for example, optimization techniques and Bhansali and Wise (2005) method for forecasting portfolio risk in normal and stressed markets in order to limit the impact of changes in the prevailing environment that may affect macro-factors correlation; i.e. changes in the correlation between level and slope during a monetary tightening or between corporate spreads and treasuries[1] or an increased likelihood of a change in risk aversion that may affect all correlations and risk levels simultaneously for an increase in the probability of a recession. Macro-factors' risk budgeting can be performed for, say, a 6 months horizon and it allows to "put economics (back) into quantitative models" (Bhansali, 2005) as the impact of a changing environment can be traduced into tracking error upper limits for an active management optimization of the factor-portfolios. Amman and Zimermann (2001) found that the size of deviations from the benchmark relates to the statistical tracking error and that tracking error restrictions should both restrict the tactical ranges of the individual asset classes and the tracking of the individual asset classes. This is of utmost importance if the factor-portfolios returns have non-zero correlations and unintended coupled exposure to related risk factors (e.g. credit risk slope and credit risk barbell factor portfolios) is to be avoided, i.e. if the zero correlation assumption of the law of active management cannot be maintained (Grinold, 1989) and correlations are not static.

The macro-factors' tracking error allocation can be obtained by assigning an expected information ratio $E[IR_j]$ for each strategy j, an expected consistent correlation

[1] Empirical evidence (Morris, Neal and Rolph, 1998) shows that correlation of returns between treasuries and corporate spreads is negative but cointegration is positive.

matrix F (see Bhansali and Wise, 2005), an overall tracking error to distribute TE_M and, if desired, a maximum tracking error for each macro-factor κ_j :

$$\max(\alpha_M = \sum_{j=1}^{Q} E[IR_j] * TE_j) \tag{6}$$

Subject to:

$$TE_M = \xi \tag{7}$$

$$TE_j \leq \kappa_j \text{ with } j \in \text{ universe of macro-factors} \tag{8}$$

TE_M is the tracking error to be allocated to the Q macro-factors given their expected information ratios $E[IR_j]$ and their expected correlation matrix F .

$$F = \begin{bmatrix} E[c_{1,1}], E[c_{1,h}], \ldots, E[c_{1,q}] \\ E[c_{2,1}], E[c_{2,h}], \ldots, E[c_{2,q}] \\ \vdots \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ E[c_{q,1}], E[c_{q,2}], \ldots, E[c_{q,q}] \end{bmatrix}$$

ξ is the desired overall tracking error to be allocated.

Say, for simplicity, that the portfolio manager has a benchmark that includes treasuries and an eligible universe of instruments for active management that comprises treasuries, GSE agencies, corporate and Asset Backed securities. All these instruments are exposed to yield curve dynamics (parallel shift, slope and curvature changes) but all are exposed to differing risks, e.g. credit, liquidity risks and optionality. Instead of taking active positions across asset classes, factor-portfolios can be constructed to gain indirect exposure to selected risk factors such as parallel shifts, slope and curvature changes of the yield curve, changes in spreads for all instruments or credit risk positioning (long AA, short A and AAA, spread duration and duration neutral). Also, using the macro risk budgeting allocation, the overall risk to credit exposures can be limited – e.g. aggregate tracking error can be limited for positions in agencies and corporate spread duration and barbells or butterflies in the spreads' space.

After constructing the factor portfolios and allocating macro-factor risk budgets, tracking error must be allocated to each factor portfolio. Macro-factor risk budgets are used to limit the aggregated exposure of factors that in normal or special conditions under specific environments – increase in risk aversion or fall in prices and collateral feedbacks that affect liquidity across various markets simultaneously or change in the monetary stance – can generate significant loses in the portfolio. Hence, the optimization problem includes restrictions on tracking error space of combinations of factor portfolios. We propose the use of genetic algorithms (GA) as optimization tool for three reasons: the solution's landscape can be discontinuous and rugged; tracking error restrictions can easily be included in the fitness function (see below) and GA can be used to obtain populations of portfolios that are conditioned to specific environments (Reveiz, 2008).

Application of Genetic Algorithms to portfolio optimization

The vocabulary used by the evolutionary algorithms community is borrowed from natural genetics. Individuals in a population, the genotypes or structures, usually represent a potential solution to a problem. They are also called strings or "chromosomes", and have L units or "genes". The positions of each gene, which are arranged in a linear succession, are its "loci". Each gene can take one of θ values, often referred as “alleles”. The representation therefore has θ^L possible strings. In genetic algorithms, a binary representation is normally used yielding 2^L possible strings.

A Genetic Algorithm (GA) is an analogue procedure to the evolutionary process first applied by Holland (1975) that has been shown to successfully solve linear and non-linear problems in which the solution space is not “well behaved”, i.e. non-differentiable, non-continuous, etc. The strength of the genetic algorithm derives from the application of search and selection operators to a subset, or population, of candidate solutions.

Exploration of all areas of the solution space and exponential exploitation of promising areas is done through mutation, reproduction and selection operators applied to individuals in the population. Of particular advantage is the synchronized exploration of several areas of the solution space and the simultaneous evaluation of fitness of various candidate solutions (Houck et al., 1996).

An initial population is generated, in general at random, such that it is uniformly distributed in the search space. After every iteration of the search - a generation - the objects in the population are evaluated using a [context-dependent] fitness measure. A subset of the population is selected on the strength of its relative fitness, according to its worth in some environment, to be the basis of the following population generation. These objects are called parents, whist objects created by the application of the reproduction, and optionally the mutation, operators to the parents are called “offspring”. The latter substitute objects in the populations that were not selected for reproduction. The iteration is repeated until a given halting criteria is fulfilled - either a given fitness has been reached or a maximum number of generations has been attained.

GA belongs to the class of probabilistic algorithms and has the advantage of modeling the environment as a separate evaluation function, the fitness function. The benefits of evolutionary algorithms are characterized by Angeline (1993):

> *While the search progresses, the population preserves the best solutions found and attempts additional manipulations. As the population becomes filled with progressively better members, the search is constrained into areas of the search space that are dense with features previously found to be applicable to the task. Thus, empirical credit assignment allows evolutionary algorithms to adapt its search in the problem space dynamically… Because no explicit knowledge exists in the evolutionary algorithm, this knowledge must emerge from the interaction of the simple problem solver and the task environment.*

In genetic algorithms, by manipulating the structure of the object independently of its interpretation in order to perform the search, structural features of the parents are preserved in their offspring[2]. However, their behavior can differ markedly from their parents as no emphasis is placed on reproducing behavior. An adequate definition of the structure is essential as in some cases individual positions of the string may have specific

[2] For details on schemata theory see Angeline (1993).

static interpretations. For GA, binary representation seems best suited, bearing in mind how the algorithm works and for mathematical consistency (Angeline, 1993).

As mentioned earlier, an adequate representation in which each individual is made up out of a sequence of genes from a certain alphabet that range from binary (Holland's (1975) original representation) to any more natural representations (symbols, matrices, etc.) must be constructed in order to apply genetic algorithms to problem solving. For instance, suppose we want to compute the portfolio with highest information ratio for changes in the tracking error allocation TE_j for each factor portfolio j.

At the mechanistic level (Spears, 2000; Michalewicz, 1996; Forrest, 1990; Koza, 1992; Koza et al., 1999) the genetic algorithm consists of:

Step 1: Solution Representation. In a GA procedure, the parameters $\frac{TE_j}{\sigma_j}$ are converted to genotypes by means of a mapping procedure. For simplicity in this example we will use a binary representation and we will divide the search space into N spaces evenly divided in terms of the bounds defined for each variable, namely $-1 \leq \frac{TE_j}{\sigma_j} \leq 1$. For illustrative purposes, suppose we divide the search space into 8 parts for each variable for a total of 8^2 prospective solutions. For instance, as shown in figure 2, a candidate solution for a 2 factor portfolios optimization is represented by a concatenated

6-bit string such that the solution $s_l = [-0.67;0.33]$ is mapped by the function $g(TE_l, \sigma_l)$ to b_l =[010101]:

$$g(TE_l, \sigma_l) \rightarrow b_l \tag{9}$$

Figure 1 – Solution representation in binary space for 2 factor portfolios

Binary	Decimal							
[1 0 0]	-1.00	$Sol_{1,1}$	$Sol_{1,2}$	$Sol_{1,3}$	$Sol_{1,4}$	$Sol_{1,5}$	$Sol_{1,6}$	$Sol_{1,7}$
[0 1 0]	-0.67	$Sol_{2,1}$	$Sol_{2,2}$	$Sol_{2,3}$	$Sol_{2,4}$	$Sol_{2,5}$	$Sol_{2,6}$	$Sol_{2,7}$
[1 1 0]	-0.33	$Sol_{3,1}$	$Sol_{3,2}$	$Sol_{3,3}$	$Sol_{3,4}$	$Sol_{3,5}$	$Sol_{3,6}$	$Sol_{3,7}$
[0 0 1]	0.00	$Sol_{4,1}$	$Sol_{4,2}$	$Sol_{4,3}$	$Sol_{4,4}$	$Sol_{4,5}$	$Sol_{4,6}$	$Sol_{4,7}$
[1 0 1]	0.33	$Sol_{5,1}$	$Sol_{5,2}$	$Sol_{5,3}$	$Sol_{5,4}$	$Sol_{5,5}$	$Sol_{5,6}$	$Sol_{5,7}$
[0 1 1]	0.67	$Sol_{6,1}$	$Sol_{6,2}$	$Sol_{6,3}$	$Sol_{6,4}$	$Sol_{6,5}$	$Sol_{6,6}$	$Sol_{6,7}$
[1 1 1]	1.00	$Sol_{7,1}$	$Sol_{7,2}$	$Sol_{7,3}$	$Sol_{7,4}$	$Sol_{7,5}$	$Sol_{7,6}$	$Sol_{7,7}$
	Binary	[1 0 0]	[0 1 0]	[1 1 0]	[0 0 1]	[1 0 1]	[0 1 1]	[1 1 1]
	Decimal	-1.00	-0.67	-0.33	0.00	0.33	0.67	1.00

Step 2: Randomly select an initial population of bit strings of size pop_size, an initial set of guesses where z represents the bit length of a given guess, from the entire set of possible bit strings of the [parameterized] solution landscape.

Step 3: Define a (problem specific) fitness function f that assigns a numerical fitness value to every prospective solution, i.e. each individual of the selected population. We can define this fitness function as:

$$f(b_l, \Psi, \Omega, \mathrm{A}, \mathrm{B}, ..., \mathrm{H}) = \max(IR_p + h(\Gamma, \Omega, \mathrm{A}, \mathrm{B}, ..., \mathrm{H})) \tag{10}$$

$$\begin{cases} h() = 1 & if \;\; \sqrt{\Gamma.\Omega.\Gamma} \le a \;\wedge\; \sqrt{\Gamma.\Omega.\Gamma} \le d \;\wedge\; \sqrt{\Gamma.\Omega.\Gamma} \ge r \;\wedge\; ... \;\wedge\; \sqrt{\Gamma.\Omega.\Gamma} \le v \\ h() = -1 & if \;\; \sqrt{\Gamma.\Omega.\Gamma} \ge a \;\vee\; \sqrt{\Gamma.\Omega.\Gamma} \le d \;\vee\; \sqrt{\Gamma.\Omega.\Gamma} \ge r \;\vee\; ... \;\vee\; \sqrt{\Gamma.\Omega.\Gamma} \le v \end{cases} \tag{11}$$

Where IR_p is the information ratio and $\Gamma = b_l.\Psi$ is the vector of tracking errors for the factor portfolios of prospective solution b_l computed with variances Ψ; a, d, r

and v are tracking error restrictions for the overall portfolio or subsets of factor-portfolio[3] which may or not come from the macro-risk factors exercise; the first step of the investment decision process described throughout this document. Ω is the correlation matrix of the factor-portfolios' returns.

The function is fully defined in the sense that it is capable of evaluating any prospective solution it might encounter (Koza et al., 1999).

Step 4: Determine a scheme for differentially reproducing the population based on the fitness measure. This "survival of the fittest" approach, in which the properties of the best solutions (generation t) are passed onto the offspring (generation $t+1$), results in an improvement of the collective properties of the individuals of the population over time. Several schemes can be used for this task. The most widely used are roulette wheel (proportionate) selection, tournament selection and ranking selection. In order to maximize $f()$, we arbitrarily choose a normalized geometric ranking method.

Step 5: Define a set of "genetic" operators that modify individuals in order to produce new prospective solutions. Crossover takes two individuals and produces two new offspring, providing random exchange of information. Mutation modifies an individual's genotype, preventing genetic drift and providing an additional random search while the population converges. Many variants of these two basic operations are used in the literature (Spears, 2000; Winston, 1992; Michod, 1999; Koza et al., 1999; Angeline et al., 1997; Michalewicz, 1996) and their specific use depends on the representation chosen.

[3] Which may or not come from the macro-risk factors step of the investment decision process described throughout this document.

Step 6: Perform many iterations (generations) measuring fitness of candidate solutions, differentially reproducing through selection operators and applying "genetic" operators until one of the following occurs: a maximum number of generations is reached, no further improvement of the best solution is attained or a target value for the fitness function is reached.

For illustrative purposes suppose we perform an active management over, say, the 1-3 US bond index published by Merrill Lynch. As a first step we construct 9 factor portfolios:

FP_1: Exposure to a parallel shift of the US curve in the 1-3 sector by constructing a factor-portfolio of treasuries' bonds or futures that has equal key rate duration exposures to the 1, 2 and 3 year spot key rates.

FP_2: Exposure to an slope shift factor of the spot curve US using the 2 year and 10 year treasuries bonds and futures to obtain a duration neutral factor-portfolio that has a positive 10 year and negative 2 year key rate durations.

FP_3: Exposure to changes in the spread of bullet agencies by constructing a factor-portfolio that is duration neutral and long spread duration using US bullet Agencies of the 1-3 sector and treasuries bonds or futures.

FP_4: Exposure to changes in Agencies' spreads slope constructing a portfolio with US bullet Agencies of the 1-5 sector and treasuries bonds or futures to obtain a duration and spread duration neutral factor-portfolio.

FP_5: Exposure to changes in the spread of Corporates by constructing a factor portfolio that is duration neutral and long spread Duration using US Corporates of the 1-3

Sector with the same rating distribution of the Merryl Lynch A/AA/AAA corporate index and treasuries bonds or futures.

FP_6: Exposure to changes in Corporates' spreads slope constructing a factor portfolio with US Corporates of the 1-5 sector and treasuries bonds or futures to obtain a duration and spread duration neutral factor-portfolio.

FP_7: Exposure to relative changes between the AA and A/AA ratings (Credit Butterfly) by constructing a portfolio that is long spread duration for AA rated bonds and short spread duration for AAA and A rated bonds with US Corporates of the 1-5 sector and treasuries bonds or futures to obtain a duration and spread duration neutral factor-portfolio.

FP_8: Exposure to relative changes between AAA Financial and Industrial Corporate with US Corporates of the 1-5 sector and treasuries bonds or futures to obtain a duration and spread duration neutral factor-portfolio.

FP_9: Exposure to changes in spreads of AAA ABS in the 1-3 Sector with US Asset Backed Securities and treasuries bonds or futures to obtain a duration and spread duration neutral factor-portfolio.

Figure 2 shows the correlations between these factor-portfolios. Selective indirect exposure to factors yields low and negative correlations that will result in an adequate diversification benefit of the active deviations’ portfolio.

Figure 2 – Factor-portfolios correlation matrix

Factor Portfolio	FP_1	FP_2	FP_3	FP_4	FP_5	FP_6	FP_7	FP_8	FP_9
FP_1	1.00	0.38	-0.15	-0.03	0.46	-0.10	0.11	0.19	0.10
FP_2	0.38	1.00	0.26	-0.16	0.23	0.11	0.14	-0.02	0.23
FP_3	-0.15	0.26	1.00	-0.12	-0.11	0.15	-0.15	-0.26	0.00
FP_4	-0.03	-0.16	-0.12	1.00	0.02	0.07	0.04	0.15	0.09
FP_5	0.46	0.23	-0.11	0.02	1.00	-0.64	0.05	0.56	0.41
FP_6	-0.10	0.11	0.15	0.07	-0.64	1.00	-0.07	-0.37	-0.26
FP_7	0.11	0.14	-0.15	0.04	0.05	-0.07	1.00	0.09	0.01
FP_8	0.19	-0.02	-0.26	0.15	0.56	-0.37	0.09	1.00	-0.10
FP_9	0.10	0.23	0.00	0.09	0.41	-0.26	0.01	-0.10	1.00

We compute α_j for each factor-portfolio j using Grinold's (1989) law of active management

$$\alpha_j = IC_j . \sigma_j . S_j \quad (12)$$

Where IC_j is the information coefficient, σ_j is the standard deviation and S_j is the score for each factor-portfolio. Next, we determine the tracking error constraints – see figure 3.

Figure 3 – Tracking error constraints for GA optimization

Factor Portfolio (1)	Restriction A (2)	Restriction B (3)	Restriction C (4)	Restriction D (5)	Restriction E (6)
FP_1	X	X			
FP_2	X	X			
FP_3	X	X	X		X
FP_4	X	X	X		X
FP_5	X	X		X	X
FP_6	X	X		X	X
FP_7	X	X		X	X
FP_8	X	X		X	X
FP_9	X	X		X	X
Restriction Type	Higher Than	Lower Than	Lower Than	Lower Than	Lower Than
Restriction Limit (B.P.)	10	27	15	9	15

Inputs and tracking error allocations are presented in figure 4.

Figure 4 – Results of factor-portfolios GA optimization

Factor Portfolio (1)	Std. Dev. (B.P.) (3)	IC (4)	SCORE (5)	Alpha (α) (6)	T.E. Max (B.P.) (7)	T.E. Alloc. (B.P.) (8)	Position Long (+) /Short(-) (9)
FP_1	104	0.22	2	46	20	17	+
FP_2	42	0.09	-2	-8	10	1	+
FP_3	14	0.22	2	6	10	10	+
FP_4	28	0.09	-1	-3	0	6	-
FP_5	43	0.13	1	5	10	4	-
FP_6	27	0.09	-2	-5	5	5	-
FP_7	25	0.13	-2	-6	5	5	-
FP_8	65	0.09	2	12	5	5	+
FP_9	53	0.22	-2	-23	5	5	-

The active portfolio has an expected alpha α_p of 18 b.p., a tracking error $TE_{p\langle\Omega\rangle}$ of 20 b.p., an information ratio IR_p of 0.88 and a diversification benefit of 0.64. The latter is defined as:

$$BD_p = 1 - \left(\frac{TE_{p\langle\Omega\rangle}}{TE_{p\langle\Phi\rangle}} \right) \tag{13}$$

Where $TE_{p\langle\Omega\rangle}$ is the tracking error of the portfolio using correlation matrix Ω and $TE_{p\langle\Phi\rangle}$ is the tracking error of the portfolio using the stressed correlation matrix Φ. The diversification benefit BD_p is defined in the range [0,1].

Conclusions

The portfolio management decision process and technique presented has many advantages. First, assets' exposures to macro-risk factors can be controlled and managed.

Second, changes in the environment can be reflected in the macro-factors' medium term correlation matrix or expected information ratios in order to limit overall exposure but without affecting directly the risk budget factor-portfolio GA optimization[4].

Third, factor-portfolios can be constructed with low or negative correlations and the number of potential positions or "bets" can be increased, both resulting in a higher expected information ratio.

Fourth, because the specific assets that constitute the factor portfolio can be bought or sold in the appropriate amounts, the transfer coefficient (TC) can be managed actively as changes in the size of a tracking error of a (factor-portfolio) position can be implemented immediately, and its impact in terms of overall risk and diversification benefits is known. This flexibility allows to take advantage from changes in the correlations of the factor-portfolios.

Finally the asset manager can take profits or loses arising from factor dynamics without the need to optimize the portfolio to obtain the new composition that reflect the new desired exposures[5].

[4] For the GA optimization actual Factor-portfolios' returns and correlations are used.

[5] Transactions can be performed rapidly, limiting the execution risk.

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