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Relevance of the collateral constraint form in the analysis of financial crisis interventions

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Relevance of the collateral constraint form in the analysis of financial crisis interventions

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Abstract

We combine two modifications to the standard (current and total income) collateral constraint that has been commonly used in models that analyze financial crisis interventions. Specifically, we consider an alternative constraint stated in terms of future and disposable income. We find that in this case a state-contingent debt tax (effective during crisis only, as opposed to a macroprudential tax) increases debt capacity and lowers the probability of crisis. This shows one more instance to call the attention of academics and policymakers to the fact that the specific form of the borrowing constraint is crucial in determining the appropriate crisis intervention.

Keywords: collateral constraint, financial crises, macroprudential tax, ex-post intervention

JEL Classification: E44, F34, F41, G01, H21

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(R) The order of authors has been randomly chosen.

Relevancia de la forma de la restricción de endeudamiento en el análisis de las intervenciones en crisis financieras

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Resumen

Combinamos dos modificaciones a la restricción crediticia estándar (i.e., en términos de los ingresos corrientes y totales) que se ha utilizado comúnmente en los modelos que analizan las intervenciones en crisis financieras. Específicamente, consideramos una restricción alternativa expresada en términos de ingresos futuros y disponibles. Encontramos que, en este caso, un impuesto a la deuda dependiente del estado de la economía (efectivo solo durante las crisis, a diferencia de un impuesto macroprudencial) aumenta la capacidad de endeudamiento y reduce la probabilidad de crisis. Este resultado representa un ejemplo más para llamar la atención de académicos y formuladores de políticas sobre el hecho de que la forma específica de la restricción de endeudamiento es crucial para determinar la intervención de crisis adecuada.

Palabras clave: restricción crediticia, crisis financieras, impuesto macroprudencial, intervención ex post

Códigos JEL: E44, F34, F41, G01, H21

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1 Introduction

The form of the financial constraint matters for policy advice, and for the effectiveness and desirability of different policy interventions (e.g. macroprudential or ex-post policies). In this document, we show one more instance in which this distinction becomes relevant. More specifically, we combine two modifications to the standard financial constraint that have been analyzed separately by previous literature.

Vargas and Parra-Polania (2021) show the difference in policy advice between considering the commonly used total income financial constraint or the alternative disposable income financial constraint, both specified in terms of current income. Specifically, the ex-post policies (i.e. interventions implemented during crisis - e.g., a subsidy on non-tradable consumption as in Benigno et al., 2016) that are effective under the standard financial constraint become utterly ineffective under the alternative one. Instead, under the alternative constraint, macroprudential (i.e. ex-ante) interventions are still effective in mitigating the negative effect on welfare of financial crises.

In a recent paper, Ottonello, Perez and Varraso (2021) show the difference in policy advice between economies with the standard total income financial constraint (stated in current income) and those with a total income constraint stated in future income only. They show that no inefficiencies arise when the total income financial constraint is stated in terms of future income and, therefore, there is no need for macroprudential interventions.

In the present document, we show the difference in policy advice between an economy with a future total income financial constraint and a future disposable income financial constraint.

We use a standard small open economy with tradable and nontradable goods subject to a financial constraint. When we consider the future total income financial constraint, the result shown by Ottonello, Perez and Varraso (2021) applies and we find that the allocation of the decentralized (DC) economy is the same as that of the social planner (SP). We show the theoretical curiosity that, although not needed for equalizing allocations, an appropriate ex-post tax on debt (effective only during crisis) can equalize the DC and SP equilibria, including the shadow value of borrowing. In other words, in a situation where future total income is the relevant one for the assessment of debt capacity, the ex-post tax on debt is a mere theoretical curiosity with no practical effects. Then, we consider an economy with a future disposable income financial constraint. In this case, we find that the ex-post tax on debt has practical and very relevant effects: it increases debt capacity and lowers the probability of crisis of the economy.

As aforementioned, this is one more instance to call the attention of academics and policymakers to the fact that the specific form of the borrowing constraint is crucial in determining the appropriate crisis intervention.

2 The Model and Results

We use a very standard theoretical framework which represents a small open economy subject to an occasionally-binding financial constraint. For the analysis of the present paper we specifically consider a next-period income financial constraint.

There is a representative household that seeks to maximize its lifetime expected utility function expressed by

$$U = E_1 \left[\sum_{t=1}^{\infty} \beta^t u\left(C_t\right) \right], \tag{1}$$

where $E\left[\cdot\right]$ is the expectations operator, $u\left(\cdot\right)$ is the well-behaved period utility function, β is the discount factor and C_t is the consumption index which aggregates tradable (T) and nontradable (N) goods:

$$C_t = C\left(C_t^T, C_t^N\right). \tag{2}$$

Every period, each household receives a stochastic (and exogenous) bundle of tradable and nontradable goods, Y_t^T and Y_t^N, and has access to international credit markets through one-period loans B_{t+1} at an interest rate r ($R \equiv 1 + r$). The budget constraint, expressed in units of tradable goods, is

$$C_t^T + P_t^N C_t^N + RB_t = Y_t^T + P_t^N Y_t^N + B_{t+1}, \tag{3}$$

where P_t^N is the price of nontradables.

There is access to credit up to a fraction κ of expected (next-period) total income:

$$B_{t+1} \leq \kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N \right]. \tag{4}$$

We refer to this specific form as the "future total income financial constraint".

The market-clearing conditions for nontradables and tradables are, respectively:

$$C_t^N = Y_t^N \tag{5}$$

$$C_t^T + RB_t = Y_t^T + B_{t+1} \tag{6}$$

We denote by λ_t and μ_t the Lagrange multipliers associated to the financial and budget constraints, respectively. The first-order conditions for this DC economy are determined by the following equation system, in addition to Equations (5) and (6):

$$u_{T,t} = \mu_t \tag{7}$$

$$\mu_t = R\beta E_t \mu_{t+1} + \lambda_t \tag{8}$$

$$P_t^N = \frac{u_{N,t}}{u_{T,t}} \tag{9}$$

$$\lambda_t \left(\kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N\right] - B_{t+1}\right) = 0 \tag{10}$$

where $u_{T,t} \equiv u'(C_t)\left(\partial C_t / \partial C_t^T\right)$ and $u_{N,t} \equiv u'(C_t)\left(\partial C_t / \partial C_t^N\right)$. This six-equation system provides a solution for C_t^T, C_t^N, μ_t, λ_t, B_{t+1} and P_t^N for given values of $\left\{B_t, Y_t^T, Y_t^N\right\}$ and the (consistent) expected values of future variables.

If the economy is financially unconstrained in period t, $B_{t+1} \leq \kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N\right]$ and hence $\lambda_t = 0$, from Equation (10). If, instead, the economy is constrained, $\lambda_t \geq 0$ and $B_{t+1} = \kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^N\right]$. As in the related literature, we interpret financially constrained periods as 'crisis' periods and the financially unconstrained ones as 'normal' periods.

2.1 Social planner's equilibrium

Since each household has an insignificant impact on the market, it takes prices as given. Instead, a SP subject to the same financial constraint, internalizes the effect of borrowing and consumption decisions on prices. We follow the constrained-efficiency criterion (i.e., we assume the SP is constrained by the same pricing rule of the DC equilibrium). The first-order conditions for the SP equilibrium are given by the following equation system, in addition to Equation (5) and the pricing rule (9):

$$C_t^{T,SP} + RB_t^{SP} = Y_t^T + B_{t+1}^{SP} \tag{11}$$

$$u_{T,t}^{SP} = \mu_t^{SP} \tag{12}$$

$$\mu_t^{SP} = R\beta E_t \mu_{t+1}^{SP} + \lambda_t^{SP}\left(1 - E_t \psi_{t+1}^{SP}\right) \tag{13}$$

$$\lambda_t^{SP}\left(\kappa E_t \left[Y_{t+1}^T + Y_{t+1}^N P_{t+1}^{N,SP}\right] - B_{t+1}^{SP}\right) = 0 \tag{14}$$

where $E_t \psi_{t+1}^{SP} \equiv \kappa E_t \left[Y_{t+1}^N \left(\partial P_{t+1}^{N,SP} / \partial C_{t+1}^{T,SP}\right)\left(\partial C_{t+1}^{T,SP} / \partial B_{t+1}^{SP}\right)\right] \leq 0$.[1]

Notice that if there were no crisis periods (and therefore $\lambda_t = 0$, $\forall t$), both the DC and the SP equilibria would be exactly the same. In contrast, in the presence of crisis periods those two first-order-condition systems may produce different solutions. However, as shown by Ottonello, Perez and Varraso (2021), the only difference between these two set of solutions occurs in the values of λ_t and λ_t^{SP}.[2] This can be seen as follows. Assume we find specific values for C_t^T, C_t^N, μ_t, B_{t+1}, P_t^N and λ_t that solve the DC system of

[1] This follows from: (i) we assume that $\partial P_{t+1}^{N,SP} / \partial C_{t+1}^{T,SP} > 0$. This holds for standard composite index forms (e.g. Cobb-Douglas or CES). A sufficient condition is that $\partial^2 C_t^{SP} / \left(\partial C_t^{T,SP}\right)^2 < 0$ and $\partial^2 C_t^{SP} / \left(\partial C_t^{T,SP} \partial C_t^{N,SP}\right) > 0$. (ii) $\partial C_{t+1}^{T,SP} / \partial B_{t+1}^{SP} \leq 0$: when the financial constraint is binding, $C_{t+1}^{T,SP}$ is determined by Equation (11) and $\partial C_{t+1}^{T,SP} / \partial B_{t+1}^{SP} = -R$. When it is not binding the derivative is negative as well via the effect that the current level of debt has on future debt and, in turn, on the future valuation of liquidity, which affects the optimal level of consumption during normal periods.

[2] Their result is obtained for a more general financial constraint form, which may include the entire path of future incomes.

Equations (5)-(10) for given values of $\left\{B_t,\, Y_t^T,\, Y_t^N\right\}$ and the (consistent) expected values of future variables. It is easy to verify that the same values for the first five variables satisfy the pricing rule (9) and the SP Equations (5), (11), (12) and (14) and that $\lambda_t^{SP} = \lambda_t / \left(1 - E_t \psi_{t+1}^{SP}\right)$ satisfies Equation (13).

2.2 Implementing the SP equilibrium with an ex-post policy

The result highlighted in subsection 2.1 implies that the DC equilibrium allocation (i.e., C_t^T, C_t^N, B_{t+1}) and the one of the SP are equal without the need for any intervention; that is to say, this is a constrained-efficient economy. However, for the subsequent discussion it is useful to show that there is a policy intervention that equalizes all values of both equilibria (DC and SP), i.e., including the Lagrange multipliers λ_t^{SP} and λ_t.

To this purpose, we assume the government in a DC economy imposes a state-contingent tax rate ω_t on debt that applies only during crisis ($\omega_t = 0$ in normal times). We also assume that the government's budget is balanced period by period (as it is standard in the related literature), and hence the tax is returned to the consumers, in the same period, as a lump-sum transfer; that is:

$$T_t = -\omega_t B_{t+1}. \tag{15}$$

Consequently, the agent's budget constraint becomes

$$C_t^T + P_t^N C_t^N + RB_t = Y_t^T + P_t^N Y_t^N + B_{t+1}\left(1 - \omega_t\right) - T_t, \tag{16}$$

and the first order condition with respect to B_{t+1}:

$$\mu_t \left(1 - \omega_t\right) = \lambda_t + R\beta E_t \mu_{t+1}. \tag{17}$$

Proposition 1 *In the DC economy represented by utility (1), consumption index (2), future total income financial constraint (4), lump-sum transfer (15) and budget constraint (16), there is a value of a state-contingent tax rate on debt ω_t (effective during crisis periods only and returned to households via a lump-sun transfer) such that the SP equilibrium is implemented.*

Proof. As mentioned above and shown by Ottonello, Perez and Varraso (2021), the only difference between the DC and the SP equilibria (in the absence of taxes/subsidies) occurs in the values of λ_t and λ_t^{SP}. Then to implement the SP equilibria it will be enough to equalize Equations (13) and (17) since these are the ones that determine λ_t^{SP} and λ_t, respectively, and the other equations of both systems have the same form. It can be easily verified that the following tax rate fulfills that purpose:

$$\omega_t = -\frac{\lambda_t E_t \psi_{t+1}}{u_{T,t}} \geq 0 \tag{18}$$

where ψ refers to ψ^{SP} evaluated in the equilibrium values (also remember SP and DC equilibria are equalized).

Notice that during normal times (i.e., $\lambda_t = 0$) the tax on debt is nil. ■

In this case, the implementation of this debt tax that equalizes the SP and the DC equilibria seems to be a theoretical curiosity with no practical relevance since the SP and DC allocations are already equal and the only difference between both equilibria is the fact that the shadow value of borrowing for the SP is a rescaled version of that for the DC economy. However, the next subsection shows that the debt tax may have practical effects when the relevant income in the financial constraint is the disposable income rather than the total one.

2.3 Disposable income financial constraint

If lenders assess borrowing capacity taking into account next-period disposable income, the financial constraint becomes

$$B_{t+1} \leq \kappa \left(Y_{t+1}^T + Y_{t+1}^N E_t P_{t+1}^N - E_t T_{t+1} \right), \tag{19}$$

where T, as mentioned before, is the lump-sum transfer through which the debt tax is returned to households. We refer to this specific form as the "future disposable income financial constraint". In this case, as the following proposition shows, the debt tax reduces the severity and probability of crisis.[3]

Proposition 2 *In the DC economy represented by utility (1), consumption index (2), lump-sum transfer (15), budget constraint (16), future disposable income financial constraint (19) and continuous probability distributions for exogenous variables Y^T and Y^N, the implementation of the state-contingent tax rate (18) (effective only during crisis periods and returned to households via a lump-sun transfer) increases debt capacity and reduces the probability of crisis. Furthermore, the higher the value of the tax rate on debt, the higher the debt capacity of borrowers (hence, the lowest probability of crisis is reached when $\omega = 1$).*

Proof. As abovementioned, the resources collected through the debt tax are returned to consumers as a lump-sum transfer. Such a transfer affects disposable income. Thus, the fact that there might be a crisis event in the next period affects the expected future disposable income which is the relevant one for the current financial constraint.[4] The presence of a expected positive transfer ($-E_t T_{t+1} = E_t \omega_{t+1} B_{t+2} > 0$) increases both

[3] If future income is the relevant one for the financial constraint, and the government for any reason intends to mitigate crises using a subsidy on nontradable consumption (as in Benigno et al., 2016), it is easy to see that such a subsidy would have to be promised to be implemented right after every crisis period, distorting consumers' decisions during normal periods. Furthermore, in the presence of the future disposable income financial constraint such a subsidy would be ineffective to reduce the severity of crises: an analogous result to the one shown by Vargas and Parra-Polania (2021) for the case with the current disposable income financial constraint.

[4] This requires a positive probability of crisis in the next period (even if the current one is a crisis period as well) and the credible promise that if it happens the debt tax will apply. Notice that a benevolent government does not have incentives to break that promise since implementing the tax during $t+1$ causes no harm (althought it does not bring benefits either) to the economy in that period.

current debt capacity and the realized level of debt B_{t+1} during crisis (since the latter is determined by the binding financial constraint). Increasing debt capacity (in those periods in which there would be crisis in the absence of the expected debt tax) mitigates the severity of the crisis and in some occasions it could even avoid the crisis (that is, it reduces the crisis probability).

We assume continuous distributions for Y^T and Y^N to avoid an inconsistency problem. With discrete distributions it might be the case that there were not consistent value for $-E_t T_{t+1}$ due to discrete changes in the probability of crisis (i.e. for two adjacent values of crisis probability, the lower one may produce an inconsistently low value of $-E_t T_{t+1}$ and the higher one an inconsistently high value).

The second part of the proposition (i.e., the higher the value of ω, the higher the debt capacity of borrowers) follows from the fact that (as a result of a higher ω) the probability of crisis would not fall up to the point of decreasing the expected transfer because it would imply lower debt capacity and hence higher crisis probability (a contradiction). Notice that a **corollary** of this part is that it is not possible for ω, even at its highest level, to avoid all crises (i.e. to implement the never-constrained equilibrium) as it would reduce the crisis probability to zero but then the expected transfer (since the tax is effective only during crisis) would be nil as well. ■

3 Conclusion

In this document we show one more instance in which the form of the financial constraint matters for policy advice, and for the effectiveness and desirability of different policy interventions (e.g. macroprudential or ex-post policies). We combine two modifications to the standard (current and total income) collateral constraint that has been commonly used in models that analyze financial crisis interventions. Specifically, we consider an alternative constraint stated in terms of future and disposable income.

When we consider the future total income financial constraint, the result shown by Ottonello, Perez and Varraso (2021) applies, i.e., the allocation of the decentralized (DC) economy is the same as that of the social planner (SP). We show the theoretical curiosity that, although not needed for equalizing allocations, an appropriate ex-post tax on debt (effective only during crisis) can equalize the DC and SP equilibria, including the shadow value of borrowing. Then, we consider an economy with a future disposable income financial constraint. In this case, we show that the ex-post tax on debt, which was a mere theoretical curiosity before, increases debt capacity and lowers the probability of crisis of the economy.

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