Box I
Characterizing and Communicating the Balance of Risks of Macroeconomic Forecasts: A Predictive Densities Approach for Colombia[1]

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Developing and evaluating macroeconomic forecasts are crucial elements of a central bank's monetary policy agenda, especially in economies operating under a target inflation scheme (Svensson, 2010). *Banco de la República*'s approach in this regard consists of process that involves a qualified technical staff, modeling tools with theoretical foundations, and critical assessment of empirical features in order to forecast major macroeconomic variables over an eight-quarter policy horizon (see González et al. [2019] and González et al. [2020]). The resulting forecasts help to inform the decisions of *Banco de la República*'s board of directors and are summarized in the *Monetary Policy Report*.

These forecasts are conditional to an evaluation of the current and future state of the economy, to predictions of external variables, and a view of the endogenous monetary policy response designed to move inflation to the target and stabilize output and employment. Monetary policy, however, operates in an environment with uncertainty (Friedman (1972), Batini and Nelson (2001), and Goodhart (2001)), and weighing the balance of risks to the macroeconomic forecast thus represents a key element in the policymaking process.

The balance of risks thus incorporates an assessment of the prospective shocks that the economy might face over the forecast horizon, and which might affect the expected behavior of macroeconomic variables. This is a complex task for monetary policy authorities, requiring a characterization of the potential origin of these shocks, their nature (if they are permanent or temporary), and the degree to which their effects might persist, as well as the best manner in which these elements and their results would be communicated to the public.

The economic literature suggests that central banks use four main tools to characterize and communicate their prospective balance of risks: qualitative evaluation, symmetric fan charts, asymmetric fan charts, and predictive densities[2].

Qualitative evaluations offer an exhaustive description of the future state of the economy and likely risks using a narrative approach, without providing an explicit quantitative explanation about different potential sources of risk and their magnitudes.

Fan charts characterize the balance of risks to the macroeconomic forecast through a probability distribution created separately from forecasting models and then superimposed on the central forecast path. The construction of these charts follows the classic estimation of confidence intervals based on the historical volatility of forecast errors and an assumption over their density function. Symmetric fan charts (Blix and Sellin, 1999) suppose normal distributions that allow for the characterization of a balanced risk, while asymmetric fan charts (Britton et al., 1998) consider a two-piece normal distribution, allowing for the description of a skewed balance of risks.

Fan charts are the result of a statistical methodology that does not account for the economic structure of the model, nor does it consider the general equilibrium relationships on which the central forecast is based. Furthermore, the probability distributions for each variable are independent, which does not guarantee macroeconomic consistency between the fan charts for each of the variables considered in the model.

The three tools described above have been used by *Banco de la República* in its characterization and communication of forecast risks. Until 2018, the bank used an asymmetric fan chart for GDP growth and headline inflation in its *Inflation Report*, making the risk factors for these variables explicit. In 2019, with the new forecasting process reflected in its *Monetary Policy Report*, the bank adopted symmetric fan charts, reflecting the volatility of the forecast implicit in historical prediction errors. In 2020, as a consequence of the high uncertainty generated by COVID-19 and the difficulty of presenting a probability distribution that would portray the effects of this shock on the forecast, *Banco de la República* suspended the publication of its fan chart and adopted a qualitative evaluation of the risks.

Starting with the current quarterly Monetary Policy Report, the bank will now characterize and communicate the prospective balance of risks of its macroeconomic forecast using a Predictive Density (PD) approach. This supplement thus has two objectives: First, to briefly present the technical

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2 For example, the central banks of Sweden and France, as well as the European Central Bank, have adopted symmetrical fan charts, while the central banks of England, Hungary, Brazil, and Peru have preferred asymmetrical fan charts. Qualitative evaluation is used explicitly by the Bank of Japan and the U.S. Federal Reserve, and complements the analysis of central banks that use quantitative tools. The characterization of risk and its communication with predictive densities has been considered by central banks in Norway, Israel, and Canada, and by the technical staff of the New York Federal Reserve.

aspects of the PD methodology, and second to illustrate the results of the PD methodology referenced in this report.

1. Predictive densities

Predictive density methodology aims to characterize, quantify, and communicate the prospective balance of risks. This task requires generating a probability distribution of the forecasts of all economic variables included in the general equilibrium models (PATACON and 4GM). Using this methodology, the probability distribution of the forecasts preserves the transmission channels intrinsic to the economic structure of the model, and thus maintains its macroeconomic consistency and the general equilibrium dynamics.

More specifically, the probability distribution obtained using PD is based on the structure of the models and shock sequences, allowing for the inclusion of external information to guide the mode and variance of its distributions, as well as an asymmetric balance of risks. These characteristics offer a more robust macroeconomic projection, making clear the sensitivity of the forecast to future risks and allowing for the quantification of its effects, thus contributing to a more complete monetary policy recommendation.

1.1 Description of the methodology

Banco de la República's technical staff constructs its macroeconomic forecast and policy recommendation using results from the PATACON and 4GM monetary policy models. These are rational expectations models that capture transmission mechanisms for a small, open, oil-exporting economy and whose parameters are estimated using data from the Colombian economy.

The general solution of these models can be represented by the system[3]:

$$Y_t = Z(\theta)S_t + H(\theta)v_t \quad (1)$$

$$S_t = T(\theta)S_{t-1} + R(\theta)\epsilon_t \quad (2)$$

where (1) and (2) are denominated measurement and transition equations, respectively. The measurement variables Y_t are informed by observable data (e.g. inflation and GDP growth), while the state variables S_t are latent (not observable) and come from the dynamics of the model itself (e.g. output gap, potential output). Equation (1) establishes a relationship between the observed variables Y_t and the state variables S_t and includes a vector of measurement errors v_t (or data revisions). Equation (2) defines the dynamic of change over the course of time for the model variables. This equation also accounts for structural shocks ϵ_t (or innovations) that are exogenous components of the models, but that affect the dynamic of its variables (e.g. a demand shock).

3 The matrices Z, H, T and R characterize the solution of the model and are a function of parameters θ.

The system of equations (1) and (2) generates forecasts whose dynamic is explained by the economic structure of the model, its transmission channels, and the structural shocks faced by the economy (Smets and Wouters, 2003 and 2007; Christiano et al., 2003). In the Bayesian statistics context, the analysis lies on the probability distributions of the forecast, also called the predictive density. The predictive density reflects the probability assigned to each one of the future possible outcomes of a variable, conditional to a set of observable data (Geweke and Whiteman, 2006).

Following Del Negro and Schorfheide (2013), the one-period ahead predictive density quantifies the probability of having a forecast Y_{T+1} given the set of observed information Y_T:

$$P\left(Y_{T+1}|Y_{1:T}\right) = \int P\left(Y_{T+1}|\theta, S_{T+1}\right) P\left(S_{T+1}, S_T|\theta, Y_{1:T}\right) d\left(S_{T+1}, S_T\right) \quad (3)$$

Equation (3) captures two sources of uncertainty[4]: First, some state variables are estimated based on the structure of the model and, as a result, are stochastic variables (e.g. output gap). Second, the equation captures the uncertainty over the exogenous risk factors that would affect the economy in the future (for example, structural shocks ϵ_t)

The following illustrates the steps followed by the technical staff in the construction of the PD for macroeconomic forecasts:

- Using available data, a diagnostic of the current state of the economy is defined (e.g. output gap and real exchange rate gap) and a central forecast for the macroeconomic variables is created.
- A qualitative balance of risks to the central forecast scenario is generated. The prospective risk factors identified in this analysis are characterized by using the probability distribution (mode, variance, and skewness) of the structural shocks of the models.
- A combination of shocks is drawn from these distributions as input for the PATACON and 4GM models. Each model thus generates a forecast path consistent with these shocks. This simulation exercise is repeated a considerable number of times, resulting in a set of forecast paths for each variable.
- For each model and prediction period, the set of forecasts is represented with a distribution that assigns probabilities to the projections for each variable. The densities on the PATACON and 4GM forecasts are combined to obtain a unified predictive density. This combination is done giving equal weight to the predictive densities of each model[5].

4 A more exhaustive analysis incorporates uncertainty in the parameter θ through a probability distribution. Nevertheless, in this case a point estimate is used, as is common practice.

5 Uses a linear pooling methodology proposed by Stone (1961). The literature has shown that a combination of forecasts that assigns the same weight to its components tends to outperform the forecast capacity of more sophisticated combinations (Graefe et al., 2014).

1.2 Characterizing the distribution of shocks

The construction of the balance of risks implies the prospective identification of the factors that could affect the economy in the future and the expected performance of macroeconomic variables. In the first instance, this balance provides a qualitative evaluation that exhaustively describes each of the elements that could affect the forecast.

The characterization of the prospective risk factors is translated in terms of the distribution of shocks used in the PATACON and 4GM models and, specifically, its considerations about the mode, variance, and skewness of these shocks on the forecast horizon.

The representation of risks in terms of the distribution of shocks has at least two advantages. First, it allows for the quantification of the marginal effects of each factor of uncertainty in the construction of the predictive density. Second, it offers the possibility of including exogenous information in the forecast within the models, in such a way that the general equilibrium dynamics and the macroeconomic consistency are mantained.

1.2.1 Mode

First, we do considerations about the mode of distribution for different shocks to condition the projection models on exogenous information.

For example, assumptions about external variables on the forecast horizon, such as oil prices or the U.S. Federal Reserve interest rate, are derived from the analysis and combination of projections using different sources of information. These trajectories are included in the models through shocks, whose distribution implicitly has a non-zero mode to condition the assumed value. These shocks have their own effect on the probability distributions of the other variables through the economic structure and the implicit channels of transmission in the models.

1.2.2 Variance

Second, the variance of the distribution of the shocks is adjusted to characterize different magnitudes of risk over the forecast horizon. This characterization allows for the communication of different levels of uncertainty over the forecast horizon, in line with the prospective risk factors from the qualitative analysis.

An example of this would be the quantification of risk associated with climate factors and, in particular, the possibility of an El Niño weather pattern given exogenous information regarding the likelihood of observing this event during the forecast horizon.

Historically, El Niño weather patterns have implied increases in uncertainty associated with the dynamics of the food basket. As such, considerations of the variance of the shocks associated with prices in this basket allow for an adequate reflection of this prospective risk.

The variance of the distribution of the distinct shocks can be informed using data from external sources, or by recurring to the shocks estimated in the models.

1.2.3 Skewness

Third, the distribution of shocks is modified to characterize the asymmetry present in the analysis of the prospective risk factors. This analysis implies abandoning the assumption of symmetry of the normal distribution but allows that the Technical Staff incorporates risk elements reflecting a higher probability of obtaining macroeconomic forecast paths above (or below) the central projection. An example of this would be the characterization of risks on the future dynamic of economic activity as a consequence of the COVID-19 health crisis and subsequent social distancing measures. In this case, the distribution of the demand-side shocks would exhibit a negative skew, capturing the higher probability that GDP growth forecasts are below the central forecast than above it, consistent with a more negative output gap.

2. Characterization of the balance of forecast risks from the July 2021 report

In this section we will qualify and inform the Technical Staff's prospective balance of risks on the macroeconomic forecast of July 2021. The exercise was developed using PD methodology, allowing for the construction of a probability distribution for the forecasts of each relevant variable, incorporating risk factors considered and the transmission of its effects, in light of the economic structure implicit in the PATACON and 4GM models, their general equilibrium relationships and the monetary policy response.

In this exercise, the balance of risks accounts for external and internal factors. The latter includes shocks on prices and economic activity.

The external risk factors consider the possibility of less favorable international conditions than those reflected in the central forecast scenario. These risks can be grouped into four categories. First, in the second half of 2021 the propagation of new strains of COVID-19 and the persistence of global supply chain disruptions, represents a downward risk to the growth of trade partners. This risk would be consistent with a downward skew in the price of oil in this period. Since 2022 the risks on both variables are considered to be balanced.

Second, the uncertainty associated with the inflationary effects, to international and local level, of the disruption of global supply chains, higher transportation costs, elevated commodities and food prices, and the reopening of the economy.

Third, the possibility of a normalization of monetary policy in the United States faster than anticipated in the central projection, for example in response to persistent inflationary pressures that affect compliance with the target (2% on average) and economic growth or a recovery in employment in coming years stronger than expected. In consequence, the PD for the Fed interest rate and the natural U.S. interest rate are positively skewed.

Fourth, fiscal uncertainty in Colombia could be reflected in more restrictive international financing conditions than those considered in the central forecast, captured in upward risks to the risk premium and its medium- and longterm components. In line with this, both the exchange rate and the real neutral interest rate for Colombia would also have a positive skewness.

This balance of external risks implies financial conditions with a higher probability of being unfavorable than presented in the central forecast, contributing to skew inflation upward and GDP growth downward.

Regarding the risk factors linked directly to prices in Colombia, elements of each of the component groups of the consumer price index (CPI) are characterized for analytic purposes. Food basket incorporates an upward risk until the fourth quarter of 2022, explained primarily by three factors. First, the probability of higher prices in commodities and inputs due to the disruption of global supply chains. Second, increased upward pressures on international food prices associated with the risk of increased demand from China. Third, the possibility of a slower recovery from the deterioration of the agricultural chain, which began during the pandemic and was accentuated by roadblocks in May.

With regard to regulated basket, the PD suggests the risk of more significant adjustment in fuel prices, added to the possibility of higher energy rates and the indexation of public services and regulated education to higher inflation than in the central forecast. These risks would be present until the fourth quarter of 2022.

For core inflation, measured as the CPI excluding foods and regulated items, associated risk factors suggest a positive skewness on the forecast horizon. This would be explained by a goods basket that is expected to have an upward bias until the middle of 2022, reflecting the possibility of interruptions in global supply chains and domestic value chains that have more persistent effects on prices of this basket. Services show a negative skewness given the risk of more pronounced negative demand pressures, lower mobile telecommunications services prices for the rest of 2021, and lower rental housing prices given the ample supply observed over the course of the pandemic and that would be expected to persist in 2021 and the first half of 2022.

The macroeconomic consistency of the PD methodology suggests a positive skewness in headline inflation. Nevertheless, despite the risk factors and the skews mentioned, both core inflation and headline inflation would be expected to remain between 2% and 4% in 2022 with a probability above 70%.

In relation to economic activity, the PD reflects a positive skewness in the GDP nowcast (second quarter of 2021) to capture the risk of an improved economic performance compared to the central forecast, in line with recent results from the monthly economic tracking indicator (ISE). For the rest of the forecast horizon the projection suggests a negative skewness in economic activity, as the consequence of a possible worsening of the health crisis due to the appearance of new strains of the virus, and from political and fiscal uncertainty and its possible effects on consumption and investment decisions.

Panels A-D in Graph B1.1 present the probability distribution for the annual GDP growth forecast, headline inflation, inflation excluding food and regulated items, and the output gap, respectively.

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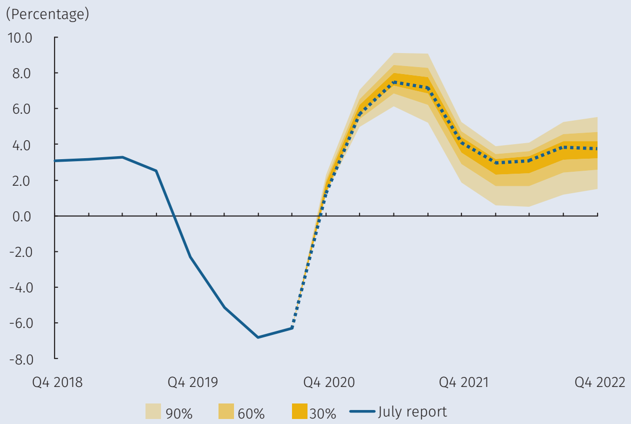
Graph B1.1

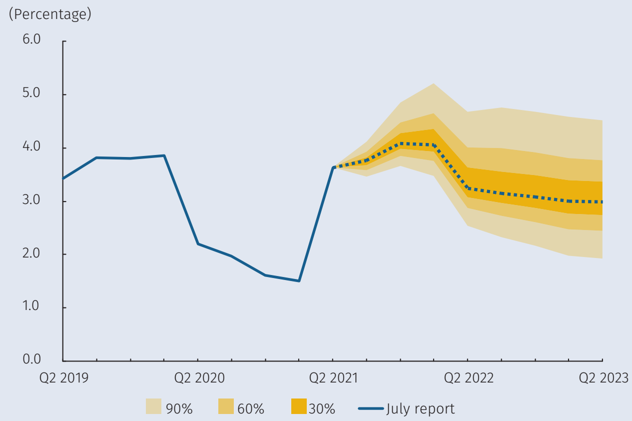
A: Gross domestic product, four-quarter accumulation [a/ b/ c/]
(annual change)



	Growth at 12 months		
	Q4 2021	Q4 2022 (percentage)	Q2 2023
<2.00	0.0	37.4	10.6
2.00 to 3.50	0.0	46.7	38.5
3.50 to 5.00	0.1	13.0	38.7
5.00 to 6.50	11.8	0.6	10.7
6.50 to 8.00	54.7	0.0	0.7
>8.00	33.0	0.0	0.0

a/ Seasonally adjusted and corrected for calendar effects
b/ The graph presents the probability distribution and its most likely path on an eight-quarter forecast horizon. Densities characterize the balance of potential risks with areas of 30%, 60% and 90% probability around the central forecast (mode), using a combination of densities from the PATACON and 4GM models.
c/ The probability distribution corresponds to the forecast exercise in the July report.
Source: DANE; calculations and projections by *Banco de la República*

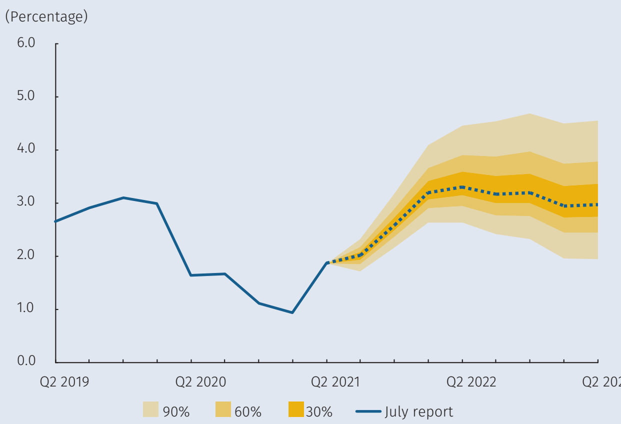
B: Consumer price index [a/ b/]
(annual change, end-of-period)



	Headline inflation		
	Q4 2021	Q4 2022 (percentage)	Q2 2023
<2.0	0.0	3.2	6.0
2.0 to 3.0	0.0	26.7	32.8
3.0 to 4.0	25.3	47.8	45.1
4.0 to 5.0	72.5	19.9	14.4
>5.0	2.2	2.5	1.7
2.0 to 4.0	25.3	74.5	77.9

a/ The graph presents the probability distribution and its most likely path on an eight-quarter forecast horizon. Densities characterize the balance of potential risks with areas of 30%, 60% and 90% probability around the central forecast (mode), using a combination of densities from the PATACON and 4GM models.
b/ The probability distribution corresponds to the forecast exercise in the July report.
Source: DANE; calculations and projections by *Banco de la República*

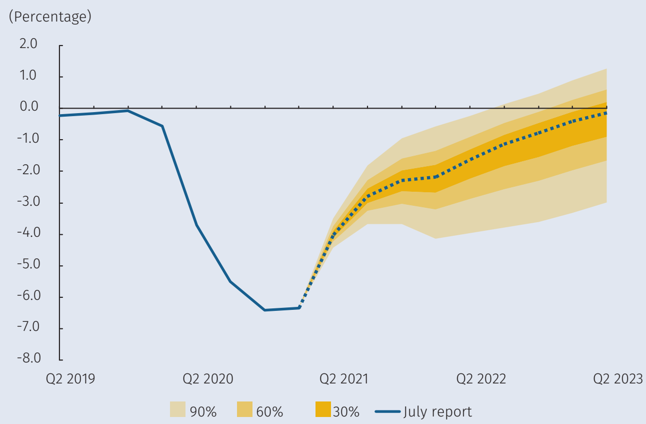
C: CPI excluding foods and regulated items [a/ b/]
(annual change, end-of-period)



	Inflation excluding food and regulated items		
	Q4 2021	Q4 2022 (percentage)	Q2 2023
<2.0	1.5	1.8	5.7
2.0 to 3.0	84.0	21.7	31.7
3.0 to 4.0	14.3	51.7	45.0
4.0 to 5.0	0.0	22.7	15.8
>5.0	0.2	2.1	1.8
2.0 to 4.0	98.4	73.4	76.7

a/ The graph presents the probability distribution and its most likely path on an eight-quarter forecast horizon. Densities characterize the balance of potential risks with areas of 30%, 60% and 90% probability around the central forecast (mode), using a combination of densities from the PATACON and 4GM models.
b/ The probability distribution corresponds to the forecast exercise in the July report.
Source: DANE; calculations and projections by *Banco de la República*

D: Output gap [a/ b/ c/]
(accumulated for 4 quarters)



	Annual gap		
	Q4 2021	Q4 2022 (percentage)	Q2 2023
<-3.00	21.7	12.5	4.7
-3.00 to -2.00	43.6	24.3	14.0
-2.00 to -1.00	28.4	30.7	27.4
-1.00 to 0.00	5.7	21.8	27.9
>0.00	0.1	10.0	24.7

a/ The historical estimate of the output gap is calculated as the difference between observed GDP (four-quarter accumulation) and potential GDP (trend; four-quarter accumulation) from the 4GM model; for the forecast it is calculated as the difference between the technical staff's GDP estimate (four-quarter accumulation) and potential GDP (trend; four-quarter accumulation) from the 4GM model.
b/ The graph presents the probability distribution and its most likely path on an eight-quarter forecast horizon. Densities characterize the balance of potential risks with areas of 30%, 60% and 90% probability around the central forecast (mode), using a combination of densities from the PATACON and 4GM models.
c/ The probability distribution corresponds to the forecast exercise in the July report.
Source: DANE; calculations and projections by *Banco de la República*