The Exchange Rate and Oil Prices in Colombia: A High Frequency Analysis

By: Juan Manuel Julio-Román
Fredy Gamboa-Estrada
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Juan Manuel Julio-Román†  
Fredy Gamboa-Estrada‡

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Abstract

We study the relationship between daily oil prices and nominal exchange rates between 1995 and 2019 in Colombia through a Time-Varying Vector Auto-Regressions with residual Stochastic Volatility, TV-VAR-SV, model. For this task we also employ co-integration, Univariate Auto-Regressions with residual Stochastic Volatility, UAR-SV-TV, and De-trended Cross Correlation, DCC analyses. We found that a stable long-run relationship between the two processes is lacking. We also found significant time variation in residual volatility and co-volatility. More specifically, we found that both periods of time, the international financial crisis and the oil price drop of 2015, behave conspicuously different from other “more normal” times. These results are consistent with a shift in the features of the DCC at the start of the crisis. Before the crises the DCCs are positive but weak for different windows sizes, turning negative and significant after it. The latter DCCs and their significance increase with the window size. These results are concurrent, also, with two clearly differentiated periods of time; one when oil production was not financially feasible, and thus production, exports and oil related currency inflows were small, and the other when oil production became feasible because of the price increase, which led to a boom in exploration, production, exports and oil related currency inflows.

Keywords: Nominal Exchange Rate, Oil prices, Small Open Economy, Co-Volatility  
JEL: C22, C51, F31, F41, G15

*First draft for comments.  
†jjulioro@banrep.gov.co Senior Researcher, Research Unit, Economics Research VP, BANCO DE LA REPUBLICA, and part time Associate Professor, School of Economics, Universidad Nacional de Colombia, Bogotá, Colombia.  
‡fgamboes@banrep.gov.co Researcher, Monetary and International Investments VP, BANCO DE LA REPUBLICA, Bogotá, Colombia.
La Tasa de Cambio Nominal y el Precio del Petróleo en Colombia: Un Análisis de Alta Frecuencia

Juan Manuel Julio-Román\textsuperscript{1}
Fredy Gamboa-Estrada\textsuperscript{2}

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Resumen

Estudiamos la evolución de la relación entre los precios diarios del petróleo y la tasa de cambio nominal Colombiana entre 1995 y 2019 a través de un modelo de Vectores Auto-Regresivos Tiempo-Variantes con Volatilidad Estocástica Residual. Para esto empleamos también técnicas de co-integración, Auto-Regresiones Univariadas con Volatilidad Estocástica Residual, y Correlaciones Cruzadas Des-tendenciadas. Se encontró que no existe una relación estable de largo plazo entre estos dos procesos. También hallamos evidencia de variación temporal de la volatilidad y co-volatilidad residual. Más específicamente, encontramos que tanto la crisis financiera global como la reducción de los precios de petróleo de 2015 son periodos particularmente distintos de otros periodos “más normales”. Estos resultados son consistentes con un cambio en el comportamiento de las DCC al inicio de la crisis financiera de 2008. En efecto, antes de la crisis estas correlaciones eran positivas pero poco significativas para diferentes tamaños de la ventana de estimación, pero después de la crisis se tornaron negativas y significativas. Las últimas DCCs y su significancia se incrementaron a mayores tamaños de la ventana. Estos resultados coinciden con dos periodos de tiempo claramente diferenciados en Colombia, uno en el cual la producción petrolera no era financieramente factible, y en consecuencia la producción, exportación y flujos entrantes de divisas por petróleo eran pequeños, y otro donde la producción fue factible, conduciendo a un boom en la exploración, producción, exportación y en los flujos entrantes de divisas relacionados con petróleo.

Palabras Clave: Tasa de Cambio Nominal, Precios del Petróleo, Economía Pequeña Abierta, Co-Volatilidad

JEL: C22, C51, F31, F41, G15

\textsuperscript{1}Primera versión para comentarios.
\textsuperscript{1}jjulioro@banrep.gov.co Investigador Principal, Unidad de Investigaciones, Sub-Gerencia de Estudios Económicos, BANCO DE LA REPÚBLICA y Profesor Asociado de tiempo parcial, Escuela de Economía, Universidad Nacional de Colombia. Bogotá D. C., Colombia.
\textsuperscript{2}fgamboes@banrep.gov.co Investigador, Sub-Gerencia Monetaria y de Inversiones Internacionales, BANCO DE LA REPÚBLICA, Bogotá D. C., Colombia.
1 Introduction and Motivation

The dynamics of oil prices during the last decade led to an escalation in the research on the relationship between oil prices and nominal exchange rates using a variety of methodologies. For instance, based on non-linear techniques, [Akram (2004)](#) identified a relatively strong relationship between the Norwegian exchange rate and oil prices when they are falling and are below 14 dollars per barrel. [Chen and Chen (2007)](#) found a strong relationship and significant forecasting power of real oil prices on the real exchange rates in G7 countries. However, their empirical results seem to point to bi-directional causality at very short ranges of time. [Reboredo (2012)](#) detected weak dependence between oil prices and exchange rates, although the dependence increased noticeably in the aftermath of the global financial crisis. Using wavelet multi-resolution analysis, [Reboredo and Rivera-Castro (2013)](#) found evidence of no dependence between oil prices and exchange rates before the global financial crisis. But, they found evidence of contagion and negative relationship after the beginning of the crisis, and a bi-directional causality during the crisis. [Reboredo, Rivera-Castro, and Zebende (2014)](#), the closest to our study, used De-trended Cross-Correlation, DCC, analysis to find that before the recent financial crisis oil prices and exchange rates had a negative weak dependence at different time scales. After the crisis they found a significant dependence between oil prices and exchange rates in the short as well as the long-run. [De Truchis and Keddad (2016)](#) analysed the co-volatility of oil prices with four exchange rates in developed countries in the short and long-run. They revealed mixed results regarding the long-run dependence, and that dependence is more responsive to market conditions in the short-run. [Beckmann, Czudaj, and Arora (2017)](#) studied the theoretical transmission channels between oil prices and exchange rates, focusing on bidirectional causality. They found that the evidence varies according to the country and the empirical method used. However, there common patterns between countries seem to arise. For instance, there is a strong link between those variables over the long-run, but the impact is time-varying.

Colombia has not been alien to this trend. The following three papers report analyses of the relationship between oil prices and the exchange rate in this country, to the best of our knowledge. [Melo-Becerra, Ramos-Forero, Parrado-Galvis, and Zarate-Solano (2016)](#) study the impact of oil price and production shocks on the real exchange rate, private investment, private consumption and public debt. Using a similar empirical strategy as the one in this paper but sampled at a lower frequency, they find that positive oil price shocks do not have any significant effect on the real exchange rate, except in January 2004, while negative oil shocks have significant and permanent effects on the real exchange rate. [Francis and Restrepo-Angel (2018)](#) study the impact of oil price shocks in the Colombian economy between 2000 and 2017. Using structural time-invariant vector autoregressive models and local projections modelling, they find that a positive oil price shock appreciates the exchange rate. Finally, [Gomez-Gonzalez, Hirs-Garzon, and Uribe (2019)](#) analyse
the relationship between oil prices and exchange rates for six oil producers and six oil importing countries, including Colombia. They compute connectedness between oil prices and exchange rate returns using forecast error variance decomposition from time-invariant vector auto-regressions, and bidirectional causality using rolling windows. For oil producers, their results show a positive trend in global connectedness between 2001 and 2018, and bidirectional causality between exchange rate returns and oil prices, which exhibits considerable time variation. Causality relations are more recurrent from oil to exchange rates.

Furthermore, the dynamics of international oil prices have had a strong influence on oil exploration, production and revenue inflows to the country since 2000. Figures B.1 and B.2 show that between 2000 and 2019 the economy experienced three important episodes in the oil industry: first, oil production fell between 2000-2003 reaching a trough of 541,000 barrels per day; second, rising oil prices and the creation of the National Hydrocarbons Agency (ANH) in 2003 gave rise to a significant boost in exploration and production between 2009-2013, breaking through the one million barrels a day mark; third, international oil prices fell by more than 40% in 2014, reaching a second trough in 2015 and leading to a reduction in oil production in Colombia by about 12% in 2016. However, by the end of 2017 oil prices rose above $60 per barrel, leading the Colombian Petroleum Company (ECOPETROL) to a new period of growth in oil reserves discoveries and production.

As a result, oil became the prime Colombian export over more traditional products such as coffee, reaching roughly 7% of GDP in 2017, thus having a profound effect on the Colombian economy. First, different factors in the oil activity such as the cycle of each of the stages of production, the domestic demand, the foreign share in the business, and international oil prices affect the dynamics of exports, imports and the revenue of foreign firms, which has an important effect on the country’s current account, López, Montes, Garavito, and Collazos (2013). Before the oil prices drop of 2014, oil sector transactions had largely offsetting effects on the current account balance related to the relative increase in oil exports and international oil prices. For instance, the average current account in 2010 and 2011 was -3.0% of GDP, which was compensated by the surplus oil current account of 4.2% of GDP, López et al. (2013). Furthermore, the dependence of the Colombian economy on oil prices was evident after their fall in 2014 as the share of the oil sector within exports fell to 33% in 2016 after a record level of 55% in 2013. As a result, the current account deteriorated, increasing its deficit from 3.2% of GDP in 2013 to 4.3% in 2016.

Second, oil activity plays an important role in the foreign exchange market due to the net supply of foreign exchange originated in the monetization of foreign direct investment inflows and foreign exchange flows related to oil exports, López et al. (2013). The excess supply (demand) of foreign exchange has sometimes coincided with an appreciation (depreciation) of the exchange rate, and a close relationship with oil prices seem to have
Third, this relationship seems to be accompanied by co-movement between their volatilities, with greater uncertainty around 2008 and after the second semester of 2014, Figure B.3.

Fourth, there is evidence that a permanent fall in oil revenues due to negative shocks in international oil prices can cause that the nominal exchange rate depreciation drives inflation away from its target in Colombia, Hamann, Bejarano, and Rodríguez (2015). However, this result will depend on the degree of openness of the economy, as well as the degree of price stickiness of the non-tradable sector.

And fifth, the literature evidence that there is a positive relationship between oil price changes and GDP growth, Perilla (2010). The direction of this effect is asymmetric as rising oil prices do not have a statistical impact on GDP growth, while a reduction in prices slows it down. The effects of changes in commodity prices were so severe that “Dutch Disease” seem to have arisen in Colombia, Poncela, Senra, and Sierra (2017), as increases in commodity prices between 1972 and 2013 led to an appreciation, which had a negative impact on the competitiveness of the non-energy exporting sector.

In this context, the relationship between oil price and the nominal exchange rate plays a key role. In fact, an inflation targeting, IT, central bank in a small open economy, SOE, whose main export becomes a commodity is interested in answering the following questions about this relationship: Does the international oil price (or its volatility), help forecast the nominal exchange rate(or its volatility)? Does the opposite holds?

In this paper we study relationship between daily oil prices and nominal exchange rates in Colombia through the use of Time-Varying Vector Auto-Regressions with residual Stochastic Volatility, TV-VAR-SV. This model has the following features. First, the fact that this model’s responses are “fully” time-varying allows accommodating smooth, sudden and even threshold response transitions between periods of time, as well as differentiated responses according to the shock size. Second, the residual SV assumption allows us controlling for and addressing the dependency of the responses on heteroskedasticity and co-volatility, factors that may lead to estimation biases when not accounted for. And third, it avoids the endogeneity issues arising in single equation studies. As a result, this exploratory rather than pre-identified approach may provide useful information on the relationship between exchange rates and oil prices at high frequencies.

Our results point out to the non-existence of lagged but contemporaneous Granger causality between the Colombian nominal exchange rate and oil price returns. We also rule out any long-term effects between these indicators due to the following facts; co-integration is lacking and responses are significant only on impact. Furthermore, risk shocks relate to increased correlation between these variables, which is a part of the general co-volatility observed. Furthermore, when oil prices are higher than 60 USD and this change is
permanent and persistent, the relationship with the exchange rate strengthens. We finally found that the financial crisis is a turning point regarding the nature of the relationship between oil prices and exchange rate volatilities as they reached higher levels after 2008. Volatilities were on average higher after the global financial crisis than in the pre-crisis period, with peak levels occurring in 2009 and 2016.

These results are consistent with negative covariance and correlation during the sample analysed, with lowest levels observed in 2009 and 2016. Finally, the de-trended cross correlations evidence two different patterns in the pre-crisis and post-crisis periods. For instance, the correlation coefficient shows positive weak dependence between oil prices and the exchange rate for different windows sizes during the pre-crisis period, while there is negative significant dependence between these variables after the global financial crisis, which increased at longer time scales.

This document consists of four sections aside from the introduction. The second summarizes the conceptual framework of the paper. The third describes the data and methodology. The fourth contains the results. And the last summarizes the main findings.

2 Conceptual Framework

The theoretical relationship between oil prices and exchange rates have been examined in a important number of studies. The literature identifies two main channels through which an oil price shock affects *exchange rates*: the *terms of trade*, and the wealth and portfolio reallocation channel. See Habib, Büitzer, and Stracca (2016), for instance.

Regarding the terms of trade channel, Amano and van Norden (1998) propose a model with tradable and non-tradable sectors, which use both oil as a tradable input and labour, which is non-tradable. The price of traded goods are fixed internationally, which determine domestic wages for both sectors. A positive oil price shock that affects positively the terms of trade, increases the price of the non-traded good in the domestic economy and appreciates the exchange rate. However, the effect depends on whether the non-tradable sector is more energy intensive than the tradable one in the domestic economy. Golub (1983) and Krugman (1980) developed a three country framework with two oil-importing economies and one oil exporter. A positive shock in oil prices is related to a wealth transfer from oil importing countries to the oil-exporting one. This has a direct effect on the current account (short-run effect) and leads to portfolio reallocation (medium and long-run effects). Golub (1983) finds that the dollar tended to appreciate in 1973-1974 when oil prices increased unexpectedly, and depreciated in 1979 following news of rising oil prices. The increase in the dependence of the United States on OPEC oil was the main factor explaining that pattern. Krugman (1980), in turn, shows that exchange rates in oil exporting countries tend to depreciate in the short run following an increase in oil prices,
assuming a stronger preference of these economies for dollar-denominated assets. The effect of higher oil prices on exchange rates in oil importing economies depends on whether the share of their oil imports is similar to their share of exports to OPEC. Although these models tend to predict that a positive shock in oil prices causes a real depreciation (appreciation) in oil-importing (exporting) economies, there are potential offsetting factors that may reduce the effect of oil price shocks on exchange rates. Habib et al. (2016). For instance, monetary authorities in Latin America may counter exchange rate pressures derived from commodity terms of trade shocks accumulating or reducing foreign exchange reserves, Aizenman, Edwards, and Riera-Crichton (2012).

3 Data and Methods

3.1 Data

Daily USD WTI oil prices and COP/USD nominal exchange rate records between January 1995 and January 2019 are studied in this paper. The former were obtained from the Federal Reserve FRED database, while the latter comes from Banco de la República’s, the central bank of Colombia, web page.

To deal with the conditional heteroskedasticity observed in oil price and COP/USD exchange rate data, as well as to reduce its effect on relevant tests, such as co-integration, we implement univariate and multivariate Stochastic Volatility, SV, models. These models have important advantages over traditional deterministic volatility ones such as ARCH or GARCH.

3.2 Univariate Auto-Regressive Model with Stochastic Volatility Noise

We start with univariate analyses of oil price and COP/USD exchange rate returns volatilities. To this end, we fit Univariate Auto-Regressions, UAR, whose noise volatility is stochastic, that is UAR-SV models. This model is described by the following equations

$$Y_t | \bar{Y}_{t-1} \sim N(\beta_0 + \beta_1 Y_{t-1} + \cdots + \beta_p Y_{t-p}, \exp h_t)$$

(1)

$$h_t | h_{t-1}, \mu, \phi, \sigma_\eta \sim N(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2)$$

(2)

These advantages derive from two facts. On the one hand, by definition volatility is stochastic in SV models, and thus it may be subject to shocks as opposite to other well known alternatives where volatility is deterministic. And on the other, SV models have a strong relationship with the types of models used in financial theory, that is, SV models are more likely to have theoretical counterparts in financial mathematics than other popular alternatives such as the ARCH GARCH family. See Shephard (1996), Kim, Shephard, and Chib (1997) and Shephard (2005).
for \( t \in \mathbb{N} \), and
\[
\begin{align*}
h_0 \mid \mu, \phi, \sigma_\eta & \sim N(\mu, \sigma_\eta^2/(1-\phi^2)) \\
\end{align*}
\]
where \( \Theta = (\beta_0, \ldots, \beta_p, \mu, \phi, \sigma_\eta)’ \) is the vector of time invariant parameters, \( \mathbf{X}_t \) is the information set up to time \( t \), \( Y_t \) is the observable variable, \( \mu \) is the level of log-variance, \( \phi \) determines the log-variance persistence, and \( \sigma_\eta \) is the volatility of the log-variance.

Estimation of model (1), (2) and (3) is carried out by Bayesian MCMC simulation techniques. More specifically, a standard simulation technique for \( \beta_0, \ldots, \beta_p \) is coupled with one specifically designed for the rest of parameters, including the unobserved log-variance. Details on the prior distributions, the simulation algorithm for the second component, along with its properties may be found in Kastner and Frühwirth-Schnatter (2014).

3.3 Co-integration Tests

We employ co-integration tests for the relationship between the log COP/USD nominal exchange rate and the log USD WTI oil prices. To this, we use different co-integration approaches such as the Engle-Granger test, the Gregory-Hansen test with regime shifts, the Hansen parameter instability test, and ARDL tests. The Engle and Granger (1987) test is a residual-based approach as unit root tests are applied to the residuals obtained from the static OLS (SOLS) estimator. This test uses a parametric (ADF) approach where the null hypothesis of no co-integration corresponds to residual nonstationarity. The Gregory-Hansen approach with regime shifts is a residual-based test which evaluates the null hypothesis of no co-integration with a single break at an unknown point in time. According to Gregory and Hansen (1996) structural changes can take different forms. We consider four cases in the co-integration relationship. A level shift model, a level shift with trend model, a regime shift model where the equilibrium relation rotates with parallel shift, and a regime shift with trend model. Hansen (1992) develops a test where the alternative hypothesis of no co-integration is consistent with the evidence of parameter instability. He proposes a test statistic to evaluate the parameter stability and arises from the theory of lagrange multiplier tests. This test relies on estimates from the original equation in contrast to the residuals based co-integration approaches. We employ bound tests according to M. H. Pesaran, Shin, and Smith (2001). This approach tests the existence of a level relationship between a dependent variable and a set of explanatory variables irrespective of whether they are purely I(1), purely I(0), or mutually co-integrated. These tests are based on standard F and t-statistics which are used to tests the significance of the lags of the dependent and the regressors in an equilibrium correction regression.

If no evidence of co-integration between the nominal exchange rate in Colombia and WTI oil prices arises, we analyse their returns and volatilities.
3.4 The Time-Varying Vector Auto-Regressive Model with Stochastic Volatility Noise

The dynamic relationship between the log USD oil price and the log COP/USD nominal exchange rate has usually been studied through VAR models, which have several drawbacks in this particular case as their impulse-response functions are time invariant, do not take into account the presence of heteroskedasticity and the evolution of co-volatility.

Instead, Primiceri (2005) proposed a TV-VAR-SV of the form

\[ Y_t = c_t + \sum_{j=1}^{k} B_{jt} Y_{t-j} + u_t \quad t \in \mathbb{Z} \]  

(4)

where \( B_{jt} \) for \( j = 1, 2, \ldots, k \) are \( n \times n \) unknown parameter matrices, and \( Y_t, c_t \) and \( u_t \) are \( n \times 1 \) vectors of time series, time-varying unknown parameters and unobserved reduced form time-varying heteroskedastic shocks, respectively, such that \( u_t \) are independent \( (0, \Omega_t) \) for \( t \in \mathbb{Z} \) and

\[ A_t \Omega_t A_t' = \Sigma_t \Sigma_t' \]  

(5)

for a lower triangular matrix

\[
A_t = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
\alpha_{21,t} & 1 & 0 & \cdots & 0 \\
\alpha_{31,t} & \alpha_{32,t} & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & 0 \\
\alpha_{n1,t} & \alpha_{n2,t} & \cdots & \alpha_{n-n+1,t} & 1
\end{bmatrix}
\]  

(6)

and a diagonal matrix

\[
\Sigma_t = \begin{bmatrix}
\sigma_{1,t} \\
& \ddots \\
& & \sigma_{n,t}
\end{bmatrix}
\]  

(7)

containing time-varying unknown parameters.

Primiceri (2005) and Negro and Primiceri (2015) write this model in a familiar way as follows

\[
Y_t = X_t' B_t + A_t^{-1} \Sigma_t \epsilon_t
\]  

(8)

\[
B_t = B_{t-1} + \nu_t
\]  

(9)

\[
\alpha_t = \alpha_{t-1} + \zeta_t
\]  

(10)

\[
\log \sigma_t = \log \sigma_{t-1} + \eta_t
\]  

(11)
where \( \epsilon_t = A_t^{-1} \Sigma_t u_t \) for \( t \in \mathbb{Z} \), \( B_t \) are all the parameters to the right hand side of Equation (8) stacked in a vector, \( X_t \) is determined by

\[
X_t = I_n \otimes \left[ 1, Y_{t-1}', \ldots, Y_{t-k}' \right],
\]

\( \alpha_t \) is a vector containing the unknown elements in \( A_t \), \( \sigma_t \) contains the diagonal elements in \( \Sigma_t \), and \( \nu_t, \zeta_t \) and \( \eta_t \) are vectors of unobserved noises. Finally, all the noises in model (8)-(11) have a variance co-variance matrix

\[
V = \operatorname{Var} \left[ \begin{bmatrix} \epsilon_t \ 
\nu_t \\ \zeta_t \\ \eta_t \end{bmatrix} \right] = \operatorname{diag}(I_n, Q, S, W) \tag{12}
\]

where \( S \) is usually assumed to be block diagonal, and Equation (11) is usually known as a "geometric random walk" process.

Given a set of sample of information, \( \{Y_t | t = 1, 2, \ldots, T\} \), estimation is carried out by Bayesian methods. There are several issues that prevent the use of standard estimation procedures in this setting. One one hand, Equations (9), (10) and (11) help define the stochastic behaviour of an important number of unobserved time-varying components. And on the other, Equations (8)-(11) define a non-linear system. The high dimension and non-linearity in this problem pose important problems to standard estimation techniques in contrast to the Bayesian ones, which are known to be well suited under these circumstances. See Primiceri (2005) and Negro and Primiceri (2015).

The posterior of the state, i.e. the elements in Equations (9), (10) and (11), as well as the parameters in matrix \( V \) (Equation (12)), was found by MCMC simulation. More precisely, the posterior of the state elements corresponds to the smoothing density. For a detailed description of the assumptions on the prior distributions, the simulation algorithm and its properties see Primiceri (2005) and Negro and Primiceri (2015).

### 3.5 Impulse Response Analysis

We perform impulse responses analyses as a way to summarize the dynamic joint behaviour of log oil prices and exchange rates as well as to study the relationship between their returns. In this particular case, no economic theory provides restrictions to identify structural shocks, so we apply H. H. Pesaran and Shin (1998) generalized impulse responses, from model (8)-(11), to selected dates. The scaled generalized response of \( Y_{t+n} \), to a impulse in \( Y_{jt} \) is defined as

\[
\psi^g_{j}(n) = \sigma^{-1/2}_{jj} A_n \Sigma e_j \tag{13}
\]
where \( \sum_{n=0}^{\infty} A_n L^n = (\sum_{n=0}^{\infty} B_n L^n)^{-1} \), and \( e_j \) is a vector full of zeros except at position \( j \) where it is one.

The main advantage of this approach is that it does not require orthogonalization and is, therefore, invariant to the ordering of the variables in the VAR. See H. H. Pesaran and Shin (1998) for further details.

### 3.6 The De-Trended Cross-Correlation Analysis

Zebende (2011) proposed a measure of correlation among two series that is resistant to the existence of complexity such as self-affinity and conditional heteroskedasticity coupled with high tails. The De-trended Cross-Correlation Analysis, DCCA, proposed by Zebende (2011) is a generalization of the De-trended Fluctuation Analysis, DFA, of Peng et al. (1994), which has been successful in estimating long-range auto-correlations under power law in auto-covariances. i.e. slowly decreasing correlations. See Vassoler and Zebende (2012) also.

DFA is a method to detect power laws in the auto-covariance function of a time series, which may be used to study the behaviour of power law correlations between two series. The calculation is as follows. First, the two integrated series are built, \( R_t = \sum_{j=1}^{t} X_j \), \( S_t = \sum_{j=1}^{t} Y_j \) for \( t = 1, \ldots, T \). Second, the sample is split into \( T - n \) overlapping sub-samples of size \( n+1 \). The \( i \)-th sub-sample starts at observation \( i \) and ends at \( i+n \). Then the local trends \( \tilde{R}_{ki} \) and \( \tilde{S}_{ki} \) for each observation \( i \leq k \leq i+n \) of the \( i \)-th sample is estimated for each \( 1 \leq i \leq T-n \). The local trend might be estimated through an OLS fit of the integrated series against a linear trend, for each sub-sample. Finally, the co-variance of the residuals in each sub-sample is calculated, \( f_{DCCA}^2(n) = 1/(n+1) \sum_{k=1}^{i+n} (R_k - \tilde{R}_{ki})(S_k - \tilde{S}_{ki}) \). The variances are calculated in a similar manner. Finally, the de-trended covariance function is calculated as the average of the co-variances

\[
F_{DCCA}^2(n) = \frac{\sum_{i=1}^{T-n} f_{DCCA}^2(n)}{T - n} \tag{14}
\]

The de-trended DCCA cross-correlation, \( \rho_{DCCA}(n) \) between \( X_t \) and \( Y_t \) is the ratio between the de-trended covariance function \( F_{DCCA}^2(n) \) and the de-trended standard deviation functions \( F_{DFA(X_t)}(n) \) and \( F_{DFA(Y_t)}(n) \), i.e.,

\[
\rho_{DCCA}(n) = \frac{F_{DCCA}^2(n)}{F_{DFA(X_t)}(n)F_{DFA(Y_t)}(n)} \tag{15}
\]

In the same way we can generalize beyond contemporary cross-correlations by calculating \( f_{DCCA,m}^2(n) = 1/(n+1-m) \sum_{k=1}^{i+n} (R_k - \tilde{R}_{(ki)})(S_{k-m} - \tilde{S}_{k-m,i}) \), and the corresponding
\( \rho_{DCCA,m}(n) \), for \( m = 1, 2, \ldots, M(n) \), where \( M(n) \) is an increasing function of \( n \) that provides a good estimator behaviour to \( \rho_{DCCA,m}(n) \). That is, \( M(n) \leq M \) a small number of lags \( \forall n \geq n_0(T) \) a constant depending on the total sample size.

These estimators may be computed from both, log figures and returns as well. They provide a glance to alternative features of the relationship between nominal exchange rates and oil prices.

4 Results

Our results arise from several analyses. First, we fit separate UAR-SV models to log oil prices and log exchange rate processes to determine the existence and significance of volatility changes and unit roots. Second, we conduct co-integration tests to rule out the existence of a long-run stable relationship between log oil prices and nominal exchange rate figures. Afterwards, we fit a TV-VAR-SV model to determine the joint dynamics of oil price and exchange rate returns processes, to estimate the time-varying responses as well as to determine the time-varying residual volatilities and co-volatilities. Finally, we employ a DCC analysis to further explore the nature of their dependency at different time horizons.

Co-integration and UAR-SV analyses provide the basic features of the processes, which are required for further modelling. The existence of unit roots might be studied from the posterior distribution of \( \beta_1 + \beta_2 \), integrated volatility through \( \phi \)'s posterior, mean jumps and breaks through the shape of the posterior distribution of \( \mu \), stochastically varying volatility may be characterized by \( h_t \) dynamics, and the existence of power law residual tails may be explored through the behaviour of the standardized residuals\(^5\). These features derive from Equations (1)-(3).

TV-VAR-SV, Equations (8)-(11), fits help us characterize the dynamics of the relationship between returns processes. This analysis yields the more important results of this paper; the response of the nominal exchange rate return, NER, to an oil price impulse and the response of the oil price return to a NER impulse, as well as the residual co-variance and standard deviations at each date in the sample.

We complement these analyses by estimating DCCs to abstract from self-affinity, i.e. power law cross correlation, to further characterize the dependence between oil prices and NERs across different horizons.

UAR-SV and TV-VAR-SV models are estimated through Bayesian methods based on simulation. Thus, the validity of our conclusions depend on on the convergence properties.

---

\(^5\)These COP/USD exchange rate stylized facts were established by \( \text{Julio (2017)}. \)
of our simulations to the posterior distributions of the parameters and states. As a result, prior to presenting UAR-SV and TV-VAR-SV results, we study the convergence of our simulations.

Log NER and log oil price Co-integration test results may be found in Table A.1. The main results of the UAR-SV are contained in Figures B.5 and B.8, while Figures B.4, B.6, B.7 and B.9, as well as Tables A.2 and A.3 show the fit features. In turn, Figures B.10-B.16 contain the results of the TV-VAR-SV analysis. Finally, Figures B.17-B.20 depict the DCC estimation results.

A preliminary assessment of return data, Figure B.3, suggests co-volatility, especially after 2008. In fact, this figure suggests that ascending and descending episodes in oil prices have came along with periods of considerable volatility not only in oil prices but in the nominal exchange rate.

4.1 UAR-SV Analyses

4.1.1 The Log COP/USD Nominal Exchange Rate Process

To uncover the features of the log COP/USD nominal exchange rate process a UAR-SV model, Equations 1-3, with \( p = 2 \) was fitted. The Bayesian posterior densities for the model's parameters, \( \Theta = (\beta_0, \beta_1, \beta_2, \mu, \phi, \sigma_\eta) \)' as well as for the highest auto-regressive root \( \beta_1 + \beta_2 \), may be found in Table A.2. Figure B.4, in turn, displays the simulation traces and posterior density estimates from which converge to the posterior distribution may be inferred. The estimated residual volatility from the model may be found in Figure B.5, and the standardized residuals in Figure B.6. From these Figures the existence of conditional stochastic heteroskedasticity and the performance of these estimates may also be abstracted. From these Table and Figures the following results are obtained.

First, the log nominal exchange rate process has a unit root. The Highest Probability Density, HPD, interval of the posterior of the biggest autoregressive root in Equation 1, \( \beta_1 + \beta_2 \), covers comfortably the unit according to the results in Table A.2. As a result, the long-run component of the log nominal exchange rate process is non-stationary.

Second, its volatility is highly persistent, but not integrated. Indeed, \( \phi \)'s posterior HPD in Table A.2 does not cover the unit but ranges between 0.96 and 0.98. Therefore, risk shocks may have some important effect the days after they happen, but they die out eventually.

Third, the log nominal exchange rate process is significantly heteroskedastic. Figure B.5 shows very strong evidence of residual heteroskedasticity as the 16 and 84 posterior HPD band is extremely narrow. Furthermore, the estimated log-volatility NER process
shows two important episodes of prolonged volatility increases related to the global financial crisis, 2007-2010, and the oil price drop, 2015-2016.

Fourth, there is no evidence of trend breaks or jumps in the log NER process. In fact, Figure B.4 depicts a fairly stable simulation trace from $\mu$’s posterior, which leads to a symmetric uni-modal density, thus suggesting that no jumps or trend breaks exist in the NER process.

4.1.2 Log WTI Oil Price

The results about the properties of the log international oil price process are similar to the ones above as Table A.3 and Figures B.7,B.8 and B.9 reveal.

In contrast with the estimated NER residual volatility, the estimated oil price log-volatility process in Figure B.8 show a slightly less prolonged volatility shift at the global financial crisis, but a similar one at the oil price fall of 2015. There is, furthermore, a remarkable coincidence in the most important volatility spikes of 2009, 2015, 2016 and 2017.

4.2 Co-integration Analyses

Once we have established that log oil prices and log NER have unit roots, the possibility that a stable long-term relationship, i.e. co-integration, arises.

Several co-integration tests strongly suggest that a stable long-term relationship between log figures is absent. In fact, Table A.1 reports the co-integration tests results. Using the ADF t-statistic, the Engle-Granger approach does not reject the hypothesis of no co-integration between the log NER and the log WTI oil prices. The Gregory-Hansen test with regime shifts rules out the existence of a long-run stable relationship between these variables, except for the model with level shift and trend. In addition, the Hansen parameter instability test rejects the hypothesis of co-integration at the 1 percent significance level, and the ARDL test does not reject the null of no co-integration with 3 and 2 (optimal) lags for the log NER and the log WTI oil prices, respectively. As a result the co-integration hypothesis is rejected, thus suggesting that the relationship between oil prices and the NER must be studied in differences, i.e. returns.
4.3 TV-VAR-SV on the Nominal Exchange Rate and WTI Oil Price Returns

The results on the generalized impulse response function of the TV-VAR-SV model on the NER depreciation and WTI oil price returns may be found in figures B.10 to B.15. Figures B.10 to B.13 depict the lagged responses at selected dates while Figures B.14 and B.15 display the cross responses at every period in the sample including the response on impact.

Figures B.10 to B.13 reveal that the lagged response of depreciation to oil price return shocks is negative but non significant for the selected dates, while the response of oil prices returns is positive and non significant as well. Furthermore, these Figures also exhibit important but non-significant variation across dates. More precisely, these figures show that the lagged response of exchange rate returns to oil price shocks non significantly increases when oil prices are higher than 60 USD.

However, Figures B.14 and B.15, where responses on impact are included, indicate that the contemporaneous responses of oil price returns and depreciations are negative and bigger than lagged responses.

More precisely, oil price return shocks affect depreciation for just three days at the start of the sample, but by its end the response becomes more persistent and dies out after four days. In the same way, the corresponding contemporary response, Figure B.14 at lag 0, is more revealing as it shows a reducing long-term trend response with spikes at the financial crisis and the oil price drop of 2015, which may account, through bigger correlation, for the increased co-movement of the series observed during these times.

In the same way, the generalized response of oil price returns to one standard deviation shocks to depreciation in Figure B.15 lasts just two days. However, the corresponding generalized response on impact, Figure B.15 at lag 0, shows increasing responses that are bigger than the responses of the NER.

The dynamics of the residual variance-covariance/correlation mode matrices from the TV-SV-VAR(1) model are shown in Figure B.16. The residuals of the oil prices returns and exchange rate returns equations evidence higher variance in 2009 and 2016. These results are consistent with the evidence found by the estimated residual volatility from SV-AR(2) models. More interestingly, the volatility of oil prices and exchange rate returns shocks was on average higher since the post-financial crisis period than in the pre-crisis one. These results are in line with the residual covariance and correlation between oil prices returns and exchange rate returns. In fact, we find evidence of negative covariance and correlation during the period analysed, with the lowest levels observed in 2009 and 2016.
4.4 De-trended Cross Correlations

The results for the de-trended cross correlations between oil price returns at time $t$ and exchange rate returns at time $t + k$ for different time scales are shown in Figures B.17 to B.20. The DCCA evidence a negative weak dependence between oil prices and the Colombian peso for the full sample, with correlations close to $-0.1$ for different windows sizes, B.17. The correlation coefficient before 2008 shows a positive weak dependence between oil prices and exchange rate returns for different time scales, with correlations close to 0.1, B.18. However, after the recent financial crisis, there is a significant and negative dependence between these variables, which increased from 150 days on, B.19. The evidence points out to lead and lag effects between oil price and exchange rate returns that are probably the result of the financial contagion observed with the beginning of the global financial crisis, Reboredo et al. (2014). Between June 2006 and 2009, the correlation coefficient shows a positive but weak dependence for a window size of 50 days, while from 100 days on, the correlation became negative and more significant, B.20. These results are consistent with the findings in Reboredo et al. (2014), as their cross-correlation analysis for a wide set of currencies indicate negative significant dependence between oil price and exchange rates that increased after the onset of the global financial crisis.

5 Conclusion

Using data from 1995 to 2019, we study the relationship between oil prices and exchange rate volatilities in Colombia. We use different methodologies such as co-integration, Time-Varying Vector Auto-Regression with Stochastic Volatility, Univariate Auto-Regressions with residual Stochastic Volatility, and De-trended Cross-Correlations to examine this relationship. Our approach avoids sample partition and causality biases that could arise using other methodologies. Hence, it allows for a better description and analysis of oil and nominal exchange rate dependence.

Our results points out to the non-existence of lagged but contemporaneous Granger causality between oil price and exchange rate results. They also rule out any long-term effects between these indicators due to the following facts; there is no co-integration and responses are significant only on impact. Furthermore, risk shocks relate to increased correlation between these variables, which is a part of the general co-volatility observed. Furthermore, when oil prices are higher than 60 USD and this change is permanent and persistent, the relationship with the exchange rate strengthens.

Our results suggest that the global financial crisis marks a turning point in oil prices and the nominal exchange rate dependence in Colombia. The estimated contemporary residual volatility from SV-AR models indicate that on average oil prices and exchange
rate volatilities were higher after the global financial crisis. The residuals estimated from the TV-SV-VAR model evidence higher volatility in 2009 and 2016, concurring with lowest levels of negative covariance and correlation in the same years.

When we study the de-trended cross correlations between oil prices and exchange rate returns, the evidence shows two patterns. First, in the pre-crisis period there was a positive but weak dependence between oil prices and the Colombian peso. Second, after the global financial crises there is negative significant interdependence between these variables, which increased at longer time scales. These results are concurrent with two clearly differentiated sub-samples; one when oil production was not financially feasible in general, and the other when oil production became feasible because of the price increase, which led to a boom in exploration, production, exports and oil related currency inflows.

Regarding the policy implications, in an oil-exporting country like Colombia our results have important monetary and fiscal implications. According to the data reported by the U.S. Energy Information Administration the percentage share of Colombia’s crude oil exports only accounted for 1.2% of total world exports in 2018. However, the dependence of Colombia on oil revenues makes it vulnerable to oil shocks. In line with Reboredo and Rivera-Castro (2013) and Reboredo et al. (2014) our results suggest that monetary policy should be time-varying and take into account the time scale at which decisions are taken to control oil inflationary effects. This caution is especially important in the aftermath of the global crisis when an increased interdependence between oil prices and the nominal exchange rate arose. These authors explain that as oil price and exchange rate dependence, for a range of currencies, increased after the global financial crisis, monetary policy could be used more passively to control inflation pressures derived from a positive oil price shock, as inflationary effects can be partially offset by USD depreciation. In Colombia, our results would indicate an active monetary policy at short time scales as oil and exchange rate correlation is weaker at that time frame than at longer time scales. However, in an oil-exporting country, oil inflationary effects depend on exchange rate pass-through levels, that will partially be compensated if the elasticity of the current account to exchange rate movements is high.

Oil prices and exchange rate dependence has important implications for the fiscal policy in Colombia. Our results indicate that before the global financial crisis government spending was exposed to oil revenues as there was a weak dependence between oil prices and the Colombian peso. The increasing dependence since the onset of the global financial crisis would imply that a negative oil shock could be partly offset by USD appreciation and vice versa. Reboredo and Rivera-Castro (2013) and Reboredo et al. (2014). However, Colombia’s exports depend on its commodities, especially on crude oil, which means that a negative oil price shock may have negative effects on fiscal revenues due to the absence of export diversification, and exchange rate depreciation may not be able to totally offset those effects. To isolate government spending from oil price volatility, the government
should diversify its revenues changing the economic model investing in infrastructure and developing the industrial sector.

Finally, high interdependence between oil prices and exchange rate returns volatilities could determine investor hedging strategies as they can obtain cheaper oil contracts if they are expecting a weaker USD, De Truchis and Keddad (2016).
References


Habib, M. M., Büützer, S., & Stracca, L. (2016, August). *Global Exchange Rate Configura-


Perilla, J. (2010). El impacto de los precios del petróleo sobre el crecimiento económico


Appendices

A  Tables
Table A.1: Co-integration tests for the relationship between the log nominal exchange rate and log WTI oil price in USD\textsuperscript{a}

<table>
<thead>
<tr>
<th>Test</th>
<th>ADF</th>
<th>Lc</th>
<th>F</th>
<th>Lag</th>
<th>Breakpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engle-Granger test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho: no cointegration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with trend</td>
<td>-2.23</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without trend</td>
<td>-1.91</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gregory-Hansen test with regime shifts</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho: no cointegration with break at unknown time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level shift model</td>
<td>-3.60</td>
<td>3</td>
<td>7/14/1999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level shift with trend model</td>
<td>-5.09**</td>
<td>3</td>
<td>7/15/1999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime shift model</td>
<td>-4.13</td>
<td>3</td>
<td>7/14/1999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime and trend shift model</td>
<td>-4.97</td>
<td>3</td>
<td>3/11/2003</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Hansen parameter instability test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho: cointegration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with trend</td>
<td>32.28***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without trend</td>
<td>30.91***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ARDL test</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho: No cointegration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with trend</td>
<td>4.34 (3,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without trend</td>
<td>3.97 (3,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a} Source: Author’s Calculation

\textsuperscript{b} The Engle-Granger test and the Gregory-Hansen test lag specification are based on the Schwartz and Bayesian information criteria, respectively. Lags in the ARDL test are chosen using the Akaike information criterion. The Hansen test is based on the FMOLS (Fully Modified Least Squares) estimator with d.f. correction for the long-run variance and coefficient covariance estimates. Lc is a test statistic which arises from the theory of Lagrange multiplier tests for parameter instability, and its distribution is non-standard. F is the F-bound statistic, and its critical values are estimated according to M. H. Pesaran et al. \textit{(2001)}. (3,2) lags in the ARDL test are the optimal lags for the dependent (LCOP) variable, and the regressor (LWTI), respectively. *, ** and *** indicates significance at the 10%, 5% and 1% levels, respectively.

\textsuperscript{c} All test results reject the co-integration hypothesis at 5%
Table A.2: Bayesian Estimation Results SV-AR(2) fit to log COP/USD Nominal Exchange Rate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \mu )</th>
<th>( \phi )</th>
<th>( \sigma )</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_1 + \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
</tr>
<tr>
<td>Median</td>
<td>-11.0649</td>
<td>0.96941</td>
<td>0.291086</td>
<td>0.003198</td>
<td>1.18567</td>
<td>-0.21432</td>
<td>0.971351</td>
</tr>
<tr>
<td>84% Perc.</td>
<td>-10.9063</td>
<td>0.97366</td>
<td>0.308364</td>
<td>0.004016</td>
<td>1.199705</td>
<td>-0.20026</td>
<td>0.99945</td>
</tr>
<tr>
<td>Mean</td>
<td>-10.7498</td>
<td>0.977521</td>
<td>0.325791</td>
<td>0.004814</td>
<td>1.213841</td>
<td>-0.18623</td>
<td>1.027615</td>
</tr>
<tr>
<td>Lower HPD</td>
<td>-10.9058</td>
<td>0.974875</td>
<td>0.308632</td>
<td>0.004003</td>
<td>1.199706</td>
<td>-0.20023</td>
<td>0.999475</td>
</tr>
<tr>
<td>Upper HPD</td>
<td>-10.5777</td>
<td>0.981466</td>
<td>0.343244</td>
<td>0.005525</td>
<td>1.227466</td>
<td>-0.17329</td>
<td>1.054175</td>
</tr>
</tbody>
</table>

* Source: Author’s Calculation
* Perc. is Percentile
* HPD is the Highest Probability Density

Table A.3: Bayesian Estimation Results SV-AR(2) fit to log WTI Oil Price USD

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \mu )</th>
<th>( \phi )</th>
<th>( \sigma )</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_1 + \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
<td>16% Perc.</td>
</tr>
<tr>
<td>Median</td>
<td>-7.82554</td>
<td>0.967605</td>
<td>0.164718</td>
<td>0.001009</td>
<td>0.968461</td>
<td>0.002424</td>
<td>0.970885</td>
</tr>
<tr>
<td>84% Perc.</td>
<td>-7.72951</td>
<td>0.973771</td>
<td>0.181589</td>
<td>0.002635</td>
<td>0.982523</td>
<td>0.017</td>
<td>0.999523</td>
</tr>
<tr>
<td>Mean</td>
<td>-7.63172</td>
<td>0.978913</td>
<td>0.200799</td>
<td>0.004223</td>
<td>0.997047</td>
<td>0.031079</td>
<td>1.028126</td>
</tr>
<tr>
<td>Lower HPD</td>
<td>-7.72865</td>
<td>0.973306</td>
<td>0.182567</td>
<td>0.002622</td>
<td>0.982626</td>
<td>0.01688</td>
<td>0.999506</td>
</tr>
<tr>
<td>Upper HPD</td>
<td>-7.91179</td>
<td>0.96196</td>
<td>0.14808</td>
<td>-0.00052</td>
<td>0.954497</td>
<td>-0.01028</td>
<td>0.944213</td>
</tr>
</tbody>
</table>

* Source: Author’s Calculation
* Perc. is Percentile
* HPD is the Highest Probability Density
Figure B.1: Colombia’s Daily Oil Production

Source: Colombian Mining and Energy Planning Unit, UPME.
Figure B.2: International Oil Price and COP/USD Nominal Exchange Rate

Source: Banco de la República and Bloomberg.
Figure B.3: Daily International Oil Price and COP/USD Nominal Exchange Rate Returns

Source: Authors calculation.
Figure B.4: MCMC Simulation Traces and Posterior Distributions from SV-AR(2) fit for the log Nominal Exchange Rate

Source: Authors calculation.

The log Nominal Exchange Rate

Figure B.4: MCMC Simulation Traces and Posterior Distributions from SV-AR(2) fit for the log Nominal Exchange Rate

Source: Authors calculation.
Figure B.5: Estimated Residual Volatility from SV-AR(2) fit for the log Nominal Exchange Rate

<table>
<thead>
<tr>
<th>Date</th>
<th>Log Exchange Rate Residual Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Authors' calculation.
Figure B.6: Standardized Residuals from SV-AR(2) fit for the log Nominal Exchange Rate

Source: Authors calculation.
Figure B.7: MCMC Simulation Traces and Posterior Distributions from SV-AR(2) fit for the log WTI Oil Price in USD

Source: Authors calculation.
<table>
<thead>
<tr>
<th>Date</th>
<th>Log WTI Oil Price Residual Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>0.00</td>
</tr>
<tr>
<td>1996</td>
<td>0.00</td>
</tr>
<tr>
<td>1997</td>
<td>0.02</td>
</tr>
<tr>
<td>1998</td>
<td>0.04</td>
</tr>
<tr>
<td>1999</td>
<td>0.06</td>
</tr>
<tr>
<td>2000</td>
<td>0.08</td>
</tr>
<tr>
<td>2001</td>
<td>0.10</td>
</tr>
<tr>
<td>2002</td>
<td>0.10</td>
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<td>2003</td>
<td>0.10</td>
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<td>2004</td>
<td>0.10</td>
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<td>2006</td>
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<td>2007</td>
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<td>2008</td>
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<td>2009</td>
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<td>2010</td>
<td>0.10</td>
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<td>2011</td>
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<td>2013</td>
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<td>2014</td>
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<td>2015</td>
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</tr>
<tr>
<td>2016</td>
<td>0.10</td>
</tr>
<tr>
<td>2017</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Source: Authors calculation.
Figure B.9: Standardized Residuals from SV-AR(2) fit for the log WTI Oil Price in USD

Source: Authors calculation.
Figure B.10: Lagged Generalized Response to a Standard Deviation of the Date Impulse

RESPONSE AT 2006−01−24

Oil Price

IMPULSE

Days

−0.003 −0.001

−0.003

−0.001

Days

2 4 6 8 10

2 4 6 8 10

Source: Authors calculation.
Figure B.11: Lagged Generalized Response to a Standard Deviation of the Date Impulse

Source: Authors calculation.
Figure B.12: Lagged Generalized Response to a Standard Deviation of the Date Impulse

RESPONSE AT 2013–02–06

Oil Price

Oil Price

Exchange Rate

Exchange Rate

Source: Authors calculation.
Figure B.13: Lagged Generalized Response to a Standard Deviation of the Date Impulse

RESPONSE AT 2015–10–26

Oil Price

Exchange Rate

Source: Authors calculation.
Figure B.14: Generalized Daily Lagged Responses of Nominal Exchange Rate Returns to One Standard Deviation Oil Price Return along the Sample

Source: Authors calculation.

Responses were multiplied by 100,000

Lagged responses are not significantly different from zero at all dates
Figure B.15: Generalized Daily Lagged Responses of Oil Price Return to One Standard Deviation Nominal Exchange Rate Return along the Sample

Source: Authors calculation.

Responses were multiplied by 1.000

Lagged responses are not significantly different from zero at all dates
**Figure B.16: Residual Variance-Covariance and Correlation Matrices**

<table>
<thead>
<tr>
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Source: Authors calculation.
Figure B.17: De-Trended Cross Correlations Between Oil Price and Exchange Rate Returns for windows of size 50, 100, 150 and 200 Days, Full Sample

Source: Authors calculation.
Figure B.18: De-Trended Cross Correlations Between Oil Price and Exchange Rate Returns for windows of size 50, 100, 150 and 200 Days, Before 2008

Source: Authors calculation.
Figure B.19: De-Trended Cross Correlations Between Oil Price and Exchange Rate Returns for windows of size 50, 100, 150 and 200 Days, After 2009

Source: Authors calculation.
Figure B.20: De-Trended Cross Correlations Between Oil Price and Exchange Rate Returns for windows of size 50, 100, 150 and 200 Days, Between June-2006 and 2009

Source: Authors calculation.